PART–A

1. Which of the following is CORRECT with respect to grammar and usage?

   Mount Everest is ______.

   (A) The highest peak in the world
   (B) Highest peak in the world
   (C) One of highest peak in the world
   (D) One of the highest peak in the world

2. The policeman asked the victim of a theft,

   "What did you ____?"

   (A) Loose
   (B) Lose
   (C) Loss
   (D) Louse

3. Despite the new medicine's _______in treating diabetes, it is not ____widely.

   (A) effectiveness --- prescribed
   (B) availability --- used
   (C) prescription --- available
   (D) acceptance ---- proscribed

4. The unruly crowd demanded that the accused be _____without trail.

   (A) Hanged
   (B) Hanging
   (C) Hankering
   (D) Hung

5. Select the most suitable Synonym for the word 'ABSTRACT'.

   (A) Peculiar
   (B) Summary
   (C) Normal
   (D) Diagnostic

6. Select the most suitable Synonym for the word 'STALWART'.

   (A) Watchful
   (B) Sturdy
   (C) Delicate
   (D) Encomium

7. Select the most suitable Antonym for the word 'AGGRAVATE'.

   (A) Segregate
   (B) Adulterate
   (C) Ameliorate
   (D) Accommodate

8. Select the most suitable Antonym for the word 'CONSIDERATE'.

   (A) Comprehensive
   (B) Atrocious
   (C) Malignant
   (D) Indifferent

9. Select the pair which shows the same relationship as ADVANCE : RETREAT

   (A) BUY: SELL
   (B) PUSH : PULL
   (C) CREATE : DESTROY
   (D) FORWARD : ONWARD

10. Choose the appropriate set of words that makes the sentence most meaningful:

    He was _______ very clever, but he _____ performed excellently.

    (A) certainly, obviously
    (B) never, also
    (C) not, always
    (D) rarely, seldom

11. Which type of output device creates coloured images which look and feel like photographs?

    (A) Electrostatic plotter
    (B) Laser printer
    (C) Dye sublimation printer
    (D) Inkjet plotter

12. What does GPRS stand for?

    (A) Global Position Reading System
    (B) General Packet Radio Service
    (C) Global Packet Radio Service
    (D) General Position Reading System

13. IPR protects the use of information and ideas that are of
(A) Ethical value
(B) Moral value
(C) Social value
(D) Commercial value

14. India became a member of which one of the following in 2016?
(A) Non-Proliferation Treaty
(B) Missile Technology Control Regime
(C) Nuclear Suppliers Group
(D) Wassenaar Arrangement

15. Which one of the following elements is used as a timekeeper in atomic clocks?
(A) Potassium
(B) Caesium
(C) Calcium
(D) Magnesium

16. Find the odd man out:
(A) Mercury
(B) Mars
(C) Moon
(D) Venus

17. How many times, the two hands of a clock will be at 30° with each other in a day?
(A) 36
(B) 40
(C) 44
(D) 48

18. By selling 30 articles, a shopkeeper gained the selling price of 10 articles. Find the profit percentage.
(A) 20%
(B) 30%
(C) 50%
(D) 40%

19. If a man runs at 6 meters per second, what distance (in km) will he cover in 3 hours and 45 minutes?
(A) 81
(B) 96
(C) 91
(D) 27

20. Complete the series 17, 19, 23, 29, 31, 37,…?
(A) 41
(B) 43
(C) 40
(D) 42

21. Complete the series 64, 125, 216, 343, ….?
(A) 64
(B) 424
(C) 317
(D) 512

22. If $x + y = 4$, find the maximum/minimum possible value of $x^2 + y^2$.
(A) Minimum, 8
(B) Maximum, 8
(C) Maximum, 16
(D) Minimum, 16

23. If k is natural number and $(k^2 - 3k + 2)(k^2 - 7k + 12) = 120$, find k.
(A) 7
(B) 6
(C) 5
(D) 9

24. Find the nature of the roots of the quadratic equation $2x^2 + 6x - 5 = 0$
(A) Complex conjugates
(B) Real and equal
(C) Conjugate surds
(D) Unequal and rational

25. Complete the series GKF, IPC, LTY, PWT, UYN, …?
(A) ABZ
(B) XBX
(C) XAH
(D) AZG

PART–B

26. The system of equations 2x+y=5, x-3y = -1, 3x+4y = k is consistent when k is:
   (A) 1
   (B) 10
   (C) 5
   (D) 2

27. If the eigen values of the matrix $A = \begin{pmatrix} 2 & 3 \\ x & y \end{pmatrix}$ are 4 and 8, then:
   (A) x = 4, y = 10
   (B) x = 5, y = 8
   (C) x = -3, y = 9
   (D) x = -4, y = 10

28. If A is a 5×5 matrix with trace 15 and if 2 and 3 are eigen values of A, each with algebraic multiplicity 2, then the determinant of A is equal to:
   (A) 180
   (B) 24
   (C) 120
   (D) 0

29. Consider the equation AX = B where $A = \begin{pmatrix} -1 & 2 \\ 2 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$, then:
   (A) The equation has no solution
   (B) $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ is the solution
   (C) There exists a non-zero unique solution
   (D) The equation has infinitely many solutions

30. If the application of Taylor's theorem gives $\log(1+x) = 2ax - bx^2 + \frac{cx^3}{(1+\theta x)^3}$, where $0 < a, b, c\ \theta < 1$, then the value of a + b + c is:
   (A) 1/3
   (B) 2/3
   (C) 4/3
   (D) 9/17

31. If 0<a<b<1, then the signs of expressions $(\tan^{-1} b - \tan^{-1} a) - \frac{b-a}{1+a^2}$ and $(\tan^{-1} b - \tan^{-1} a) - \frac{b-a}{1+b^2}$ are:
   (A) (+, +)
   (B) (+, –)
   (C) (–, +)
   (D) (–, –)

32. Let *A* be a 4×4 non-singular matrix and *B* be the matrix obtained from *A* by adding to its third row thrice the first row. Then det $(3A^{-1}B)$ equals
   (A) 81
   (B) 27
   (C) 9
   (D) 3

33. If A is non-singular matrix of size 4×4 and $|A| = \frac{1}{4}$, then value of $\left|\mathrm{Adj}\left(2\left(\mathrm{Adj}(2A)\right)\right)\right|$ is equal to:
   (A) 2^{30}
   (B) 2^{48}
   (C) 2^{66}
   (D) 2^{76}

34. By changing the order of integration, the integral $\int_1^e \int_{\ln y}^1 f(x, y)\, dx\, dy$ can be expressed as
   (A) $\int_0^1 \int_1^{\ln x} f(x, y)\, dy\, dx$
   (B) $\int_0^1 \int_0^{\ln x} f(x, y)\, dy\, dx$
   (C) $\int_0^1 \int_{e^x}^{e} f(x, y)\, dy\, dx$
   (D) $\int_0^1 \int_1^{e^x} f(x, y)\, dy\, dx$

35. $\lim_{x\to\infty}\left(\dfrac{a^{\frac{1}{x}}-b^{\frac{1}{x}}}{\log\dfrac{x}{x-1}}\right)=$

(A) $\log\dfrac{a}{b}$ (B) $-\log\dfrac{a}{b}$ (C) $2\log\dfrac{a}{b}$ (D) $\dfrac{1}{2}\log\dfrac{a}{b}$

36. Let $T:R^2 \to R^2$ be a linear transformation such that T(1, 2) = (2, 3) and T(0, 1) = (1, 4). Then the matrix associated with the linear transformation is:

(A) $\begin{bmatrix}1 & 0\\ -5 & 4\end{bmatrix}$ (B) $\begin{bmatrix}0 & 1\\ -5 & 4\end{bmatrix}$ (C) $\begin{bmatrix}0 & 4\\ -5 & 1\end{bmatrix}$ (D) $\begin{bmatrix}-5 & 1\\ 0 & 4\end{bmatrix}$

37. If A is any non-singular matrix of size 4×4 and its characteristic equation is $\lambda^4-10\lambda^3+7\lambda^2-23\lambda+190=0$, then the value of $\dfrac{|A|}{\text{Trace}(A)}$ is

(A) 1
(B) 0.7
(C) 2.3
(D) 19

38. If $\iiint_{x^2+y^2+z^2\le 1} x^2\,dx\,dy\,dz = \dfrac{4\pi}{15}$, then value of integral $\iiint_{x^2+y^2+z^2\le 1} (\pi-x)^2\,dx\,dy\,dz$ is equal to:

(A) $\dfrac{4\pi}{3}\left(\pi^2+\dfrac{1}{3}\right)$
(B) $\dfrac{4\pi}{3}\left(\pi^2+\dfrac{1}{5}\right)$
(C) $\dfrac{4\pi}{3}\left(\pi^2+\dfrac{1}{9}\right)$
(D) $\dfrac{4\pi}{3}\left(\pi^2-\dfrac{1}{9}\right)$

39. The volume of the solid generated by revolving the region bounded by the lines x = 0, x = 2 and the curve $y^2 = x$ about the line y = 0 is equal to:

(A) 2π
(B) $\dfrac{\pi}{2}$
(C) $\dfrac{\pi}{3}$
(D) $\dfrac{\pi}{9}$

40. If $U_{n+1}=-\dfrac{1}{2}U_n+\dfrac{9}{2}$ n ≥1 and $U_1=\dfrac{2}{5}$, then the value of U_n is:

(A) $3+\left(-\dfrac{1}{2}\right)^n$
(B) $3^n-\dfrac{1}{2}$
(C) $2+\left(\dfrac{1}{2}\right)^n$
(D) $5-\left(\dfrac{5}{2}\right)^n$

41. The area bounded by the curve $y^2 = x$ and $y^2 = 2x - 1$ is:

(A) $\dfrac{11}{3}$
(B) $\dfrac{2}{3}$
(C) $\dfrac{5}{2}$
(D) $\dfrac{1}{12}$

42. If $A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$, then the value of A^{100} is:

(A) I
(B) A
(C) $A + I$
(D) $A - I$

43. The nullity of the matrix $\begin{bmatrix} 3 & 1 & 4 & 0 \\ 0 & 2 & 2 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix}$ is:

(A) 4
(B) 1
(C) 3
(D) 2

44. If $u_n = \frac{n^2}{3^n}$, then the value of $\lim_{i \to \infty} \frac{u_{n+1}}{u_n}$ and the nature of the series $\sum_{n=1}^{\infty} u_n$ are:

(A) 1/3, Divergent
(B) 1/3, Convergent
(C) 1/4, Convergent
(D) Infinity, divergent

45. If $u_n = \left(1 + \frac{1}{\sqrt{n}}\right)^{-n^{3/2}}$, then the value of $\lim_{n \to \infty} (u_n)^{\frac{1}{n}}$ and the nature of the series $\sum_{n=1}^{\infty} u_n$ are:

(A) 1/e, Convergent
(B) 3/2e, Convergent
(C) 1/e, Divergent
(D) Infinity, divergent

46. The solution of the linear differential equation $\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 9x = 0$ is:

(A) $x = (c_1 + c_2 t)e^{-3t}$
(B) $x = (1 + c_1 + c_2 t)e^{-3t}$
(C) $x = (c_1 + c_2 t^2)e^{-3t}$
(D) $x = (c_1 + c_2 t)e^{-t}$

47. The value of particular integral $\frac{1}{(D+2)(D-1)^2}(e^{-2x})$, where $D = \frac{d}{dx}$, is:

(A) $\frac{(x+1)e^{-2x}}{9}$
(B) $\frac{xe^{-2x}}{4}$
(C) $\frac{x^2 e^{-2x}}{9}$
(D) $\frac{xe^{-2x}}{9}$

48. The solution of differential equations $\frac{dy}{dx} = 2y + z$ and $\frac{dz}{dx} = 3y$ satisfies the relation :

(A) $(y + z)\, e^{3x} = k$
(B) $(3y + z)\, e^{-3x} = k$
(C) $(y - z)\, e^{3x} = k$
(D) $(3y + z)\, e^{3x} = k$

49. The general solution of the differential equation $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$ is:

(A) $\alpha e^{-2y} + \beta = x$
(B) $\alpha e^{y} + \beta = x^2$
(C) $\alpha e^{y} + \beta = x$
(D) $\alpha e^{-y} + \beta = x$

50. In three independent throws of a fair die, let X denote the number of upper faces showing six. Then the value of $E\,(3–X)^2$ is

(A) $\frac{20}{3}$ (B) $\frac{2}{3}$ (C) $\frac{5}{2}$ (D) $\frac{5}{12}$

51. Let X_1, X_2, ..., X_{21} be a random sample of size 21 from a Normal distribution having variance 5. Let $\overline{X} = \frac{1}{21}\sum_{i=1}^{n} X_i$ and $S = \sum_{i=1}^{n}(X_i - \overline{X})^2$. Then the value of $E\,(S)$ is

(A) 5
(B) 100
(C) 0.25
(D) 105

52. Let X be Poisson(2) and Ybe Binomial $\left(10, \frac{3}{4}\right)$ random variables. If X and Y are independent, then $P(XY = 0)$ is:

(A) $e^{-2} + \left(\frac{1}{4}\right)^{10}\left(1 - e^{-2}\right)$
(B) $e^{-2} + \left(\frac{1}{4}\right)^{10}\left(1 - 2e^{-2}\right)$
(C) $e^{-2} + \left(\frac{1}{4}\right)^{10}\left(1 + e^{-2}\right)$
(D) $e^{-2} + 1 - \left(\frac{1}{4}\right)^{10}$

53. Let $f(x) = \frac{k\,|x|}{(1+|x|)^4} - \infty < x < \infty$

Then the value of k for which $f(x)$ is a probability density function is

(A) $\frac{1}{6}$ (B) $\frac{1}{2}$ (C) 3 (D) 6

54. Let the joint probability density function of X and Y be $f(x, y) = \begin{cases} e^{-x}, if\ 0 \le y \le x < \infty \\ 0, \quad otherwise \end{cases}$. Then $E\,(X)$ is

(A) 0.5
(B) 1
(C) 2
(D) 6

55. Let X and Y be independent standard normal random variables. Then the distribution of

$U = \left(\frac{X - Y}{X + Y}\right)^2$ is

(A) Chi-square with 2 degrees of freedom
(B) Chi-square with 1 degree of freedom
(C) F with (2,2) degrees of freedom
(D) F with (1,1) degrees of freedom

56. If $X_1, X_2, ..., X_n$ is a random sample from a normal population with mean μ and variance σ^2 (unknown), then the sampling distribution of $\frac{\sqrt{n}\left(\overline{X} - \mu\right)}{S}, S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2$, is:

(A) $N\left(\mu, \frac{\sigma^2}{n}\right)$
(B) t with n-1 degrees of freedom when n is small (≤ 30)
(C) t with n degrees of freedom when n is small (≤ 30)
(D) t with n-1 degrees of freedom for all n.

57. Let $X_1, X_2, \ldots$ be a sequence of independent and identically distributed Chi-square random variables, each having 4 degrees of freedom. Define $S_n = \sum_{i=1}^{n} X_i^2, n = 1, 2, \ldots$. If $\frac{S_n}{n} \xrightarrow{p} \mu$, as $n \to \infty$, then μ is equal to:

(A) 8 (B) 16
(C) 24 (D) 32

58. The mean difference between 9 paired observations is 15.0 and the standard deviation of differences is 5.0. The value of t-statistic is:

(A) 27 (B) 9
(C) 3 (D) 0

59. Let X and Y be jointly distributed random variables such that the conditional distribution of Y, given X = x, is uniform on the interval (x – 1, x + 1). Suppose E(X) = μ. Then the mean of the random variable Y is:

(A) $\frac{1}{2} + \mu$ (B) μ
(C) $\frac{3}{2} + \mu$ (D) $2 + \mu$

60. Suppose that, due to an earthquake, there is a chance for a 5 years old building to collapse, whether the design is faulty or not. The probability that the design is faulty is 15%. The probability that the building collapses is 90% if the design is faulty and otherwise it is 40%. If an earthquake occurs, then the probability that the building collapses is:

(A) 0.475 (B) 0.340
(C) 0.284 (D) 0.135

61. Let $X_1, X_2, \ldots, X_n$ be a random sample from a Bernoulli distribution with probability of success p (unknown). If $T_n = \frac{1}{(n+1)} \sum_{i=1}^{n} X_i$ is an estimator of p, then T_n is:

(A) Consistent but biased for p
(B) Unbiased for p
(C) Not consistent for p
(D) Minimum variance unbiased estimator for p

62. Let X follow a normal distribution with unknown mean μ and known variance σ_0^2. If it is desired to have confidence interval for μ with confidence coefficient 0.95 and length $2\sigma_0$, then the sample size required to achieve this is:

(A) 1 (B) 2
(C) 3 (D) 4

63. The following is the joint probability mass function of X and Y:

X = x / Y = y	0	1	2
0	3/28	9/28	3/28
1	3/14	3/14	0
2	1/28	0	0

Then

(A) Cov (X, Y) = 0 (B) Cov (X, Y) = 3/14
(C) X and Y are independent (D) X and Y are not independent

64. If X is a random variable with p.d.f. $f(x)=\begin{cases}\frac{2}{x^3}, & x\geq 1\\ 0, & \text{otherwise}\end{cases}$, then :

(A) E(X) does not exist
(B) Var(X) exists
(C) E(X) exists but Var(X) does not exist
(D) Both E(X) and Var(X) exist

65. If the correlation coefficient between X and Y is + 0.73, then the correlation coefficient between 3 – 2X and 5 – 3Y is:

(A) – 0.73
(B) + 0.73
(C) $(0.73)^2$
(D) 1

66. Let A, B and C be three events which are mutually independent with probabilities a, b, c respectively. Let the random variable N denote the number of events which occur among A, B, C. Then, the probability that N = 2 is:

(A) ab + bc + ca – abc
(B) ab +bc +ca -3abc
(C) 2(a + b + c) –abc
(D) ab + bc +ca

67. Let X_1, X_2 be i. i. d. Poisson variates with common parameter λ. If we consider the following statistics for λ:

(i) $T_1 = X_1 + X_2$
(ii) $T_2 = X_1 + 2X_2$

Then:

(A) T_1 is sufficient, T_2 is not sufficient
(B) T_1 is sufficient, T_2 is sufficient
(C) T_1 is not sufficient, T_2 is sufficient
(D) T_1 is not sufficient, T_2 is not sufficient

68. A box contains two coins, one of which is "fair" and the other is "two headed". One coin is chosen at random and tossed twice. If two heads appear, then the probability that the chosen coin was "two headed" is

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{1}{3}$ (D) $\frac{4}{5}$

69. A beta variable of the first kind with parameters (1, 1) is:

(A) Exponential variable with mean 1
(B) Beta variable of second kind
(C) N(0,1)
(D) Uniform variable over (0, 1)

70. Let X_1, X_2, ..., X_n be a random sample from $N(\mu,1)$, μ unknown. Then for testing $H_0{:}\ \mu=\mu_0$ against $H_1{:}\ \mu>\mu_0$, the UMP critical region W_0 of size α is:

(A) $W_0=\left\{\underset{\sim}{X} \,\middle|\, \sqrt{n}\left(\bar{x}-\mu_0\right) > z_\alpha\right\}$
(B) $W_0=\left\{\underset{\sim}{X} \,\middle|\, \bar{x}-\mu_0 > z_\alpha\right\}$
(C) $W_0=\left\{\underset{\sim}{X} \,\middle|\, \sqrt{n}\ \bar{x} > z_\alpha\right\}$
(D) None of the above

71. If the area (under a normal density curve) to the left of the point x_1 is 0.4 and to the right of the point x_2 is 0.3, then x_1 and x_2 are such that:

(A) $x_1 < x_2$
(B) $x_1 > x_2$
(C) $x_1 = x_2$
(D) None of these

72. Let X and Y have the joint probability mass function:

$$P(X=x, Y=y) = \frac{1}{2^{y+2}(y+1)}\left(\frac{2y+1}{2y+2}\right)^x, \; x, y = 0, 1, 2, \dots .$$

Then the marginal distribution of Y is

(A) Poisson with parameter $\lambda = \frac{1}{4}$

(B) Poisson with parameter $\lambda = \frac{1}{2}$

(C) Geometric with parameter $P = \frac{1}{4}$

(D) Geometric with parameter $P = \frac{1}{2}$

73. Consider families with two children, and assume that all four possible distributions of sex: BB, BG, GB, GG, where B stand for boy and G for girl, are equally likely. If the events M and N be

M = { a randomly chosen family has at most one girl }
N = { the family has children of both sexes},

then the events M and N are:

(A) Mutually exclusive and independent
(B) Mutually exclusive but not independent
(C) Independent but not Mutually exclusive
(D) Neither mutually exclusive nor independent

74. If $X_1, X_2, \dots, X_n$ is a random sample from a distribution with p. d. f. $f(x,\theta) = \begin{cases} \frac{1}{\theta}, & 0 \le x \le \theta \\ 0, & \text{otherwise} \end{cases}$ then an unbiased estimator of θ is:

(A) $\frac{1}{\bar{X}}$
(B) $\frac{2}{\bar{X}}$
(C) $\bar{X}$
(D) $2\bar{X}$

75. An urn contains 6 red balls and 5 blue balls. A ball is drawn randomly, its colour is noted and then replaced in the urn. This process is continued till the first red ball is drawn. Then the probability that the first red ball will be drawn after the fifth draw is:

(A) $\left(\frac{5}{11}\right)^5$
(B) $\left(\frac{6}{11}\right)^5$
(C) $\left(\frac{5}{11}\right)^5 \times \left(\frac{6}{11}\right)$
(D) $\left(\frac{6}{11}\right)^5 \times \left(\frac{5}{11}\right)$

76. If x = 4y + 5 and y = kx + 4 are the lines of regression of x on y, and of y on x respectively, then which one of the following is true?

(A) $k \ge 1$
(B) $k \le -1$
(C) $-1 \le k \le 0$
(D) $0 \le k \le \frac{1}{4}$

77. The test used to test the equality of two normal population variances is:

(A) t – test
(B) F – test
(C) Z – test
(D) χ^2 – test

78. Match List-I with List-II and select the correct answer using the codes given below the lists:

List-I (*Probability distribution*)	List-II (m.g.f.)
a. an exponential distribution	1. $\exp(t^2)$
b. a degenerate distribution	2. $\frac{(e^t - e^{-t})}{2t}$, $(t \neq 0)$
c. a normal distribution	3. $\frac{1}{1-t}$, $t < 1$
d. a uniform distribution	4. $\exp(t)$

Codes:

(A) a b c d — 4 3 1 2

(B) a b c d — 3 4 2 1

(C) a b c d — 2 1 3 4

(D) a b c d — 3 4 1 2

79. If X has a Poisson distribution such that 2P(X = 2) = 3P(X = 3), then E(X) is :

(A) 1
(B) 2
(C) 3
(D) 4

80. Let $Y_1, Y_2, ..., Y_n$ be a random sample from a distribution with p.d.f. $f(y, \theta) = \frac{2}{\theta^2}(\theta - y)$, $0 \leq y \leq \theta$. If $\overline{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i$, then the estimator of θ by the method of moments is:

(A) $3\overline{Y}$
(B) $4\overline{Y}$
(C) $\overline{Y}$
(D) $2\overline{Y}$

81. If the number of items produced in a factory during a week is a random variable with mean 100 and variance 400, then the probability that this week's production will be at least 130 is:

(A) $\frac{1}{2}$
(B) $\frac{4}{9}$
(C) $\leq \frac{1}{2}$
(D) $\leq \frac{4}{9}$

82. On the basis of a random sample $X_1, X_2, ..., X_n$ of size n from $N(0, \sigma^2)$, σ^2 unknown, the critical region of size α of the likelihood ratio test for testing $H_0 : \sigma^2 = \sigma_0^2$ against $H_1 : \sigma^2 \neq \sigma_0^2$ is:

(A) $\sum_{i=1}^{n} x_i^2 \leq \chi^2_{n, 1-\frac{\alpha}{2}} \cdot \sigma_0^2$ or $\sum_{i=1}^{n} x_i^2 \geq \chi^2_{n, \frac{\alpha}{2}} \cdot \sigma_0^2$

(B) $\sum_{i=1}^{n} x_i^2 \leq \chi^2_{n-1, 1-\frac{\alpha}{2}} \cdot \sigma_0^2$ or $\sum_{i=1}^{n} x_i^2 \geq \chi^2_{n-1, \frac{\alpha}{2}} \cdot \sigma_0^2$

(C) $\sum_{i=1}^{n} \left(x_i - \overline{x}\right)^2 \leq \chi^2_{n-1, 1-\frac{\alpha}{2}} \cdot \sigma_0^2$ or $\sum_{i=1}^{n} \left(x_i - \overline{x}\right)^2 \geq \chi^2_{n-1, \frac{\alpha}{2}} \cdot \sigma_0^2$

(D) $\sum_{i=1}^{n} \left(x_i - \overline{x}\right)^2 \leq \chi^2_{n, 1-\frac{\alpha}{2}} \cdot \sigma_0^2$ or $\sum_{i=1}^{n} \left(x_i - \overline{x}\right)^2 \geq \chi^2_{n, \frac{\alpha}{2}} \cdot \sigma_0^2$

83. If X be a uniform random variable defined on (-k,k), then the value of k so that $P(|X|<1) = P(|X|>2)$ is:

(A) 1 (B) 2

(C) 3 (D) 4

84. In case of small sample from a normal population, if population variance is unknown, then to test the hypothesis $H_0 : \mu = \mu_0$ appropriate test is:

(A) t – test (B) Z – test

(C) F – test (D) χ^2 – test

85. If $X_1, X_2, ..., X_n$ be a random sample from a p.d.f. $f(x;\theta) = \begin{cases} \frac{1}{\theta}, x \in \left[-\frac{\theta}{2}, \frac{\theta}{2}\right] \\ 0, \text{ otherwise} \end{cases}$, $\theta > 0$ (unknown) then:

(A) $\min_{1 \le i \le n} X_i$ is sufficient for θ

(B) $\max_{1 \le i \le n} X_i$ is sufficient for θ

(C) $\left(\min_{1 \le i \le n}, \max_{1 \le i \le n} X_i\right)$ is sufficient for θ

(D) neither $\min_{1 \le i \le n} X_i$ nor $\max_{1 \le i \le n} X_i$ is sufficient for θ

86. The mean marks in Mathematics of a random sample of 25 students from a school are 60. Assuming that the marks in Mathematics of all students in that school are distributed as $N(\mu, \sigma^2)$, μ unknown and $\sigma^2 = 100$, the 95% confidence interval for μ is:

(A) (56.08, 63.92)

(B) (56.72, 63.28)

(C) (56, 64)

(D) (57, 63)

87. In which one of the following cases, is the minimum variance bound as specified by Cramer – Rao inequality for the estimators NOT attained ?

(A) $\frac{1}{n}\sum_{i=1}^{n} X_i$ as an estimator of μ in $N(\mu, \sigma^2)$

(B) $\frac{1}{n}\sum_{i=1}^{n} X_i$ as an estimator of μ in $N(\mu, 1)$

(C) $\frac{1}{n}\sum_{i=1}^{n} X_i^2$ as an estimator of σ^2 in $N(0, \sigma^2)$

(D) $\frac{1}{n}\sum_{i=1}^{n} \left(X_i - \overline{X}\right)^2$ as an estimator of σ^2 in $N(\mu, \sigma^2)$

88. Let $X_1, X_2, ..., X_6$ be independent random variables such that $P(X_i = -1) = P(X_i = 1) = \frac{1}{2}$ Then $P\left[\sum_{i=1}^{6} X_i = 4\right]$ is equal to:

(A) $\frac{3}{32}$ (B) $\frac{3}{4}$ (C) $\frac{3}{64}$ (D) $\frac{3}{16}$

89. The Central Limit Theorem is important in Statistics because:
(A) For a large n, it says the population is approximately normal
(B) For any population, it says the sampling distribution of the sample mean is approximately normal, regardless of the sample size
(C) For a large n, it says the sampling distribution of the sample mean is approximately normal, regardless of the shape the population
(D) For any sized sample, it says the sampling distribution of the sample mean is approximately normal.

90. A sequence $\{X_n\}$ of random variables is said to converge in probability to a constant a, if for any $\varepsilon > 0$:
(A) $\lim_{n\to\infty} P\{|X_n - a| < \varepsilon\} = 1$
(B) $\lim_{n\to\infty} P\{|X_n - a| < \varepsilon\} = 0$
(C) $\lim_{n\to\infty} P\{|X_n - a| \geq \varepsilon\} = 1$
(D) $\lim_{n\to\infty} P\{|X_n - a| \geq \varepsilon\} = 0.95$

91. On the basis of a random sample of size n from a normal population with known variance σ^2, the critical region of size α of the likelihood ratio test for testing $H_0 : \mu = \mu_0$ against $H_1 : \mu \neq \mu_0$, $\left(\text{assume } Z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}\right)$, is:
(A) $|Z| \leq z_{\alpha/2}$
(B) $|Z| \geq z_{\alpha/2}$
(C) $Z \leq z_{\alpha/2}$
(D) $Z \geq z_{\alpha/2}$

92. Let $X_1, X_2, ..., X_n$ be i. i. d. N $(\theta, 1)$, X_1 is unbiased for θ and $\bar{X}$ is a complete sufficient for θ. Then $E(X_1 | \bar{X})$ is the UMVUE of θ by:
(A) Only Rao – Blackwell theorem
(B) Only Lehmann – Scheffe theorem
(C) Both Rao – Blackwell and Lehmann – Scheffe theorems
(D) Neither Rao – Blackwell theorem nor Lehmann – Scheffe theorem

93. If $\{X_n\}$ is a sequence of i. i. d. random variables with finite mean and variance, then $\{X_n\}$ satisfies :
(A) Central limit theorem but not necessarily weak law of large numbers
(B) Weak law of large numbers but not necessarily central limit theorem
(C) Both central limit theorem and weak law of large numbers
(D) Neither central limit theorem nor weak law of large numbers

94. If the two lines of regression are :
$20x + 9y + 107 = 0$
$4x + 5y - 33 = 0$,
then the correlation coefficient between X and Y is:
(A) $\frac{9}{25}$
(B) $\frac{3}{5}$
(C) $-\frac{3}{5}$
(D) $-\frac{9}{25}$

95. On the basis of a single observation x from f (x; β) = $\frac{1}{\beta} e^{-x/\beta}$, x>0, β>0, to test $H_0 : \beta = 2$ against $H_1 : \beta = 3$, if the critical region is { x >1 }, then the power of the test is:
(A) $e^{-1/3}$
(B) $e^{-2/3}$
(C) e^{-1}
(D) e

96. In testing of hypothesis, type II error is committed when a:

(A) True H_0 is rejected
(B) True H_1 is accepted
(C) False H_0 is accepted
(D) False H_1 is rejected

97. If $\Phi(x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{1}{2}\theta^2}\, d\theta$ and $\Phi(3)=0.9987$, then the value of $\lim_{n\to\infty}\left[\frac{1}{2^{\frac{n}{2}}\Gamma\left(\frac{n}{2}\right)}\int_{n+3\sqrt{2n}}^{\infty} e^{-t/2}t^{(n/2)-1}dt\right]$ equals to

(A) 0.9973
(B) 0.1587
(C) 0.0027
(D) 0.0013

98. Two independent events E and F are such that $P(E \cap F) = \frac{1}{6}$, $P(E^c \cap F^c) = \frac{1}{3}$ and $P(E) > P(F)$. Then $P(E) - P(F)$ is

(A) $\frac{1}{2}$
(B) $\frac{1}{6}$
(C) $\frac{1}{3}$
(D) $\frac{1}{4}$

99. Let X be a continuous random variable with density $f(x) = \frac{1}{2}e^{-|x|}, -\infty < x < \infty$. Then $P(-1 < X < 2)$ equals

(A) $\frac{1}{2}(2-e^{-1}-e^{-2})$
(B) $\frac{1}{2}(2+e^{-1}-e^{-2})$
(C) $\frac{1}{2}(2-e^{-1}+e^{-2})$
(D) $\frac{1}{2}(2+e^{-1}+e^{-2})$

100. The value of the $\lim_{n\to\infty} e^{-n}\sum_{j=n}^{4n}\frac{n^j}{j!}$ equals

(A) $\frac{1}{6}$
(B) $\frac{1}{4}$
(C) $\frac{1}{2}$
(D) $\frac{3}{4}$