# Markov Chain Practice Set - 1

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## Practice / Asked Que - Set

Assam PSC-19

(1) Consider a Markov chain with state space $\{0,1,2\}$ and the transition prob. matrix
$$P = \begin{bmatrix} 0 & 1/2 & 1/2 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$
The the period of 0 is —

(A). 0 (B). 1

(C). 2 (D). None

Ans : 2

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(2) [4.75] Let $\{X_n\}$ be a markov chain on $\{0,1,2,\ldots\}$ with $P_{00} = 2/3$, $P_{01} = 1/3$, $P_{i,i+1} = 2/3$, $P_{i,i-1} = 1/3$, $i \geq 1$, $P_{ij} = 0$ otherwise. W.o.t.f statements are correct

(a) $\{x_n\}$ is recurrent

(b) $\{x_n\}$ is transient.

(c). $P(\lim_{n\to\infty} X_n = 0) > 0$

(d). $P(\lim_{n\to\infty} X_n = +\infty) > 0$

Ans : 2, 4

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(3). W.o.t.f statements are correct ?

(4.75) (a). For a finite state markov chain there is at least one transient state.

(b). For a finite state markov chain there is at least one stationary distn.

(c). For a countable state Markov chain, every state can be transient.

(d). For an aperiodic countable state Markov chain there is at least one stationary distn.

Ans : 2, 3

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(4). An aperiodic Markov chain with stationary distn prob. on the state space $\{1,2,3,4,5\}$ must have at least one

(a). null recurrent state (b). positive recurrent state

(c). +ve recc. & null recc. (d). transient state.

Ans : (b)

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(5). Let $\Pi = (\pi_1, \pi_2, \pi_3, \pi_4)$ be the stationary dist$^n$ for a markov chain on the state space $\{1,2,3,4\}$ with transition prob. matrix $P$. Suppose that states 1 & 2 are transient & states 3 & 4 form a communicatory class. w.o.t.f is/are true?

(1) $\pi P^3 = \pi P^5$ (2). $\pi_1 = 0$ & $\pi_2 = 0$

(3) $\pi_3 + \pi_4 = 1$ (4). one of $\pi_3$ & $\pi_4$ is 0.

Ans: 2, 3

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(6) ~~w.o.t.f are c.d.f of random~~

Let $\{X_n\}$ be stationary ~~dist$^n$~~ Markov Chain s.t —
$P(X_{i+1} = 1 \mid X_i = 1) = p_1 = 1 - P(X_{i+1} = 0 \mid X_i = 1)$,
$P(X_{i+1} = 1 \mid X_i = 0) = p_0 = 1 - P(X_{i+1} = 0 \mid X_i = 0)$ & $P(X_i = 1) = \pi_1 = 1 - P(X = 0)$. Then ——

(a) $\pi_1 = p_1$ (b). $\pi_1 = p_0$

(c). $\pi_1 = p_0 / (1 - p_0 + p_1)$ (d). $\pi_1 = 1/2$.

Ans: All

(1). Let $(X_n)_{n\geq 0}$ be a Markov chain on the state space $S := \{1, 2, \cdots, 23\}$ with transition probability given by

$$p_{i,i+1} = p_{i,i-1} = \frac{1}{2} \quad \forall\, 2 \leq i \leq 22$$
$$p_{1,2} = p_{1,23} = \frac{1}{2}$$
$$p_{23,1} = p_{23,22} = \frac{1}{2}.$$

Then, which of the following statements are true?

1. $(X_n)_{n\geq 0}$ has a unique stationary distribution.
2. $(X_n)_{n\geq 0}$ is irreducible.
3. $\mathbb{P}(X_n = 1) \longrightarrow \frac{1}{23}$.
4. $(X_n)_{n\geq 0}$ is recurrent.

(2). Consider a Markov chain $\{X_n \mid n \geq 0\}$ with state space $\{1, 2, 3\}$ and transition matrix

$$P = \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}.$$ Then $P\left(X_3 = 1 \,\middle|\, X_0 = 1\right)$ equals

1. 0
2. $\frac{1}{4}$
3. $\frac{1}{2}$
4. $\frac{1}{8}$

(3). Consider a Markov Chain with state space $S = \{0, 1, 2, 3\}$ and with transition probability matrix P given by

$$P = \begin{array}{c c} & \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \end{array} & \begin{pmatrix} 2/3 & 0 & 1/3 & 0 \\ 1 & 0 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 \\ 1/4 & 1/4 & 1/4 & 1/4 \end{pmatrix} \end{array}$$

Then

1. 1 is a recurrent state.
2. 0 is a recurrent state.
3. 3 is a recurrent state.
4. 2 is a recurrent state.

(4). Consider a Markov chain with five states {1,2,3,4,5} and transition matrix

$$P = \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{7} & 0 & 0 & \frac{6}{7} \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \\ \frac{1}{3} & 0 & 0 & \frac{2}{3} & 0 \\ 0 & \frac{5}{8} & 0 & 0 & \frac{3}{8} \end{pmatrix}$$

Which of the following are true?

1. 3 and 1 are in the same communicating class
2. 1 and 4 are in the same communicating class
3. 4 and 2 are in the same communicating class
4. 2 and 5 are in the same communicating class