Probability Practice Set- 1

Practice set by P. Kalika

Asam PSC-19

(1) If x is a discrete r.V & $E(ax+b) = aE(x)+b$, where a & b are constants, then the variance $ax+b$ is —

(a). $a\,Var(x)$ (b). $a^2 Var(x)$ Assam PSC-19

(c). $b\,Var(x)$ (d). $ab\,Var(x)$.

NET-17D

(2) Let X & Y be independent exp. r.V. If $E(X)=1$ & $E(Y)=1/2$. Then $P(X>2Y \mid X>Y)$ is —

(a). $1/2$ (b). $1/3$ (c). $2/3$ (d). $3/4$

Ans: (d)

NET-16D

(3) A & B play a game of tossing a fair coin. A starts the game by tossing the coin once & B then tosses the coin twice followed by A tossing the coin once & B tossing the coin twice & hits until a head turns up. whoever gets the first head wins the game. Then —

(1). $P(B\text{ wins}) > P(A\text{ wins})$
(2). $P(B\text{ wins}) = 2P(A\text{ wins})$
(3). $P(A\text{ wins}) > P(B\text{ wins})$
(4). $P(A\text{ wins}) = 1 - P(B\text{ wins})$

Ans: 3, 4

NET-16D

(4) For any two events A & B, w.o.t.f always holds?

(1). $P^2(A\cap B^c) + P^2(A\cap B) + P^2(A^c) \geq 1/3$
(2). $P^2(A\cap B^c) + P^2(A\cap B) + P^2(A^c) = 1/3$
(3). " " " $= 1$
(4). " " " $\leq 1/3$

Ans: 1

NET-2018 D

(5) Suppose $X \sim \text{Cauchy}(0,1)$. Then the dist^n of $\frac{1-X}{1+X}$ is ____

(1). Uniform (0,1) (2). Normal (0,1)
(3). Double Exp. (0,1) (4). Cauchy (0,1)

NET-2018 J

(6). In a data set with mean 2.5 & std. deviation 0.5. The median must be

(1) bigger than 2.5 (2). smaller than 2.5
(3). smaller than 3 (4). bigger than 2.

Ans: 3, 4

NET-2018 J

(7). Let X & Y be two random variables satisfying for $X, Y \geq 0$, $E(X) = 3$, $V(X) = 9$, $E(Y) = 2$ and $V(Y) = 4$, w.o.t.f statements are correct?

(1). $0 \leq Cov(X,Y) \leq 4$ (2). $E(XY) \leq 6$
(3). $V(X+Y) \leq 25$ (4). $E(X+Y)^2 \geq 25$

Ans: 3, 4

NET-2018 J

(8) Let X & Y be two r.v with joint p.d.f –

$$f(x,y) = \begin{cases} 1/\pi & ; 0 \leq x^2+y^2 \leq 1 \\ 0 & ; \text{o.w} \end{cases}$$

Then w.o.t.f statements are true?

(1). X & Y are independent (2). $P(X>0) = 1/2$
(3). $E(Y) = 0$ (4). $Cov(X,Y) = 0$

Ans: 2, 3, 4

(By: P. Kalika (maths.whisperer@gmail.com))

(9). NET-18J

A std. fair die is rolled until some face other than 5 or 6 turns up. Let X denote the face value of the last roll and $A = \{X \text{ is even}\}$ & $B = \{X \text{ is at most } 2\}$. Then—

(1). $P(A \cap B) = 0$ (2). $P(A \cap B) = 1/6$

(3). $P(A \cap B) = 1/4$ (4). $P(A \cap B) = 1/3$

Ans: 3

(10). NET–18J

A simple random sample of size n will be drawn from a class of 125 students, and the mean maths score of the sample will be computed. If the std. error of the sample mean for "with replacement sampling" is twice as much as the std. error of the sample mean for "without replacement" sampling the value of n is —

(1). 32 (2). 63 (3). 79 (4). 94

Ans:

NET 2011-June

(11). w.o.t.f is/are c.d.f of random variables —

(1). $F_1(x) = \begin{cases} 0 & ; x \le 0 \\ e^{-x} & ; x > 0 \end{cases}$ (2). $F_2(x) = \begin{cases} 0 & ; x \le 0 \\ 1-e^{-x} & ; x > 0 \end{cases}$

(3). $F_3(x) = \begin{cases} 0 & ; x \le 0 \\ 1 & ; x > 0 \end{cases}$ (4). $F_4(x) = \begin{cases} 0 & ; x < 0 \\ 1/2 & ; 0 \le x < 1 \\ 1 & , x \ge 0 \end{cases}$

NET–2011J

Ans: 1

(12) Let X be a r.v taking values in set E. Let $P(X > a+b \mid X > a) = P(X > b)$ $\forall\, a, b \in E$. Then w.o.t.f is a possible distn of X ?

(1). Poisson (2). Geometric

(3). Log-normal (4). Exponential.

Ans:

Probability Practice Set- 2

(1). Five numbers 10, 7, 5, 4 and 2 are to be arranged in a sequence from left to right following the directions given below:

1. No two odd or even numbers are next to each other.
2. The second number from the left is exactly half of the left-most number.
3. The middle number is exactly twice the right-most number.

GATE 2019

Which is the second number from the right?

(A) 2 (B) 4 (C) 7 (D) 10

(2). Forty students watched films A, B and C over a week. Each student watched either only one film or all three. Thirteen students watched film A, sixteen students watched film B and nineteen students watched film C. How many students watched all three films?

GATE 2018

(A) 0 (B) 2 (C) 4 (D) 8

(3). Let X and Y have joint probability density function given by

$$f_{X,Y}(x,y) = \begin{cases} 2, & 0 \le x \le 1-y,\ 0 \le y \le 1 \\ 0, & \text{otherwise.} \end{cases}$$

If f_Y denotes the marginal probability density function of Y, then $f_Y(1/2) =$ 1 .

(4). Let the cumulative distribution function of the random variable X be given by

$$F_X(x) = \begin{cases} 0, & x < 0, \\ x, & 0 \le x < 1/2, \\ (1+x)/2, & 1/2 \le x < 1, \\ 1, & x \ge 1. \end{cases}$$

Then $\mathbb{P}(X = 1/2) =$ 1/4 .

(5). Let $\{X_j\}$ be a sequence of independent Bernoulli random variables with $\mathbb{P}(X_j = 1) = 1/4$ and let $Y_n = \frac{1}{n}\sum_{j=1}^{n} X_j^2$. Then Y_n converges, in probability, to 1/4 .

(6).

Let X be the number of heads in 4 tosses of a fair coin by Person 1 and let Y be the number of heads in 4 tosses of a fair coin by Person 2. Assume that all the tosses are independent. Then the value of $\mathbb{P}(X = Y)$ correct up to three decimal places is .273.

(7).

Let X_1 and X_2 be independent geometric random variables with the same probability mass function given by $\mathbb{P}(X = k) = p(1-p)^{k-1}$, $k = 1, 2, \ldots$ Then the value of $\mathbb{P}(X_1 = 2 | X_1 + X_2 = 4)$ correct up to three decimal places is 1/3.

(8).

An urn contains four balls, each ball having equal probability of being white or black. Three black balls are added to the urn. The probability that five balls in the urn are black is

(A) 2/7 (B) 3/8 (C) 1/2 (D) 5/7

GATE 2018

(9).

An unbiased coin is tossed six times in a row and four different such trials are conducted. One trial implies six tosses of the coin. If H stands for head and T stands for tail, the following are the observations from the four trials:

(1) HTHTHT (2) TTHHHT (3) HTTHHT (4) HHHT__ __.

Which statement describing the last two coin tosses of the fourth trial has the highest probability of being correct?

(A) Two T will occur.

(B) One H and one T will occur.

(C) Two H will occur.

(D) One H will be followed by one T.

GATE 2018