9, 18, 20

Sr. No. 4656

MEDIUM

E

SUBJECT CODE BOOKLET CODE

4 **C**

2018 (II)
MATHEMATICAL SCIENCES
TEST BOOKLET

Time : 3:00 Hours *Maximum Marks: 200*

INSTRUCTIONS

1. **You have opted for English as medium of Question Paper.** This Test Booklet contains one hundred and twenty (20 Part'A'+40 Part 'B' +60 Part 'C') Multiple Choice Questions (MCQs). You are required to answer a maximum of 15, 25 and 20 questions from part 'A' 'B' and 'C' respectively. If more than required number of questions are answered, only first 15, 25 and 20 questions in Parts 'A' 'B' and 'C' respectively, will be taken up for evaluation.
2. **OMR** answer sheet has been provided separately. Before you start filling up your particulars, please ensure that the booklet contains requisite number of pages and that these are not torn or mutilated. If it is so, you may request the Invigilator to change the booklet of the same code. Likewise, check the **OMR** answer sheet also. Sheets for rough work have been appended to the test booklet.
3. Write your Roll No., Name and Serial Number of this Test Booklet on the **OMR** Answer sheet in the space provided. Also put your signatures in the space earmarked.
4. **You must darken the appropriate circles with a black ball pen related to Roll Number, Subject Code, Booklet Code and Centre Code on the OMR answer sheet. It is the sole responsibility of the candidate to meticulously follow the instructions given on the OMR Answer Sheet, failing which, the computer shall not be able to decipher the correct details which may ultimately result in loss, including rejection of the OMR answer sheet.**
5. Each question in Part 'A' carries 2 marks, Part 'B' **3** marks and Part 'C' **4.75** marks respectively. There will be negative marking @ 0.5 marks in Part 'A' and @ 0.75 marks in Part 'B' for each wrong answer and no negative marking for Part 'C'.
6. Below each question in Part 'A' and 'B', four alternatives or responses are given. Only one of these alternatives is the "correct" option to the question. You have to find, for each question, the correct or the best answer. In Part 'C' each question may have **'ONE'** or **'MORE'** correct options. Credit in a question shall be given only on identification of **'ALL'** the correct options in Part 'C'.
7. Candidates found copying or resorting to any unfair means are liable to be disqualified from this and future examinations.
8. Candidate should not write anything anywhere except on **OMR** answer sheet or sheets for rough work.
9. Use of calculator is not permitted.
10. **After the test is over, at the perforation point, tear the OMR answer sheet, hand over the original OMR answer sheet to the invigilator and retain the carbonless copy for your record.**
11. Candidates who sit for the entire duration of the exam will only be permitted to carry their Test booklet.

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I have verified all the information filled in by the candidate.

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Signature of the Invigilator

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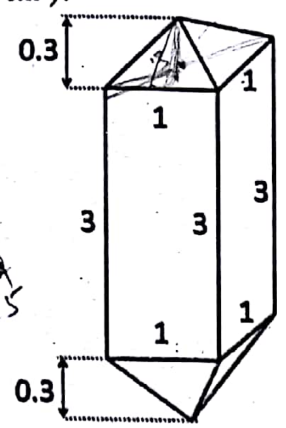
S/27 CISR/ 1(–

PART - A

1. The diagram shows the dimensions (in cm) of a zircon crystal having a square prism and two identical square pyramids. What is the volume of this crystal (in cm^3)?

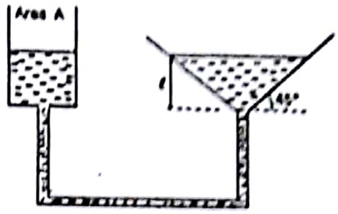


1. 3.2   2. 3.6
3. 6.4   4. 7.2

2. A boy throws a ball with a speed v at a vehicle that is approaching him with a speed V. After bouncing from the vehicle, the ball hits the boy with a speed
   1. v 2. $v+V$
   3. $v+2V$ 4. $v+4V$

3. Four friends were sharing a pizza. They decided that the oldest friend will get an extra piece of pizza. Bahu is two months older than Kattappa, who in turn is three months younger than Bhalla. Devsena is one month older than Kattappa. Who should get the extra piece of pizza?
   1. Bahu   2. Devsena
   3. Bhalla   4. Kattappa

4. A funnel is connected to a cylindrical vessel of cross sectional area A as shown, to make an interconnected system of vessels. Water is poured in the cylinder such that the height of water in the funnel is l as shown. If the level of water in the cylindrical vessel is pushed down by a distance $x \ll l$, the level of water in the funnel:



   1. remains unchanged
   2. rises by $\frac{Ax}{\pi l^2}$
   3. rises by $\frac{\pi l^2}{Ax}$
   4. rises by $\frac{A^2x}{\pi^2 l^4}$

5. Marks (out of 30) of seven students in an examination are 4, 15, 6, 7, 5, a and b, where a (>0) is a multiple of 4 and b is a prime. What is the maximum possible value of the range of marks (i.e. maximum mark – minimum mark)?
   1. 25   2. 26
   3. 27   4. 29

6. Two persons A and B start walking in opposite directions from a point. A travels twice as fast as B. The speed at which B travels is 1 km/h. If A travels 2 km and turns back and starts walking towards B, at what distance from the starting point will A cross B?
   1. 2 km   2. 4 km
   3. 6 km   4. 8 km

7. A person wanted to travel from Charbag to Alambag with an average speed of 60 km/h by car. The distance between Charbag and Alambag is 2 km. Due to heavy traffic, he could travel at 30 km/h for the first kilometre of his journey. What should his speed be for the remaining journey to achieve his average speed target of 60 km/h?
   1. Cannot achieve his target with any finite speed.
   2. 60 km/h
   3. 90 km/h
   4. 120 km/h

(https://pkalika.in/category/download/question-paper/)

10 s
10 m

8. The average rainfall over a given place during the three-year period of 2003-2005 was 65 cm. During the three-year period 2002-2004 the average rainfall was 63 cm. The actual rainfall during 2005 was 60 cm. What was the rainfall in 2002?

1. 55 cm
2. 60 cm
3. 54 cm
4. 53 cm

9. In a four consecutive day schedule, four pilots flew flights each on a different day. Mr. A was scheduled to work on Monday, but he traded with Ms. B who was originally scheduled to work on Wednesday. Ms. C traded with Mr. D, who was originally scheduled to work on Thursday. After all the switching was done, who worked on Tuesday?

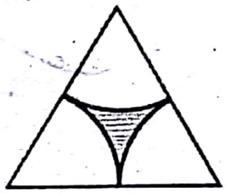
34/40

1. Mr. A
2. Mr. D
3. Ms. B
4. Ms. C

10. After 6 g of carbon is completely burnt in an atmosphere of 40 g of oxygen, the percentage oxygen left is:

1. 80
2. 60
3. 40
4. 20

11. What fraction of the equilateral triangle shown below with three identical sectors of a circle is shaded?



a

1. $1-\frac{\pi}{2\sqrt{3}}$
2. $\frac{\pi}{2\sqrt{3}}$
3. $1-\frac{2\pi}{\sqrt{3}}$
4. $1-\frac{\sqrt{3}\pi}{2}$

12. How many different salads can be made from cucumber, tomatoes, onions, beetroot and carrots?

1. 16
2. 28
3. 31
4. 32

C, T, O, B, C

$5+(4+3+2+1)+(3+$

$5+10+10+5+1$

$=31$

13. A bottle of perfume is opened and a person at a distance of 10 m gets the smell after 10 seconds. The time taken for a person 20 m away to get the smell is about

1. 20s
2. 40s
3. 14s
4. 80s

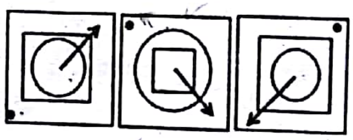
14. A mineral contains a cubic and a spherical cavity. The length of the side of the cube is the same as the diameter of the sphere. If the cubic cavity is half filled with a liquid and the spherical cavity is completely filled with liquid, what is the approximate ratio of the volume of liquid in the cubic cavity to that in the spherical cavity?

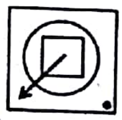
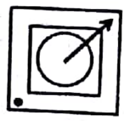
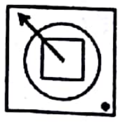
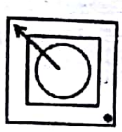
1. 2:1
2. 1:1
3. 1:2
4. 1:4

15. Out of 6 unbiased coins, 5 are tossed independently and they all result in heads. If the 6th is now independently tossed, the probability of getting head is

1. 1
2. 0
3. 1/2
4. 1/6

16. What could the fourth figure in the sequence be?

 ?

1. 
2. 
3. 
4. 

17. The average age of A, B and C, whose ages are integers x, y and z respectively $(x \le y \le z)$, is 30. If the age of B is exactly 5 more than that of A, what is the minimum possible value of z?

1. 31
2. 33
3. 35
4. 37

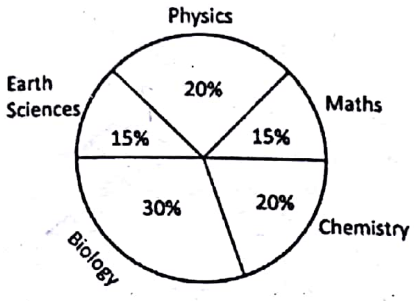
$\frac{x+y+z}{3} = 90$

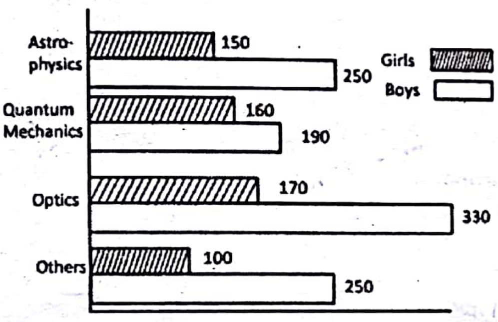
$A+(A+5)+z=90$

(https://pkalika.in/category/download/question-paper/)

5 .

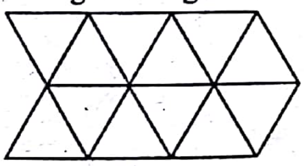
18. Percentage-wise distribution of all science students in a university is given in the pie-diagram. The bar chart shows the distribution of physics students in different sub-areas, where a student takes one and only one sub-area. What percentage of the total science students is girls studying quantum mechanics?





1. 10 2. 1
3. 0.2 4. 2

19. What is the total number of parallelograms in the given diagram?

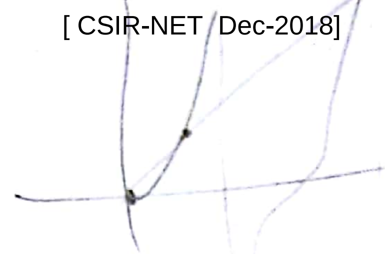


1. 27 2. 24
3. 22 4. 14

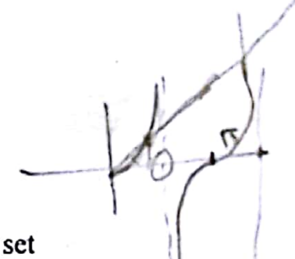
20. Election results of a city, which contains 3 segments (A, B and C) are given in the Table. Percentage votes obtained by parties X, Y and Z are also shown. Which party won the election?

Segment	Total Voters	% of voting	X	Y	Z
A	2,00,000	60	30	30	40
B	2,50,000	70	40	30	30
C	3,00,000	80	30	40	30

1. Y
2. X
3. Z
4. It was a tie between X and Y



PART - B



Unit-1

21. Consider the function $\tan x$ on the set $S = \left\{x \in \mathbb{R} : x \geq 0,\ x \neq k\pi + \frac{\pi}{2} \text{ for any } k \in \mathbb{N} \cup \{0\}\right\}$.

We say that it has a fixed point in S if $\exists\, x \in S$ such that $\tan x = x$. Then

1. there is a unique fixed point.
2. there is no fixed point.
3. there are infinitely many fixed points.
4. there are more than one but finitely many fixed points.

3.14
$\pi/4 = 1$

22. Define $f(x) = \frac{1}{\sqrt{x}}$ for $x > 0$. Then f is uniformly continuous

1. on $(0, \infty)$
2. on $[r, \infty)$ for any $r > 0$.
3. on $(0, r]$ for any $r > 0$.
4. only on intervals of the form $[a, b]$ for $0 < a < b < \infty$.

$x^{-1/2}$
$-\frac{1}{2}x^{-3/2}$
$= \frac{-1}{2x\sqrt{x}}$

23. Consider the subspaces W_1 and W_2 of $\mathbb{R}^3$ given by

$W_1 = \{(x, y, z) \in \mathbb{R}^3 : x + y + z = 0\}$

and

$W_2 = \{(x, y, z) \in \mathbb{R}^3 : x - y + z = 0\}$.

If W is a subspace of $\mathbb{R}^3$ such that

(i) $W \cap W_2 = \text{span}\{(0,1,1)\}$

(ii) $W \cap W_1$ is orthogonal to $W \cap W_2$ with respect to the usual inner product of $\mathbb{R}^3$, then

1. $W = \text{span}\{(0,1,-1), (0,1,1)\}$
2. $W = \text{span}\{(1,0,-1), (0,1,-1)\}$
3. $W = \text{span}\{(1,0,-1), (0,1,1)\}$
4. $W = \text{span}\{(1,0,-1), (1,0,1)\}$

(https://pkalika.in/category/download/question-paper/)

6

24. Let $C = \left[\begin{pmatrix}1\\2\end{pmatrix}, \begin{pmatrix}2\\1\end{pmatrix}\right]$ be a basis of $\mathbb{R}^2$ and $T : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $T\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}x+y\\x-2y\end{pmatrix}$. If $T[C]$ represents the matrix of T with respect to the basis C then which among the following is true?

1. $T[C] = \begin{bmatrix}-3 & -2\\ 3 & 1\end{bmatrix}$
2. $T[C] = \begin{bmatrix}3 & -2\\ -3 & 1\end{bmatrix}$
3. $T[C] = \begin{bmatrix}-3 & -1\\ 3 & 2\end{bmatrix}$
4. $T[C] = \begin{bmatrix}3 & -1\\ -3 & 2\end{bmatrix}$

25. Let $W_1 = \{(u,v,w,x) \in \mathbb{R}^4 \mid u+v+w=0, 2v+x=0, 2u+2w-x=0\}$ and $W_2 = \{(u,v,w,x) \in \mathbb{R}^4 \mid u+w+x=0, u+w-2x=0, v-x=0\}$. Then which among the following is true?

1. $\dim(W_1) = 1$
2. $\dim(W_2) = 2$
3. $\dim(W_1 \cap W_2) = 1$
4. $\dim(W_1 + W_2) = 3$

26. Let A be an $n \times n$ complex matrix. Assume that A is self-adjoint and let B denote the inverse of $A + iI_n$. Then all eigenvalues of $(A - iI_n)B$ are

1. purely imaginary
2. of modulus one
3. real
4. of modulus less than one

27. Let $\{u_1, u_2, \dots, u_n\}$ be an orthonormal basis of $\mathbb{C}^n$ as column vectors. Let $M = (u_1, \dots, u_k)$, $N = (u_{k+1}, \dots, u_n)$ and P be the diagonal $k \times k$ matrix with diagonal entries $\alpha_1, \alpha_2, \dots, \alpha_k \in \mathbb{R}$. Then which of the following is true?

1. Rank $(MPM^*) = k$ whenever $\alpha_i \neq \alpha_j$ $1 \le i,j \le k$.
2. Trace $(MPM^*) = \sum_{i=1}^{k} \alpha_i$
3. Rank $(M^*N) = \min(k, n-k)$
4. Rank $(MM^* + NN^*) < n$

28. Let $B : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ be the function $B(a,b) = ab$.

Which of the following is true?

1. B is a linear transformation
2. B is a positive definite bilinear form
3. B is symmetric but not positive definite
4. B is neither linear nor bilinear

29. Consider the map $f: \mathbb{Q} \to \mathbb{R}$ defined by
(i) $f(0) = 0$
(ii) $f(r) = \frac{p}{10^q}$ where $r = \frac{p}{q}$ with $p \in \mathbb{Z}$, $q \in \mathbb{N}$ and gcd $(p,q) = 1$.
Then the map f is

1. one-to-one and onto
2. not one-to-one, but onto
3. onto but not one-to-one
4. neither one-to-one nor onto

30. Let x be a real number such that $|x| < 1$. Which of the following is FALSE?

1. If $x \in \mathbb{Q}$, then $\sum_{m\ge0} x^m \in \mathbb{Q}$
2. If $\sum_{m\ge0} x^m \in \mathbb{Q}$ then $x \in \mathbb{Q}$
3. If $x \notin \mathbb{Q}$ then $\sum_{m\ge0} m x^{m-1} \notin \mathbb{Q}$
4. $\sum_{m\ge1} \frac{x^m}{m}$ converges in $\mathbb{R}$

31. Suppose that $\{x_n\}$ is a sequence of real numbers satisfying the following For every $\varepsilon > 0$, there exists n_0 such that $|x_{n+1} - x_n| < \varepsilon \;\forall\; n \ge n_0$.
The sequence $\{x_n\}$ is

1. bounded but not necessarily Cauchy
2. Cauchy but not necessarily bounded
3. convergent
4. not necessarily bounded

32. Let $A(n) = \int_n^{n+1} \frac{1}{x^3}\,dx$ for $n \ge 1$.
For $c \in \mathbb{R}$ let $\lim_{n\to\infty} n^c A(n) = L$. Then

1. $L = 0$ if $c > 3$
2. $L = 1$ if $c = 3$
3. $L = 2$ if $c = 3$
4. $L = \infty$ if $0 < c < 3$

4-C-E

Unit-2

33. Consider the polynomials $p(z), q(z)$ in the complex variable z and let

$$I_{p,q} = \oint_\gamma p(z)\overline{q(z)}\,dz$$

where γ denotes the closed contour $\gamma(t) = e^{it}, 0 \le t \le 2\pi$. Then

1. $I_{z^m,z^n} = 0$ for all positive integers m, n with $m \ne n$
2. $I_{z^n,z^n} = 2\pi i$ for all positive integers n
3. $I_{p,1} = 0$ for all polynomials p
4. $I_{p,q} = p(0)\ \overline{q(0)}$ for all polynomials p, q

34. Let $\gamma(t) = 3e^{it}, 0 \le t \le 2\pi$ be the positively oriented circle of radius 3 centred at the origin. The value of λ for which

$= -\frac{1}{3} \times 2\pi i$

$$\oint_\gamma \frac{\lambda}{z-2}dz = \oint_\gamma \frac{1}{z^2 - 5z + 4}dz$$

$= \frac{1}{(z-4)(z-1)}$

is

1. $\lambda = -1/3$
2. $\lambda = 0$
3. $\lambda = 1/3$
4. $\lambda = 1$

35. The number of group homomorphisms from the alternating group A_5 to the symmetric group S_4 is:

1. 1
2. 12
3. 20
4. 6

36. Let $p \ge 23$ be a prime number such that the decimal expansion (base 10) of $\frac{1}{p}$ is periodic with period $p-1$ (that is, $\frac{1}{p} = 0 \cdot \overline{a_1 a_2 \ldots a_{p-1}}$) with $a_i \in \{0,1,\ldots,9\}$ for all i and for any $m, 1 \le m < p-1, \frac{1}{p} \ne 0 \cdot \overline{a_1 a_2 \ldots a_m}$).

Let $(\mathbb{Z}/p\mathbb{Z})^*$ denote the multiplicative group of integers modulo p. Then which of the following is correct?

1. The order of $10 \in (\mathbb{Z}/p\mathbb{Z})^*$ is a proper divisor of $(p-1)$.
2. The order of $10 \in (\mathbb{Z}/p\mathbb{Z})^*$ is $\frac{(p-1)}{2}$.
3. The element $10 \in (\mathbb{Z}/p\mathbb{Z})^*$ is a generator of the group $(\mathbb{Z}/p\mathbb{Z})^*$.
4. The group $(\mathbb{Z}/p\mathbb{Z})^*$ is cyclic but not generated by the element 10.

37. Given integers a and b, let $N_{a,b}$ denote the number of positive integers $k \le 100$ such that $k \equiv a \pmod 9$ and $k \equiv b \pmod{11}$. Then which of the following statements is correct?

1. $N_{a,b} = 1$ for all integers a and b.
2. There exist integers a and b satisfying $N_{a,b} > 1$.
3. There exist integers a and b satisfying $N_{a,b} = 0$.
4. There exist integers a and b satisfying $N_{a,b} = 0$ and there exist integers c and d satisfying $N_{c,d} > 1$.

38. Let X be a topological space and U be a proper dense open subset of X. Pick the correct statement from the following:

1. If X is connected then U is connected.
2. If X is compact then U is compact.
3. If $X \setminus U$ is compact then X is compact.
4. If X is compact, then $X \setminus U$ is compact.

39. Let R denote the radius of convergence of the power series

$$\sum_{k=1}^{\infty} kx^k.$$

Then

1. $R > 0$ and the series is convergent on [-R, R].
2. $R > 0$ and the series converges at $x = -R$ but does not converge at $x = R$.
3. $R > 0$ and the series does not converge outside $(-R, R)$.
4. $R = 0$.

40. Let $f: \mathbb{C} \to \mathbb{C}$ be a non-constant entire function and let

Image$(f) = \{w \in \mathbb{C} : \exists z \in \mathbb{C}$ such that $f(z) = w\}$.

Then

1. The interior of Image(f) is empty.
2. Image (f) intersects every line passing through the origin.
3. There exists a disc in the complex plane, which is disjoint from Image (f)
4. Image(f) contains all its limit points.

Unit-3

41. Let $u(x,t)$ be a function that satisfies the PDE
$u_{xx} - u_{tt} = e^x + 6t,\ x\epsilon\mathbb{R}, t > 0$ and the initial conditions
$u(x,0) = \sin(x), u_t(x,0) = 0$
for every $x \in \mathbb{R}$.
Here subscripts denote partial derivatives corresponding to the variables indicated.
Then the value of $u\left(\frac{\pi}{2},\frac{\pi}{2}\right)$ is

1. $e^{\pi/2}\left(1+\frac{1}{2}e^{\pi/2}\right)+\left(\frac{\pi^3+4}{8}\right)$
2. $e^{\pi/2}\left(1+\frac{1}{2}e^{\pi/2}\right)+\left(\frac{\pi^3-4}{8}\right)$
3. $e^{\pi/2}\left(1-\frac{1}{2}e^{\pi/2}\right)-\left(\frac{\pi^3+4}{8}\right)$
4. $e^{\pi/2}\left(1-\frac{1}{2}e^{\pi/2}\right)-\left(\frac{\pi^3-4}{8}\right)$

42. Let $u(x,t)$ satisfy the IVP:
$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2},\ x\epsilon\mathbb{R}, t > 0$

$u(x,0) = \begin{cases} 1, & 0 \le x \le 1 \\ 0, & \text{elsewhere.} \end{cases}$

Then the value of $\lim_{t\to 0+} u(1,t)$ equals

1. e
2. π
3. $1/2$
4. 1

43. Let $f(x)$ be a polynomial of unknown degree taking the values

x	0	1	2	3
$f(x)$	2	7	13	16

All the fourth divided differences are $-1/6$. Then the coefficient of x^3 is

1. $1/3$
2. $-2/3$
3. 16
4. -1

44. Consider the functional

$$J[y] = \int_0^2 (1-y'^2)^2 dx$$

defined on $\{y\epsilon C[0,2]: y$ is piecewise C^1 and $y(0) = y(2) = 0\}$. Let y_e be a minimizer of the above functional. Then y_e has

1. a unique corner point
2. two corner points
3. more than two corner points
4. no corner points

45. If φ is the solution of

$$\int_0^x (1-x^2+t^2)\varphi(t)dt = \frac{x^2}{2},$$

then $\varphi(\sqrt{2})$ is equal to

1. $\sqrt{2}e^{\sqrt{2}}$
2. $\sqrt{2}e^2$
3. $\sqrt{2}e^{2\sqrt{2}}$
4. $2e^4$

46. Consider the two dimensional motion of a mass m attached to one end of a spring whose other end is fixed. Let k be the spring constant. The kinetic energy T and the potential energy V of the system are given by
$T = \frac{1}{2}m\left(\dot{r}^2 + (r\dot{\theta})^2\right)$ and $V = \frac{1}{2}kr^2$,
where $\dot{r} = \frac{dr}{dt}$ and $\dot{\theta} = \frac{d\theta}{dt}$ with t as time.
Then which of the following statements is correct?

1. r is an ignorable coordinate
2. θ is not an ignorable coordinate
3. $r^2\dot{\theta}$ remains constant throughout the motion
4. $r\dot{\theta}$ remains constant throughout the motion

47. If $y_1(x)$ and $y_2(x)$ are two solutions of the differential equation
$(\cos x)y'' + (\sin x)y' - (1+e^{-x^2})y = 0\ \ \forall x \in \left(\frac{-\pi}{2},\frac{\pi}{2}\right)$
with $y_1(0) = \sqrt{2}, y_1'(0) = 1, y_2(0) = -\sqrt{2},\ y_2'(0) = 2,$
then the Wronskian of $y_1(x)$ and $y_2(x)$ at $x = \frac{\pi}{4}$ is

1. $3\sqrt{2}$
2. 6
3. 3
4. $-3\sqrt{2}$

48. The critical point $(0,0)$ for the system
$x'(t) = x - 2y + y^2\sin(x)$
$y'(t) = 2x - 2y - 3y\cos(y^2)$
is a

1. stable spiral point
2. unstable spiral point
3. saddle point
4. stable node

$\begin{vmatrix} 1 & -2 \\ 2 & -2 \end{vmatrix}$

$x^2 + x + 2 = 0$

$x = \frac{-1 \pm \sqrt{1-8}}{2}$

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$C \to \frac{x^2}{2} + x^2 C \Big|_0^x +$ $\quad \phi = x + 2x \cdot \left(\frac{1}{2} + \frac{1}{1-x^2}\right)$

Unit-4 $\Rightarrow C(1-x^2) = x^2/2$

49. To test the hypotheses H_0 against H_1 using the test statistic T, the proposed test procedure is not to support H_0 if T is large. Based on a given sample, the p-value of the test statistic is computed to be 0.05 assuming that the distribution of T is N(0, 1) under H_0. If the distribution of T under H_0 is the t-distribution with 10 degrees of freedom instead, the p-value will be

1. 0.05
2. $< 0.05 - \frac{1}{100}$
3. $0.05 - \frac{1}{100}$
4. > 0.05

50. Let $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$ be n independent observations from a bivariate continuous distribution. Let r_p be the product moment correlation coefficient and r_s be the rank correlation coefficient computed based on these n observations. Which of the following statements is correct?

1. $r_p \geq 0$ implies $r_s \geq 0$
2. $r_s \geq 0$ implies $r_p \geq 0$
3. $r_p = 1$ implies $r_s = 1$
4. $r_s = 1$ implies $r_p = 1$

51. Consider a linear model

$Y_i = \theta_1 + \theta_2 + \varepsilon_i$ for $i = 1, 2$ and

$Y_i = \theta_1 - \theta_3 + \varepsilon_i$ for $i = 3, 4$, where ε_i's are independent with $\mathbb{E}(\varepsilon_i) = 0$, $Var(\varepsilon_i) = \sigma^2 > 0$ for $i = 1, \ldots, 4$, and $\theta_1, \ldots \theta_3 \in \mathbb{R}$. Which of the following parametric functions is estimable?

1. $\theta_1 + \theta_3$
2. $\theta_2 - \theta_3$
3. $\theta_2 + \theta_3$
4. $\theta_1 + \theta_2 + \theta_3$

52. If $X \sim N_p(0, I)$ and $A_{p \times p}$ is an idempotent matrix with rank $(A) = k < p$, then which of the following statements is correct?

1. $\frac{X'AX}{X'X} \sim \frac{k}{p} F_{k,p}$
2. $\frac{X'AX}{X'X} \sim \frac{k}{p-k} F_{k,p-k}$
3. $\frac{X'AX}{X'X} \sim Beta\left(\frac{k}{2}, \frac{p}{2}\right)$
4. $\frac{X'AX}{X'X} \sim Beta\left(\frac{k}{2}, \frac{p-k}{2}\right)$

53. A sample of size $n\ (\geq 2)$ is drawn from a population of $N (\geq 3)$ units using $PPSWR$ sampling scheme, where p_i is the probability of selecting i^{th} unit in a draw, $0 < p_i < 1\ \forall i = 1, \ldots, N$, and $\sum_{i=1}^{N} p_i = 1$. Then the inclusion probability π_{ij} is

1. $1 - p_i^n - p_j^n + (p_i + p_j)^n$
2. $1 - (p_i + p_j - p_i p_j)^n$
3. $1 - (1 - p_i)^n - (1 - p_j)^n - (p_i + p_j)^n$
4. $1 - (1 - p_i)^n - (1 - p_j)^n + (1 - p_i - p_j)^n$

54. In a 2^4 experiment with two blocks and factors A, B, C and D, one block contains the following treatment combinations $a, b, c, ad, bd, cd, abc, abcd$. Which of the following effects is confounded?

1. ABC
2. ABD
3. BCD
4. $ABCD$

55. In an airport, domestic passengers and international passengers arrive independently according to Poisson processes with rates 100 and 70 per hour, respectively. If it is given that the total number of passengers (domestic and international) arriving in that airport between 9:00 AM and 11:00 AM on a particular day was 520, then what is the conditional distribution of the number of domestic passengers arriving in this period?

1. Poisson (200)
2. Poisson (100)
3. Binomial $\left(520, \frac{10}{17}\right)$
4. Binomial $\left(520, \frac{7}{17}\right)$

56. Let $X \geq 0$ be a random variable on $(\Omega, \mathcal{F}, P)$ with $\mathbb{E}(X) = 1$. Let $A \in \mathcal{F}$ be an event with $0 < P(A) < 1$. Which of the following defines another probability measure on $(\Omega, \mathcal{F})$?

1. $Q(B) = P(A \cap B) \quad \forall\ B \in \mathcal{F}$
2. $Q(B) = P(A \cup B) \quad \forall\ B \in \mathcal{F}$
3. $Q(B) = \mathbb{E}(X I_B) \quad \forall\ B \in \mathcal{F}$
4. $Q(B) = \begin{cases} P(A|B) & if\ P(B) > 0 \\ 0 & if\ P(B) = 0 \end{cases}$

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57. Let X and Y be $i.i.d$ random variables uniformly distributed on $(0,4)$. Then $P(X > Y | X < 2Y)$ is

1. $\frac{1}{3}$
2. $\frac{5}{6}$
3. $\frac{1}{4}$
4. $\frac{2}{3}$

$1/2$

58. Suppose $\{X_n\}$ is a Markov Chain with 3 states and transition probability matrix

$$\begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Then which of the following statements is true?

1. $\{X_n\}$ is irreducible
2. $\{X_n\}$ is recurrent
3. $\{X_n\}$ does not admit a stationary probability distribution
4. $\{X_n\}$ has an absorbing state

59. Suppose $X \sim$ Cauchy$(0,1)$. Then the distribution of $\frac{1-X}{1+X}$ is

1. Uniform $(0,1)$
2. Normal $(0,1)$
3. Double exponential $(0,1)$
4. Cauchy $(0,1)$

60. Given the observations 0.8, 0.71, 0.9, 1.2, 1.68, 1.4, 0.88, 1.62 from the uniform distribution on $(\theta - 0.2, \theta + 0.8)$ with $-\infty < \theta < \infty$, which of the following is a maximum likelihood estimate for θ?

1. 0.7
2. 0.9
3. 1.1
4. 1.3

$T^3 = T^2 - T$

$= (T - I_n) - T$

$= -I_n$

$T + I = 0$

PART - C

Unit-1

61. Let $f: \mathbb{R}^2 \to \mathbb{R}^2$ be a function given by $f(x,y) = (x^3 + 3xy^2 - 15x - 12y, x + y)$. Let $S = \{(x,y) \in \mathbb{R}^2 : f$ is locally invertible at $(x,y)\}$. Then

1. $S = \mathbb{R}^2 \setminus \{(0,0)\}$
2. S is open in $\mathbb{R}^2$
3. S is dense in $\mathbb{R}^2$
4. $\mathbb{R}^2 \setminus S$ is countable

62. Let $X = \mathbb{N}$, the set of positive integers. Consider the metrics d_1, d_2 on X given by

$d_1(m,n) = |m - n|, m, n \in X$

$d_2(m,n) = \left|\frac{1}{m} - \frac{1}{n}\right|, m, n \in X$ $= \frac{n-m}{mn}$

Let X_1, X_2 denote the metric spaces $(X, d_1), (X, d_2)$ respectively. Then

1. X_1 is complete
2. X_2 is complete
3. X_1 is totally bounded
4. X_2 is totally bounded

63. Let $T: \mathbb{R}^n \to \mathbb{R}^n$ be a linear map that satisfies $T^2 = T - I_n$. Then which of the following are true?

1. T is invertible
2. $T - I_n$ is not invertible
3. T has a real eigen value
4. $T^3 = -I_n$

$\lambda^2 = \lambda - 1$

$\lambda^2 - \lambda + 1 = 0$

$\lambda = \frac{1 \pm \sqrt{1-4}}{2}$

64. Let $M = \begin{bmatrix} 2 & 0 & 3 & 2 & 0 & -2 \\ 0 & 1 & 0 & -1 & 3 & 4 \\ 0 & 0 & 1 & 0 & 4 & 4 \\ 1 & 1 & 0 & 0 & 1 & 1 \end{bmatrix}$,

4-eqn 6-var

$b_1 = \begin{bmatrix} 5 \\ 1 \\ 1 \\ 4 \end{bmatrix}$ and $b_2 = \begin{bmatrix} 5 \\ 1 \\ 3 \\ 3 \end{bmatrix}$. Then which of the following are true?

1. both systems $MX = b_1$ and $MX = b_2$ are inconsistent
2. both systems $MX = b_1$ and $MX = b_2$ are consistent
3. the system $MX = b_1 - b_2$ is consistent
4. the systems $MX = b_1 - b_2$ is inconsistent

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65. Let $M = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & 4 \\ -2 & 1 & -4 \end{bmatrix}$. Given that 1 is an eigenvalue of M, then which among the following are correct?
1. The minimal polynomial of M is $(X-1)(X+4)$
2. The minimal polynomial of M is $(X-1)^2(X+4)$
3. M is not diagonalizable
4. $M^{-1} = \frac{1}{4}(M+3I)$

66. Let A be a real matrix with characteristic polynomial $(X-1)^3$. Pick the correct statements from below:
1. A is necessarily diagonalizable
2. If the minimal polynomial of A is $(X-1)^3$, then A is diagonalizable
3. Characteristic polynomial of A^2 is $(X-1)^3$
4. If A has exactly two Jordan blocks, then $(A-1)^2$ is diagonalizable

67. Let P_3 be the vector space of polynomials with real coefficients and of degree at most 3. Consider the linear map $T: P_3 \to P_3$ defined by
$$T(p(x)) = p(x+1) + p(x-1)).$$
Which of the following properties does the matrix of T (with respect to the standard basis $B = \{1, x, x^2, x^3\}$ of P_3) satisfy?
1. $\det T = 0$
2. $(T-2I)^4 = 0$ but $(T-2I)^3 \neq 0$
3. $(T-2I)^3 = 0$ but $(T-2I)^2 \neq 0$
4. 2 is an eigenvalue with multiplicity 4

68. Let M be an $n \times n$ Hermitian matrix of rank $k, k \neq n$. If $\lambda \neq 0$ is an eigenvalue of M with corresponding unit column vector u, with $Mu = \lambda u$, then which of the following are true?
1. rank $(M\text{-}\lambda uu^*) = k-1$
2. rank $(M\text{-}\lambda uu^*) = k$
3. rank $(M\text{-}\lambda uu^*) = k+1$
4. $(M - \lambda uu^*)^n = M^n - \lambda^n uu^*$

69. Define a real valued function B on $\mathbb{R}^2 \times \mathbb{R}^2$ as follows. If $v = (x_1, x_2), w = (y_1, y_2)$ belong to $\mathbb{R}^2$ define $B(u, w) = x_1y_1 - x_1y_2 - x_2y_1 + 4x_2y_2$. Let $v_0 = (1, 0)$ and let $W = \{v \in \mathbb{R}^2 : B(v_0, v) = 0\}$. Then W
1. is not a subspace of $\mathbb{R}^2$
2. equals $\{(0,0)\}$
3. is the y axis
4. is the line passing through (0,0) and (1, 1)

70. Consider the Quadratic forms
$Q_1(x, y) = xy$
$Q_2(x, y) = x^2 + 2xy + y^2$
$Q_3(x, y) = x^2 + 3xy + 2y^2$
on $\mathbb{R}^2$. Choose the correct statements from below:
1. Q_1 and Q_2 are equivalent
2. Q_1 and Q_3 are equivalent
3. Q_2 and Q_3 are equivalent
4. all are equivalent

71. Let $\{u_n\}_{n\geq1}$ be a sequence of real numbers satisfying the following conditions:
(1)$(-1)^n u_n \geq 0$, for all $n \geq 1$
(2)$|u_{n+1}| < \frac{|u_n|}{2}$, for all $n \geq 13$
Which of the following statements are necessarily true?
1. $\sum_{n\geq1} u_n$ does not converge in $\mathbb{R}$.
2. $\sum_{n\geq13} u_n$ converges to zero.
3. $\sum_{n\geq13} u_n$ converges to a non-zero real number.
4. If $|u_{n-1}| < \frac{|u_n|}{2}$, for all $2 \leq n \leq 13$, then $\sum_{n\geq1} u_n$ is a negative real number.

72. Let S be an infinite set. Which of the following statements are true?
1. If there is an injection from S to $\mathbb{N}$, then S is countable
2. If there is a surjection from S to $\mathbb{N}$, then S is countable
3. If there is an injection from $\mathbb{N}$ to S, then S is countable
4. If there is a surjection from $\mathbb{N}$ to S, then S is countable

73. Let p_n denote the n-th prime number, when we enumerate the prime numbers in the increasing order. For example, $p_1 = 2, p_2 = 3, p_3 = 5$, and so on. Let $S = \{s_n = p_{n+1} - p_n | n \in \mathbb{N}, n \geq 1\}$. Then which of the following are correct?

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1. $\sup S = \infty$
2. $\limsup_{n\to\infty} s_n = \infty$
3. $\inf S < \infty$ and $\inf S = 1$
4. $\liminf_{n\to\infty} s_n \geq 2$

74. For $n \geq 1$, consider the sequence of functions
$f_n(x) = \frac{1}{2nx+1}, g_n(x) = \frac{x}{2nx+1}$ on the open interval $(0, 1)$. Consider the statements:
(I) The sequence $\{f_n\}$ converges uniformly on (0, 1)
(II) The sequence $\{g_n\}$ converges uniformly on (0, 1)
Then,
1. (I) is true
2. (I) is false
3. (I) is false and (II) is true
4. Both (I) and (II) are true

75. Suppose that $\{f_n\}$ is a sequence of continuous real valued functions on [0,1] satisfying the following:
(A) $\forall x \in \mathbb{R}$, $\{f_n(x)\}$ is a decreasing sequence.
(B) the sequence $\{f_n\}$ converges uniformly to 0.
Let $g_n(x) = \sum_{k=1}^{n} (-1)^k f_k(x) \quad \forall x \in \mathbb{R}$.
Then
1. $\{g_n\}$ is Cauchy with respect to the sup norm.
2. $\{g_n\}$ is uniformly convergent
3. $\{g_n\}$ need not converge pointwise
4. $\exists M > 0$ such that $|g_n(x)| \leq M, \ \forall n \in \mathbb{N}, \ \forall x \in \mathbb{R}$

76. Given $f: \left[\frac{1}{2}, 2\right] \to \mathbb{R}$, a strictly increasing function, we put $g(x) = f(x) + f(1/x), x \in [1, 2]$. Consider a partition P of $[1, 2]$ and let $U(P, g)$ and $L(P, g)$ denote the upper Riemann sum and lower Riemann sum of g. Then
1. for a suitable f we can have $U(P, g) = L(P, g)$
2. for a suitable f we can have $U(P, g) \neq L(P, g)$
3. $U(P, g) \geq L(P, g)$ for all choices of f
4. $U(P, g) < L(P, g)$ for all choices of f

77. Let f be a real valued continuously differentiable function of $(0, 1)$. Set $g = f' + if$, where $i^2 = -1$ and f' is the derivative of f. Let $a, b \in (0, 1)$ be two consecutive zeros of f. Which of the following statements are necessarily true?
1. If $g(a) > 0$, then g crosses the real line from upper half plane to lower half plane at a
2. If $g(a) > 0$, then g crosses the real line from lower half plane to upper half plane at a
3. If $g(a)g(b) \neq 0$, then $g(a), g(b)$ have the same sign
4. If $g(a)g(b) \neq 0$, then $g(a), g(b)$ have opposite signs

78. Let A be an invertible real $n \times n$ matrix. Define a function $F: \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ by $F(x, y) = \langle Ax, y\rangle$ where $\langle x, y\rangle$ denotes the inner product of x and y. Let $DF(x, y)$ denote the derivative of F at (x, y) which is a linear transformation from $\mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$. Then
1. If $x \neq 0$, then $DF(x, 0) \neq 0$
2. If $y \neq 0$, then $DF(0, y) \neq 0$
3. If $(x, y) \neq (0,0)$ then $DF(x, y) \neq 0$
4. If $x = 0$ or $y = 0$, then $DF(x, y) = 0$

Unit-2

79. For any group G, let Aut(G) denote the group of automorphisms of G. Which of the following are true?
1. If G is finite, then Aut(G) is finite.
2. If G is cyclic, then Aut(G) is cyclic
3. If G is infinite, then Aut(G) is infinite
4. If Aut(G) is isomorphic to Aut(H), where G and H are two groups, then G is isomorphic to H

80. Let G be a group with the following property: Given any positive integers m, n and r there exist elements g and h in G such that order$(g) = m$, order$(h) = n$ and order$(gh) = r$. Then which of the following are necessarily true?
1. G has to be an infinite group
2. G cannot be a cyclic group
3. G has infinitely many cyclic subgroups
4. G has to be a non-abelian group

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$\frac{az+b}{cz+d}$

ad - bc

81. Let R be the ring $\mathbb{C}[x]/(x^2+1)$. Pick the correct statements from below:

1. $\dim_{\mathbb{C}} R = 3$
2. R has exactly two prime ideals
3. R is a UFD
4. (x) is a maximal ideal of R

82. Let $f(x) = x^7 - 105x + 12$. Then which of the following are correct?

1. $f(x)$ is reducible over $\mathbb{Q}$
2. There exists an integer m such that $f(m) = 105$
3. There exists an integer m such that $f(m) = 2$
4. $f(m)$ is not a prime number for any integer m

83. Let $\alpha = \sqrt[5]{2} \in \mathbb{R}$ and $\xi = exp\left(\frac{2\pi i}{5}\right)$. Let $K = \mathbb{Q}(\alpha\xi)$. Pick the correct statements from below:

1. There exists a field automorphism σ of $\mathbb{C}$ such that $\sigma(K) = K$ and $\sigma \neq id$
2. There exists a field automorphism σ of $\mathbb{C}$ such that $\sigma(K) \neq K$
3. There exists a finite extension E of $\mathbb{Q}$ such that $K \subseteq E$ and $\sigma(K) \subseteq E$ for every field automorphism σ of E
4. For all field automorphisms σ of K, $\sigma(\alpha\xi) = \alpha\xi$

84. Let $X = \{(x_i)_{i\geq 1} : x_i \in \{0,1\}$ for all $i \geq 1\}$ with the metric $d\big((x_i),(y_i)\big) = \sum_{i\geq 1}|x_i - y_i|2^{-i}$. Let $f: X \to [0,1]$ be the function defined by $f(x_i)_{i\geq 1} = \sum_{i\geq 1} x_i 2^{-i}$. Choose the correct statements from below:

1. f is continuous
2. f is onto
3. f is one-to-one
4. f is open

85. Let A be a subset of $\mathbb{R}$ satisfying $A = \bigcap_{n\geq 1} V_n$, where for each $n \geq 1$, V_n is an open dense subset of $\mathbb{R}$. Which of the following are correct?

1. A is a non-empty set
2. A is countable
3. A is uncountable
4. A is dense in $\mathbb{R}$

86. Let H denote the upper half plane, that is,
$H = \{z = x + iy : y > 0\}$
For $z \in H$, which of the following are true?

1. $\frac{1}{z} \in H$
2. $\frac{1}{z^2} \in H$
3. $\frac{-z}{z+1} \in H$
4. $\frac{z}{2z+1} \in H$

87. Let $f: \mathbb{C} \to \mathbb{C}$ be an analytic function. Then which of the following statements are true?

1. If $|f(z)| \leq 1$ for all $z \in \mathbb{C}$, then f' has infinitely many zeros in $\mathbb{C}$
2. If f is onto, then the function $f(\cos z)$ is onto
3. If f is onto, then the function $f(e^z)$ is onto
4. If f is one-one, then the function $f(z^4 + z + 2)$ is one-one

88. Consider the entire functions $f(z) = 1 + z + z^{20}$ and $g(z) = e^z, z \in \mathbb{C}$. Which of the following statements are true?

1. $\lim_{|z|\to\infty}|f(z)| = \infty$
2. $\lim_{|z|\to\infty}|g(z)| = \infty$
3. $f^{-1}(\{z \in \mathbb{C} : |z| \leq R\})$ is bounded for every $R > 0$
4. $g^{-1}(\{z \in \mathbb{C} : |z| \leq R\})$ is bounded for every $R > 0$

89. Which of the following statements are true?

1. $\tan z$ is an entire function
2. $\tan z$ is a meromorphic function on $\mathbb{C}$
3. $\tan z$ has an isolated singularity at ∞
4. $\tan z$ has a non-isolated singularity at ∞

90. Let $a_1 < a_2 < \cdots < a_{51}$ be given distinct natural numbers such that $1 \leq a_i \leq 100$ for all $i = 1,2,\ldots,51$. Then which of the following are correct?

1. There exist i and j with $1 \leq i < j \leq 51$ satisfying a_i divides a_j.
2. There exists i with $1 \leq i \leq 51$ such that a_i is an odd integer.
3. There exists j with $1 \leq j \leq 51$ such that a_j is an even integer.
4. There exist $i < j$ such that $|a_i - a_j| > 51$.

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Unit-3

91. Let $u(x,t)$ be a function that satisfies the PDE : $u_t + uu_x = 1, x \in \mathbb{R}, t > 0$, and the initial condition $u\left(\frac{t^2}{4}, t\right) = \frac{t}{2}$. Then the IVP has

1. only one solution
2. two solutions
3. an infinite number of solutions
4. solutions none of which is differentiable on the characteristic base curve

92. Let $f: [0,1] \to [0,1]$ be twice continuously differentiable function with a unique fixed point $f(x_*) = x_*$. For a given $x_0 \in (0,1)$ consider the iteration $x_{n+1} = f(x_n)$ for $n \geq 0$.
If $L = \max_{x\in[0,1]} |f'(x)|$, then which of the following are true?

1. If $L < 1$, then x_n converges to x_*
2. x_n converges to x_* provided $L \geq 1$
3. The error $e_n = x_n - x_*$ satisfies $|e_{n+1}| < L|e_n|$
4. If $f'(x_*) = 0$, then $|e_{n+1}| < C|e_n|^2$ for some $C > 0$

93. Let $u(x)$ satisfy the boundary value problem

$$(BVP) \quad \begin{cases} u'' + u' = 0, & x\epsilon(0,1) \\ u(0) = 0 \\ u(1) = 1. \end{cases}$$

Consider the finite difference approximation to (BVP)

$$(BVP)_h \begin{cases} \frac{U_{j+1}-2U_j+U_{j-1}}{h^2} + \frac{U_{j+1}-U_{j-1}}{2h} = 0, j = 1, \dots, N-1 \\ U_0 = 0 \\ U_N = 1 \end{cases}$$

Here U_j is an approximation to $u(x_j)$ where $x_j = jh, j = 0, \dots, N$ is a partition of [0,1] with $h = 1/N$ for some positive integer N. Then which of the following are true?

1. There exists a solution to $(BVP)_h$ of the form $U_j = ar^j + b$ for some $a, b \in \mathbb{R}$ with $r \neq 1$ and r satisfying $(2+h)r^2 - 4r + (2-h) = 0$
2. $U_j = (r^j - 1)/(r^N - 1)$ where r satisfies $(2+h)r^2 - 4r + (2-h) = 0$ and $r \neq 1$
3. u is monotonic in x
4. U_j is monotonic in j.

94. Consider the functional $J[y] = \int_0^1 [(y')^2 - (y')^4]\, dx$ subject to $y(0) = 0, y(1) = 0$. A broken extremal is a continuous extremal whose derivative has jump discontinuities at a finite number of points. Then which of the following statements are true?

1. There are no broken extremals and $y = 0$ is an extremal
2. There is a unique broken extremal
3. There exist more than one and finitely many broken extremals
4. There exist infinitely many broken extremals

95. The extremals of the functional $J[y] = \int_0^1 [720x^2y - (y'')^2]dx$, subject to $y(x) = y'(0) = y(1) = 0, y'(1) = 6$, are

1. $x^6 + 2x^3 - 3x^2$
2. $x^5 + 4x^4 - 5x^3$
3. $x^5 + x^4 - 2x^3$
4. $x^6 + 4x^3 - 6x^2$

96. If φ is the solution of $\varphi(x) = 1 - 2x - 4x^2 + \int_0^x [3 + 6(x-t) - 4(x-t)^2]\, \varphi(t) dt$, then $\varphi(\log 2)$ is equal to

1. 2
2. 4
3. 6
4. 8

97. A characteristic number and the corresponding eigenfunction of the homogeneous Fredholm integral equation with kernel

$$K(x,t) = \begin{cases} x(t-1), & 0 \leq x \leq t \\ t(x-1), & t \leq x \leq 1 \end{cases} \quad \text{are}$$

1. $\lambda = -\pi^2, \varphi(x) = \sin\pi x$
2. $\lambda = -2\pi^2, \varphi(x) = \sin 2\pi x$
3. $\lambda = -3\pi^2, \varphi(x) = \sin 3\pi x$
4. $\lambda = -4\pi^2, \varphi(x) = \sin 2\pi x$

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98. Consider a point mass of mass m which is attached to a mass-less rigid rod of length a. The other end of the rod is made to move vertically such that its downward displacement from the origin at time t is given by $z(t) = z_0 \cos(\omega t)$. The mass is moving in a fixed plane and its position vector at time t is given by

$$\bar{r}(t) = (a \sin\theta(t), z(t) + a\cos\theta(t)).$$

Then the equation of motion of the point mass is

1. $a\frac{d^2\theta}{dt^2} + (g + z_0\omega^2 \cos(\omega t))\sin\theta = 0$
2. $a\frac{d^2\theta}{dt^2} + (g - z_0\omega \cos(\omega t))\sin\theta = 0$
3. $a\frac{d^2\theta}{dt^2} + (g + z_0^2\omega^2 \cos(\omega t))\cos\theta = 0$
4. $a\frac{d^2\theta}{dt^2} + \left(g - z_0\ \omega^2 \cos(\omega t)\right)\cos\theta = 0$

99. Three solutions of a certain second order non-homogeneous linear differential equation are

$y_1(x) = 1 + xe^{x^2}, y_2(x) = (1 + x)e^{x^2} - 1, y_3(x) = 1 + e^{x^2}$.

Which of the following is (are) general solution(s) of the differential equation?

1. $(C_1 + 1)y_1 + (C_2 - C_1)y_2 - C_2y_3$, where C_1 and C_2 are arbitrary constants
2. $C_1(y_1 - y_2) + C_2(y_2 - y_3)$, where C_1 and C_2 are arbitrary constants
3. $C_1(y_1 - y_2) + C_2(y_2 - y_3) + C_3(y_3 - y_1)$, where C_1, C_2 and C_3 are arbitrary constants
4. $C_1(y_1 - y_3) + C_2(y_3 - y_2) + y_1$, where C_1 and C_2 are arbitrary constants

100. The method of variation of parameters to solve the differential equation $y'' + p(x)y' + q(x)y = r(x)$, where $x \in I$ and $p(x), q(x), r(x)$ are non-zero continuous functions on an interval I, seeks a particular solution of the form $y_p(x) = v_1(x)y_1(x) + v_2(x)y_2(x)$, where y_1 and y_2 are linearly independent solutions of $y'' + p(x)y' + q(x)y = 0$, and $v_1(x)$ and $v_2(x)$ are functions to be determined. Which of the following statements are necessarily true?

1. The Wronskian of y_1 and y_2 is never zero in I.
2. v_1, v_2 and $v_1y_1 + v_2y_2$ are twice differentiable
3. v_1 and v_2 may not be twice differentiable, but $v_1y_1 + v_2y_2$ is twice differentiable
4. The solution set of $y'' + p(x)y' + q(x)y = r(x)$ consists of functions of the form $ay_1 + by_2 + y_p$ where $a, b \in \mathbb{R}$ are arbitrary constants.

101. Consider the eigenvalue problem

$$y'' + \lambda y = 0 \quad \text{for} \quad x \in (-1,1)$$
$$y(-1) = y(1)$$
$$y'(-1) = y'(1).$$

Which of the following statements are true?

1. All eigenvalues are strictly positive.
2. All eigenvalues are non-negative.
3. Distinct eigenfunctions are orthogonal in $L^2[-1,1]$.
4. The sequence of eigenvalues is bounded above.

102. Consider the IVP:

$$xu_x + tu_t = u + 1, x \in \mathbb{R}, t \geq 0$$
$$u(x,t) = x^2,\ t = x^2.$$

Then

1. the solution is singular at (0,0)
2. the given space curve $(x,t,u) = (\xi, \xi^2, \xi^2)$ is not a characteristic curve at (0,0).
3. there is no base-characteristic curve in the (x,t) plane passing through (0,0).
4. a necessary condition for the IVP to have a unique C^1 solution at $(0,0)$ does not hold.

Unit-4

103. Let $X_1, X_2, \cdots, X_n$ be independent random variables following a common continuous distribution F, which is symmetric about 0. For $i = 1, 2, \cdots, n$, define

$$S_i = \begin{cases} 1 & if\ X_i > 0 \\ -1 & if\ X_i < 0 \\ 0 & if\ X_i = 0 \end{cases} \text{ and}$$

R_i = rank of $|X_i|$ in the set $\{|X_1|, \cdots |X_n|\}$.
Which of the following statements are correct?

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1. $S_1, S_2, \cdots, S_n$ are independent and identically distributed
2. $R_1, R_2, \cdots, R_n$ are independent and identically distributed
3. $\boldsymbol{S} = (S_1, \cdots, S_n)$ and $\boldsymbol{R} = (R_1, \cdots, R_n)$ are independent
4. The distribution of $T = \sum_{i=1}^{n} S_i R_i$ does not depend on the functional form of F

104. Suppose $Y|\theta \sim$ Poisson $(\theta), \theta > 0$ and prior density τ of θ is given by $\tau(\theta) \propto e^{-\alpha\theta}\theta^{\beta-1}$, where $\alpha > 0$ and $\beta > 0$ are hyper-parameters. Which of the following are true?
1. Marginal distribution of Y is hypergeometric
2. Posterior distribution of θ given $Y = y$ is Gamma
3. τ is a conjugate prior
4. Bayes' estimate of θ for squared error loss function is $\frac{\beta+y}{\alpha+1}$

105. Consider a linear model $\boldsymbol{Y}_{4\times1} = X_{4\times4}\boldsymbol{\beta}_{4\times1} + \boldsymbol{\varepsilon}_{4\times1}$, where $\text{Disp}(\varepsilon) = \sigma^2 I_4$ for some $\sigma^2 > 0$. One needs to choose the design matrix X such that its elements take values in the set $\{-1, 0, 1\}$. Now, consider the following three choices of X

$$X_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$X_2 = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \text{ and}$$

$$X_3 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix}.$$

Which of the following statements are true?
1. For all three choices of X, $\boldsymbol{\beta}=(\beta_1, \beta_2, \beta_3, \beta_4)'$ is estimable
2. For all three choices of X, $\hat{\beta}_i$ and $\hat{\beta}_j$, the least squared estimates of β_i and β_j, are uncorrelated for all $i \neq j$
3. X_2 is a better choice than X_1
4. X_2 is a better choice than X_3

106. Suppose that $X_1, X_2, \cdots X_{10}$ are $i.i.d.\, N(0,1)$. Which of the following statements are correct?
1. $P\{X_1 > X_2 + X_3 + \cdots + X_{10}\} = \frac{1}{2}$
2. $P\{X_1 > X_2 X_3 \cdots X_{10}\} = \frac{1}{2}$
3. $P\{\sin(X_1) > \sin(X_2) + \sin(X_3) + \cdots + \sin(X_{10})\} = \frac{1}{2}$
4. $P\{\sin(X_1) > \sin(X_2 + X_3 + \cdots + X_{10})\} = \frac{1}{2}$

107. Consider a classification problem between two uniform distributions $U(0,2)$ and $U(1,5)$. Let π $(0 < \pi < 1)$ be the prior probability of the class having $U(0,2)$ distribution. If we consider the $0-1$ loss function, which of the following statements are correct?
1. For $\pi < 1/3$, the Bayes' risk (i.e., the average misclassification probability of the Bayes classifier) is smaller than 1/6
2. For $\pi > 1/3$, the Bayes' risk is smaller than 1/6
3. For $\pi = 1/3$, the Bayes' risk is 1/6
4. For all choices of π, the Bayes' classifier is unique

108. Suppose $n(\geq 2)$ units are drawn from a population of $N(> n)$ units sequentially as follows. A random sample $U_1, U_2, \cdots, U_N$ of size N is drawn from $U(0,1)$. The k-th population unit is selected if $U_k < \frac{n-n_k}{N-k+1}, k = 1, 2, \cdots, N$, where $n_1 = 0$ and n =number of units selected out of first $k-1$ units for each $k = 2, 3, \cdots, N$. Then
1. The probability of inclusion of the 2nd unit in the sample is $\frac{n}{N}$
2. The probability of inclusion of the 1st and 2nd unit in the sample is $\frac{n(n-1)}{N(N-1)}$
3. The probability of not including the 1st unit and including the 2nd unit in the sample is $\frac{n(N-n)}{N(N-1)}$
4. The probability of including the 1st unit but not including the 2nd unit in the sample is $\frac{n(n-1)}{N(N-1)}$

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109. Consider a block design with three blocks and four treatments A, B, C and D where only A and B are allotted to block-1, only A, B and D are allotted to block-2 and only C is allotted to block-3. Then the resulting block design is

1. incomplete and not connected
2. incomplete and not balanced
3. balanced and connected
4. neither balanced nor connected

110. Suppose X is a positive random variable with the following probability density function

$f(x) = \left(\alpha x^{\alpha-1} + \beta x^{\beta-1}\right)e^{-x^{\alpha}-x^{\beta}}; x > 0,$ for $\alpha > 0$ and $\beta > 0$. Then the hazard function of X for some choices of α and β can be

1. an increasing function
2. a decreasing function
3. a constant function
4. a non-monotonic function

111. A parallel system has $n(\geq 1)$ identical components. The lifetimes of the n components are independent identically distributed exponential random variables with mean 1. If the lifetime of the system is denoted by X, then which of the following statements are true?

1. The mode of X is 0 for some n.
2. The mode of X is less than or equal to n for all n.
3. The mean of X is greater than equal to 1 for all n.
4. The median of X is greater than 100 for some n.

112. Suppose ABC is a triangle on the xy-plane with centroid D. Which of the following points can NEVER be a minimizer of the function $7x - 10y + 1$ as (x, y) runs over the triangle ABC?

1. A
2. B
3. C
4. D

113. Suppose $X_1, X_2, \cdots, X_n$ is a random sample from the uniform distribution on $(0, 2)$ and $M_n = \max\{X_1, X_2, \cdots, X_n\}$ for every positive integer n. Then which of the following statements are true?

1. $M_n \to 2$ almost surely
2. $M_n \to 2$ in probability
3. $M_n \to 2$ in distribution
4. $\frac{M_n - 2}{\sqrt{n}}$ converges in distribution to normal distribution

114. Let $X_1, X_2, \cdots$ be $i.i.d.$ $N(0,1)$ random variables. Let $S_n = X_1^2 + X_2^2 + \cdots + X_n^2$, $\forall n \geq 1$. Which of the following statements are correct?

1. $\frac{S_n - n}{\sqrt{2}} \sim N(0,1)$ for all $n \geq 1$.
2. For all $\varepsilon > 0$, $P\left(\left|\frac{S_n}{n} - 2\right| > \varepsilon\right) \to 0$ as $n \to \infty$.
3. $\frac{S_n}{n} \to 1$ with probability 1.
4. $P\left(S_n \leq n + \sqrt{n}x\right) \to P(Y \leq x)\ \forall x \in \mathbb{R}$, where $Y \sim N(0,2)$.

115. Let $\{X_n\}$ be a Markov chain with state space S. For any $i, j \in S$, let $p_{ij}^{(n)}$ denote the n-step transition probability of going from i to j. Let $d(i)$ denote the period of state i $(i \in S)$. Which of the following statements are correct?

1. If $d(i) = d(j)$ then $\lim_{n\to\infty} p_{ij}^{(n)} > 0$.
2. If $d(i) = d(j)$ then $p_{ij}^{(n)} > 0$ and $p_{ji}^{(m)} > 0$ for some $n, m \geq 1$.
3. If $p_{ij}^{(n)} > 0$ and $p_{ji}^{(m)} > 0$ for some $n, m \geq 1$, then $d(i) = d(j)$.
4. $\lim_{n\to\infty} p_{ij}^{(n)} > 0$ implies $d(i) = d(j)$.

116. Consider a Markov chain with transition probability matrix P given by

$$P = \begin{pmatrix} 1/2 & 1/2 & 0 \\ 0 & 1/2 & 1/2 \\ 1/3 & 1/3 & 1/3 \end{pmatrix}.$$

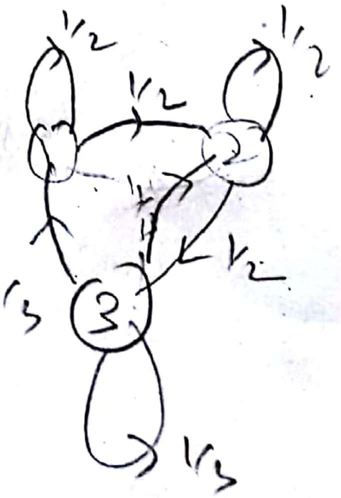
For any two states i and j, let $p_{ij}^{(n)}$ denote the n-step transition probability of going from i to j. Identify correct statements.

X 1. $\lim_{n\to\infty} p_{11}^{(n)} = 2/9$.

X 2. $\lim_{n\to\infty} p_{21}^{(n)} = 0$.

X ~~3.~~ $\cancel{\lim_{n\to\infty}} p_{32}^{(n)} = 1/3$.

4. $\lim_{n\to\infty} p_{13}^{(n)} = 1/3$.



$\frac{1}{2} * \frac{1}{2} * \frac{1}{2} + \frac{1}{3}$

$\frac{1}{3} * \frac{1}{3} + \frac{1}{6}$

$\frac{1}{2} + \frac{1}{12} + \frac{1}{36}$

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117. Suppose that (X_1, X_2) follows a bivariate distribution with common marginal distribution F and $Corr(X_1, X_2) = 0$. Then which of the following statements are correct?

1. $F = \text{Uniform}(0,1) \Rightarrow X_1$ and X_2 are independent.
2. $F = \text{Bernoulli}(\theta) \Rightarrow X_1$ and X_2 are independent.
3. $F =$ Discrete uniform $\{-1, 0, 1\} \Rightarrow X_1$ and X_2 are independent.
4. $F = N(0,1) \Rightarrow X_1$ and X_2 are independent.

118. Let $X_1, X_2, \cdots, X_n$ be a random sample from the distribution with p.d.f.

$$f_\theta(x) = \begin{cases} \frac{1}{\theta} \cdot x^{\frac{1-\theta}{\theta}}, & 0 < x < 1, \\ 0, & \text{otherwise,} \end{cases}$$

where $\theta > 0$. Then which of the following are true?

1. $\prod_{i=1}^{n} X_i$ is sufficient for θ.
2. $-\frac{1}{n}\sum_{i=1}^{n} \ln X_i$ is sufficient for θ.
3. $\prod_{i=1}^{n} X_i$ is a maximum likelihood estimate for θ.
4. $-\frac{1}{n}\sum_{i=1}^{n} \ln X_i$ is a maximum likelihood estimate for θ.

119. X is a discrete random variable on $\{-2, -1, 1, 2\}$ with probability mass functions $P_\theta[X = x]$, $\theta \in \{\theta_0, \theta_1\}$ given below

X	-2	-1	1	2
$\theta = \theta_0$	0.05	0.6	0.3	0.05
$\theta = \theta_1$	0.2	0.4	0.2	0.2

The aim is to test $H_0: \theta = \theta_0$ against $H_1: \theta = \theta_1$. Which of the following statements are correct?

1. The test procedure with critical region $\{x = 2\}$ is a most powerful test of size 0.05
2. The test procedure with critical region $\{x = -2\}$ is a most powerful test of size 0.05
3. The test procedure with critical region $\{x = -1\}$ is not a most powerful test of its size
4. The test procedure with critical region $\{x = 1\}$ is not a most powerful test of its size

120. Suppose $X_1, X_2, \cdots, X_n$ is a random sample from uniform distribution on $(\theta, \theta + 1)$, where $\theta \in \mathbb{R}$ is an unknown parameter. Let $X_{(1)} < X_{(2)} < \cdots < X_{(n)}$ be the corresponding order-statistics. Which of the following are $100(1-\alpha)\%$ confidence intervals for θ?

1. $(-\infty, X_{(n)} - \alpha^{1/n})$
2. $(X_{(1)} + \alpha^{1/n} - 1, \infty)$
3. $\left(X_n + \frac{\alpha}{2} - 1,\ X_n - \frac{\alpha}{2}\right)$
4. $(-\infty, X_1 - \alpha)$

FINAL ANSWER KEYS OF JOINT CSIR-UGC TEST FOR JUNIOR RESEARCH FELLOWSHIP (JRF) AND ELIGIBILITY FOR LECTURESHIP (NET) HELD ON 16-12-2018.

SUBJECT: **MATHEMATICAL SCIENCES – SET-C (BILINGUAL & ENGLISH)**

Question No.	Key	Question No.	Key	Question No.	Key
1	1	41	3	81	2
2	3	42	3	82	4
3	3	43	1	83	1,2,3,4
4	2	44	*	84	1,2
5	2	45	2	85	1,3,4
6	2	46	3	86	4
7	1	47	3	87	1,2
8	3	48	3	88	1,3
9	2	49	4	89	2,4
10	2	50	3	90	1,2,3
11	1	51	3	91	2,4
12	3	52	4	92	**1**
13	2	53	4	93	1,2,3,4
14	2	54	1	94	**4**
15	3	55	3	95	*
16	3	56	3	96	1
17	2	57	1	97	1,4
18	4	58	4	98	1
19	1	59	4	99	1,4
20	1	60	2	100	1,3,4
21	3	61	2,3	101	2,3
22	2	62	1,4	102	1,2,3,4
23	1	63	1,4	103	1,3,4
24	3	64	1,3	104	234
25	3	65	2,3	105	1,2,3
26	2	66	3,4	106	1,2,3,4
27	2	67	4	107	1,2,3
28	2	68	1,4	108	1,2,3
29	4	69	*****	109	1,2,4
30	3	70	2	110	1,2,3,4
31	4	71	3,4	111	1,2,3,4
32	2	72	1,4	112	4
33	3	73	1,2,3,4	113	1,2,3
34	1	74	2,3	114	3,4
35	1	75	1,2,4	115	3
36	3	76	1,2,3	116	1,4
37	1	77	2,4	117	2
38	4	78	1,2,3	118	1,2,4
39	3	79	1	119	1,2,3,4
40	2	80	*****	120	1,2,3,4

*Benefit of marks to those who have attempted this question
Change of key indicated in bold