

CUCET Mathematics MSc Questions Paper

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- CUCET-2019 Question Paper
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- CUCET-2017 Question Paper
- CUCET-2016 Question Paper
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Test Paper Series Code

C

Question Booklet No. :

2276163

Test Date: 26 May 2019

PG-QP-27

Time : 09:00 AM To 11:00 AM

Entrance Test for the Course(s) : M.A./M.Sc. (Mathematics) [CUJAM], [CUKAS], [CUMGB], [CUSBR], M.Sc. (Mathematics) [CUKER], [CUHAR], [CUPUN], [CUKNK], [CURAJ], [CUJHD], M.Sc. B.Ed. (Mathematics) [CURAJ].

Roll Number :

2 1 8 8 1 3

Test Center Code :

1 8 8

Name of the Candidate :

NEHA KUMARI.

Candidate's Signature: Neha Kumari

Invigilator's Signature: ..

....

Instructions to Candidates

1. Do NOT open the Question Booklet until the Hall Superintendent gives the signal for the commencement of the examination.
2. Write your Name, Roll Number and Test Center Code (as given in the Admit Card) and sign in the space provided above.
3. After the commencement of the examination, open the Question Booklet. If the Question Booklet or the OMR Answer Sheet or both are not in good condition, then ask for immediate replacement. No replacement will be made 5 minutes after the commencement of the examination.
4. In the ANSWER SHEET (OMR) fill up/shade the required entries (Roll Number, Test Center Code, Test Paper Code, Question Booklet Number etc. in the space provided) using black/blue ball point pen.
5. Part-A of the Question Booklet contains 25 Questions. Part-B of the Question Booklet contains 75 Questions. A candidate is required to answer all the questions.
6. All questions are in MCQ Pattern. There is only one most appropriate correct answer for each question.
7. All questions carry equal marks. There will be negative marking. Each correct answer carries 01 mark and for each wrong/incorrect answer 0.25 mark will be deducted. Question not attempted will not be assessed.
8. Darken only one circle for each question. If you darken more than one circle for the question, it will be deemed as wrong/incorrect answer. Any change in the answer once marked is NOT allowed.
9. Use the Answer Sheet (OMR) carefully. No spare Answer Sheet will be given.
10. Do not make stray marks on the OMR Sheet.
11. After completion of examination, a candidate will be allowed to take Question Booklet and Candidate's copy of OMR answer sheet with him/her. However, each candidate must ensure to handover original copy of OMR sheet to the invigilator. In case a candidate takes away the original OMR answer sheet, his/her examination will be treated as cancelled.
12. No candidate will be allowed to leave the examination hall before completion of Entrance Test. Total time allowed for the paper is 2 Hours.
13. Calculator, Tables or any other Calculating Devices, Mobiles, Pagers, Booklets, Papers etc. are strictly prohibited.
14. Rough work should be done on the blank space provided in this Question Booklet. No extra paper will be provided.

P.T.O.

SEAL

PART-A

1. If the difference between simple interests for 3 years and 4 years at 5% annual rate is 42, then the amount will be,
 (A) Rs. 210 (B) Rs 280
 (C) Rs. 750 (D) Rs. 840
2. The sum of three consecutive even integer is 54. What is the smallest number? $1+2+3$
 (A) 18 (B) 14
 (C) 16 (D) 12
3. Area of circle and a square is equal. Ratio of one side of the square to radius of the circle will be,
 (A) $1:\sqrt{\pi}$ (B) $\sqrt{\pi}:1$
 (C) $1:\pi$ (D) $\pi:1$
4. Fill in the blank to complete the series: 181, 174, 178, _____, 175, 182.
 (A) 174 (B) 176
 (C) 178 (D) 180
5. 'Tree' is related to 'Forest' in the same way as 'Soldier' is related to
 (A) Battle (B) Army
 (C) Gun (D) General
6. Pointing to a gentleman, Deepak said. "His only brother is the father of my daughter's father." How is that gentleman related to Deepak?
 (A) Father (B) Grandfather
 (C) Brother-in-law (D) Uncle
7. Complete the series BEP, CIQ, DOR, FUS, GAT,?
 (A) HEV (B) HIT
 (C) IET (D) IEU
8. Convert 36 km/hr into meters per second.
 (A) 10 (B) 12
 (C) 15 (D) 20
9. 'Wings of Fire' was written by _____.
 (A) APJ Abdul Kalam (B) Salman Rushdie
 (C) Amitav Ghosh (D) Shashi Tharoor
10. 'Chhau' dance is associated with which of the following states?
 (A) Punjab (B) Maharashtra
 (C) Jammu Kashmir (D) Jharkhand
11. Mineral rich 'Jharia' is located in which of the following states?
 (A) Bihar (B) West Bengal
 (C) Utter Pradesh (D) Gujrat
12. Jhansi was annexed by which of the following Governor General?
 (A) Lord Bentinck (B) Lord Dalhausie
 (C) Lord Cornwallis (D) Lord Clive
13. Who among the following personalities stated "Swaraj is my birth right and I am going to have it."
 (A) Bal Gangadhar Tilak
 (B) Subhas Chandra Bose
 (C) Mahatma Gandhi
 (D) Jawahar Lal Nehru

14. Choose the correct word to fill in the blank. The students _____ the teacher on teacher's day for twenty years of dedicated teaching.
- (A) Facilitated (B) Felicited
(C) Fantasized (D) Facillitated
15. Choose the correct word to fill in the blank. Dhoni as well as the other team members of Indian team _____ present on the occasion
- (A) were (B) was
(C) has (D) have
16. Choose the word most similar in meaning: Awkward
- (A) Inept (B) Careful
(C) Suitable (D) Dread full
17. Choose the correct verb to fill in the blank below
Let us _____.
- (A) Introvent (B) Alternate
(C) Atheist (D) Altruist
18. Select the most suitable Synonym for the word 'RESILIENT'.
- (A) Stretchable (B) Spirited
(C) Rigid (D) Buoyant
19. Select the most suitable Synonym for the word 'ZEST'.
- (A) Humour (B) Keen Interest
(C) Attitude (D) Liking
20. Select the most suitable Antonym for the word 'ROBUST'.
- (A) Sturdy (B) Ridiculous
(C) Muscular (D) Feeble
21. Select the most suitable Antonym for the word 'DULL'.
- (A) Monstrous (B) Horrid
(C) fascinating (D) Ghastly
22. Select the pair which shows the same relationship as CANE : BAMBOO
- (A) Wood : Woodpecker
(B) Timber : Tree
(C) Rubber : Malaysia
(D) South Africa : Apartheid
23. Why were you absent _____ your dance classes yesterday?
- (A) for (B) from
(C) in (D) to
24. A man is facing towards South. He take 135° anticlock wise, 180° clockwise rotation then what was facing side of the man?
- (A) North-East (B) North-West
(C) South-East (D) South-West
25. If the value of "x" is 25% less than the value of "y". How much % y's is more than that of x's ?
- (A) $33\frac{1}{3}\%$ (B) 25%
(C) 75% (D) $66\frac{2}{3}\%$

PART - B

26. Solution of the differential equation $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$ is
 (A) $e^y = x + e^x + c$ (B) $e^y = x^2/2 + e^x + c$
 (C) $e^y = x^3/3 + e^x + c$ (D) $e^y = x^4/4 + e^x + c$
27. The integrating factor of the differential equation $(1 - x^2)dy/dx + 2xy = x\sqrt{1 - x^2}$ is
 (A) $\frac{1}{1-x}$ (B) $\frac{1}{1-x^2}$ (C) $1 - x^2$ (D) $1 - x$
28. The solution of differential equation $\frac{d^2y}{dx^2} + 4y = 0$ with initial conditions $y = 2$ and $dy/dx = 0$ when $x = 0$ is
 (A) $y = 2 \sin 2x$ (B) $y = 2 \cos 2x$ (C) $y = \sin 4x$ (D) $y = \tan x$
29. Which of the following is a particular integral of $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{5x}$?
 (A) $\frac{1}{12}e^{5x}$ (B) e^{-5x} (C) e^x (D) e^{x^2}
30. Let $D =: d/dx$. Then the value of $\left\{ \frac{1}{xD+1} \right\} x^{-1}$ is
 (A) $\log x$ (B) $\frac{\log x}{x}$ (C) $\frac{\log x}{x^2}$ (D) $\frac{\log x}{x^3}$
31. If $y_1(x)$ and $y_2(x)$ are two solutions of $\frac{d^2y}{dx^2} + 4y = 0$, then the value of Wronskian is
 (A) 0 (B) 1 (C) 2 (D) 3
32. Differential equation of the family of parabola $y^2 = 4ax$, where a is an arbitrary constant is
 (A) $y = 2x(dy/dx)$ (B) $y = dy/dx$ (C) $y = 2x + dy/dx$ (D) $dy/dx + y^2 = x^2$
33. The orthogonal trajectory of the hyperbola $xy = a$ is
 (A) $x^2 - y^2 = a$ (B) $x^2 = ay^2$ (C) $x^2 + y^2 = a$ (D) $x = ay^2$
34. The order of differential equation $\frac{dy}{dx} = \sqrt{x} + \sqrt{y}$ is
 (A) 1 (B) 2 (C) 3 (D) 4
35. Solution of the initial value problem $e^x(\cos y dx - \sin y dy) = 0$ with $y(0) = 0$ is
 (A) $e^x \cos y + 1 = 0$ (B) $e^x \cos y - 1 = 0$
 (C) $e^y \cos x + 1 = 0$ (D) $e^y \cos x - 1 = 0$
36. If $F(x, y, z) = xy^2 + 3x^2 - z^3$, then the value of $\nabla F(x, y, z)$ at $(2, -1, 4)$ is equal to
 (A) $13i - 4j - 48k$ (B) $i - 4j - k$ (C) $13i + j - 6k$ (D) $-13i + 4j - 6k$

37. The directional derivative of the function $F(x, y, z) = xy^2 - 4x^2y + z^2$ at $(1, -1, 2)$ in the direction of $6i + 2j + 3k$ is
 (A) $1/7$ (B) $2/7$ (C) $54/7$ (D) 7
38. If $\vec{F} = xi + xj + yk$, then $\text{curl } \vec{F}$ is
 (A) $i + j + k$ (B) 0 (C) $i - j - k$ (D) $2i + j - 2k$
39. Let F be a finite field. Then which of the following may be the possible cardinality of F ?
 (A) 15 (B) 20 (C) 25 (D) 30
40. Every subgroup of an abelian group is
 (A) abelian (B) cyclic
 (C) non abelian (D) none of the above.
41. Let $G = \left\{ \begin{bmatrix} a & a \\ a & a \end{bmatrix} \mid a \in \mathbb{R} \setminus \{0\} \right\}$ be a group with binary operation defined by usual matrix multiplication. Then the inverse of $\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$ is
 (A) $\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$ (B) $\begin{bmatrix} 1/2 & -1/2 \\ -1/2 & 1/2 \end{bmatrix}$ (C) $\begin{bmatrix} 1/4 & 1/4 \\ 1/4 & 1/4 \end{bmatrix}$ (D) $\begin{bmatrix} 1/8 & 1/8 \\ 1/8 & 1/8 \end{bmatrix}$
42. Let H and K be subgroups of G . Then which of the following is necessarily a subgroup of G ?
 (A) HK (B) KH (C) $H \cap K$ (D) $H \cup K$
43. Let S_5 be the permutation group on five symbols $\{1, 2, 3, 4, 5\}$. Then order of permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 3 & 5 & 1 \end{pmatrix}$ is equal to
 (A) 5 (B) 4 (C) 3 (D) 6
44. Let G be a group and $a, b, c \in G$ are non-identity elements. Which of the following solves the equation $axb = c$ for x ?
 (A) acb^{-1} (B) $a^{-1}b^{-1}$ (C) $a^{-1}cb^{-1}$ (D) cb^{-1}
45. Let H be a subgroup of a noncyclic group G . Then which of the following is correct?
 (A) H is always noncyclic (B) H is always cyclic
 (C) H is always nonabelian (D) None of the above
46. Let S_6 be the permutation group on six symbols $\{1, 2, 3, 4, 5, 6\}$. Which of the following is not an even permutation?
 (A) $(1\ 3\ 5\ 6\ 2)$ (B) $(1\ 2\ 3)(4\ 5)(4\ 5)$
 (C) $(2\ 6\ 3\ 4\ 5\ 1)$ (D) $(1\ 2)(1\ 4)(2\ 3)(4\ 5)$

47. Which of the following is correct?
- (A) Every integral domain is a field.
 (B) Every finite integral domain is a field.
 (C) There is an integral domain with characteristic equal to 10.
 (D) None of the above.
48. Let J be an ideal of commutative ring with unity and let u be a unit element of R such that $u \in J$. Then
- (A) The multiplicative identity $1 \notin J$
 (B) J is a proper ideal of R such that $J \neq R$
 (C) $J = R$
 (D) There is a minimal ideal M such that $J \subset M \subseteq R$
49. Which of the following is a prime ideal of $(\mathbb{Z}, +, \cdot)$?
- (A) $6\mathbb{Z}$ (B) $2\mathbb{Z} \cap 4\mathbb{Z}$ (C) $7\mathbb{Z}$ (D) $4\mathbb{Z} \cap 8\mathbb{Z}$
50. If $Z = 2 - 3i$, then $|Z|$ equals
- (A) 13 (B) $\sqrt{13}$ (C) -13 (D) -1
51. $\int_0^1 ze^{2z} dz$ equals
- (A) $e^2 + 1$ (B) $(e^2 + 1)/4$ (C) $(e^2 - 1)/4$ (D) $e^2 - 1$
52. $\lim_{z \rightarrow i} \frac{z^{10} + 1}{z^6 + 1}$ equals
- (A) $3/5$ (B) $2/5$ (C) $5/3$ (D) $1/3$
53. The integral $\int_{3i}^{1-i} 4z dz$ equals
- (A) $18 - 4i$ (B) $-4i$ (C) i (D) $-i$
54. If $f(z)$ is analytic in a simply connected domain D and $f'(z)$ is continuous in D , then $\oint_C f(z) dz$ equals
- (A) 0 (B) 1 (C) $2\pi i$ (D) $-2\pi i$
55. The value of the integral $\int_{|z-2|=2} \frac{5z+7}{z^2+2z-3} dz$ is equal to
- (A) πi (B) $2\pi i$ (C) $3\pi i$ (D) $6\pi i$
56. If $f(z) = u(x, y) + iv(x, y)$ is analytic in a domain D , then
- (A) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ and $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0$ (B) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ and $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \neq 0$
 (C) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \neq 0$ and $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0$ (D) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \neq 0$ and $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \neq 0$

57. An entire function is
 (A) infinitely differentiable (B) finitely differentiable
 (C) not differentiable (D) identically zero
58. Which of the following is incorrect statement?
 (A) If $f(z)$ is entire and bounded in complex plane, then $f(z)$ is constant.
 (B) If $f(z)$ is analytic at z_0 , then $f'(z)$ is also analytic at z_0 .
 (C) Analytic function is entire.
 (D) Entire function is analytic.
59. The complex line integral is
 (A) path dependent (B) independent of end points
 (C) path independent (D) none of these
60. The set of all feasible solutions to a linear programming problem (LPP) is
 (A) a concave set (B) a convex set
 (C) a bounded set (D) an infinite set only
61. A basic feasible solution to a LPP, in which at least one of the basic variables is zero is
 (A) degenerate (B) infeasible (C) non-degenerate (D) unbounded
62. The optimal solution of the LPP: Maximize $Z = 4x_1 + x_2$, such that $x_1 + x_2 \leq 50$,
 $3x_1 + x_2 \geq 90$, $x_1, x_2 \geq 0$, is
 (A) $x_1 = 30, x_2 = 0$ (B) $x_1 = 20, x_2 = 30$
 (C) $x_1 = 0, x_2 = 0$ (D) $x_1 = 0, x_2 = 50$
63. Which of the following is incorrect statement?
 (A) Arbitrary intersection of convex sets is a convex set.
 (B) Hyperplane is a convex set.
 (C) Union of two convex sets need not to be a convex set.
 (D) Union of two convex sets is a convex set.
64. In a linear programming problem constraints are
 (A) nonlinear (B) linear
 (C) linear as well as nonlinear (D) none of the above
65. The sequence $\left\{ \frac{1}{n} \right\}$ is
 (A) convergent (B) divergent (C) oscillatory (D) unbounded
66. $\lim_{n \rightarrow \infty} \frac{2n-3}{n+1}$ equals
 (A) 0 (B) 1 (C) 2 (D) e

67. The series $\sum_{n=1}^{\infty} \frac{n+1}{n^p}$ is convergent for
 (A) $0 < p < 1$ (B) $1 < p < 2$ (C) $p = 2$ (D) $p > 2$
68. The series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$ is
 (A) convergent (B) divergent
 (C) conditionally convergent (D) absolutely convergent
69. $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$ equals
 (A) e (B) $\frac{1}{e}$ (C) 0 (D) 1
70. Which of the following statements is false?
 (A) Every bounded sequence is convergent.
 (B) Every convergent sequence is bounded.
 (C) Every bounded sequence has a limit point.
 (D) Every convergent sequence has a unique limit.
71. If a series $\sum_{n=0}^{\infty} a_n$ converges, then
 (A) $\lim_{n \rightarrow \infty} a_n = 0$ (B) $\lim_{n \rightarrow \infty} a_n = \infty$ (C) $\lim_{n \rightarrow \infty} a_n = 1$ (D) $\lim_{n \rightarrow \infty} a_n = 10$
72. If $f : \mathbb{R} \rightarrow \mathbb{R}$ is defined by $f(x) = |x - c|$, for all $x \in \mathbb{R}$; then
 (A) f is discontinuous
 (B) f is differentiable
 (C) f is continuous but not differentiable
 (D) f is continuously differentiable
73. The function $f(x) = \begin{cases} x \sin 1/x, & \text{when } x \neq 0 \\ 0, & \text{when } x = 0 \end{cases}$ is
 (A) continuous at $x = 0$ (B) derivable at $x = 0$
 (C) discontinuous at $x = 0$ (D) infinitely differentiable at $x = 0$
74. If Rolle's theorem holds for $f(x) = x^3 + ax^2 + bx$ on $[-2, 2]$ at $x = 1$, then
 (A) $a = 1/2, b = -4$ (B) $a = 2, b = -4$
 (C) $a = -1/2, b = 4$ (D) $a = 4, b = 1/2$
75. The local maxima of $x^3 - 3x + 3$ is attend at
 (A) $x = -1$ (B) $x = 1$ (C) $x = 0$ (D) $x = 3$
76. The function $f(x) = \sin 3x, x \in [0, \pi/2]$ is increasing in the interval
 (A) $(0, \pi/6)$ (B) $(\pi/6, \pi/2)$ (C) $(0, \pi/2)$ (D) $(\pi/3, \pi/2)$

77. The function $f(x) = x^2$ is not uniformly continuous on the interval
 (A) $[-1, 1]$ (B) $[1, 2]$ (C) $[0, \infty)$ (D) $[0, 1]$
78. Every compact set of real numbers is
 (A) open (B) closed
 (C) closed and bounded (D) open and bounded
79. The set \mathbb{R} of real numbers is
 (A) closed (B) bounded
 (C) countable (D) none of the above
80. The upper limit of the sequence $\{(-1)^n\}$ is
 (A) 1 (B) -1 (C) 0 (D) 2
81. If $f(x, y)$ is a homogeneous function of degree n in x and y and has continuous partial derivatives, then $x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y}$ is equal to
 (A) f (B) nf (C) 0 (D) $n(n-1)f$
82. $\lim_{(x,y) \rightarrow (2,1)} (x^2 + 2x - y^2)$ equals
 (A) 0 (B) -7 (C) 7 (D) -1
83. The radius of convergence of the series $1 + 2x + 3x^2 + 4x^3 + \dots$ is
 (A) 0 (B) 1 (C) ∞ (D) 2
84. The value of the integral $\int_0^1 \int_0^x e^{y/x} dx dy$ is
 (A) $\frac{(e-1)}{2}$ (B) $\frac{(e+1)}{2}$ (C) e (D) e^2
85. The value of the surface integral $\int \int_S (x^3 dy dz + y^3 dz dx + z^3 dx dy)$ over the sphere $x^2 + y^2 + z^2 = a^2$ is
 (A) $\frac{12}{5}\pi a^5$ (B) πa^5 (C) $\frac{5}{12}\pi a^5$ (D) πa^2
86. Which of the following sets forms a basis of \mathbb{R}^2 ?
 (A) $\{(1,1), (3,1)\}$ (B) $\{(0,1), (0,-3)\}$
 (C) $\{(2,1), (1,-1), (3,0)\}$ (D) $\{(1,0), (2,0)\}$
87. Rank of the matrix $\begin{pmatrix} 2 & 1 & 1 \\ 0 & 3 & 0 \\ 3 & 1 & 2 \end{pmatrix}$ is equal to
 (A) 1 (B) 2 (C) 3 (D) 4

88. Which of the following functions $F : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is not a linear transformation ?
- (A) $F(x, y) = (x + y, x - y)$ (B) $F(x, y) = (x + y, x)$
 (C) $F(x, y) = (2x - y, x)$ (D) $F(x, y) = (x, 1 + y)$
89. The dimension of the vector space of all 3×3 real symmetric matrices is
- (A) 9 (B) 6 (C) 3 (D) 4
90. The determinant of $\begin{pmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{pmatrix}$ is
- (A) $(z - x)(z - y)(y - x)$ (B) $(z - x)^2(z - y)(y - x)$
 (C) $(z^2 - x^2)(z^2 - y^2)(y^2 - x^2)$ (D) $(z - x)^2(z - y)^2(y - x)^2$
91. If $M = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$, then M^{2019} equals
- (A) $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ (B) $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ (C) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (D) $\begin{pmatrix} 1 & 2019 \\ 0 & 1 \end{pmatrix}$
92. Which of the following matrix is singular?
- (A) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (B) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ (C) $\begin{pmatrix} 1 & 4 \\ 2 & 10 \end{pmatrix}$ (D) $\begin{pmatrix} 2 & 2 \\ 3 & 3 \end{pmatrix}$
93. If $M = \begin{pmatrix} 4 & 0 \\ 2 & 3 \end{pmatrix}$, then the eigenvalues of M are
- (A) -4 and -3 (B) 4 and 3 (C) 2 and 0 (D) 3 and -3
94. Let $F : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be a linear transformation defined by $F(x, y) = (2x + 3y, 4x - 5y)$. Then the matrix representation of the linear transformation relative to basis $B = \{(1, 0), (0, 1)\}$ is
- (A) $\begin{pmatrix} 2 & 3 \\ 4 & -5 \end{pmatrix}$ (B) $\begin{pmatrix} 0 & -3 \\ 4 & 5 \end{pmatrix}$ (C) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (D) $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$
95. The eigenvalues of a skew-symmetric matrix are
- (A) always pure imaginary (B) always zero
 (C) either zero or imaginary (D) always real
96. If $M = \begin{pmatrix} 2 & -2 \\ -2 & 5 \end{pmatrix}$ and $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, which of the following is a zero matrix ?
- (A) $M^2 - 7M - 6I$ (B) $M^2 - 7M + 6I$ (C) $M^2 - 6M - 7I$ (D) $M^2 - 6M - 7I$
97. Let $T : V_n(F) \rightarrow V_m(F)$, where $V_n(F)$ and $V_m(F)$ are finite dimensional vector spaces. Then
- (A) $\text{rank}(T) + \text{nullity}(T) = \dim(V_n(F))$ (B) $\text{rank}(T) = \text{nullity}(T)$
 (C) $\text{rank}(T) - \text{nullity}(T) = \dim(V_n(F))$ (D) $\text{rank}(T) - \text{nullity}(T) = \dim(V_m(F))$

98. The singleton set $\{x\}$ is linearly dependent if

- (A) $x = 0$ (B) $x \neq 0$ (C) x is a scalar (D) none of these

99. The eigenvalues of an orthogonal matrix are

- (A) zero (B) imaginary (C) always negative (D) of unit modulus

100. Degree of the differential equation $dy = (y + \sin x)dx$ is

- (A) 1 (B) 2 (C) 3 (D) 4

CUCET-2018

MSc Mathematics

1. Which of the following best expresses the meaning of 'Exasperate'?
A) Elevate B) Irritate C) Distrust D) Transcend
2. Which of the following is opposite in meaning to the word 'Captivate'?
A) Canvass B) Fascinate C) Offend D) Campaign
3. Which of the alternatives best expresses the meaning of the underlined phrase in the following sentence?
Sheetal is in the habit of taking French leave very often.
A) Taking sick leave B) Taking extra ordinary leave
C) Taking leave on medical grounds D) Taking leave without permission
4. Below are given three statements, such as P, Q, and R, followed by four conclusions. You have to take the given statements to be true even if they appear to be at variance with commonly known facts and then decide which of the conclusions logically follow(s) from the given statements.
Statements
P. All books are notes.
Q. Some notes are watches.
R. No watch is a pencil.
Conclusions
I. Some watches are books. II. Some notes are pencils.
III. No watch is a book. IV. Some notes are not pencils.
A) I and either II or IV follow B) I, III and IV follow
C) I, II and III follow D) Either I or III and IV follow
5. At which of the following places is the Indian National Defence University being set up?
A) Hyderabad, Telangana B) Bhubaneswar, Odisha
C) Gurgaon, Haryana D) Jodhpur, Rajasthan
6. Who was the last Hindu king of North India?
A) Pushyabhuti B) Harshavardhana
C) Pushyamitra D) Skandagupta
7. Which one of the following travelers visited India during the Gupta period?
A) Hiuen-Tsang B) Fa-Hien C) Marco Polo D) Nicolo Conti
8. The 'International Day of Older Persons' is observed every year on
A) 1st October B) 2nd October C) 3rd October D) 4th October
9. Santosh Trophy is related to
A) Cricket B) Hockey C) Football D) Badminton
10. What is the full form of HTTP in data communication?
A) Hardware Test Trial Protocol B) Hyper Text Transfer Package
C) Hyper Text Transfer Protocol D) Hyphenated Text Transfer Protocol
11. Language of the Preamble of the Indian Constitution has been borrowed from
A) US B) Canada
C) Australia D) Ireland

12. Which of the following terms is used in banking or finance?
 A) Moral Suasion
 B) Nelson
 C) Jacksonian Seizure
 D) Incarnation
13. The Nawabganj Bird Sanctuary in Uttar Pradesh has been renamed after
 A) Govind Ballabh Pant
 B) Ashfaquallah Khan
 C) Ram Prasad Bismil
 D) Chandrashekhar Azad
14. $1^3 + 7^3 + 13^3 = ?$
 A) 254
 B) 2541
 C) 2540
 D) 25400
15. If a sum of money doubles itself in 6 years, it becomes 5 times in how many years?
 A) 12 years
 B) 24 years
 C) 10 years
 D) 13 years
16. A mixture of 40 litres of milk and water contains 10% water. How much water should be added to it so that water may be 20% in the new mixture?
 A) 50
 B) 150
 C) 200
 D) 375
17. Three years ago, the average age of a family of five members was 16 years. A baby having been born, the average age of the family is now the same as before. Find the age of the baby.
 A) One year
 B) Two years
 C) Three years
 D) Four years
18. The speed of a car is increased by 2 km every one hour. If the distance travelled in the first hour was 35 km, what was the total distance travelled in 12 hours?
 A) 562 km
 B) 552 km
 C) 482 km
 D) 662 km
19. *Ashish drives his car extremely fast when there is rainfall.*
 The underlined word is an example of
 A) Noun
 B) Adverb
 C) Adjective
 D) Pronoun
20. Which of the following is correctly spelt?
 A) Commodious
 B) Commodius
 C) Commodous
 D) Commodos
21. Which part of the following sentence contains error?
 A) Never I have listened / B) to such beautiful music / C) as the piece we heard / D) on the radio last night.
22. Which of the alternatives is correct, if the following sentence is changed into passive voice?
 Open your door.
 A) Your door has opened.
 B) Has your door be opened?
 C) Let your door be opened.
 D) Let's open your door.
23. Which part of the following sentence contains error?
 A) Ganges, one of the most sacred rivers / B) to Hindus, / C) is a trans-boundary river of Asia / D) which flows through the nations of India and Bangladesh
24. He has ____ fear of heights.
 A) A
 B) An
 C) The
 D) None of the above
25. Select the correct plural of 'arch'
 A) Arches
 B) Archs
 C) Archees
 D) Arch

$$2z + y = 5$$

36. The system of equations $x - 3y = -1$ is consistent when $k =$
 $3x + 4y = k$

- A) 1
 C) 5
 B) 2
 D) 10

37. If $A = \begin{bmatrix} 3 & 2 & -1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{bmatrix}$ then the characteristic polynomial for A is

- A) $x^3 + 5x + 8x + 4$
 C) $x^3 - 5x + 8x - 4$
 B) $x^2 + 5x$
 D) None of these

38. If two vectors are linearly dependent then for some scalar c

- A) $\alpha = c\beta$
 C) $\alpha = c - \beta$
 B) $c + \beta$
 D) None of these

39. A matrix M has eigen value values 1 and 4 with corresponding eigen vectors $(1, -1)^T$ and $(2, 1)^T$ respectively. Then M is

- A) $\begin{pmatrix} -4 & -8 \\ 5 & 9 \end{pmatrix}$
 C) $\begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix}$
 B) $\begin{pmatrix} 9 & -8 \\ 5 & -4 \end{pmatrix}$
 D) $\begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix}$

$$\begin{aligned} & \begin{bmatrix} 3 & 2 & -1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 3-\lambda & 2 & -1 \\ 2 & 2-\lambda & -1 \\ 2 & 2 & -\lambda \end{bmatrix} = 0 \\ &= (3-\lambda) \{ -2\lambda + \lambda^2 + 2 \} \\ &\quad - 2 \{ -2\lambda + 2 \} \\ &\quad - 1 \{ 4 - 4 + 2\lambda \} \\ &= -6\lambda + 3\lambda^2 + 6 \\ &\quad + 2\lambda^2 - \lambda^3 - 2\lambda \\ &\quad + 4\lambda - 2 - 2 \end{aligned}$$

40. If V is the vector space of $m \times n$ matrices over the field K then $\dim V$ is

- A) n
 C) mn
 B) m
 D) $m - n$

41. If M is a 7×5 matrix of rank 3 and N is a 5×7 matrix of rank 5 then rank MN is

- A) 1
 C) 5
 B) 2
 D) 3

42. The eigen values of a skew-symmetric matrix are

- A) always zero
 C) either zero or imaginary
 B) always pure imaginary
 D) always real

43. The system of simultaneous linear equations $x + y + z = 0$ and $x - y - z = 0$ has

- A) no solution in R^3
 C) infinitely many solutions in R^3
 B) a unique solution R^3
 D) more than 2 but finitely many solutions in R^3

44. If $A = \begin{bmatrix} 2 & 1 \\ 3 & -1 \end{bmatrix}$ and I is the 2×2 identity matrix then which of the following the zero matrix ?

- A) $A^2 - A - 5I$
 C) $A^2 + A - I$
 B) $A^2 + A - 5I$
 D) $A^2 - 3A + 5I$

$$= -\lambda^3 + 5\lambda^2 +$$

$$A I =$$

45. The rank of the linear transformation $T : R^3 \rightarrow R^2$ defined by $T(x \ y \ z) = (y \ 0 \ z)$ is

- A) 0
 C) 2
 B) 1
 D) 3

$$\begin{bmatrix} 7 & 1 \\ 3 & 4 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 3 & -1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 4+3 & 2-1 \\ 6-3 & 3+1 \end{bmatrix} = \begin{bmatrix} 7 & 1 \\ 3 & 4 \end{bmatrix}$$

46. Let $(Z, *)$ be an algebraic structure, where Z is the set of integers and the operation "*" is a binary operation defined by $n * m = \max\{n, m\}$. Then $(Z, *)$ is a
- A) groupoid
B) semigroup
C) monoid
D) group
47. Let $(G, *)$ be an algebraic structure where G is the set of all non-zero real numbers and "*" is a binary operation defined by $a * b = \frac{ab}{4}$ for all $a, b \in G$. Then the inverse of 'a' in G is
- A) $\frac{a}{4}$
B) $16a$
C) $\frac{16}{a}$
D) $\frac{4}{a}$
48. If (G, o) be a group and for all $a, b \in G$, $(aob)^2 = a^2ob^2$ then (G, o) is a
- A) normal sub group
B) abelian group
C) quotient group
D) lagrange group
49. Every sub group of an Abelian group 'G' is a
- A) conjugate group
B) associative group
C) normal sub group
D) lagrange group
50. If H, K are two subgroups of a group G then HK is a subgroup of G iff
- A) $HK \neq KH$
B) $HK \subset KH$
C) $HK \supset KH$
D) $HK = KH$
51. The inverse of an even permutation is
- A) odd permutation
B) even permutation
C) even or odd permutation
D) none of these
52. The product of permutations $(1\ 2\ 3)(2\ 4\ 3)(1\ 3\ 4)$ is
- A) $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 2 & 1 \end{pmatrix}$
B) $\begin{pmatrix} 1 & 2 & 5 & 3 \\ 1 & 6 & 5 & 4 \end{pmatrix}$
C) $\begin{pmatrix} 1 & 2 & 5 & 3 \\ 1 & 2 & 3 & 4 \end{pmatrix}$
D) I
53. The order of identity element in an additive group of integers is
- A) zero
B) infinity
C) one
D) two
54. A ring R is an integral domain if
- A) R is a commutative ring
B) R is a commutative ring with zero divisor
C) R is a commutative ring without zero divisor
D) R is a ring with zero divisor
55. If the number of left cosets of a subgroup H in a group G is m and the number of right cosets of H in G is n then
- A) $m \geq n$
B) $m \leq n$
C) $m = n$
D) $m \neq n$

74. If $\vec{A} = (3xz^2)\vec{i} - (yz)\vec{j} + (x+2z)\vec{k}$ then $\text{curl}(\text{curl } \vec{A}) =$
- A) $6x\vec{i} + 6y\vec{j} - 6z\vec{k}$
 B) $6x\vec{i} + (6y-1)\vec{j}$
 C) $-6x\vec{i} + (6z-1)\vec{k}$
 D) none of these
75. $\nabla \cdot (\nabla \times \vec{v}) =$
- A) $\nabla \times (\nabla \cdot \vec{v})$
 B) $\nabla \cdot (\nabla \cdot \vec{v})$
 C) 0
 D) none of these
76. The series $\frac{2}{1^2} + \frac{3}{2^2} + \frac{4}{3^2} + \frac{5}{4^2} + \frac{6}{5^2} + \dots$ is
- A) conditionally convergent
 B) absolutely convergent
 C) absolutely convergent
 D) none of these
77. The radius of convergence of the series $1 - x^2 + x^4 - x^6 + \dots$ is
- A) 0
 B) 1
 C) 2
 D) none of these
78. If (G, o) is a group of order 24 then G can have a subgroup of order
- A) 5
 B) 7
 C) 8
 D) 9
79. PI of the ODE $\frac{d^2y}{dx^2} + \frac{dy}{dx} = x^2 + 2x + 4$ is
- A) $\frac{x^2}{3} + 4x$
 B) $\frac{x^3}{3} + 4$
 C) $\frac{x^3}{3} + 4x$
 D) $\frac{x^2}{3} + 4$
80. The relative cost $z_j - c_j$ for a non-basic variable in a simplex table is zero then there exists an alternate optimal solution, provided
- A) it is starting simplex table
 B) it is optimal simplex table
 C) it can be any simplex table
 D) none of these

81. If a series $\sum_{n=0}^{\infty} a_n$ converges then the sequence $\{a_n\}_1^{\infty}$
- A) diverges
 B) converges to zero
 C) converges to any number
 D) None of these
82. If a sequence is not a Cauchy sequence then it is a
- A) divergent sequence
 B) convergent sequence
 C) bounded sequence
 D) none of these
83. $\lim_{n \rightarrow \infty} \frac{1}{n} \left(1 + 2^{\frac{1}{2}} + 3^{\frac{1}{3}} + \dots + n^{\frac{1}{n}} \right)$ is
- A) 1
 B) 2
 C) 0
 D) none of these
84. If $f(x) = \begin{cases} -x^{\frac{1}{3}} & , -1 \leq x \leq 0 \\ x^{\frac{1}{3}} & , 0 \leq x \leq 1 \end{cases}$, then
- A) Rolle's theorem applies to f in $[-1, 1]$
 B) Rolle's theorem does not apply to f in $[-1, 1]$
 C) f is not continuous at $x=0$
 D) $f'(0)=0$
85. The function $f(x) = \frac{|x|}{x}$, $x \neq 0$ may be continuous at the origin, if
- A) $f(0) = 0$
 B) $f(0) = -1$
 C) $f(0) = \infty$
 D) cannot be continuous for any value of $f(0)$
86. The function $f(x) = \frac{1}{x}$, $x > 0$ is
- A) continuous but not uniformly continuous
 B) discontinuous everywhere
 C) neither continuous nor uniformly continuous
 D) uniformly continuous but not continuous
87. The polynomial $2x^3 - 15x^2 + 36x + 1$ is decreasing in the interval
- A) $(-\infty, 2)$
 B) $(3, \infty)$
 C) $(2, 3)$
 D) none of these
88. For any complex number $z = (x, y)$ in C , if $z\bar{z} = z$ then $\bar{z} =$
- A) $(0, 0)$
 B) $(1, 0)$
 C) $(0, 1)$
 D) $(1, 1)$
89. An analytic function is
- A) infinitely differentiable
 B) finitely differentiable
 C) not differentiable
 D) none of these

$$6x^2 - 30x + 36 = 0$$

$$12x^2 - 30x = 0$$

$$48 - 30 = 18 = 0$$

$$39 - 30 = 9$$

90. A non-empty set of real numbers which is bounded below has
 A) supremum
 B) infimum
 C) no upper bound
 D) no lower bound
91. If F is an open covering of a closed and bounded set A then
 A) There exist an infinite sub collection of A which covers A
 B) There exist an uncountable sub collection of A which covers A
 C) There exist a finite sub collection of A which covers A
 D) None of these
92. Singleton set $\{x_0\}$ of \mathbb{R} is
 A) open
 B) closed
 C) neither open nor closed
 D) None of these
93. Every compact set of real numbers is
 A) closed and bounded
 B) open
 C) open and bounded
 D) closed
94. The whole set $X = \mathbb{R}$ and ϕ are both
 A) open
 B) closed
 C) neither open nor closed
 D) open and closed
95. Every finite subset R of real numbers has
 A) exactly one limit point
 B) all its points are limit points
 C) no limit point
 D) None of these
96. If $f(z)$ is analytic in a simply connected domain D then for every closed path C in D
 A) $\oint_C f(z) dz = 0$
 B) $\oint_C f(z) dz = 1$
 C) $\oint_C f(z) dz \neq 0$
 D) $\int_C f(z) dz \neq 1$
97. The Cauchy-Riemann equations are
 A) both necessary and sufficient condition for a complex function to be analytic
 B) only a necessary condition for a complex function to be analytic
 C) only a sufficient condition for a complex function to be analytic
 D) None of these
98. The complex line integral is
 A) path dependent
 B) path independent
 C) independent of end points
 D) None of these
99. An analytic function is
 A) infinitely differentiable
 B) finitely differentiable
 C) not differentiable
 D) None of these
100. If $f(z)$ is analytic in a simply connected domain D then for any point z_0 in D enclosed by a rectifiable Jordan C and $f(z)$ is continuous on C then for any point z_0 in D , we have $f(z_0)$ is equal to
 A) $\frac{1}{2\pi} \oint_C \frac{f(z)}{z-z_0} dz$
 B) $\frac{1}{2\pi i} \oint_C \frac{f(z)}{z-z_0} dz$
 C) $2\pi i \oint_C \frac{f(z)}{z-z_0} dz$
 D) $2\pi \oint_C \frac{f(z)}{z-z_0} dz$

CUCET-2017

MSc Mathematics

CUCET MSc Math 2017 Que paper

PART - A

1. Choose the correct homophone for Ascent.
A) Accent
B) Assent
C) Axent
D) Axant
2. Which of the following is an Indian Grammarian ?
A) Bhash
B) Bharata
C) Panini
D) Prakasam
3. Find the correct expression.
A) Between you and I
B) Between you and me
C) Between you and my
D) Between you and mine
4. Pair with harp from the following.
A) on
B) at
C) upon
D) in
5. Who is the Indian Nobel Laureate for Literature ?
A) Jatin Kumar Naik
B) Hargobind Khurana
C) Rabindranath Tagore
D) Mother Teresa
6. Who among the following wrote The Jungle Book?
A) Mark Twaine
B) R.K. Narayan
C) Rudyard Kipling
D) Rabindranath Tagore
7. Fill appropriate preposition in the blank :
The bread is made _____ wheat flour.
A) of
B) from
C) in
D) on
8. Find the appropriate homonym for Altar.
A) Alter
B) Altor
C) Altur
D) Altair
9. A Simile is a
A) Contrast
B) Parallel
C) Combination
D) Comparison

10. Find the correct idiom.
- A) Better safe than sad
B) Better safe than serious
C) Better safe than sorry
D) Better safe than regretful
11. A can do a piece of work in 80 days. He works at it for 10 days and then B alone finishes the remaining work in 42 days. In how much time will A and B working together, finish the work ?
- A) 40 days
B) 35 days
C) 50 days
D) 30 days
12. Choose the missing term out of the given alternatives Y, W, U, S, Q, ?, ?
- A) M, L
B) J, R
C) L, M
D) O, M
13. In a certain language, PEN is written as QDM, then how BOOK will be written in that code?
- A) CMJN
B) CNNJ
C) CNLS
D) NMJP
14. After deducting a commission of 5%, a T.V. set costs Rs. 9595. Its marked price is.
- A) Rs. 10,000
B) Rs. 10,075
C) Rs. 10,100
D) Rs. 10,500
15. At what rate percent per annum will a sum of money double in 16 years ?
- A) 6.25% p.a.
B) 6.00% p.a.
C) 6.75% p.a.
D) 6.50% p.a.
16. In an exam two papers maths and chemistry, 60% of the students pass in maths and 70% pass in chemistry. What is minimum percentage of students who could have failed in both the subjects ?
- A) 0%
B) 30%
C) 40%
D) None of these
17. B, the son of A was married to C, whose sister D was married to E, the brother of B. How D is related to A ?
- A) Sister
B) Daughter-in-law
C) Sister-in-law
D) Cousin



18. Statements :
- I. The farmers have decided against selling their Kharif crops to the Government agencies.
 - II. The Government has reduced the procurement price of Kharif crops starting from the last month to the next six months.
- A) Statement I is the cause and statement II is its effect
 - B) Statement II is the cause and statement I is its effect
 - C) Both the statements I and II are independent causes
 - D) Both the statements I and II are effects of independent causes
19. Indiscreet is related to imprudent in the same way as Indisposed is related to
- A) Concerned
 - B) Crucial
 - C) Clear
 - D) Reluctant
20. Rangaswamy Cup is associated with
- A) Archery
 - B) Cricket
 - C) Football
 - D) Hockey
21. Who is the father of Geometry ?
- A) Aristotle
 - B) Euclid
 - C) Pythagoras
 - D) Kepler
22. Shivaji's war strategy used against the Mughals was
- A) Alert Army
 - B) Political Supremacy
 - C) Large Army
 - D) Guerilla Warfare
23. Marginal utility, a consumer derives from a good, is
- A) Change in his total utility as a result of adding one unit to his stock of a good
 - B) Utility derived from a particular good
 - C) Change in utility derived as a result of a change in the price of a good
 - D) Change in his total utility when he buys extra units of a good
24. Joint Military Exercise Nomadic Elephant 2017 is being held between India and
- A) Vietnam
 - B) Mongolia
 - C) Sri Lanka
 - D) Thailand
25. $1014 \times 986 = ?$
- A) 998804
 - B) 998814
 - C) 998904
 - D) 999804

PART - B

26. The solution of the differential equation $\frac{dy}{dx} = \frac{1-x}{y}$ represents a family of

- A) circle with centre at (1,0)
- B) circle with centre at (0, 0)
- C) circle with centre at (-1,0)
- D) straight line with slope -1

$$y dy = (1-x) dx$$

$$\frac{y^2}{2} = x - \frac{x^2}{2} + C$$

$$y^2 = 2x - x^2 + C$$

27. Suppose $\alpha = \lim_{(x,y) \rightarrow (0,0)} \frac{\sin(x^2 + y^2)}{x^2 + y^2}$; $\beta = \lim_{(x,y) \rightarrow (0,0)} \frac{x^2 - y^2}{x^2 + y^2}$ then which of

the following statements is true ?

- A) α exists but β does not exist
- B) α does not exist but β exists
- C) α, β do not exist
- D) both α, β exist

$$y^2 + x^2 - 2x = C_2$$

$$+4 = 4$$

$$y^2 + (x-2)^2 =$$

28. The set $U = \left\{ x \in \mathbb{R} : \sin x = \frac{1}{2} \right\}$ is

- A) open
- B) closed
- C) both open and closed
- D) neither open nor closed

$$\frac{x^2 - mx^2}{x^2 + mx^2} = \frac{(x-1)^2}{1+m}$$

$$= 1$$

29. Let $\{a_n\}$ and $\{b_n\}$ be sequences of real numbers defined as $a_1 = 1$ and for $n \geq 1$,

$$a_{n+1} = a_n + (-1)^n 2^{-n}, \quad b_n = \frac{2a_{n+1} - a_n}{a_n}. \text{ Then}$$

- A) $\{a_n\}$ converges to zero and $\{b_n\}$ is a Cauchy sequence
- B) $\{a_n\}$ converges to a non-zero number and $\{b_n\}$ is a Cauchy sequence
- C) $\{a_n\}$ converges to zero and $\{b_n\}$ is not a convergent sequence
- D) $\{a_n\}$ converges to a non-zero number and $\{b_n\}$ is not a convergent sequence

$$a_2 = 1 + (-1)^1 2^{-1} = 1 - \frac{1}{2} = \frac{1}{2}$$

$$a_3 = \frac{1}{2} + (-1)^2 2^{-2} = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$

$$a_4 = \frac{3}{4} + (-1)^3 2^{-3} = \frac{3}{4} - \frac{1}{8} = \frac{5}{8}$$

$$a_5 = \frac{5}{8} + (-1)^4 2^{-4} = \frac{5}{8} + \frac{1}{16} = \frac{11}{16}$$

30. The matrix equation $AX = B$ has a unique non-zero solution if
- A is singular
 - A is non-singular
 - A is non-singular and B is not a null matrix
 - A is non-singular and B is a null matrix
31. The sequence $\left\{(-1)^n\right\}_{n=1}^{\infty}$ is
- bounded and convergent
 - convergent and unbounded
 - bounded and divergent
 - divergent and unbounded
32. If sequences of real numbers $\{a_n\}_{n=1}^{\infty}$, $\{b_n\}_{n=1}^{\infty}$ and $\{c_n\}_{n=1}^{\infty}$ are such that, $b_n = a_{2n}$ and $c_n = a_{2n+1}$, then $\{a_n\}_{n=1}^{\infty}$ is convergent implies
- $\{b_n\}_{n=1}^{\infty}$ is convergent but $\{c_n\}_{n=1}^{\infty}$ need not be convergent
 - $\{c_n\}_{n=1}^{\infty}$ is convergent but $\{b_n\}_{n=1}^{\infty}$ need not be convergent
 - both $\{b_n\}_{n=1}^{\infty}$ and $\{c_n\}_{n=1}^{\infty}$ are convergent
 - both $\{b_n\}_{n=1}^{\infty}$ and $\{c_n\}_{n=1}^{\infty}$ are divergent
33. Consider the statements
- ✓ a. The series $\sum \sin \frac{1}{n}$ is convergent
- b. The series $\frac{1.2}{3^2 \cdot 4^2} + \frac{3.4}{5^2 \cdot 6^2} + \frac{5.6}{7^2 \cdot 8^2} + \dots$ is convergent.
- Then
- both the statements (a) and (b) are true
 - (a) is true and (b) is false
 - (a) is false and (b) is true
 - neither (a) nor (b) is true

4. The series $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$ is convergent for

- A) all real values of x
- B) $|x| < 1$ only
- C) $|x| < 1$
- D) $-1 < x < 1$

$$\lim_{n \rightarrow \infty} \frac{x^{n+1}}{n+1} \cdot \frac{n}{x^n}$$

$$= x \cdot \frac{n}{n+1} < 1.$$

$$= x \cdot \frac{1}{1 + \frac{1}{n}} = 12$$

x^n
 $\frac{n}{n+1}$
 $1 + \frac{1}{n} = \frac{1}{x}$
 $x < 1$

5. $\lim_{n \rightarrow \infty} (2^n + 3^n)$ is equal to

- A) 2
- B) 3
- C) 5
- D) 6

$$= \frac{xy^2}{2} + \frac{x^3y^2}{2} + \frac{xy^4}{6} - \frac{2x^2y^2}{5} - \frac{2xy^3}{5}$$

6. The value of $\iiint xyz dx dy dz$ over the domain bounded by $x = 0, y = 0, z = 0, x + y + z = 1$ is

- A) $\frac{1}{360}$
- B) 360
- C) $\frac{1}{720}$
- D) 720

$$\iiint_{z=0}^{1-x-y} xyz dx dy dz$$

$$= \iint xy \frac{(1-x-y)^2}{2} dy dx$$

$$= \frac{1}{2} \int \frac{x(1+x^2+y^2-2xy)}{2} dx$$

37. The value of the integral $\int_{-1}^1 \frac{|x|}{x} dx$ is equal to

- A) 2
- B) $\frac{1}{2}$
- C) $\frac{1}{2}$
- D) 0

$$\int_{-1}^1 \frac{|x|}{x} dx = \int_{-1}^0 -1 dx + \int_0^1 1 dx = -x \Big|_{-1}^0 + x \Big|_0^1 = 1 + 1 = 2$$

$$\frac{x^2}{2} + \frac{x^4}{4} + 4x^3 - 2x^2$$

38. The integral $\int_0^\pi xF(\sin x)dx$ is equal to

- A) $\frac{\pi}{4} \int_0^\pi F(\sin x)dx$
 B) $\frac{\pi}{2} \int_0^\pi F(\sin x)dx$
 C) $\pi \int_0^\pi F(\sin x)dx$
 D) $\int_0^\pi F(\sin x)dx$

$$\frac{\pi}{2} \int_0^\pi F(\sin x) dx = \int_0^\pi \int_0^\pi F(\sin x) dx$$

$$= \pi \int_0^\pi F(\sin x) dx$$

$$\int_0^\pi (\pi-x) F(\sin x) dx$$

39. If G is a group and H is a subgroup of index 2 in G , then which of the following is a correct statement ?

- A) H is a normal subgroup of G
 B) H is not a normal subgroup of G
 C) H is a subgroup of G
 D) None of these

$$2H = \int_0^\pi \pi F(\sin x) dx = \frac{\pi}{2} \int_0^\pi F(\sin x) dx$$

40. If $a, b \in G$, where G is a group of order m , then order of ab and ba are

- A) equal to m
 B) same
 C) unequal
 D) not defined

$$\begin{matrix} a^m = e \\ b^m = e \end{matrix} \quad ab$$

41. Which amongst the following statements is not true ?

- A) A sequence cannot converge to more than one limit
 B) Every convergent sequence is bounded
 C) Every bounded sequence is convergent
 D) Limit of a convergent sequence is unique

42. If $u_n = \sqrt{n+1} - \sqrt{n}$ and $v_n = \sqrt{n^4+1} - n^2$, then

- A) $\sum_{n=1}^{\infty} u_n$ converges but $\sum_{n=1}^{\infty} v_n$ diverges
 B) $\sum_{n=1}^{\infty} u_n$ diverges but $\sum_{n=1}^{\infty} v_n$ converges
 C) $\sum_{n=1}^{\infty} u_n$ and $\sum_{n=1}^{\infty} v_n$ both converges
 D) $\sum_{n=1}^{\infty} u_n$ and $\sum_{n=1}^{\infty} v_n$ both diverges

43. The sequence $a_n = \frac{1}{n^2} + \frac{1}{(n+1)^2} + \dots + \frac{1}{(2n)^2}$

A) converges to 0

B) converges to $\frac{1}{2}$

C) converges to $\frac{1}{4}$

D) does not converge

44. Let $a_n = \sin \frac{1}{n^2}$, $n = 1, 2, \dots$, then

A) $\lim_{n \rightarrow \infty} a_n = 1$

B) $\sum_{n=1}^{\infty} a_n$ converges

C) $\lim_{n \rightarrow \infty} \sup a_n \neq \lim_{n \rightarrow \infty} \inf a_n$

D) $\sum_{n=1}^{\infty} a_n$ diverges

45. The derivative of a periodic function with period t is

A) a constant function

B) a periodic function with period t

C) a non-periodic function

D) none of the above

$$\frac{\sqrt{n+1} - \sqrt{n}}{\sqrt{n+1} + \sqrt{n}}$$

$$\frac{(n+1) - n}{\sqrt{n+1} + \sqrt{n}}$$

$$\frac{1}{\sqrt{n+1} + \sqrt{n}}$$

$$\frac{1}{\infty + \infty} = \frac{1}{\infty}$$

$$\frac{1}{(n+m)^2}$$

$$\frac{1}{2} = \frac{1}{2}$$

$$\frac{1}{2n^2} + \frac{1}{2n^2} < \frac{1}{2n^2} + \frac{1}{2n^2} + \frac{1}{2n^2} + \dots$$

$$\frac{1}{2n^2} < \frac{1}{2n^2} + \frac{1}{2n^2} + \frac{1}{2n^2} + \dots$$

$$\frac{1}{2n^2} < \frac{1}{2n^2} + \frac{1}{2n^2} + \frac{1}{2n^2} + \dots$$

$$\frac{1}{2n^2} < \frac{1}{2n^2} + \frac{1}{2n^2} + \frac{1}{2n^2} + \dots$$



46. Let A and B be any $n \times n$ real matrices, then which of the following statements is true ?

- A) $\text{rank}(A + B) = \text{rank}(A) + \text{rank}(B)$
 B) $\text{rank}(A + B) \leq \text{rank}(A) + \text{rank}(B)$
 C) $\text{rank}(A + B) = \text{rank}(A) \cdot \text{rank}(B)$
 D) $\text{rank}(A + B) \geq \text{rank}(A) + \text{rank}(B)$

Handwritten notes:
 $(1 \ 0 \ 0 \ 0) + (1 \ 0 \ 0 \ 0)$
 $(2 \ 0 \ 0 \ 0)$
 $\text{rank} = 1$
 $(1 \ 0 \ 0 \ 0) + (0 \ 1 \ 0 \ 0)$
 $(1 \ 1 \ 0 \ 0)$
 $\text{rank} = 2$

47. If $E = \{e^{2x}, e^{3x}\}$, $x \in \mathbb{R}$ then the set E is

- A) linearly independent over \mathbb{R}
 B) linearly dependent over \mathbb{R}
 C) linearly independent over any interval (a, b) , only when 0 does not belong to (a, b)
 D) none of the above

48. Which one of the following statements is correct ?

- A) There is no vector space of dimension 1
 B) Any three vectors of a vector space of dimension 3 are linearly independent
 C) There is one and only one basis of a vector space of finite dimension
 D) If a non zero vector space V is generated by a finite set S, then V can be generated by a linearly independent subset of S

49. If $V_1 = (1, 2, 0, 3, 0)$, $V_2 = (1, 2, -1, -1, 0)$, $V_3 = (0, 0, 1, 4, 0)$, $V_4 = (2, 4, 1, 0, 1)$ and $V_5 = (0, 0, 0, 0, 1)$, then the dimension of the linear span of $\{V_1, V_2, V_3, V_4, V_5\}$ is

- A) 2
 B) 3
 C) 4
 D) 5

50. The dimension of the vector space $V = \{A = (a_{ij})_{m \times n} : a_{ij} \in \mathbb{C}, a_{ij} = -a_{ji}\}$ is

A) n^2

B) $n^2 - 1$

C) $\frac{n^2 - n}{2}$

D) $\frac{n^2}{2}$

Handwritten matrix:
 $\begin{pmatrix} 1 & 2 & 0 & 3 & 0 \\ 0 & 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

Handwritten matrix:
 $\begin{pmatrix} 1 & 2 & 0 & 3 & 0 \\ 1 & 2 & -1 & -1 & 0 \\ 0 & 0 & 1 & 4 & 0 \\ 2 & 4 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$

51. Using Rolle's theorem, the equation $a_0x^n + a_1x^{n-1} + \dots + a_n = 0$ has atleast one root between 0 and 1, if

A) $\frac{a_0}{n} + \frac{a_1}{n-1} + \dots + a_{n-1} = 0$

B) $\frac{a_0}{n-1} + \frac{a_1}{n-2} + \dots + a_{n-2} = 0$

C) $\frac{a_0}{n+1} + \frac{a_1}{n} + \dots + a_n = 0$

D) $na_0 + (n-1)a_1 + \dots + a_{n-1} = 0$

52. $\lim_{x \rightarrow 0} \frac{(1+x)^{\frac{1}{2}} - e + \frac{1}{2}ex}{x^2}$ is equal to

A) $\frac{24}{11}e$

B) $\frac{11}{24}e$

C) $\frac{1}{11}e$

D) $\frac{1}{24}e$

53. A monotonic function

A) is always continuous

B) is continuous only, if it satisfies intermediate value property

C) can be nowhere continuous

D) can be discontinuous at infinitely many points

54. The set of points where $f(x) = |\sin x|$ is not differentiable is

A) empty

B) $\{0\}$

C) $\{k\pi; k \in \mathbb{Z}\}$

D) $\left\{ \frac{k\pi}{2}; k \in \mathbb{Z} \right\}$

55. Let $P_n(x)$ be a Taylor's polynomial of degree $n \geq 0$ for the function e^x about $x = 0$. Then, the error in this approximation is

A) $\frac{x^n}{n!} e^t$ for some $t, 0 < t < x$

B) $\frac{x^n}{(n+1)!} e^t$ for some $t, 0 < t < x$

C) $\frac{x^{n+1}}{n!} e^t$ for some $t, 0 < t < x$

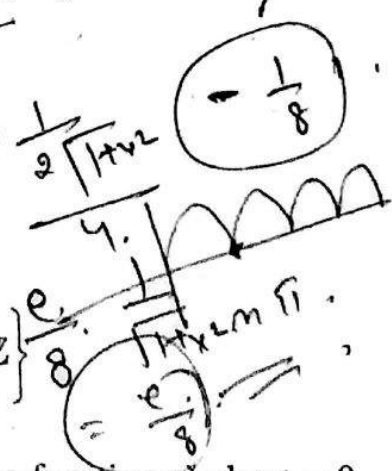
D) $\frac{x^{n+1}}{(n+1)!} e^t$ for some $t, 0 < t < x$

$n a_0 x^{n-1} + \dots + \frac{1}{2} \sqrt{1+x} + \frac{1}{2} e$

$\frac{1}{2\sqrt{1+x}} + \frac{1}{2} e$

$\frac{1}{2} \left(\frac{1}{2} \right) \frac{1}{(1+x)^{3/2}}$

$\frac{1+e\sqrt{1+x}}{4x}$



56. Consider the matrix $M = \begin{bmatrix} 0 & 1 & 2 & 0 \\ 1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 2 \\ 0 & 0 & 2 & 0 \end{bmatrix}$ then

- A) M has no real eigen values
- B) All real eigen values of M are positive
- C) All real eigen values of M are negative
- D) M has both positive and negative real eigen values

$$\begin{bmatrix} -\lambda & 1 & 2 & 0 \\ -1 & -\lambda & 1 & 0 \\ 2 & 1 & -\lambda & 2 \\ 0 & 0 & 2 & -\lambda \end{bmatrix} = \begin{bmatrix} -\lambda & 1 & 2 & 0 \\ 1 & -\lambda & 1 & 0 \\ 0 & 1+2\lambda & -\lambda-2 & 0 \\ 0 & 0 & 2 & -\lambda \end{bmatrix}$$

3 5 -
1 5
2 0 1
5 0 2

57. If the nullity of the matrix $\begin{bmatrix} k & 1 & 2 \\ 1 & -1 & -2 \\ 1 & 1 & 4 \end{bmatrix}$ is 1, then the value of k is

- A) -1
- B) 0
- C) 1
- D) 2

$$\begin{bmatrix} 2 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 4 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

58. Let $M = \begin{bmatrix} 4 & 2 & 1 & 3 \\ 6 & 3 & 4 & 7 \\ 2 & 1 & 0 & 1 \end{bmatrix}$ then the rank of M is

- A) 4
- B) 3
- C) 2
- D) 1

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & 2 \\ 1 & -1 & -2 \\ 1 & 1 & 4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$k(-4+2) - 1(4+2) + 2(1+1)$
 $-2k - 6 + 4 = 0$
 $-2k - 2 = 0$
 $k = -1$

59. Following system of linear equations

$x + 4y + 3z = 0$
 $x + 3y + 4z = 0$
 $x + 2y + 5z = 0$ does have

- A) no solution
- B) infinitely many solutions
- C) more than one but finitely many solutions
- D) exactly one solution

$$\begin{bmatrix} 1 & 4 & 3 \\ 1 & 3 & 4 \\ 1 & 2 & 5 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 4 & 3 \\ 0 & -1 & 1 \\ 0 & -2 & 2 \end{bmatrix}$$

$k(-4+2) - 1(4+2) + 2(1+1) = 0$
 $-2k - 6 + 4 = 0$
 $-2k - 2 = 0$
 $k = -1$

60. Let A be a 3×3 complex matrix, whose characteristic polynomial is given by $f(t) = t^3 + c_2 t^2 + c_1 t + c_0$, then

A) $\det(A) = c_2$

B) $\det(A) = c_0$

C) $\det(A) = -c_2$

D) $\det(A) = -c_0$

61. For the function $f: \mathbb{R}^2 \rightarrow \mathbb{R}$ defined by $f(x, y) = x^3 + y^3 - 3x - 12y + 20$, which of the following is true ?

A) f has maximum at (1, 2)

B) f has minimum at (-1, -2)

C) f has maximum at (1, 2) and minimum at (-1, -2)

D) The saddle points of f are (-1, 2) and (1, -2)

$f_x = 3x^2 - 3 = 0 \Rightarrow x = 1$
 $f_y = 3y^2 - 12 = 0 \Rightarrow y = 2$
 $f_{xx} = 6x > 0$
 $f_{yy} = 6y > 0$
 $f_{xy} = 0$
 $\Delta = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{xy} & f_{yy} \end{vmatrix} = \begin{vmatrix} 6 & 0 \\ 0 & 12 \end{vmatrix} = 72 > 0$
 Since $\Delta > 0$ and $f_{xx} > 0$, (1, 2) is a local minimum.

62. $\int \sqrt{1 + 2 \tan x (\tan x + \sec x)} dx$ is equal to

A) $-\log(1 + \sin x) + c$

B) $\log(1 - \sin x) + c$

C) $-\log(1 - \sin x) + c$

D) $\log(1 + \sin x) + c$

$1 + 2 \tan x (\tan x + \sec x) = 1 + 2 \tan^2 x + 2 \tan x \sec x$
 $= \sec^2 x + \tan^2 x + 2 \tan x \sec x = (\sec x + \tan x)^2$
 $\int \sqrt{(\sec x + \tan x)^2} dx = \int (\sec x + \tan x) dx = \log|\sec x + \tan x| + \sin x + c$

63. The value of $\int_0^\infty \log\left(x + \frac{1}{x}\right) \frac{dx}{1+x^2}$ is

A) 0

C) $\log 2$

$\int_0^\infty \log\left(x + \frac{1}{x}\right) \frac{dx}{1+x^2} = \int_0^\infty \log\left(\frac{x^2+1}{x}\right) \frac{dx}{1+x^2}$
 $= \int_0^\infty \frac{\log(x^2+1)}{1+x^2} dx - \int_0^\infty \frac{\log x}{1+x^2} dx$
 The second integral is 0. The first integral is $\pi \log 2$.

B) ∞

D) $\pi \log 2$

64. If $I_n = \int_0^{\pi/4} \tan^n x dx$ then

A) $I_n - I_{n-2} = \frac{1}{n-1}$

B) $I_n + I_{n+2} = \frac{1}{n-1}$

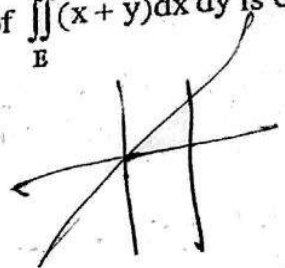
C) $I_n + I_{n-2} = \frac{1}{n}$

D) $I_n - I_{n-2} = \frac{1}{n-2}$



65. If $E = \{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq 1, 0 \leq y \leq x\}$, then the value of $\iint_E (x+y) dx dy$ is equal to

- A) -1
- B) 0
- C) 1
- D) $\frac{1}{2}$



66. If $f(x) = (x + |x|) |x|$, for all $x \in \mathbb{R}$, then which of the following is incorrect ?

- A) f is continuous
- B) f is not differentiable for some x
- C) f' is continuous
- D) f' is differentiable

Handwritten notes for Q66: $f(x) = (x+|x|)|x| = 2x|x|$. Calculations show $f'(x) = 2|x|$ for $x > 0$ and $f'(x) = -2|x|$ for $x < 0$. A graph shows a V-shaped curve passing through the origin. A double integral $\int_0^1 \int_0^x (x+y) dy dx$ is also shown.

67. The Maclaurin's expansion of $\tan^{-1} x$ in powers of x is valid in the interval

- A) $(-\infty, \infty)$
- B) $(-\frac{3\pi}{2}, \frac{3\pi}{2})$
- C) $(-\pi, \pi)$
- D) $(-\frac{\pi}{2}, \frac{\pi}{2})$

Handwritten notes for Q67: A graph of $y = \tan^{-1} x$ is shown with asymptotes at $x = \pm \frac{\pi}{2}$. Calculations show the series expansion $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \dots$ is valid for $|x| < \frac{\pi}{2}$.

68. Consider the function $f: \mathbb{R}^2 \rightarrow \mathbb{R}$ of two variables defined by

$$f(x, y) = \frac{xy}{x^2 + y^2}, x^2 + y^2 \neq 0$$

$$0, x^2 + y^2 = 0$$

Handwritten notes for Q68: $\frac{xy}{x^2+y^2} = \frac{m}{1+m^2}$ where $y = mx$.

Determine which one of the following facts about f is true.

- A) f is continuous at $(0, 0)$
- B) f has removable discontinuity at $(0, 0)$
- C) f is not differentiable at $(0, 0)$
- D) none of the above

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69. If $f: \mathbb{R}^2 \rightarrow \mathbb{R}$ be defined by $f(x, y) = x^2 + y^2$ if x and y are rational
0 otherwise

then which of the following statements is true ?

- A) f is not continuous at $(0, 0)$
 B) f is continuous at $(0, 0)$ but not differentiable at $(0, 0)$
 C) f is differentiable only at $(0, 0)$
 D) f is differentiable everywhere

70. Consider the function $f: \mathbb{R}^2 \rightarrow \mathbb{R}$ defined by $f(x, y) = xy$. Determine which of the following is true for f .

- A) $f_x(a, 0) = 0$ for any constant a
 B) $f_y(e, 0) = 0$
 C) $f_{xy}(1, 0) = 0$
 D) $f_{yx}(1, 1) = 1$

71. The number of elements of order 5 in a symmetric group S_5 is

- A) 5
 B) 20
 C) 24
 D) 12

72. The set M of square matrices (of same order) with respect to matrix multiplication is

- A) group
 B) semi-group
 C) monoid
 D) rank

73. The number of generators in a cyclic group of order 10 are

- A) 1
 B) 2
 C) 3
 D) 4

74. The dimension of the vector space of all 3×3 real symmetric matrices is

- A) 3
 B) 4
 C) 6
 D) 9



75. If U is a 3×3 complex Hermitian matrix, which is unitary, then the distinct eigen values of U are

A) $i, -i$

B) $1+i, 1-i$

C) $1, -1$

D) $\frac{1+i}{2}, \frac{1-i}{2}$

76. If E is a connected subset of \mathbb{R} with at least two elements, then the number of elements in E is

A) exactly two

B) more than two but finite

C) countably infinite

D) uncountable

77. Define $f: \mathbb{R}^2 \rightarrow \mathbb{R}$ by $f(x, y) = 1$ if $xy = 0$
2 otherwise.

If $S = \{(x, y) : f \text{ is continuous at the point } (x, y)\}$, then

A) S is open

B) $S = \emptyset$

C) S is connected

D) S is closed

78. Let Q be the set of rational numbers and E be the set of all rationals p , such that $2 < p^2 < 3$, then E is

A) closed and bounded in Q

B) closed and unbounded in Q

C) not compact in Q

D) compact in Q

79. Consider the following subsets of \mathbb{R}

$$E = \left\{ \frac{n}{n+1} : n \in \mathbb{N} \right\}, F = \left\{ \frac{n}{x+1} : 0 \leq x \leq 1 \right\}, \text{ then}$$

A) Both E and F are closed

B) E is closed and F is not closed

C) E is not closed and F is closed

D) Neither E nor F is closed

Handwritten notes and a circled '1' are present in the bottom right area of the page.

80. If G is a cyclic group of order 8, then the order of the group of automorphisms of G is

- A) 2
- B) 4
- C) 6
- D) 8

$\phi(8) = 2^3$

81. $\int_0^{\pi/2} \frac{\sin^{3/2} x}{\sin^{3/2} x + \cos^{3/2} x} dx$ is equal to

- A) 0
- B) 1
- C) $\frac{\pi}{4}$
- D) $\frac{\pi}{2}$

Handwritten solution for Q81:
 $\int_0^{\pi/2} \frac{\sin^{3/2} x}{\sin^{3/2} x + \cos^{3/2} x} dx$
 Let $x = \frac{\pi}{2} - u$
 $\int_0^{\pi/2} \frac{\sin^{3/2} x}{\sin^{3/2} x + \cos^{3/2} x} dx = \int_{\pi/2}^0 \frac{\sin^{3/2}(\frac{\pi}{2}-u)}{\sin^{3/2}(\frac{\pi}{2}-u) + \cos^{3/2}(\frac{\pi}{2}-u)} (-du)$
 $= \int_0^{\pi/2} \frac{\cos^{3/2} u}{\cos^{3/2} u + \sin^{3/2} u} du$
 $\int_0^{\pi/2} \frac{\sin^{3/2} x}{\sin^{3/2} x + \cos^{3/2} x} dx + \int_0^{\pi/2} \frac{\cos^{3/2} x}{\sin^{3/2} x + \cos^{3/2} x} dx = \int_0^{\pi/2} 1 dx = \frac{\pi}{2}$

82. The entire length of the curve whose equation is $x^{2/3} + y^{2/3} = r^{2/3}$ is equal to

- A) $\frac{3}{2}r$
- B) $2\sqrt{3}$
- C) $6r$
- D) none of these

Handwritten notes for Q82:
 $\frac{2}{3}r$
 $\frac{2}{3}r$
 $\frac{2}{3}r$

83. The value of $\int_0^{\infty} \int_{1/y}^{\infty} x^4 e^{-x^3 y} dx dy$ is equal to

- A) 1/4
- B) 1/3
- C) 1/2
- D) 1

Handwritten solution for Q83:
 $\int_0^{\infty} \int_{1/y}^{\infty} x^4 e^{-x^3 y} dx dy$
 Let $u = x^3 y$
 $\frac{du}{dy} = 3x^2$
 $x^4 dx = \frac{1}{3} du$
 $\int_0^{\infty} \int_{1/y}^{\infty} x^4 e^{-x^3 y} dx dy = \int_0^{\infty} \int_{1/y}^{\infty} \frac{1}{3} e^{-u} du dy$
 $= \frac{1}{3} \int_0^{\infty} \left[-e^{-u} \right]_{1/y}^{\infty} dy = \frac{1}{3} \int_0^{\infty} e^{-1/y} dy$
 Let $v = 1/y$
 $dy = -\frac{1}{v^2} dv$
 $\int_0^{\infty} e^{-1/y} dy = \int_{\infty}^0 e^{-v} \left(-\frac{1}{v^2}\right) dv = \int_0^{\infty} \frac{e^{-v}}{v^2} dv$
 $= \left[-\frac{e^{-v}}{v} - \int \frac{e^{-v}}{v} dv \right]_0^{\infty} = 0 - \left(-\frac{1}{0} - \int_0^{\infty} \frac{e^{-v}}{v} dv \right)$
 $= \int_0^{\infty} \frac{e^{-v}}{v} dv$
 This integral is known to be $\frac{1}{2}$.

84. Which of the following is not an integrating factor of the differential equation $dy - y dx = 0$?

- A) $\frac{1}{x^2}$
- B) $\frac{1}{x^2 + y^2}$
- C) $\frac{1}{xy}$
- D) $\frac{x}{y}$

Handwritten solution for Q84:
 For A) $\frac{1}{x^2}$: $\frac{1}{x^2} dy - \frac{y}{x^2} dx = 0$
 $\frac{1}{x^2} dy - \frac{y}{x^2} dx = 0$
 $\frac{1}{x^2} dy - \frac{y}{x^2} dx = 0$
 For B) $\frac{1}{x^2 + y^2}$: $\frac{1}{x^2 + y^2} dy - \frac{y}{x^2 + y^2} dx = 0$
 $\frac{1}{x^2 + y^2} dy - \frac{y}{x^2 + y^2} dx = 0$
 For C) $\frac{1}{xy}$: $\frac{1}{xy} dy - \frac{y}{xy} dx = 0$
 $\frac{1}{xy} dy - \frac{1}{x} dx = 0$
 $\frac{1}{y} dy - \frac{1}{x} dx = 0$
 $\int \frac{1}{y} dy - \int \frac{1}{x} dx = 0$
 $\ln y - \ln x = 0$
 $\ln \frac{y}{x} = 0$
 $\frac{y}{x} = 1$
 $y = x$
 For D) $\frac{x}{y}$: $\frac{x}{y} dy - \frac{y}{y} dx = 0$
 $\frac{x}{y} dy - dx = 0$
 $\frac{x}{y} dy - dx = 0$
 $\frac{x}{y} dy - dx = 0$

85. The orthogonal trajectory to the family of circles $x^2 + y^2 = 2cx$ (c arbitrary) is described by the differential equation

A) $(x^2 + y^2)y' = 2xy$

B) $(x^2 - y^2)y' = 2xy$

C) $(y^2 - x^2)y' = xy$

D) $(y^2 - x^2)y' = 2xy$

~~$2x + y \frac{dy}{dx} = c$~~

$x^2 + y^2 = 2 \left(x + y \frac{dy}{dx} \right) x$

$x^2 + y^2 = 2 \left(x - y \frac{dy}{dx} \right) x$

$= \frac{x^2 + y^2}{x} = 2 \left(x - \frac{y}{y'} \right)$

86. The maximum magnitude of the directional derivative for the surface $x^2 + xy + yz = 9$ at the point (1, 2, 3) is along the direction

A) $i + j + k$

B) $2i + 2j + k$

C) $i + 2j + 3k$

D) $i - 2j + 3k$

$\nabla F = (2x+y)i + (x+2y)j + yk$

$\frac{x^2 + y^2 - 2x}{x} = -\frac{2y}{y'}$

$\frac{x^2 + y^2 - 2x^2}{x} = \frac{2y}{y'}$

87. If \vec{F} is such that $\nabla \times \vec{F} = 0$, then \vec{F} is called

A) rotational

B) irrotational

C) solenoidal

D) rotational and solenoidal

$= 4(1+j+k) \cdot (y^2 - x^2) y' = 2xy$

$4(2+2+1) = \frac{20}{1+2+3} = \frac{20}{6}$

88. From the following, what is the value of $\int_C \vec{F} \cdot d\vec{r}$, where $\vec{F} = 2x^2y\hat{i} + 3xy\hat{j}$ and

C is $y = 4x^2$ in the plane from (0,0) to (1, 4)?

A) $\frac{104}{9}$

B) $\frac{104}{7}$

C) $\frac{104}{3}$

D) $\frac{104}{5}$

$\int_0^1 4x^2 dx$

$\frac{4x^3}{3} \Big|_0^1 = \frac{4}{3}$

$\frac{dy}{dx} = 8x dx$

$d\vec{r} = \hat{i}dx + \hat{j}dy$

$\frac{64 \times 4}{3} = \frac{256}{3}$

CO

89. What from the following is the directional derivative of $\phi = 5x^2y - 5y^2z + 2.5z^2x$ at the point $(1, 1, 1)$ in the direction of the line $\frac{x-1}{2} = \frac{y-3}{-2} = z$?

A) $\frac{11}{3}$

B) $\frac{11}{2}$

C) $11\frac{2}{3}$

D) $\frac{2}{3}$

10xy?

90. If x, y and z are positive real numbers, then the minimum value of $x^2 + 8y^2 + 27z^2$,

where $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$ is

A) 108

B) 216

C) 405

D) 1048

*$\frac{1}{2} = \frac{1}{x} - \frac{1}{y}$
 $x = \frac{2y}{y-1}$*

216

91. Which one of the following differential equations represent all circles with radius a ?

A) $1 + \left(\frac{dy}{dx}\right)^2 + \sqrt{a^2 + x^2} \frac{d^2y}{dx^2} = 0$

B) $1 + \left(\frac{dy}{dx}\right)^2 + \sqrt{a^2 + y^2} \frac{d^2y}{dx^2} = 0$

C) $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 + a^2 \left(\frac{d^2y}{dx^2}\right)^2 = 0$

D) $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 - a^2 \left(\frac{d^2y}{dx^2}\right)^2 = 0$

$2x^2y dx + 2xy dy$

$= 2x^2 \cdot 4x^2 dx + 3x^4x^2$

$8x^4 + 96x^4$

$= 104 \frac{x^4}{5} / 0$

$x^2 + y^2 = a^2$



92. The solution $y(x)$ of the differential equation $(D^2 + 4D + 4)y = 0$ satisfying the conditions $y(0) = 4, y'(0) = 8$ is

- A) $4e^{2x}$
- B) $(16x + 4)e^{-2x}$
- C) $4e^{-2x} + 16x$
- D) $4e^{-2x} + 16x4e^{2x}$

$m^2 + 4m + 4 = 0$
 $m = -2 \pm \sqrt{16 - 16}$
 $m = -2$

93. An integrating factor for the differential equation $(\cos y \sin 2x)dx + (\cos^2 y - \cos^2 x)dy = 0$ is

- A) $\sec^2 y + \sec y \tan y$
- B) $\tan^2 y + \sec y \tan y$
- C) $\frac{1}{\sec^2 y + \sec y \tan y}$
- D) $\frac{1}{\tan^2 y + \sec y \tan y}$

$(m-2)^2 = 0$
 $m = 2$

94. If c is an arbitrary constant, then the general solution of the differential equation

$\frac{dy}{dx} - \tan x \tan y = \cos x \sec y$ is

- A) $2 \sin y = (x + c - \sin x \cos x) \sec x$
- B) $\cos y = (x + c) \sin x$
- C) $\sec y = (x + c) \cos x$
- D) $\sin y = (x + c) \cos x$

$y' = \dots$
 $y = (x + c^2) e$
 $y = c$

95. The maximum value of $f(x, y, z) = xyz$ along all points lying on the intersection of the planes $x + y + z = 40$ and $z = x + y$ is

- A) 4000
- B) 3000
- C) 2000
- D) 1000

$z = 40 - x - y$
 $z = x + y$
 $x + y = 20$
 $\partial xy(x+y)$
 $y' = (x+2y)^2$
 $g = 2c_1 + c_2$

96. The differential equation $\frac{dy}{dx} = k(a-y)(b-y)$ when solved with the condition $y(0) = 0$, yields the result

- A) $\frac{b(a-y)}{a(b-y)} = e^{(a-b)kx}$
- B) $\frac{b(a-x)}{a(b-x)} = e^{(a-b)ky}$
- C) $\frac{a(b-y)}{b(a-y)} = e^{(a-b)kx}$
- D) $xy = ke$

$\frac{dy}{(a-y)(b-y)} = k dx$
 $\frac{1}{(a-y)(b-y)} = \frac{A}{a-y} + \frac{B}{b-y}$
 $\frac{1}{a-y} + \frac{1}{b-y} = \frac{1}{(a-b)(b-y)}$

$\frac{b}{a-b} + \frac{a}{b-a} = \frac{1}{b-a}$
 $\frac{b}{a-b} - \frac{a}{a-b} = \frac{1}{b-a}$
 $\frac{b-a}{a-b} = \frac{1}{b-a}$
 $A(b-y) + B(a-y) = 1$
 $aB + Ab = 1$

97. Solving by variation of parameter the differential equation $y'' - 2y' + y = e^x \log x$ the value of Wronskian W is

- A) e^{2x}
- B) 2
- C) e^{-2x}
- D) e

$$\int e^{-x}$$

98. The differential equation $2y dx - (3y - 2x)dy = 0$

- A) exact and homogeneous but not linear
- B) homogeneous and linear but not exact
- C) exact and linear but not homogeneous
- D) exact, homogeneous and linear

$$2y dx = (3y - 2x) dy$$

$$\frac{dy}{dx} = \frac{2y}{3y - 2x}$$

99. The differential equation $(\alpha xy^3 + y \cos x) dx + (x^2 y^2 + \beta \sin x) dy = 0$ is exact for the values of α and β such that

A) $\alpha = \frac{3}{2}, \beta = 1$

$$3\alpha xy^2 + \cos x = 2xy^2 + \beta \cos x$$

B) $\alpha = 1, \beta = \frac{3}{2}$

$$(3\alpha - 2)xy^2 + (1 - \beta)\cos x = 0$$

C) $\alpha = \frac{2}{3}, \beta = 1$

D) $\alpha = 1, \beta = \frac{2}{3}$

Handwritten work for Q99:

$$x^2 y^2 + 2xy^2 = 0$$

$$2xy^2 + 4x = 0$$

$$x^2 + 2xy = 0$$

$$2x + 4 = 0$$

$$2x + 4 = 0$$

$\beta = 1$

$\alpha = \frac{2}{3}$

100. The particular solution of the equation $y' \sin x = y \log y$ satisfying the initial condition

$y(\pi/2) = e$, is

- A) $e^{\tan(x/2)}$
- B) $e^{\cot(x/2)}$
- C) $\log \tan(x/2)$
- D) $\log \cot(x/2)$

$$\frac{dy}{dx} \sin x = y \log y$$

$$\frac{dy}{y \log y} = \frac{dx}{\sin x}$$

Handwritten work for Q100:

$$\log y = t$$

$$\log + \log(\log y) = \int \frac{dt}{t \cos x} + C$$

Answer Key

CUCET 2016 MSc Mathematics Entrance Exam

Q.No.	Ans	Q.No.	Ans	Q.No.	Ans
1	B	21	B	41	C
2	C	22	D	42	B
3	B	23	A	43	A
4	A	24	B	44	B
5	C	25	D	45	B
6	C	26	A	46	B
7	B	27	A	47	C
8	A	28	B	48	D
9	D	29	B	49	B
10	C	30	C	50	C
11	D	31	C	51	C
12	D	32	C	52	*
13	B	33	C	53	B
14	C	34	D	54	C
15	A	35	*	55	A
16	A	36	C	56	D
17	B	37	D	57	A
18	B	38	B	58	C
19	D	39	A	59	B
20	D	40	B	60	D

Q.No.	Ans	Q.No.	Ans
61	D	81	C
62	C	82	C
63	D	83	C
64	*	84	D
65	D	85	B
66	D	86	B
67	D	87	B
68	C	88	D
69	B	89	C
70	A or D	90	B
71	C	91	D
72	B or C	92	B
73	D	93	A
74	C	94	D
75	C	95	C
76	D	96	A
77	A	97	A
78	D	98	D
79	D	99	C
80	B	100	A

CUCET-2016

MSc Mathematics

12. Which number will come in the blank space ?

1, 2, 3, 5, 8, _

A) 9
 C) 13

B) 11
 D) 15

13. Which of the following is not a member of SAARC ?

- A) Bhutan
- B) Burma
- C) Bangladesh
- D) Maldives

14. In a group of 15 people, 7 read French, 8 read English while 3 of them read none of these two. How many of them read French and English both ?

- A) 0
- C) 4

- B) 3
- D) 5

15. How many rectangles are there in the following figure ?



A) 6
 C) 8

B) 7
 D) 9

16. Select the most suitable synonym for TACT.

- A) cunningness
- B) diplomacy
- C) intelligence
- D) discrimination

17. Select the most suitable antonym for DEPICT.

- A) misrepresent
- B) portray
- C) misunderstand
- D) sketch



18. Identify the meaning of idiom "Be in two minds".
- A) be burdened
 - B) be indifferent
 - C) be mischievous
 - D) be undecided
19. Who is the author of the book titled "The Z Factor : My Journey as the Wrong Man at the Right Time" ?
- A) Mahendra Verma
 - B) Vijay Joshi
 - C) Narayan Pandit
 - D) Subhash Chandra
20. Choose the correct option
- $$\frac{1260}{\frac{15}{7}} = ?$$
- A) 12
B) 58
C) 122
D) 588
- X-3, 22, 122, 588*
42 22
342 =
21. The average of 7 consecutive numbers is 20. The largest of these numbers is
- A) 20
 - B) 22
 - C) 23
 - D) 24
22. What percent of Rs. 2,650 is Rs. 1,987.50 ?
- A) 60%
 - B) 75%
 - C) 80%
 - D) 90%
23. A sells an article which costs him Rs. 400 to B at a profit of 20%. B then sells it to C, making a profit of 10% on the price he paid to A. How much does C pay to B ?
- A) Rs. 472
 - B) Rs. 476
 - C) Rs. 528
 - D) Rs. 532
24. If $0.75 : x :: 5 : 8$, then x is equal to
- A) 1.12
 - B) 1.20
 - C) 1.25
 - D) 1.30
25. A and B can do a piece of work in 72 days; B and C can do it in 120 days; A and C can do it in 90 days. In what time can A alone do it ?
- A) 80 days
 - B) 100 days
 - C) 120 days
 - D) 150 days

PART - B

26. Consider the set $S = \left\{ x \in \mathbb{R} : \frac{2x+1}{x+2} < 1 \right\}$, where \mathbb{R} is the set of reals. Determine which one of the following statements about S is correct.
- A) S is bounded below but not above and $\inf S = -2$
 B) S is bounded above but not below and $\sup S = 1$
 C) S is bounded both below and above with $\inf S = -2$, $\sup S = 1$
 D) S is neither bounded below nor above
27. Consider the set $S = \left\{ \frac{mn}{1+m+n} : m, n \text{ are natural numbers} \right\}$. Then determine which one of the following statements is correct.
- A) S is a bounded set
 B) S is bounded below with $\inf S = \frac{1}{3}$ but not bounded above
 C) S is bounded above with $\sup S = 1$ but not below
 D) S is neither bounded below nor above
28. Let p, q be two reals such that $p > q > 0$. Define the sequence $\{x_n\}$, where $x_1 = p + q$ and $x_n = x_1 - \frac{pq}{x_{n-1}}$ for $n \geq 2$. Then for all n , x_n is equal to one of the following and determine it.
- A) $x_n = \frac{p^{n+1} - q^{n+1}}{p^n - q^n}$ B) $x_n = \frac{p^{n+1} + q^{n+1}}{p^n + q^n}$
 C) $x_n = \frac{(pq)^n}{p^n + q^n}$ D) $x_n = \frac{(pq)^n}{p^n - q^n}$
29. Which one of the following statements is wrong ?
- A) Every convergent sequence of reals is necessarily bounded
 B) Every sequence of reals has a monotone subsequence
 C) Every monotone increasing sequence which is bounded above is convergent
 D) Every sequence which is bounded above has a convergent subsequence



30. The sequence $\left\{ \frac{n}{2^n} + 1 \right\}$

- A) Is bounded but not convergent
 B) Is convergent and converges to 0
 C) Is convergent and converges to 1
 D) Is monotone increasing

31. The minimum value of the sum $\sum_{k=1}^n a_k^2$ of reals satisfying $\sum_{k=1}^n a_k = 1$ is

- A) $\frac{1}{\sqrt{n}}$
 B) $\frac{1}{n}$
 C) $\frac{1}{n^2}$
 D) $\frac{1}{n^3}$

$S = a_1 + a_2 + \dots + a_n$
 $a_1 + a_2 + \dots + a_n$

32. Consider the sequences $\{a_n\}$ and $\{b_n\}$, where $a_n = \left(1 + \frac{1}{n}\right)^n$ and $b_n = \left(1 + \frac{1}{n}\right)^{n+1}$ for

all $n \in \mathbb{N}$. Then,

- A) both sequences are monotone increasing
 B) both sequences are monotone decreasing
 C) one of these two sequences is monotone increasing and the other one is monotone decreasing
 D) both the sequences are unbounded

33. The series $\sum_{n=1}^{\infty} \frac{n}{3 \cdot 5 \cdot 7 \dots (2n+1)}$ converges to

- A) $\frac{1}{2}$
 B) $\frac{1}{3}$
 C) $\frac{1}{4}$
 D) $\frac{1}{5}$

34. The series $\sum_{k=2}^{\infty} \frac{1}{k(\log k)^\alpha}$ where α is a real no. and $\log k = \log_e k$

- A) Converges for all α
 B) Converges only for $\alpha \leq 0$
 C) Converges only for all α satisfying $0 < \alpha \leq 1$
 D) Converges only for all $\alpha > 1$

A*

35. The $\lim_{x \rightarrow 0} \frac{\log(\cos x)}{\sin^2 x}$

A) does not exist

B) exists and its value is $-\frac{1}{2}$

C) exists and its value is 0

D) exists and its value is $\frac{1}{2}$

36. Consider the function $f(x) = \frac{1}{1 + e^{1/x}}$ for $x \neq 0$. Then

A) Left hand limit $\lim_{x \rightarrow 0^-} f(x)$ at $x = 0$ exists but the right hand limit $\lim_{x \rightarrow 0^+} f(x)$ at $x = 0$ does not exist

B) $\lim_{x \rightarrow 0^+} f(x)$ at $x = 0$ exists but $\lim_{x \rightarrow 0^-} f(x)$ does not exist

C) Both $\lim_{x \rightarrow 0^-} f(x)$ and $\lim_{x \rightarrow 0^+} f(x)$ at $x = 0$ exist and they are equal

D) Both $\lim_{x \rightarrow 0^-} f(x)$ and $\lim_{x \rightarrow 0^+} f(x)$ at $x = 0$ exist and they are not equal

37. The value of $\lim_{x \rightarrow \infty} x \left(\log \left(1 + \frac{x}{2} \right) - \log \left(\frac{x}{2} \right) \right)$ is

A) 2

B) 1

C) 0

D) -1

38. Define the function $f: \mathbb{R} \rightarrow \mathbb{R}$ by $f(x) = \begin{cases} \sin|x|, & \text{if } x \text{ is rational} \\ 0, & \text{otherwise} \end{cases}$

Then f is continuous

A) at all rational points

B) at all irrational points

C) at all $x = k\pi$, where k is any integer

D) at all $x \neq k\pi$, where k is any integer

39. Let f and $g: [a, b] \rightarrow \mathbb{R}$ be two continuous functions such that $f(a) < g(a)$ and $f(b) > g(b)$. Then
- A) there is a $c \in (a, b)$ such that $f(c) + g(c) = 0$
- B) there is a $c \in (a, b)$ such that $f(c) - g(c) = 0$
- C) for all $x \in (a, b)$, $f(x) = g(x)$
- D) for all $x \in (a, b)$, $f(x) \neq g(x)$
40. Which of the following functions is uniformly continuous on $[0, \infty]$?
- A) $f(x) = x \sin x$
- B) $g(x) = \sin x^2$
- C) $h(x) = e^x$
- D) $k(x) = \sin(\sin x)$
41. Let $f: (1, \infty) \rightarrow \mathbb{R}$ be a function defined by $f(x) = \log_x 2$. Then the derivative of f is
- A) $\frac{1}{x \log x} f(x)$
- B) $-\frac{1}{x \log x} f(x)$
- C) $\frac{1}{\log x} f'(x)$
- D) $-\frac{1}{\log x} f'(x)$
42. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function defined by $f(x) = \begin{cases} ax + b, & \text{if } x \leq 1 \\ ax^2 + c, & \text{if } 1 < x \leq 2 \\ \frac{dx^2 + 1}{x}, & \text{if } x > 2 \end{cases}$ where a, b, c, d are constants. The values of a, b, c, d so that f is differentiable on \mathbb{R} , are
- A) $a = 0, b = c = 1, d = \frac{1}{4}$
- B) $a = 0, b = c = -1, d = \frac{1}{2}$
- C) $a = 1, b = c = -1, d = \frac{1}{4}$
- D) $a = -1, b = c = 0, d = \frac{1}{2}$

43. Let $g: (0, \infty) \rightarrow \mathbb{R}$ be a differentiable function such that $g'(x) = \frac{1}{x}$ for all x . Define f on $(0, \infty)$ by $f(x) = (g(x^2))^3$. Then f is differentiable and $f'(x)$ is equal to

- A) $6x (g(x^2))^2$
 B) $6x^2 (g(x^2))^2$
 C) $6(g(x^2))^2/x$
 D) $6(g(x^2))^2/x^2$

44. Define the function f on \mathbb{R} by $f(x) = \sum_{i=1}^n (a_i - x)^2$ where a_1, a_2, \dots, a_n are real constants.

Then f has a relative extremum at the point

- A) $x = \sum_{i=1}^n a_i$
 B) $x = \frac{1}{n} \sum_{i=1}^n a_i$
 C) $x = \sum_{i=1}^n a_i^2$
 D) $x = \frac{1}{n} \sum_{i=1}^n a_i^2$

45. Let $f: (-1, 1) \rightarrow \mathbb{R}$ be a function defined by $f(x) = \begin{cases} 2x^4 + x^4 \sin \frac{1}{x} & \text{for } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases}$

Then

- A) $f(x)$ has no point of extremum in $(-1, 1)$
 B) $f(x)$ is a decreasing function
 C) $f(x)$ is an increasing function
 D) $f(x)$ is neither a decreasing nor an increasing function in $(-1, 1)$

46. The value of the integral $\int_0^{\pi/2} \frac{\sin^n x}{\sin^n x + \cos^n x} dx$, where n is a natural number is

- A) π
 B) $\frac{\pi}{2}$
 C) $\frac{\pi}{3}$
 D) $\frac{\pi}{4}$



47. Let f be a positive monotone decreasing function on $[1, \infty)$. Then the sequence $\{a_n\}$,

$$\text{where } a_n = \sum_{k=1}^n f(k) - \int_1^n f(x) dx \text{ for } n \in \mathbb{N}$$

- A) is not bounded
- B) is not a monotone sequence
- C) is convergent
- D) is oscillatory

48. Let f be a continuously differentiable function defined on an interval $[a, b]$ such that

$$f(a) = f(b) = 0 \text{ and } \int_a^b f^2(x) dx = 1. \text{ Then the value of } \int_a^b x f(x) f'(x) dx \text{ is}$$

- A) $-\frac{1}{2}$
- B) 0
- C) $\frac{1}{2}$
- D) 1

49. The improper integral $\int_1^{\infty} \frac{dx}{x^n}$

- A) converges if $n < 1$
- B) converges if $n = 1$
- C) converges if $n > 1$
- D) converges for all values of n

50. The improper integral $\int_0^{\infty} e^{-xy} \sin x dx$, where $y > 0$ satisfies which one of the following ?

- A) does not converge for some $y > 0$
- B) converges for all $y > 0$ but not uniformly on $[a, \infty)$ for any $a > 0$
- C) converges uniformly on $(0, \infty)$
- D) converges uniformly on $[a, \infty)$, where $a > 0$

A*

51. Consider the function $f: \mathbb{R}^2 \rightarrow \mathbb{R}$ of two variables defined by

$$f(x, y) = \begin{cases} 0, & \text{if } (x, y) = (0, 0) \\ \frac{xy^2}{x^2 + y^2}, & \text{if } (x, y) \neq (0, 0) \end{cases}$$

Determine which one of the following facts about f is not true.

- A) f is differentiable at $(0, 0)$
 B) f is continuous at $(0, 0)$
 C) f has directional derivative at $(0, 0)$ in the direction of any vector $u = (a, b) \neq (0, 0)$
 D) partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ at $(0, 0)$ exist

52. Let $f: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ and $g: \mathbb{R}^2 \rightarrow \mathbb{R}$ be two functions given by $f(x, y) = (x^2 + y^2, x^2 - y^2)$ and $g(x, y) = 2xy$. Define $h: \mathbb{R}^2 \rightarrow \mathbb{R}$ by $h = g \circ f$. Then h is differentiable at each $(x, y) \in \mathbb{R}^2$ and it is a 1×2 matrix given by

- A) $h'(x, y) = (2x^3 \ 2y^3)$
 B) $h'(x, y) = (8x^3 \ -8y^3)$
 C) $h'(x, y) = (-8x^3 \ 8y^3)$
 D) $h'(x, y) = (4x^3 \ 4y^3)$

53. The function $f: \mathbb{R}^2 \rightarrow \mathbb{R}$ given by $f(x, y) = xy$

- A) has a critical point at $(0, 0)$ which is a relative minimum
 B) has a critical point at $(0, 0)$ which is a relative maximum
 C) has a critical point at $(0, 0)$ which is a saddle point
 D) has no critical point

54. The area of the largest rectangle that can be inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

- A) $\frac{ab}{2}$ B) ab C) $\frac{\pi ab}{2}$ D) πab

55. The value of the integral $\int_C \frac{dx}{x+y}$ where C is the curve whose parametric representation is $x = at^2, y = 2at, 0 \leq t \leq 2$, is

A) $\log 2$

B) $\frac{1}{2} \log 2$

C) $\frac{1}{3} \log 2$

D) $2 \log 2$

56. The value of the integral $\iint_S x^2 y^2 dx dy$, where S is the region $x \geq 0, y \geq 0$ and $x^2 + y^2 \leq 1$, is

A) $\frac{\pi}{96}$

B) $\frac{\pi}{48}$

C) $\frac{\pi}{24}$

D) $\frac{\pi}{12}$

57. The value of the integral $\iiint_D (a^3 b^2 c^2 - b^2 c^2 x^2 - c^2 a^2 y^2 - a^2 b^2 z^2)^{1/2} dx dy dz$ where D

is the region $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \leq 1$, is

A) $a^2 b^2 c^2 \pi^2$

B) $\frac{1}{4} a^2 b^2 c^2 \pi^2$

C) $\frac{1}{3} a^2 b^2 c^2 \pi^2$

D) $\frac{1}{2} a^2 b^2 c^2 \pi^2$

58. The gradient vector ∇f of $f(x, y, z) = e^{xy} - x \cos(yz^2)$ at $(1, 0, 0)$ is

A) $\vec{i} + \vec{j}$

B) $\vec{i} - \vec{j}$

C) $-\vec{i} + \vec{j}$

D) $\left(-\vec{i} + \vec{j} + \vec{k} \right)$

A*

59. A unit normal to the surface $\sin xy = e^z$ at $(1, \pi/2, 0)$ is
- A) $(\vec{i} + \vec{j} + \vec{k})/\sqrt{3}$ B) \vec{i}
 C) \vec{j} D) \vec{k}
60. The equation of the tangent plane to the surface $3xy + z^2 = 4$ at $(1, 1, 1)$ is
- A) $3x + 3y + 2z = 8$ B) $3x - 3y + 2z = 8$
 C) $3x + 3y - 2z = 8$ D) $-3x + 3y + 2z = 8$
61. Suppose y is a differentiable function of x satisfying $e^{x-y} + x^2 - y = 1$. Then the value of $\frac{dy}{dx}$ at $(0, 0)$ is
- A) 0 B) $\frac{1}{2}$ C) $\frac{1}{3}$ D) $\frac{1}{4}$
62. The divergence of the vector field given by $\vec{F} = x^2y\vec{i} + z\vec{j} + xyz\vec{k}$ is
- A) xy B) $2xy$ C) $3xy$ D) $4xy$
63. The curl of the vector field $\vec{F}(x, y, z) = xy\vec{i} - \sin z\vec{j} + \vec{k}$ is
- A) $\cos z\vec{i} + x\vec{j}$ B) $\cos z\vec{i} + x\vec{k}$
 C) $\cos z\vec{j} + x\vec{k}$ D) $\cos z\vec{i} - x\vec{k}$
64. Which one of the following vector fields is not a gradient vector field ?
- A) $\vec{F}(x, y, z) = (y+z)\vec{i} + (z+x)\vec{j} + (x+y)\vec{k}$
 B) $\vec{F}(x, y, z) = y\vec{i} - x\vec{j}$
 C) $\vec{F}(x, y, z) = 2xy^2\vec{i} + 2(x^2+z^2)y\vec{j} + 2y^2z\vec{k}$
 D) $\vec{F}(x, y, z) = \frac{y}{x^2+y^2}\vec{i} - \frac{x}{x^2+y^2}\vec{j}$, where $(x, y) \neq (0, 0)$

65. Let $f(x, y, z)$ and $g(x, y, z)$ be two function defined on R^2 having second order partial derivatives with respect to x, y, z . Determine which one of the following fact about the Laplacian operator ∇^2 is true.

- A) $\nabla^2(fg) = (\nabla^2 f)(\nabla^2 g)$
 B) $\nabla^2(fg) = g\nabla^2 f + f\nabla^2 g$
 C) $\nabla^2(fg) = g\nabla^2 f + f\nabla^2 g + \nabla f \cdot \nabla g$
 D) $\nabla^2(fg) = g\nabla^2 f + f\nabla^2 g + 2(\nabla f \cdot \nabla g)$

66. The initial value problem $\frac{dy}{dx} = \sqrt{|y|}$, $y(0) = 0$ has

- A) no non-trivial solution
 B) only trivial solution
 C) two solutions
 D) more than two solutions

67. The primitive of the differential equation

$$(2x \sinh \frac{y}{x} + 3y \cosh \frac{y}{x}) dx - 3x \cosh \frac{y}{x} dy = 0 \text{ is given by}$$

- A) $x^2 = K \sinh^3 \frac{y}{x}$
 B) $x^2 = K \sinh^2 \frac{y}{x}$
 C) $x^2 = K \cosh^3 \frac{y}{x}$
 D) $x^2 = K \cosh^2 \frac{y}{x}$

Where K is an arbitrary constant.

68. An integrating factor of the differential equation $y(2xy + 1) dx + x(1 + 2xy - x^3y^3) dy = 0$ is

A) $\frac{1}{x^3y^3}$

B) $\frac{1}{x^4y^4}$

C) $\frac{1}{x^3y^3}$

D) $\frac{1}{x^2y^2}$

69. The primitive of the differential equation $6y^2 \left(\frac{dy}{dx} \right)^2 + 3x \frac{dy}{dx} - y = 0$ is given by

A) $y^3 = Kx^2 + \frac{1}{3}K^2$

B) $y^3 = Kx^2 + \frac{2}{3}K^2$

C) $y^3 = Kx^3 + \frac{1}{3}K^2$

D) $y^3 = Kx + \frac{2}{3}K^2$

Where K is an arbitrary constant.

70. Given below four sets $\{f_1, f_2, f_3\}$ of functions defined on \mathbb{R} . Determine which set is linearly dependent.

A) $\{f_1(x) = x^2, f_2(x) = x^4, f_3(x) = x^{-2}\}$

B) $\{f_1(x) = x, f_2(x) = x + 1, f_3(x) = x + 2\}$

C) $\{f_1(x) = \cos x, f_2(x) = \sin x, f_3(x) = 1\}$

D) $\{f_1(x) = e^x, f_2(x) = e^{-x}, f_3(x) = 1\}$

71. The general solution of the differential equation $x^2 y'' + xy' - y = (x^3 + 3x^2)e^x$ is

A) $y = C_1 x + C_2 x^2 + xe^x$

B) $y = \frac{C_1}{x} + C_2 x + e^x$

C) $y = \frac{C_1}{x} + C_2 x + xe^x$

D) $y = \frac{C_1}{x} + \frac{C_2}{x^2} + xe^x$

Where C_1, C_2 are arbitrary constants.

72. Let V be the vector space of all functions from the interval $[-1, 1]$ into \mathbb{R} . Determine which one of the following subsets of V is not a subspace of V .

A) $V_1 = \{f \in V : f(x^2) = f(x)^2 \text{ for all } x \in [-1, 1]\}$

B) $V_2 = \{f \in V : f(x) + f(-x) = 0 \text{ for all } x \in [-1, 1]\}$

C) $V_3 = \{f \in V : f(0) = f(1)\}$

D) $V_4 = \{f \in V : f \text{ is a continuous function}\}$



73. Determine which one of the following sets of vectors from \mathbb{R}^3 does not form a basis for \mathbb{R}^3 .
- A) $\{(1, 0, -1), (2, 5, 1), (0, -4, 3)\}$
 B) $\{(1, 2, -1), (1, 0, 2), (2, 1, 1)\}$
 C) $\{(-1, 3, 1), (2, -4, -3), (-3, 8, 2)\}$
 D) $\{(2, -4, 1), (0, 3, -1), (6, 0, -1)\}$

74. The rank of the matrix $\begin{pmatrix} 1 & 2 & 3 & 1 & 1 \\ 1 & 4 & 0 & 1 & 2 \\ 0 & 2 & -3 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$ is

A) 1
 C) 3

B) 2
 D) 4

75. Let $P = \begin{pmatrix} 1 & -2 & 1 \\ 2 & 1 & 1 \\ 0 & 5 & -1 \end{pmatrix}$ be a 3×3 matrix over \mathbb{R} . Then for a given vector

$Y = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \in \mathbb{R}^{3 \times 1}$, the vector space of all 3×1 matrices over \mathbb{R} , the system $PX = Y$ has

a solution if

- A) $y_1 - y_2 + y_3 = 0$
 C) $y_1 + y_2 - y_3 = 0$

- B) $2y_1 - y_2 + y_3 = 0$
 D) $2y_1 + y_2 - y_3 = 0$

76. Let V be a finite dimensional vector space over a field F and $T : V \rightarrow V$ be a linear operator. Which one of the following statement is true ?

- A) If T has an eigen-vector then it has infinitely many distinct eigen-vectors
 B) Sum of two eigen-values of T is an eigen-value of T
 C) Sum of two eigen-vectors of T is an eigen-vector of T
 D) Eigen-values of T are necessarily non-zero scalars

A*

81. Let $X = \left\{ \frac{1}{n} : n \in \mathbb{Z}, n \neq 0 \right\}$ be a subset of \mathbb{R} (\mathbb{Z} is the set of all integers). Determine

which one of the following properties of X is true.

- A) X is a bounded set
 B) X is an open subset of \mathbb{R}
 C) X is a closed subset of \mathbb{R}
 D) X has no limit point in \mathbb{R}

82. The set \mathbb{Z} of all integers

- A) is an open subset of \mathbb{R}
 B) is a closed subset of \mathbb{R}
 C) is a compact subset of \mathbb{R}
 D) has infinitely many limit points in \mathbb{R}

83. Determine which one of the following subsets of \mathbb{R} is connected.

- A) $(-\infty, 0) \cup (0, \infty)$
 C) $\bigcup_{n=1}^{\infty} (-n, n)$
 B) The set \mathbb{Q} of all rational numbers
 D) $\mathbb{R} \setminus \mathbb{Z}$

84. Consider the set $X = [-1, 1]$ with the subspace topology relative to \mathbb{R} . Which one of the following subsets of X is open in X and in \mathbb{R} ?

- A) $\left\{ x \in X : \frac{1}{2} \leq |x| < 1 \right\}$
 B) $\left\{ x \in X : \frac{1}{2} < |x| \leq 1 \right\}$
 C) $\left\{ x \in X : |x| > \frac{1}{2} \right\}$
 D) $\left\{ x \in X : \frac{1}{2} < |x| < 1 \right\}$

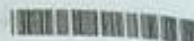
85. Which one of the following statements is true

- A) Every closed interval in \mathbb{R} is homeomorphic to \mathbb{R}
 B) Every open interval of the type (a, b) is homeomorphic to \mathbb{R}
 C) Every interval of the type $[a, b)$ is homeomorphic to \mathbb{R}
 D) Every interval of the type $(a, b]$ is homeomorphic to \mathbb{R}

86. The radius of convergence of the power series $\sum_{n=1}^{\infty} n^{2n} x^n$ is

- A) 0
 B) $\frac{1}{2}$
 C) 1
 D) ∞

A*



91. Let G be the set of all rationals $\frac{p}{q}$, where q is an odd integer. With respect to the usual multiplication of reals, G is not a group because
- The closure property does not hold
 - The associative property does not hold
 - No element of G can be an identity element
 - Not every element of G can have an inverse
92. Let G be a group and $a, b \in G$. Then which one of the following statements is not true ?
- If a, b and ab have same order then $ab = ba$
 - If $a^3 = e$, the identity element of G , and $aba^{-1} = b^2$ then the order of b is 16
 - ab and ba have same order
 - b and aba^{-1} have same order
93. Let $G = \{(a, b) \in \mathbb{R}^2 : a \neq 0\}$. On G define a binary operation by $(a, b) \circ (c, d) = (ac, bc + d)$. With respect to this operation on G , which one of the following is true ?
- G is a group with identity $(1, 1)$ and inverse of (a, b) is (a^{-1}, ba^{-1})
 - G is a group with identity $(1, 1)$ and inverse of (a, b) is $(a^{-1}, -ba^{-1})$
 - G is a group with identity $(1, 0)$ and inverse of (a, b) is $(a^{-1}, -ba^{-1})$
 - G is not a group
94. Which one of the following subsets of G , given in the problem 93, is not a subgroup of G ?
- $H_1 = \{(1, 0)\}$
 - $H_2 = \{(a, b) \in G : a = 1\}$
 - $H_3 = \{(a, b) \in G : a \text{ is rational}\}$
 - $H_4 = \{(a, b) \in G : a \text{ is irrational}\}$

A*

95. Let G be a cyclic group of order 9. Then
 A) G has nine generators
 B) G has six generators
 C) G has five generators
 D) G has three generators
96. Let P_4 be the permutation group of 4 elements. Then the order of the element $(13)(24) \in P_4$ is
 A) 6
 B) 4
 C) 3
 D) 2
97. Let G be the group of non-zero real numbers under multiplication. Determine which one of the following functions $f: G \rightarrow G$ is not a homomorphism
 A) $f(x) = x^2, x \in G$
 B) $f(x) = |x|, x \in G$
 C) $f(x) = \sqrt{|x|}, x \in G$
 D) $f(x) = 2^x, x \in G$
98. Let a_1, a_2, \dots, a_n be the roots of a polynomial $x^n + x^{n-1} + \dots + x + 1$, where $a_i \neq 1$ for $i = 1, 2, \dots, n$. Then the value of $\sum_{i=1}^n \frac{1}{1-a_i}$ is
 A) $\frac{n}{2}$
 B) $\frac{n}{3}$
 C) $\frac{n}{4}$
 D) n
99. Let P_3 be the permutation group of 3 elements. Then the number of elements in P_3 which are conjugate to $(2, 3) \in P_3$ is
 A) 1
 B) 2
 C) 3
 D) 6
100. Let G be a group of order 12. Then the maximal number of subgroups of order 4 in G can be
 A) 6
 B) 5
 C) 4
 D) 3

CUCET
Sample Question Paper
MSc Mathematics

CUCET MSc Math Sample Que Paper**PART-A**

1. Choose the correct word to fill in the blank. The students _____ the teacher on teacher's day for twenty years of dedicated teaching.
 (A) Facilitated (B) Felicitated
 (C) Fantasized (D) Facillitated
2. Choose the correct word to fill in the blank. Dhoni as well as the other team members of Indian team _____ present on the occasion
 (A) were (B) was
 (C) has (D) have
3. Choose the word most similar in meaning: Awkward
 (A) Inept (B) Careful
 (C) Suitable (D) Dread full
4. Choose the correct verb to fill in the blank below
 Let us _____.
 (A) Introvent (B) Alternate
 (C) Atheist (D) Altruist
5. Select the most suitable Synonym for the word 'RESILIENT'.
 (A) Stretchable (B) Spirited
 (C) Rigid (D) Buoyant
6. Select the most suitable Synonym for the word 'ZEST'.
 (A) Humour (B) Keen Interest
 (C) Attitude (D) Liking
7. Select the most suitable Antonym for the word 'ROBUST'.
 (A) Sturdy (B) Ridiculous
 (C) Muscular (D) Feeble
8. Select the most suitable Antonym for the word 'DULL'.
 (A) Monstrous (B) Horrid
 (C) fascinating (D) Ghastly
9. Select the pair which shows the same relationship as CANE : BAMBOO
 (A) Wood : Woodpecker (B) Timber : Tree
 (C) Rubber : Malaysia (D) South Africa : Apartheid
10. Why were you absent _____ your dance classes yesterday?
 (A) for (B) from
 (C) in (D) to
11. A man is facing towards South. He take 135° anticlock wise, 180° clockwise rotation then what was facing side of the man?
 (A) North-East (B) North-West
 (C) South-East (D) South-West
12. If the value of "x" is 25% less than the value of "y". How much % y's is more than that of x's ?
 (A) $33\frac{1}{3}\%$ (B) 25%
 (C) 75% (D) $66\frac{2}{3}\%$

13. If the difference between simple interests for 3 years and 4 years at 5% annual rate is 42, then the amount will be,
 (A) Rs. 210 (B) Rs 280
 (C) Rs. 750 (D) Rs. 840
14. The sum of three consecutive even integer is 54. What is the smallest number?
 (A) 18 (B) 14
 (C) 16 (D) 12
15. Area of circle and a square is equal. Ratio of one side of the square to radius of the circle will be,
 (A) $1:\sqrt{\pi}$ (B) $\sqrt{\pi}:1$
 (C) $1:\pi$ (D) $\pi:1$
16. Fill in the blank to complete the series: 181, 174, 178, _____, 175, 182.
 (A) 174 (B) 176
 (C) 178 (D) 180
17. 'Tree' is related to 'Forest' in the same way as 'Soldier' is related to
 (A) Battle (B) Army
 (C) Gun (D) General
18. Pointing to a gentleman, Deepak said. "His only brother is the father of my daughter's father." How is that gentleman related to Deepak?
 (A) Father (B) Grandfather
 (C) Brother-in-law (D) Uncle
19. Complete the series BEP, CIQ, DOR, FUS, GAT,?
 (A) HEV (B) HIT
 (C) IET (D) IEU
20. Convert 36 km/hr into meters per second.
 (A) 10 (B) 12
 (C) 15 (D) 20
21. 'Wings of Fire' was written by _____.
 (A) APJ Abdul Kalam (B) Salman Rushdie
 (C) Amitav Ghosh (D) Shashi Tharoor
22. 'Chhau' dance is associated with which of the following states?
 (A) Punjab (B) Maharashtra
 (C) Jammu Kashmir (D) Jharkhand
23. Mineral rich 'Jharia' is located in which of the following states?
 (A) Bihar (B) West Bengal
 (C) Utter Pradesh (D) Gujrat
24. Jhansi was annexed by which of the following Governor General?
 (A) Lord Bentinck (B) Lord Dalhausie
 (C) Lord Cornwallis (D) Lord Clive
25. Who among the following personalities stated "Swaraj is my birth right and I am going to have it."
 (A) Bal Gangadhar Tilak
 (B) Subhas Chandra Bose
 (C) Mahatma Gandhi
 (D) Jawahar Lal Nehru

PART - B

26. The sequence $\left\{ \frac{1}{n} \right\}$ is
 (A) convergent (B) divergent (C) oscillatory (D) unbounded
27. $\lim_{n \rightarrow \infty} \frac{2n-3}{n+1}$ equals
 (A) 0 (B) 1 (C) 2 (D) e
28. The series $\sum_{n=1}^{\infty} \frac{n+1}{n^p}$ is convergent for
 (A) $0 < p < 1$ (B) $1 < p < 2$ (C) $p = 2$ (D) $p > 2$
29. The series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$ is
 (A) convergent (B) divergent
 (C) conditionally convergent (D) absolutely convergent
30. $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n} \right)^n$ equals
 (A) e (B) $\frac{1}{e}$ (C) 0 (D) 1
31. Which of the following statements is false?
 (A) Every bounded sequence is convergent.
 (B) Every convergent sequence is bounded.
 (C) Every bounded sequence has a limit point.
 (D) Every convergent sequence has a unique limit.
32. If a series $\sum_{n=0}^{\infty} a_n$ converges, then
 (A) $\lim_{n \rightarrow \infty} a_n = 0$ (B) $\lim_{n \rightarrow \infty} a_n = \infty$ (C) $\lim_{n \rightarrow \infty} a_n = 1$ (D) $\lim_{n \rightarrow \infty} a_n = 10$
33. If $f : \mathbb{R} \rightarrow \mathbb{R}$ is defined by $f(x) = |x - c|$, for all $x \in \mathbb{R}$, then
 (A) f is discontinuous
 (B) f is differentiable
 (C) f is continuous but not differentiable
 (D) f is continuously differentiable
34. The function $f(x) = \begin{cases} x \sin 1/x, & \text{when } x \neq 0 \\ 0, & \text{when } x = 0 \end{cases}$ is
 (A) continuous at $x = 0$ (B) derivable at $x = 0$
 (C) discontinuous at $x = 0$ (D) infinitely differentiable at $x = 0$

35. If Rolle's theorem holds for $f(x) = x^3 + ax^2 + bx$ on $[-2, 2]$ at $x = 1$, then
 (A) $a = 1/2, b = -4$ (B) $a = 2, b = -4$
 (C) $a = -1/2, b = 4$ (D) $a = 4, b = 1/2$
36. The local maxima of $x^3 - 3x + 3$ is attend at
 (A) $x = -1$ (B) $x = 1$ (C) $x = 0$ (D) $x = 3$
37. The function $f(x) = \sin 3x, x \in [0, \pi/2]$ is increasing in the interval
 (A) $(0, \pi/6)$ (B) $(\pi/6, \pi/2)$ (C) $(0, \pi/2)$ (D) $(\pi/3, \pi/2)$
38. The function $f(x) = x^2$ is not uniformly continuous on the interval
 (A) $[-1, 1]$ (B) $[1, 2]$ (C) $[0, \infty)$ (D) $[0, 1]$
39. Every compact set of real numbers is
 (A) open (B) closed
 (C) closed and bounded (D) open and bounded
40. The set \mathbb{R} of real real numbers is
 (A) closed (B) bounded
 (C) countable (D) none of the above
41. The upper limit of the sequence $\{(-1)^n\}$ is
 (A) 1 (B) -1 (C) 0 (D) 2
42. If $f(x, y)$ is a homogeneous function of degree n in x and y and has continuous partial derivatives, then $x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y}$ is equal to
 (A) f (B) nf (C) 0 (D) $n(n-1)f$
43. $\lim_{(x,y) \rightarrow (2,1)} (x^2 + 2x - y^2)$ equals
 (A) 0 (B) -7 (C) 7 (D) -1
44. The radius of convergence of the series $1 + 2x + 3x^2 + 4x^3 + \dots$ is
 (A) 0 (B) 1 (C) ∞ (D) 2
45. The value of the integral $\int_0^1 \int_0^x e^{y/x} dx dy$ is
 (A) $\frac{(e-1)}{2}$ (B) $\frac{(e+1)}{2}$ (C) e (D) e^2
46. The value of the surface integral $\int \int_S (x^3 dy dz + y^3 dz dx + z^3 dx dy)$ over the sphere $x^2 + y^2 + z^2 = a^2$ is
 (A) $\frac{12}{5}\pi a^5$ (B) πa^5 (C) $\frac{5}{12}\pi a^5$ (D) πa^2

47. Which of the following sets forms a basis of \mathbb{R}^2 ?

- (A) $\{(1,1), (3,1)\}$ (B) $\{(0,1), (0,-3)\}$
 (C) $\{(2,1), (1,-1), (3,0)\}$ (D) $\{(1,0), (2,0)\}$

48. Rank of the matrix $\begin{pmatrix} 2 & 1 & 1 \\ 0 & 3 & 0 \\ 3 & 1 & 2 \end{pmatrix}$ is equal to

- (A) 1 (B) 2 (C) 3 (D) 4

49. Which of the following functions $F : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is not a linear transformation ?

- (A) $F(x, y) = (x + y, x - y)$ (B) $F(x, y) = (x + y, x)$
 (C) $F(x, y) = (2x - y, x)$ (D) $F(x, y) = (x, 1 + y)$

50. The dimension of the vector space of all 3×3 real symmetric matrices is

- (A) 9 (B) 6 (C) 3 (D) 4

51. The determinant of $\begin{pmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{pmatrix}$ is

- (A) $(z - x)(z - y)(y - x)$ (B) $(z - x)^2(z - y)(y - x)$
 (C) $(z^2 - x^2)(z^2 - y^2)(y^2 - x^2)$ (D) $(z - x)^2(z - y)^2(y - x)^2$

52. If $M = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$, then M^{2019} equals

- (A) $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ (B) $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ (C) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (D) $\begin{pmatrix} 1 & 2019 \\ 0 & 1 \end{pmatrix}$

53. Which of the following matrix is singular?

- (A) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (B) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ (C) $\begin{pmatrix} 1 & 4 \\ 2 & 10 \end{pmatrix}$ (D) $\begin{pmatrix} 2 & 2 \\ 3 & 3 \end{pmatrix}$

54. If $M = \begin{pmatrix} 4 & 0 \\ 2 & 3 \end{pmatrix}$, then the eigenvalues of M are

- (A) -4 and -3 (B) 4 and 3 (C) 2 and 0 (D) 3 and -3

55. Let $F : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be a linear transformation defined by $F(x, y) = (2x + 3y, 4x - 5y)$. Then the matrix representation of the linear transformation relative to basis $B = \{(1, 0), (0, 1)\}$ is

- (A) $\begin{pmatrix} 2 & 3 \\ 4 & -5 \end{pmatrix}$ (B) $\begin{pmatrix} 0 & -3 \\ 4 & 5 \end{pmatrix}$ (C) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (D) $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

56. The eigenvalues of a skew-symmetric matrix are

- (A) always pure imaginary (B) always zero
 (C) either zero or imaginary (D) always real

57. If $M = \begin{pmatrix} 2 & -2 \\ -2 & 5 \end{pmatrix}$ and $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, which of the following is a zero matrix ?
 (A) $M^2 - 7M - 6I$ (B) $M^2 - 7M + 6I$ (C) $M^2 - 6M - 7I$ (D) $M^2 - 6M - 7I$
58. Let $T : V_n(F) \rightarrow V_m(F)$, where $V_n(F)$ and $V_m(F)$ are finite dimensional vector spaces. Then
 (A) $\text{rank}(T) + \text{nullity}(T) = \dim(V_n(F))$ (B) $\text{rank}(T) = \text{nullity}(T)$
 (C) $\text{rank}(T) - \text{nullity}(T) = \dim(V_n(F))$ (D) $\text{rank}(T) - \text{nullity}(T) = \dim(V_m(F))$
59. The singleton set $\{x\}$ is linearly dependent if
 (A) $x = 0$ (B) $x \neq 0$ (C) x is a scalar (D) none of these
60. The eigenvalues of an orthogonal matrix are
 (A) zero (B) imaginary (C) always negative (D) of unit modulus
61. Degree of the differential equation $dy = (y + \sin x)dx$ is
 (A) 1 (B) 2 (C) 3 (D) 4
62. Solution of the differential equation $\frac{dy}{dx} = e^{x-y} + x^2e^{-y}$ is
 (A) $e^y = x + e^x + c$ (B) $e^y = x^2/2 + e^x + c$
 (C) $e^y = x^3/3 + e^x + c$ (D) $e^y = x^4/4 + e^x + c$
63. The integrating factor of the differential equation $(1 - x^2)dy/dx + 2xy = x\sqrt{1 - x^2}$ is
 (A) $\frac{1}{1 - x}$ (B) $\frac{1}{1 - x^2}$ (C) $1 - x^2$ (D) $1 - x$
64. The solution of differential equation $\frac{d^2y}{dx^2} + 4y = 0$ with initial conditions $y = 2$ and $dy/dx = 0$ when $x = 0$ is
 (A) $y = 2 \sin 2x$ (B) $y = 2 \cos 2x$ (C) $y = \sin 4x$ (D) $y = \tan x$
65. Which of the following is a particular integral of $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{5x}$?
 (A) $\frac{1}{12}e^{5x}$ (B) e^{-5x} (C) e^x (D) e^{x^2}
66. Let $D =: d/dx$. Then the value of $\left\{ \frac{1}{xD + 1} \right\} x^{-1}$ is
 (A) $\log x$ (B) $\frac{\log x}{x}$ (C) $\frac{\log x}{x^2}$ (D) $\frac{\log x}{x^3}$
67. If $y_1(x)$ and $y_2(x)$ are two solutions of $\frac{d^2y}{dx^2} + 4y = 0$, then the value of Wronskian is
 (A) 0 (B) 1 (C) 2 (D) 3

68. Differential equation of the family of parabola $y^2 = 4ax$, where a is an arbitrary constant is
 (A) $y = 2x(dy/dx)$ (B) $y = dy/dx$ (C) $y = 2x + dy/dx$ (D) $dy/dx + y^2 = x^2$
69. The orthogonal trajectory of the hyperbola $xy = a$ is
 (A) $x^2 - y^2 = a$ (B) $x^2 = ay^2$ (C) $x^2 + y^2 = a$ (D) $x = ay^2$
70. The order of differential equation $\frac{dy}{dx} = \sqrt{x} + \sqrt{y}$ is
 (A) 1 (B) 2 (C) 3 (D) 4
71. Solution of the initial value problem $e^x(\cos y dx - \sin y dy) = 0$ with $y(0) = 0$ is
 (A) $e^x \cos y + 1 = 0$ (B) $e^x \cos y - 1 = 0$
 (C) $e^y \cos x + 1 = 0$ (D) $e^y \cos x - 1 = 0$
72. If $F(x, y, z) = xy^2 + 3x^2 - z^3$, then the value of $\nabla F(x, y, z)$ at $(2, -1, 4)$ is equal to
 (A) $13i - 4j - 48k$ (B) $i - 4j - k$ (C) $13i + j - 6k$ (D) $-13i + 4j - 6k$
73. The directional derivative of the function $F(x, y, z) = xy^2 - 4x^2y + z^2$ at $(1, -1, 2)$ in the direction of $6i + 2j + 3k$ is
 (A) $1/7$ (B) $2/7$ (C) $54/7$ (D) 7
74. If $\vec{F} = zi + xj + yk$, then $\text{curl } \vec{F}$ is
 (A) $i + j + k$ (B) 0 (C) $i - j - k$ (D) $2i + j - 2k$
75. Let F be a finite field. Then which of the following may be the possible cardinality of F ?
 (A) 15 (B) 20 (C) 25 (D) 30
76. Every subgroup of an abelian group is
 (A) abelian (B) cyclic
 (C) non abelian (D) none of the above.
77. Let $G = \left\{ \begin{bmatrix} a & a \\ a & a \end{bmatrix} \mid a \in \mathbb{R} \setminus \{0\} \right\}$ be a group with binary operation defined by usual matrix multiplication. Then the inverse of $\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$ is
 (A) $\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$ (B) $\begin{bmatrix} 1/2 & -1/2 \\ -1/2 & 1/2 \end{bmatrix}$ (C) $\begin{bmatrix} 1/4 & 1/4 \\ 1/4 & 1/4 \end{bmatrix}$ (D) $\begin{bmatrix} 1/8 & 1/8 \\ 1/8 & 1/8 \end{bmatrix}$
78. Let H and K be subgroups of G . Then which of the following is necessarily a subgroup of G ?
 (A) HK (B) KH (C) $H \cap K$ (D) $H \cup K$

79. Let S_5 be the permutation group on five symbols $\{1, 2, 3, 4, 5\}$. Then order of permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 3 & 5 & 1 \end{pmatrix}$ is equal to
- (A) 5 (B) 4 (C) 3 (D) 6
80. Let G be a group and $a, b, c \in G$ are non-identity elements. Which of the following solves the equation $axb = c$ for x ?
- (A) acb^{-1} (B) $a^{-1}b^{-1}$ (C) $a^{-1}cb^{-1}$ (D) cb^{-1}
81. Let H be a subgroup of a noncyclic group G . Then which of the following is correct?
- (A) H is always noncyclic (B) H is always cyclic
(C) H is always nonabelian (D) None of the above
82. Let S_6 be the permutation group on six symbols $\{1, 2, 3, 4, 5, 6\}$. Which of the following is not an even permutation?
- (A) $(1\ 3\ 5\ 6\ 2)$ (B) $(1\ 2\ 3)(4\ 5)(4\ 5)$
(C) $(2\ 6\ 3\ 4\ 5\ 1)$ (D) $(1\ 2)(1\ 4)(2\ 3)(4\ 5)$
83. Which of the following is correct?
- (A) Every integral domain is a field.
(B) Every finite integral domain is a field.
(C) There is an integral domain with characteristic equal to 10.
(D) None of the above.
84. Let J be an ideal of commutative ring with unity and let u be a unit element of R such that $u \in J$. Then
- (A) The multiplicative identity $1 \notin J$
(B) J is a proper ideal of R such that $J \neq R$
(C) $J = R$
(D) There is a minimal ideal M such that $J \subset M \subseteq R$
85. Which of the following is a prime ideal of $(\mathbb{Z}, +, \cdot)$?
- (A) $6\mathbb{Z}$ (B) $2\mathbb{Z} \cap 4\mathbb{Z}$ (C) $7\mathbb{Z}$ (D) $4\mathbb{Z} \cap 8\mathbb{Z}$
86. If $Z = 2 - 3i$, then $|Z|$ equals
- (A) 13 (B) $\sqrt{13}$ (C) -13 (D) -1
87. $\int_0^1 ze^{2z} dz$ equals
- (A) $e^2 + 1$ (B) $(e^2 + 1)/4$ (C) $(e^2 - 1)/4$ (D) $e^2 - 1$
88. $\lim_{z \rightarrow i} \frac{Z^{10} + 1}{Z^6 + 1}$ equals
- (A) $3/5$ (B) $2/5$ (C) $5/3$ (D) $1/3$

89. The integral $\int_{3i}^{1-i} 4z \, dz$ equals
 (A) $18 - 4i$ (B) $-4i$ (C) i (D) $-i$
90. If $f(z)$ is analytic in a simply connected domain D and $f'(z)$ is continuous in D , then $\oint_C f(z) \, dz$ equals
 (A) 0 (B) 1 (C) $2\pi i$ (D) $-2\pi i$
91. The value of the integral $\int_{|z-2|=2} \frac{5z+7}{z^2+2z-3} \, dz$ is equal to
 (A) πi (B) $2\pi i$ (C) $3\pi i$ (D) $6\pi i$
92. If $f(z) = u(x, y) + iv(x, y)$ is analytic in a domain D , then
 (A) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ and $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0$ (B) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ and $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \neq 0$
 (C) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \neq 0$ and $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0$ (D) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \neq 0$ and $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \neq 0$
93. An entire function is
 (A) infinitely differentiable (B) finitely differentiable
 (C) not differentiable (D) identically zero
94. Which of the following is incorrect statement?
 (A) If $f(z)$ is entire and bounded in complex plane, then $f(z)$ is constant.
 (B) If $f(z)$ is analytic at z_0 , then $f'(z)$ is also analytic at z_0 .
 (C) Analytic function is entire.
 (D) Entire function is analytic.
95. The complex line integral is
 (A) path dependent (B) independent of end points
 (C) path independent (D) none of these
96. The set of all feasible solutions to a linear programming problem (LPP) is
 (A) a concave set (B) a convex set
 (C) a bounded set (D) an infinite set only
97. A basic feasible solution to a LPP, in which at least one of the basic variables is zero is
 (A) degenerate (B) infeasible (C) non-degenerate (D) unbounded
98. The optimal solution of the LPP: Maximize $Z = 4x_1 + x_2$, such that $x_1 + x_2 \leq 50$, $3x_1 + x_2 \geq 90$, $x_1, x_2 \geq 0$, is
 (A) $x_1 = 30, x_2 = 0$ (B) $x_1 = 20, x_2 = 30$
 (C) $x_1 = 0, x_2 = 0$ (D) $x_1 = 0, x_2 = 50$

99. Which of the following is incorrect statement?

- (A) Arbitrary intersection of convex sets is a convex set.
- (B) Hyperplane is a convex set.
- (C) Union of two convex sets need not to be a convex set.
- (D) Union of two convex sets is a convex set.

100. In a linear programming problem constraints are

- (A) nonlinear
- (B) linear
- (C) linear as well as nonlinear
- (D) none of the above