MATHEMATICAL SCIENCES
PAPER-I (PART-B)

41. Let $\{x_n\}$ be a sequence of non-zero real numbers. Then

1. If $x_n \to a$, then $a = \sup x_n$.
2. If $\frac{x_{n+1}}{x_n} < 1 \ \forall n$, then $x_n \to 0$.
3. If $x_n < n \ \forall n$, then $\{x_n\}$ diverges.
4. If $n \le x_n \ \forall n$, then $\{x_n\}$ diverges.

42. Let $\{x_n\}$ and $\{y_n\}$ be two sequences of real numbers such that $x_n \le y_n \le x_{n+2}$, $n = 1, 2, 3, \ldots$

1. $\{y_n\}$ is an bounded sequence.
2. $\{x_n\}$ is an increasing sequence.
3. $\{x_n\}$ and $\{y_n\}$ converge together.
4. $\{y_n\}$ is an increasing sequence.

43. Let $f:[0, 1] \to (0, \infty)$ be a continuous function. Suppose $f(0) = 1$ and $f(1) = 7$. Then

1. f is uniformly continuous and is not onto.
2. f is increasing and $f([0, 1]) = [1, 7]$.
3. f is not uniformly continuous.
4. f is not bounded.

44. Let $f: [a, b] \to [c, d]$ be a monotone and bijective function. then

1. f is continuous, but f^{-1} need not be.
2. f and f^{-1} are both continuous.
3. If $b - a > d - c$, then f is a decreasing function.
4. f is not uniformly continuous.

45. Let $\sum_{n=1}^{\infty} x_n$ be a series of real numbers. Which of the following is true?

1. If $\sum_{1}^{\infty} x_n$ is divergent, then $\{x_n\}$ does not converge to 0.
2. If $\sum_{1}^{\infty} x_n$ is convergent, then $\sum_{1}^{\infty} x_n$ is absolutely convergent.
3. If $\sum_{1}^{\infty} x_n$ is convergent, then $x_n^2 \to 0$, as $n \to \infty$.
4. If $x_n \to 0$, then $\sum_{1}^{\infty} x_n$ is convergent.

46. Let f: $\mathbb{R} \to \mathbb{R}$ be differentiable with $0 < f'(x) < 1$ for all x. Then
   1. f is increasing and f is bounded.
   2. f is increasing and f is Riemann integrable on $\mathbb{R}$.
   3. f is increasing and f is uniformly continuous.
   4. f is of bounded variation.

47. Let $f_n:[0,1] \to \mathbb{R}$ be a sequence of differentiable functions. Assume that (f_n) converges uniformly on [0, 1] to a function f. Then

   1. f is differentiable and Riemann integrable on [0, 1].
   2. f is uniformly continuous and Riemann integrable on [0, 1].
   3. f is continuous, f need not be differentiable on (0, 1) and need not be Riemann integrable on [0, 1].
   4. f need not be uniformly continuous on [0, 1].

48. Let, if possible, $\alpha = \lim\limits_{(x,y)\to(0,0)} \dfrac{\sin(x^2+y^2)}{x^2+y^2}$, $\beta = \lim\limits_{(x,y)\to(0,0)} \dfrac{x^2-y^2}{x^2+y^2}$. Then
   1. α exists but β does not.
   2. α does not exists but β exists.
   3. α, β do not exist.
   4. Both α, β exist.

49. Let $f:\mathbb{R} \to \mathbb{R}$ be a non-negative Lebesgue integrable function. Then

   1. f is finite almost everywhere.
   2. f is a continuous function.
   3. f has at most countably many discontinuities.
   4. f^2 is Lebesgue integrable.

50. Let $S = \{(x, y) \in \mathbb{R}^2 : xy = 1\}$. then
   1. S is not connected but compact.
   2. S is neither connected nor compact.
   3. S is bounded but not connected.
   4. S is unbounded but connected.

51. Consider the linear space
   X = C[0, 1] with the norm $\|f\| = \sup\{|f(t)| : 0 \le t \le 1\}$.
   Let F = $\left\{ f \in X : f(\frac{1}{2}) = 0 \right\}$ and G = $\left\{ g \in X : g(\frac{1}{2}) \neq 0 \right\}$.
   Then
   1. F is not closed and G is open.
   2. F is closed but G is not open.
   3. F is not closed and G is not open.
   4. F is closed and G is open.

52. Let V be the vector space of all n x n real matrices, $A = [a_{ij}]$ such that $a_{ij} = -a_{ji}$ for all i, j. Then the dimension of V is:

1. $\frac{n^2+n}{2}$.
2. $\frac{n^2-n}{2}$.
3. $n^2 - n$.
4. n.

53. Let n=mk where m and k are integers ≥ 2. Let $A = [a_{ij}]$ be a matrix given by $a_{ij}=1$ if for some r = 0, 1,…, m-1, $rk < i,\ j \leq (r+1)k$ and $a_{ij}= 0$, otherwise. Then the null space of A has dimension :

1. m(k – 1).
2. mk – 1.
3. k(m – 1).
4. zero.

54. The set of all solutions to the system of equations :

$(1 - i)\, x_1 - ix_2 = 0$
$2x_1 + (1 - i)x_2 = 0$

is given by:

1. $(x_1, x_2) = (0, 0)$.
2. $(x_1, x_2) = (1, 1)$.
3. $(x_1, x_2) = c\left(1, \cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)$ where c is any complex number.
4. $(x_1, x_2) = c\left(\cos\frac{3\pi}{4}, i\sin\frac{3\pi}{4}\right)$ where c is any complex number.

55. Let A be an m x n matrix where m < n. Consider the system of linear equations $A\,\underline{x} = \underline{b}$ where $\underline{b}$ is an n x 1 column vector and $\underline{b} \neq \underline{0}$. Which of the following is always true?

1. The system of equations has no solution.
2. The system of equations has a solution if and only if it has infinitely many solutions.
3. The system of equations has a unique solution.
4. The system of equations has at least one solution.

56. Let T be a normal operator on a complex inner product space. Then T is self-adjoint if and only if :

1. All eigenvalues of T are distinct.
2. All eigenvalues of T are real.
3. T has repeated eigenvalues.
4. T has at least one real eigenvalue.

57. A 2 x 2 real matrix A is diagonalizable if and only if :

1. $(\text{tr}A)^2 < 4$ Det A.
2. $(\text{tr } A)^2 > 4$ Det A.
3. $(\text{tr } A)^2 = 4$ Det A.
4. Tr A = Det A.

58. Let A be a 3 x 3 complex matrix such that $A^3 = I$ (= the 3 x 3 identity matrix). Then :

1. A is diagnonalizable.
2. A is not diagonalizable.
3. The minimal polynomial of A has a repeated root.
4. All eigenvalues of A are real.

59. Let V be the real vector space of real polynomials of degree < 3 and let T : V → V be the linear transformation defined by P(t) a Q(t) where Q(t) = P(at + b). Then the matrix of T with respect to the basis 1, t, t^2 of V is:

1. $\begin{pmatrix} b & b & b^2 \\ 0 & a & 2ab \\ 0 & 0 & a^2 \end{pmatrix}$.
2. $\begin{pmatrix} a & a & a^2 \\ 0 & b & 2ab \\ 0 & 0 & b^2 \end{pmatrix}$.
3. $\begin{pmatrix} b & b & b^2 \\ a & a & 0 \\ 0 & b & a^2 \end{pmatrix}$.
4. $\begin{pmatrix} a & a & a^2 \\ b & b & 0 \\ 0 & a & b^2 \end{pmatrix}$.

60. The minimal polynomial of the 3 × 3 real matrix $\begin{pmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & b \end{pmatrix}$ is:

1. $(X - a)(X - b)$.
2. $(X - a)^2 (X - b)$.
3. $(X - a)^2 (X - b)^2$.
4. $(X - a)(X - b)^2$.

61. The characteristic polynomial of the 3 × 3 real matrix A = $\begin{pmatrix} 0 & 0 & -c \\ 1 & 0 & -b \\ 0 & 1 & -a \end{pmatrix}$ is:

1. $X^3 + aX^2 + bX + c$.
2. $(X-a)(X-b)(X-c)$.
3. $(X-1)(X-abc)^2$.
4. $(X-1)^2 (X-abc)$.

62. Let e_1, e_2, e_3 denote the standard basis of ¡ ^{3}. Then $ae_1 + be_2 + ce_3$, e_2, e_3 is an orthonormal basis of ¡ ^{3} if and only if

1. $a \neq 0$, $a^2 + b^2 + c^2 = 1$.
2. $a = \pm 1$, $b = c = 0$.
3. $a = b = c = 1$.
4. $a = b = c$.

63. Let E = $\{z \in$ £ $: e^z = i\}$. Then E is :

1. a singleton.
2. E is a set of 4 elements.
3. E is an infinite set.
4. E is an infinite group under addition.

64. Suppose $\{a_n\}$ is a sequence of complex numbers such that $\sum_0^\infty a_n$ diverges. Then the radius of convergence R of the power series $\sum_{n=0}^{\infty} \frac{a_n}{2^n}(z-1)^n$ satisfies :

1. $R = 3$.
2. $R \leq 2$.
3. $R > 2$.
4. $R = \infty$.

65. Let f, g be two entire functions. Suppose $\left| f^2(z) + g^2(z) \right| = 1$, then

1. $f(z)f'(z) + g(z)g'(z) = 0$.
2. f and g must be constant.
3. f and g are both bounded functions.
4. f and g have no zeros on the unit circle.

66. The integral $\int_{|z|=2\pi} \frac{\sin z}{(z-\pi)^2}$ where the curve is taken anti-clockwise, equals :

1. $-2\pi i$.
2. $2\pi i$.
3. 0.
3. $4\pi i$.

67. Suppose $\{z_n\}$ is a sequence of complex numbers and $\sum_{0}^{\infty} z_n$ converges.

Let f : £ $\rightarrow$ £ be an entire function with $f(z_n) = n$, $\forall$ n = 0, 1, 2, … Then

1. $f \equiv 0$.
2. f is unbounded.
3. no such function exists.
4. f has no zeros.

68. Let $f(z) = \cos z$ and $g(z) = \cos z^2$, for $z \in$ £ . Then

1. f and g are both bounded on £ .
2. f is bounded, but g is not bounded on £ .
3. g is bounded, but f is not bounded on £ .
4. f and g are both bounded on the x-axis.

69. Let f be an analytic function and let $f(z) = \sum_{n=0}^{\infty} a_n (z-2)^{2n}$ be its Taylor series in some disc. Then

1. $f^{(n)}(0) = (2n)!a_n$
2. $f^{(n)}(2) = n!a_n$
3. $f^{(2n)}(2) = (2n)!a_n$
4. $f^{(2n)}(2) = n!a_n$

70. The signature of the permutation

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & \cdots & n \\ n & n-1 & n-2 & & 1 \end{pmatrix} \text{ is}$$

1. $(-1)^{\binom{n}{2}}$.
2. $(-1)^{n}$.
3. $(-1)^{n+1}$.
4. $(-1)^{n-1}$.

71. Let α be a permutation written as a product of disjoint cycles, k of which are cycles of odd size and m of which are cycles of even size, where $4 \leq k \leq 6$ and $6 \leq m \leq 8$. It is also known that α is an odd permutation. Then which one of the following is true?

1. k = 4 and m = 6.
2. m = 7.
3. k = 6.
4. m = 8.

72. Let p, q be two distinct prime numbers. then $p^{q-1} + q^{p-1}$ is congruent to

1. 1 mod pq.
2. 2 mod pq.
3. p–1 mod pq.
4. q–1 mod pq.

73. What is the total number of groups (upto isomorphism) of order 8?

1. only one.
2. 3.
3. 5.
4. 6.

74. Which ones of the following three statements are correct?

(A) Every group of order 15 is cyclic.
(B) Every group of order 21 is cyclic.
(C) Every group of order 35 is cyclic.

1. (A) and (C).
2. (B) and (C).
3. (A) and (B).
4. (B) only.

75. Let p be a prime number and consider the natural action of the group $GL_2(\mathbb{Z}_p)$ on $\mathbb{Z}_p \times \mathbb{Z}_p$. Then the index of the isotropy subgroup at (1, 1) is

1. $p^2 - 1$.
2. $p(p-1)$.
3. $p - 1$.
4. p^2.

76. The quadratic polynomial $X^2 + bX + c$ is irreducible over the finite field $\mathbb{Z}_5$ if and only if

1. $b^2 - 4c = 1$.
2. $b^2 - 4c = 4$.
3. either $b^2 - 4c = 2$ or $b^2 - 4c = 3$.
4. either $b^2 - 4c = 1$ or $b^2 - 4c = 4$.

77. Let K denote a proper subfield of the field $F = GF(2^{12})$ a finite field with 2^{12} elements. Then the number of elements of K must be equal to

1. 2^m where $m = 1, 2, 3, 4$ or 6.
2. 2^m where $m = 1, 2, \ldots, 11$.
3. 2^{12}.
4. 2^m where m and 12 are coprime to each other.

78. The general and singular solutions of the differential equation $y = \frac{9}{2} x p^{-1} + \frac{1}{2} px$, where $p = \frac{dy}{dx},$ are given by

1. $2cy - x^2 - 9c^2 = 0$, $3y = 2x$.
2. $2cy - x^2 + 9c^2 = 0$, $y = \pm 3x$.
3. $2cy + x^2 + 9c^2 = 0$, $y = \pm 3x$.
4. $2cy + x^2 + 9c^2 = 0$, $3y = 4x$.

79. A homogenous linear differential equation with real constant coefficients, which has $y = xe^{-3x} \cos 2x + e^{-3x} \sin 2x$, as one of its solutions, is given by:

1. $(D^2 + 6D + 13)y = 0$.
2. $(D^2 - 6D + 13)y = 0$.
3. $(D^2 - 6D + 13)^2 y = 0$.
4. $(D^2 + 6D + 13)^2 y = 0$.

80. The particular integral $y_p(x)$ of the differential equation

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=\frac{1}{x+1},\ \ x>0$$

is given by

$$y_p(x)=x\nu_1(x)+\frac{1}{x}\nu_2(x)$$

where $\nu_1(x)$ and $\nu_2(x)$ are given by

1. $x\nu_1'(x)-\frac{1}{x^2}\nu_2'(x)=0,\ \ \nu_1'(x)-\frac{1}{x^2}\nu_2'(x)=\frac{1}{x+1}$.
2. $x\nu_1'(x)+\frac{1}{x^2}\nu_2'(x)=0,\ \ \nu_1'(x)-\frac{1}{x^2}\nu_2'(x)=\frac{1}{x+1}$.
3. $x\nu_1'(x)-\frac{1}{x^2}\nu_2'(x)=0,\ \ \nu_1'(x)+\frac{1}{x^2}\nu_2'(x)=\frac{1}{x+1}$.
4. $x\nu_1'(x)+\frac{1}{x^2}\nu_2'(x)=0,\ \ \nu_1'(x)+\frac{1}{x^2}\nu_2'(x)=\frac{1}{x+1}$.

81. The boundary value problem
$y''+\lambda y=0,\ y(0)=0,\ y(\pi)+k\,y'(\pi)=0,$ is self-adjoint

1. only for $k \in \{0, 1\}$.
2. for all $k \in (-\infty,\infty)$.
3. only for $k \in [0, 1]$.
4. only for $k \in (-\infty, 1)\ \mathrm{U}(1,\infty)$.

82. The general integral of $z(xp - yq) = y^2 - x^2$ is

1. $z^2 = x^2 + y^2 + f(xy)$.
2. $z^2 = x^2 - y^2 + f(xy)$.
3. $z^2 = -x^2 - y^2 + f(xy)$.
4. $z^2 = y^2 - x^2 + f(xy)$.

83. A singular solution of the partial differential equation $z + xp - x^2y\, q^2 - x^3pq = 0$ is

1. $z = \frac{x^2}{y}$.
2. $z = \frac{x}{y^2}$.
3. $z = \frac{y}{x^2}$.
4. $z = \frac{y^2}{x}$.

84. The characteristics of the partial differential equation
$36\frac{\partial^2 z}{\partial x^2} - y^{14}\frac{\partial^2 z}{\partial y^2} - 7y^{13}\frac{\partial z}{\partial y} = 0$, are given by

1. $x + \frac{1}{y^6} = c_1,\ x - \frac{1}{y^6} = c_2$.
2. $x + \frac{36}{y^6} = c_1,\ x - \frac{36}{y^6} = c_2$.
3. $6x + \frac{7}{y^6} = c_1,\ 6x - \frac{7}{y^6} = c_2$.
4. $6x + \frac{7}{y^8} = c_1,\ 6x - \frac{7}{y^8} = c_2$.

85. The Lagrange interpolation polynomial through (1, 10), (2, –2), (3, 8), is

1. $11x^2 - 45x + 38$.
2. $11x^2 - 45x + 36$.
3. $11x^2 - 45x + 30$.
4. $11x^2 - 45x + 44$.

86. Newton's method for finding the positive square root of $a > 0$ gives, assuming $x_0 > 0$, $x_0 \neq \sqrt{a}$,

1. $x_{n+1} = \frac{x_n}{2} + \frac{a}{x_n}$.
2. $x_{n+1} = \frac{1}{2}\left(x_n + \frac{a}{x_n}\right)$.
3. $x_{n+1} = \frac{1}{\sqrt{2}}\left(x_n - \frac{a}{x_n}\right)$.
4. $x_{n+1} = \frac{1}{\sqrt{2}}\left(x_n + \frac{a}{x_n}\right)$.

87. The extremal problem

$$J[y(x)] = \int_0^{\pi} \left\{(y')^2 - y^2\right\} dx$$

$y(0)=1$, $y(\pi)=\lambda$, has

1. a unique extremal if $\lambda = 1$.
2. infinitely many extremals if $\lambda = 1$.
3. a unique extremal if $\lambda = -1$.
4. infinitely many extremal if $\lambda = -1$.

88. The functional

$$J[y] = \int_0^1 e^x (y^2 + \frac{1}{2} y'^2) dx \; ; \; y(0)=1, \; y(1)=e$$

attains

1. A weak, but not a strong minimum on e^x.
2. A strong minimum on e^x.
3. A weak, but not a strong maximum on e^x.
4. A strong maximum on e^x.

89. A solution of the integral equation

$$\int_0^x e^{x-t} \phi(t) dt = \sinh x, \text{ is}$$

1. $\phi(x) = e^{-x}$.
2. $\phi(x) = e^x$.
3. $\phi(x) = \sinh x$.
4. $\phi(x) = cosh\, x$.

90. If $\overline{\varphi}(p)$ denotes the Laplace transform of $\varphi(x)$ then for the integral equation of convolution type

$$\varphi(x) = 1 + 2\int_0^x \cos(x-t)\,\varphi(t)\,dt,$$

$\overline{\varphi}(p)$ is given by

1. $\dfrac{p^2+1}{(p-1)^2}$.
2. $\dfrac{p^2+1}{(p+1)^2}$.
3. $\dfrac{\left(p^2+1\right)}{p(p-1)^2}$.
4. $\dfrac{p^2+1}{p(p+1)^2}$.

91. The Lagrangian of a dynamical system is $L = \dot{q}_1^2 + \dot{q}_2^2 + k_1 q_1^2$, then the Hamiltonian is given by

1. $H = p_1^2 + p_2^2 - kq_1^2$.
2. $H = \frac{1}{4}\left(p_1^2 + p_2^2\right) + kq_1^2$.
3. $H = p_1^2 + p_2^2 + kq_1^2$.
4. $H = \frac{1}{4}\left(p_1^2 + p_2^2\right) - kq_1^2$.

92. The kinetic energy T and potential energy V of a dynamical system are given respectively, under usual notations, by

$$T = \frac{1}{2}\left[A(\dot{\theta}^2 + \dot{\psi}^2 \sin^2\theta) + B(\dot{\psi}\cos\theta + \dot{\phi})^2\right]$$

and V = Mgl cosθ. The generalized momentum p_ϕ is

1. $p_\phi = 2B\dot{\phi}\dot{\psi}\cos\theta + 2\dot{\phi}^2$.
2. $p_\phi = \frac{B}{2}\left(\dot{\psi}\cos\theta + \dot{\phi}\right)^2$.
3. $p_\phi = B\left(\dot{\psi}\cos\theta + \dot{\phi}\right)^2$.
4. $p_\phi = B\left(\dot{\psi}\cos\theta + \dot{\phi}\right)$.

93. Consider repeated tosses of a coin with probability p for head in any toss. Let NB(k,p) be the random variable denoting the number of tails before the k^{th} head. Then P(NB(10,p) = j 3^{rd} head occurred in 15^{th} toss) is equal to

1. P(NB (7, p) = j – 15), for j = 15, 16, $\ldots$
2. P(NB (7, p) = j – 12), for j = 12, 13, $\ldots$
3. P(NB (10, p) = j – 15), for j = 15, 16, $\ldots$
4. P(NB (10, p) = j – 12), for j = 12, 13, $\ldots$

94. Suppose X and Y are standard normal random variables. Then which of the following statements is correct?

1. (X, Y) has a bivariate normal distribution.
2. Cov (X, Y) = 0.
3. The given information does not determine the joint distribution of X and Y.
4. X + Y is normal.

95. Let F be the distribution function of a strictly positive random variable with finite expectation μ. Define

$$G(x) = \begin{cases} \frac{1}{\mu}\int_0^x (1-F(y))\,dy, & \text{if } x>0 \\ 0, & \text{otherwise} \end{cases}$$

Which of the following statements is correct?

1. G is a decreasing function.
2. G is a probability density function.
3. G (x) $\to +\infty$ as x $\to +\infty$.
4. G is a distribution function.

96. Let $X_1, X_2 \ldots$ be an irreducible Markov chain on the state space $\{1, 2, \ldots\}$. Then P(X_n = 5 for infinitely many n) can equal

1. Only 0 or 1.
2. Only 0.
3. Any number in [0, 1].
4. Only 1.

97. $X_1, X_2, \ldots, X_n$ is a random sample from a normal population with mean zero and variance σ^2. Let $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$. Then the distribution of $T = \sum_{i=1}^{n-1}(X_i - \bar{X})$ is

1. t_{n-1}
2. $N(0, (n-1)\sigma^2)$
3. $N(0, \frac{n+1}{n}\sigma^2)$
4. $N(0, \frac{n-1}{n}\sigma^2)$

98. Let $X_1, X_2, \ldots, X_n$ be independent exponential random variables with parameters $\lambda_1, \ldots, \lambda_n$ respectively. Let $Y = \min(X_1, \ldots, X_n)$. Then Y has an exponential distribution with parameter

1. $\sum_{i=1}^{n} \lambda_i$
2. $\prod_{i=1}^{n} \lambda_i$
3. $\min\{\lambda_1, \ldots, \lambda_n\}$
4. $\max\{\lambda_1, \ldots, \lambda_n\}$

99. Suppose $x_1, x_2, \ldots, x_n$ are n observations on a variable X. Then the value of A which minimizes $\sum_{i=1}^{n}(x_i - A)^2$ is

1. median of $x_1, x_2, \ldots, x_n$
2. mode of $x_1, x_2, \ldots, x_n$
3. mean of $x_1, x_2, \ldots, x_n$
4. $\frac{\min(x_1, \ldots, x_n) + \max(x_1, \ldots, x_n)}{2}$

100. Suppose $X_1, X_2, \ldots, X_n$ are i.i.d. with density function $f(x) = \frac{\theta}{x^2}, \theta < x, \theta > 0$.

Then

1. $\sum_{i=1}^{n} \frac{1}{X_i^2}$ is sufficient for θ
2. $\min_{1 \le i \le n} X_i$ is sufficient for θ.
3. $\prod_{i=1}^{n} \frac{1}{X_i^2}$ is sufficient for θ
4. $\left(\max_{1 \le i \le n} X_i, \min_{1 \le i \le n} X_i\right)$ is not sufficient for θ.

101. Suppose X is a random variable with density function f(x).
To test $H_0 : f(x) = 1, 0 < x < 1$, vs $H_1: f(x) = 2x, 0 < x < 1$, the UMP test at level $\alpha = 0.05$

1. Does not exist
2. Rejects H_0 if $X > 0.95$
3. Rejects H_0 if $X > 0.05$
4. Rejects H_0 for $X < C_1$ or $X > C_2$ where C_1, C_2 have to be determined.

102. Suppose the distribution of X is known to be one of the following:

$$f_1(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}, -\infty < x < \infty;$$
$$f_2(x) = \frac{1}{2} e^{-|x|}, -\infty < x < \infty;$$
$$f_3(x) = \frac{1}{4}, -2 < x < 2.$$

If X = 0 is observed, then the maximum likelihood estimate of the distribution of X is

1. $f_1(x)$
2. $f_2(x)$
3. $f_3(x)$
4. Does not exist.

103. Suppose X_i, $i = 1, 2, \ldots, n$, are independently and identically distributed random variables with common distribution function $F(\cdot)$. Suppose $F(\cdot)$ is absolutely continuous and the hypothesis to be tested is p^{th} ($0 < p < ½$) quantile is ξ_0. An appropriate test is

1. Sign Test
2. Mann-Whitney Wilcoxon rank sum test
3. Wilcoxon Signed rank test
4. Kolmogorov Smirnov test

104. Suppose Y ~ N (θ, σ^2) and suppose the prior distribution on θ is $N(\mu, \tau^2)$. The posterior distribution of θ is also $N\left(\frac{\tau^2}{\tau^2+\sigma^2}y+\frac{\sigma^2}{\tau^2+\sigma^2}\mu,\ \frac{\sigma^2\tau^2}{\tau^2+\sigma^2}\right)$

The Bayes' estimator of θ under squared error loss is given by

1. $\frac{\tau^2}{\tau^2+\sigma^2}y$
2. $\frac{\tau^2 y}{\tau^2+\sigma^2}$
3. $\frac{\tau^2}{\tau^2+\sigma^2}y+\frac{\sigma^2}{\tau^2+\sigma^2}\mu$
4. y.

105. Consider the model
$y_{ij} = \mu + \theta(i-1) + \beta(2-j) + \varepsilon_{ij}$, i = 1, 2; j = 1, 2,

where y_{ij} is the observation under i^{th} treatment and j^{th} block, μ is the general effect, θ and β are treatment and block parameters respectively and ε_{ij} are random errors with mean 0 and common variance σ^2. Then

1. μ, θ and β are all estimable
2. θ and β are estimable, μ is not estimable
3. μ and θ are estimable, β is not estimable
4. μ and β are estimable, θ is not estimable

106. Consider a multiple linear regression model $\underline{y} = X\underline{\beta} + \underline{\varepsilon}$
where $\underline{y}$ is a n × 1 vector of response variables, X is a n × p regression matrix, $\underline{\beta}$ is a p × 1 vector of unknown parameters and $\underline{\varepsilon}$ is a n × 1 vector of uncorrelated random variables with mean 0 and common variance σ^2. Let $\underline{\hat{y}}$ be the vector of least squares fitted values of $\underline{y}$ and $\underline{e} = (e_1 \ldots e_n)^T$ be the vector of residuals. Then

1. $\sum_{i=1}^{n} e_i = 0$ always
2. $\sum_{i=1}^{n} e_i = 0$ if one column of X is $(1, \ldots, 1)^T$
3. $\sum_{i=1}^{n} e_i = 0$ only if one column of X is $(1 \ldots, 1)^T$
4. nothing can be said about $\sum_{i=1}^{n} e_i$

107. Suppose $\underset{\sim}{X}_{p\times 1} \sim N_p\,(\underset{\sim}{0}, \Sigma)$ where

$$\Sigma = \begin{pmatrix} 1 & -1/2 & 0 & \cdots & 0 \\ -1/2 & 1 & 0 & \cdots & 0 \\ 0 & 0 & & & \\ \vdots & \vdots & & \Sigma_{22} & \\ 0 & 0 & & & \end{pmatrix}$$

and Σ_{22} is positive definite. Then

$P\,(X_1 - X_2 < 0,\ X_1 + X_2 \neq 0 \,|\, X_P > 0)$ is equal to

1. 1/8
2. 1/4
3. 1/2
4. 1

108. Suppose the variance-covariance matrix of a random vector $\underline{X}_{(3\times 1)}$ is

$$\Sigma = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 8 & 2 \\ 0 & 2 & 8 \end{pmatrix}.$$

The percentage of variation explained by the first principal component is

1. 50
2. 45
3. 60
4. 40

109. A population consists of 10 students. The marks obtained by one student is 10 less than the average of the marks obtained by the remaining 9 students. Then the variance of the population of marks (σ^2) will always satisfy

1. $\sigma^2 \geq 10$
2. $\sigma^2 = 10$
3. $\sigma^2 \leq 10$
4. $\sigma^2 \geq 9$

110. For what value of λ, the following will be the incidence matrix of a BIBD?

$$N = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & \lambda \\ 0 & 1 & 1 \end{pmatrix}$$

1. $\lambda = 0$
2. $\lambda = 1$
3. $\lambda = 4$
4. $\lambda = 3$

111. With reference to a 2^2 – factorial experiment, consider the factorial effects A, B and AB. Then the estimates of

1. Only A and B are orthogonal
2. Only A and C are orthogonal
3. Only B and C are orthogonal
4. A, B and C are orthogonal

112. Let X be a r.v. denoting failure time of a component. Failure rate of the component is constant if and only if p.d.f. of X is

1. exponential
2. negative binomial
3. weibull
4. normal

113. Consider the problem

max $6x_1 - 2x_2$
subject to $x_1 - x_2 \leq 1$
$3x_1 - x_2 \leq 6$
$x_1, x_2 \geq 0$

This problem has

1. unbounded solution space but unique optimal solution with finite optimum objective value
2. unbounded solution space as well as unbounded objective value
3. no feasible solution
4. unbounded solution space but infinite optimal solutions with finite optimum objective value

114. Consider an M/M/1/K queuing system in which at most K customers are allowed in the system with parameters λ and μ, respectively ($\rho = \lambda / \mu$). The expected steady state number of customers in the queueing system is K/2 for

1. $\rho = 1$
2. $\rho < 1$
3. $\rho > 1$
4. any ρ

115. Consider the system of equations
$P_1x_1 + P_2x_2 + P_3x_3 + P_4x_4 = b$, where

$$P_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \quad P_2 = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}, \quad P_3 = \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}, \quad P_4 = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}, \quad b = \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix}.$$

The following vector combination does not form a basis:

1. (P_1, P_2, P_3)
2. (P_1, P_2, P_4)
3. (P_2, P_3, P_4)
4. (P_1, P_3, P_4).