CSIR-NET ODE-PDE Practice Questions

ODE and PDE Practice Questions

1.

Consider a boundary value problem (BVP) $\frac{d^2y}{dx^2} = f(x)$ with boundary conditions $y(0) = y(1) = y'(1)$, where f is a real-valued continuous function on $[0,1]$. Then which of the following are true?

1. the given BVP has a unique solution for every f
2. the given BVP does not have a unique solution for some f
3. $y(x) = \int_0^x x\,t\,f(t)dt + \int_x^1 (t - x + xt) f(t)dt$ is a solution of the given BVP
4. $y(x) = \int_0^x (x - t + xt) f(t)dt + \int_x^1 xtf(t)dt$ is a solution of the given BVP

2.

. Consider the ODE on $\mathbb{R}$ $y'(x) = f(y(x))$. If f is an even function and y is an odd function, then

1. $-y(-x)$ is also a solution.
2. $y(-x)$ is also a solution.
3. $-y(x)$ is also a solution.
4. $y(x)\,y(-x)$ is also a solution.

3.

Consider the Lagrange equation $x^2\frac{\partial z}{\partial x} + y^2\frac{\partial z}{\partial y} = (x + y)z$. Then the general solution of the given equation is

1. $F\left(\frac{xy}{z}, \frac{x-y}{z}\right) = 0$ for an arbitrary differentiable function F
2. $F\left(\frac{x-y}{z}, \frac{1}{x} - \frac{1}{y}\right) = 0$ for an arbitrary differentiable function F
3. $z = f\left(\frac{1}{x} - \frac{1}{y}\right)$ for an arbitrary differentiable function f
4. $z = xy\,f\left(\frac{1}{x} - \frac{1}{y}\right)$ for an arbitrary differentiable function f

4.

. Consider the system of ODE in $\mathbb{R}^2$, $\frac{dY}{dt} = AY, Y(0) = \begin{pmatrix}0\\1\end{pmatrix}, t > 0$ where $A = \begin{bmatrix}-1 & 1\\ 0 & -1\end{bmatrix}$ and $Y(t) = \begin{pmatrix}y_1(t)\\y_2(t)\end{pmatrix}$. Then

1. $y_1(t)$ and $y_2(t)$ are monotonically increasing for $t > 0$.
2. $y_1(t)$ and $y_2(t)$ are monotonically increasing for $t > 1$.
3. $y_1(t)$ and $y_2(t)$ are monotonically decreasing for $t > 0$.
4. $y_1(t)$ and $y_2(t)$ are monotonically decreasing for $t > 1$.

5.

. The PDE $\frac{\partial^2 u}{\partial x^2} + 2\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = x$, has

1. only one particular integral.
2. a particular integral which is linear in x and y.
3. a particular integral which is a quadratic polynomial in x and y.
4. more than one particular integral.

6.

Let $y : [0,\infty) \to [0,\infty)$ be a continuously differentiable function satisfying $y(t) = y(0) + \int_0^t y(s)ds$ for $t \geq 0$. Then

1. $y^2(t) = y^2(0) + \int_0^t y^2(s)ds$.
2. $y^2(t) = y^2(0) + 2\int_0^t y^2(s)ds$.
3. $y^2(t) = y^2(0) + \int_0^t y(s)ds$.
4. $y^2(t) = y^2(0) + \left(\int_0^t y(s)ds\right)^2 + 2y(0)\int_0^t y(s)ds$.

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7.

Let $a, b \in \mathbb{R}$ be such that $a^2 + b^2 \neq 0$. Then the Cauchy problem
$a\frac{\partial u}{\partial x} + b\frac{\partial u}{\partial y} = 1;\ x, y \in \mathbb{R}$
$u(x, y) = x$ on $ax + by = 1$

1. has more than one solution if either a or b is zero
2. has no solution
3. has a unique solution
4. has infinitely many solutions

8.

Let $y : [0,\infty) \to [0,\infty)$ be a continuously differentiable function satisfying
$y(t) = y(0) + \int_0^t y(s)ds$ for $t \geq 0$.
Then

1. $y^2(t) = y^2(0) + \int_0^t y^2(s)ds.$
2. $y^2(t) = y^2(0) + 2\int_0^t y^2(s)ds.$
3. $y^2(t) = y^2(0) + \int_0^t y(s)ds.$
4. $y^2(t) = y^2(0) + \left(\int_0^t y(s)ds\right)^2 + 2y(0)\int_0^t y(s)ds.$

9.

. Consider the boundary value problem
$-u''(x) = \pi^2 u(x)\ ; x \in (0, 1)$
$u(0) = u(1) = 0.$
If u and u' are continuous on $[0, 1]$, then

1. $u'^2(x) + \pi^2 u^2(x) = u'^2(0)$
2. $\int_0^1 u'^2(x)dx - \pi^2 \int_0^1 u^2(x)dx = 0$
3. $u'^2(x) + \pi^2 u^2(x) = 0$
4. $\int_0^1 u'^2(x)dx - \pi^2 \int_0^1 u^2(x)dx = u'^2(0)$

10.

. The solution of the initial value problem

$$(x - y)\frac{\partial u}{\partial x} + (y - x - u)\frac{\partial u}{\partial y} = u,$$

$u(x, 0) = 1$, satisfies

1. $u^2(x - y + u) + (y - x - u) = 0.$
2. $u^2(x + y + u) + (y - x - u) = 0.$
3. $u^2(x - y + u) - (x + y + u) = 0.$
4. $u^2(y - x + u) + (x + y - u) = 0.$

11.

Let $y(x)$ be a continuous solution of the initial value problem
$y' + 2y = f(x), \quad y(0) = 0,$
where $f(x) = \begin{cases} 1, & 0 \leq x \leq 1 \\ 0, & x > 1 \end{cases}$
Then $y\left(\frac{3}{2}\right)$ is equal to

1. $\frac{\sinh(1)}{e^3}$
2. $\frac{\cosh(1)}{e^3}$
3. $\frac{\sinh(1)}{e^2}$
4. $\frac{\cosh(1)}{e^2}$

12.

Consider a system of first order differential equations
$\frac{d}{dt}\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} x(t) + y(t) \\ -y(t) \end{bmatrix}$
The solution space is spanned by

1. $\begin{bmatrix} 0 \\ e^{-t} \end{bmatrix}$ and $\begin{bmatrix} e^t \\ 0 \end{bmatrix}$
2. $\begin{bmatrix} e^t \\ 0 \end{bmatrix}$ and $\begin{bmatrix} \cosh t \\ e^{-t} \end{bmatrix}$
3. $\begin{bmatrix} e^{-t} \\ -2e^{-t} \end{bmatrix}$ and $\begin{bmatrix} \sinh t \\ e^{-t} \end{bmatrix}$
4. $\begin{bmatrix} e^t \\ 0 \end{bmatrix}$ and $\begin{bmatrix} e^t - \frac{1}{2}e^{-t} \\ e^{-t} \end{bmatrix}$

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13.

The initial value problem $y' = 2\sqrt{y},\ y(0) = a$, has

1. a unique solution if $a < 0$
2. no solution if $a > 0$
3. infinitely many solutions if $a = 0$
4. a unique solution if $a \geq 0$

14.

Consider the initial value problem

$$\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0, \quad u(0,y) = 4e^{-2y}.$$

Then the value of $u(1,1)$ is

1. $4e^{-2}$
2. $4e^{2}$
3. $2e^{-4}$
4. $4e^{4}$

15.

Let u(x, t) satisfy the wave equation

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}; x \in (0, 2\pi), t > 0$$

$$u(x, 0) = e^{i\omega x}$$

for some $\omega \in \mathbb{R}$. Then

1. $u(x,t) = e^{i\omega x} e^{i\omega t}$.
2. $u(x,t) = e^{i\omega x} e^{-i\omega t}$.
3. $u(x,t) = e^{i\omega x} \left(\frac{e^{i\omega t} + e^{-i\omega t}}{2}\right)$.
4. $u(x,t) = t + \frac{x^2}{2}$.

16.

Let u(x, y) be the solution of the equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, which tends to zero as $y \to \infty$ and has the value $\sin x$ when $y = 0$. Then

1. $u = \sum_{n=1}^{\infty} a_n \sin(nx + b_n)e^{-ny}$, where a_n are arbitrary and b_n are non-zero constants.
2. $u = \sum_{n=1}^{\infty} a_n \sin(nx + b_n)e^{-n^2 y}$, where $a_1 = 1$ and a_n $(n > 1)$, b_n are non-zero constants.
3. $u = \sum_{n=1}^{\infty} a_n \sin(nx + b_n)e^{-ny}$, where $a_1 = 1$, $a_n = 0$ for $n > 1$ and $b_n = 0$ for $n \geq 1$.
4. $u = \sum_{n=1}^{\infty} a_n \sin(nx + b_n)e^{-n^2 y}$, where $b_n = 0$ for $n \geq 0$ and a_n are all nonzero.

17.

Consider the differential equation

$$\frac{d^2 y}{dx^2} - 2\tan x \frac{dy}{dx} - y = 0$$

defined on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Which among the following are true?

1. there is exactly one solution $y = y(x)$ with $y(0) = y'(0) = 1$ and $y\left(\frac{\pi}{3}\right) = 2\left(1 + \frac{\pi}{3}\right)$
2. there is exactly one solution $y = y(x)$ with $y(0) = 1, y'(0) = -1$ and $y\left(-\frac{\pi}{3}\right) = 2\left(1 + \frac{\pi}{3}\right)$
3. any solution $y = y(x)$ satisfies $y''(0) = y(0)$
4. if y_1 and y_2 are any two solutions then $(ax + b)y_1 = (cx + d)y_2$ for some $a, b, c, d \in \mathbb{R}$

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18.

For $f \in C[0,1]$ and $n > 1$,
let $T(f) = \frac{1}{n}\left[\frac{1}{2}f(0) + \frac{1}{2}f(1) + \sum_{j=1}^{n-1} f\left(\frac{j}{n}\right)\right]$
be an approximation of the integral
$I(f) = \int_0^1 f(x)dx$. For which of the following functions f is $T(f) = I(f)$?

1. $1 + \sin 2\pi nx$
2. $1 + \cos 2\pi nx$
3. $\sin^2 2\pi nx$
4. $\cos^2 2\pi(n+1)x$

19.

Let $B = \{(x_1, x_2) \in \mathbb{R}^2 \mid x_1^2 + x_2^2 < 1\}$, and let
$C_{Id}^2(\bar{B}; \mathbb{R}^2) = \{u \in C^2(\bar{B}; \mathbb{R}^2) \mid u(x_1, x_2) = (x_1, x_2),\ \text{for } (x_1, x_2) \in \partial B\}$.
Let $u = (u_1, u_2)$ and define $J : C_{Id}^2(\bar{B}; \mathbb{R}^2) \to \mathbb{R}$ by

$$J(u) = \int_B \left(\frac{\partial u_1}{\partial x_1}\frac{\partial u_2}{\partial x_2} - \frac{\partial u_1}{\partial x_2}\frac{\partial u_2}{\partial x_1}\right) dx_1 dx_2$$

Then,

1. $\inf\{J(u) : u \in C_{Id}^2(\bar{B}; \mathbb{R}^2)\} = 0$
2. $J(u) > 0$, for all $u \in C_{Id}^2(\bar{B} : \mathbb{R}^2)$
3. $J(u) = 1$, for infinitely many $u \in C_{Id}^2(\bar{B}; \mathbb{R}^2)$
4. $J(u) = \pi$, for all $u \in C_{Id}^2(\bar{B}; \mathbb{R}^2)$

20.

A solution of the PDE

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 - u = 0$$

represents

1. an ellipse in the *x*-*y* plane.
2. an ellipsoid in the *xyu* space.
3. a parabola in the *u*-*x* plane.
4. a hyperbola in the *u*-*y* plane.

21.

Let $X = \{u \in C^1[0,1] \mid u(0) = 0\}$ and let $I : X \to \mathbb{R}$ be defined as

$$I(u) = \int_0^1 (u'(t)^2 - u(t)^2)dt$$

Which of the following are correct?

1. I is bounded below
2. I is not bounded below
3. I attains its infimum
4. I does not attain its infimum

Practice Questions of ODE & PDE for CSIR-NET Mathematics

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Practice Problems

1. The critical point of the system

$$x'(t) = -4x - y$$
$$y'(t) = x - 2y \qquad \textbf{(NET-June 2015)}$$

   (a) Asymptotically stable Node
   (b) Unstable node
   (c) Asymptotically stable spiral
   (d) Unstable spiral **Answer: (a)**

2. Consider the system of differential equations

$$x'(t) = 2x - 7y$$
$$y'(t) = 3x - 8y \qquad \textbf{(NET-June 2018)}$$

   Then critical point $(0, 0)$ of the system is an

   (a) Asymptotically stable Node
   (b) Unstable node
   (c) Asymptotically stable spiral
   (d) Unstable spiral **Answer: (a)**

3. Then critical point $(0, 0)$ for the system

$$x'(t) = x - 2y + y^2 Sin(x)$$
$$y'(t) = 2x - 2y - 3yCos(y^2) \qquad \textbf{(NET-Dec 2018)}$$

   (a) is a Stable spiral point
   (b) is a Unstable spiral point
   (c) is a Saddle point
   (d) is a Stable node **Answer: (c)**

4. Consider the system of differential equations [1]

$$x'(t) = x + 4y - x^2$$
$$y'(t) = 6x - y + 2xy \qquad \textbf{(Practice Que.)}$$

   Then critical point $(0, 0)$ of the system is an

(a) Asymptotically stable Node

(b) Unstable saddle point

(c) Asymptotically stable spiral

(d) Unstable spiral **Answer: (b)**

5. Then critical point $(0, 0)$ for the system

$$x'(t) = Sin(x) - 4y$$
$$y'(t) = Sin(2x) - 5y$$

(**Practice Que.**)

(a) is a Stable spiral point

(b) is a Asymptotically stable Node

(c) is a Saddle point

(d) is a Stable node **Answer: (b)**

6. Consider the system of differential equations [1]

$$x'(t) = 8x - y^2$$
$$y'(t) = -6y + 6x^2$$

(**Practice Que.**)

Then critical point $(0, 0)$ of the system is an

(a) Asymptotically stable Node

(b) Asymptotically stable spiral

(c) Unstable saddle point

(d) Unstable spiral **Answer: (c)**

Hint: There are critical points (0,0) & (2,4)
At (0,0): Unstable saddle point
At (2,4): Unstable spiral point

7. Consider the systems of differential equations [1]

$$x'(t) = -y - x^2$$
$$y'(t) = x$$

and

$$x'(t) = -y - x^3$$
$$y'(t) = x$$

(**Practice Que.**)

Find all the critical point and nature of system on the each critical points.
Hint: Nature- Centre or Spiral Point

References

[1] S. Ross, *DIFFERENTIAL EQUATIONS, 3RD ED.* Wiley India Pvt. Limited, 2007.

[2] A. Jeffrey, *Advanced engineering mathematics.* Elsevier, 2001.

[3] E. Kreyszig, "Advanced engineering mathematics, 8-th edition," 1999.

Note: *Full Notes of dynamical system with non-linear will we available soon.*

Exercise (Exact DE)

#. Find the general sol^n of followings: —

1. $x\,dx + y\,dy + \dfrac{x\,dy - y\,dx}{(x^2+y^2)} = 0$

2. $e^{x/y}\left(1 + e^{-x/y}\right)dx + \left(e^{x/y} - \frac{x}{y}e^{x/y}\right)dy = 0$

3. Value of a & b, for which diff^n eq^n is exact

$(3a^2x^2 + by\cos x)\,dx + (2\sin x - 4ay^3)\,dy = 0$

(a). $a=3, b=2$ (b). $a=2, b=3$

(c). $a=3, b=4$ (d). $a=2, b=5$

4. Consider the D.E —

$2\cos(y^2)dx - xy\sin(y^2)dy = 0$, then I.F. can be —

(a) e^x (b). e^{-x} (c). $3x$ (d). x^3

5. The eq^n $(\alpha x y^3 + y\cos x)dx + (x^2y^2 + \beta\sin x)dy = 0$ is exact if —

(a). $\alpha = 3/2,\ \beta = 1$ (b). $\alpha = 1,\ \beta = 3/2$

(c). $\alpha = 2/3,\ \beta = 1$ (d). $\alpha = 1,\ \beta = 2/3$.

---- Answer ----------------

(1). $x^2+y^2 - 2\tan^{-1}(x/y) = C_1$

(2). $x + y.e^{x/y} = C$

(3). (a)

(4). (d)

(5).

[Integrating Factor] Assignment (Exercise) P. Kalika

Solve the following differential Equations.

(1). $x\,dx + y\,dy + \dfrac{x\,dy - y\,dx}{x^2+y^2} = 0$

(2). $(3x^2y^4 + 2xy)\,dx + x^2(2xy^3 - 1)\,dy = 0$

(3). $(2xy^4e^y + 2xy^3 + y)\,dx + (x^2y^4e^y - x^2y^2 - 3x)\,dy = 0$

(Hint: type-IV)

(4). $(x^2+y^2+2x)\,dx + 2y\,dy = 0$

(5). $(6y + 2y^3 + 3x^2)\,dx + \frac{3}{2}(1+y^2)x\,dy = 0$

(6). $(xy^2 + 2x^2y^3)\,dx + (x^2y - x^3y^2)\,dy = 0$

(7). $(2x^2y - 3y^4)\,dx + (3x^3 + 2xy^3)\,dy = 0$

(8). Given that I.F of $(x^7y^2 + 3y)\,dx + (3x^8y - x)\,dy = 0$ is $x^h y^k$ then —

(a) $h = -7$, $k = 1$. (c). $h = k = 0$

(b). $h = 1$, $k = -7$ (d). $h = k = 1$ (e). NONE

(9) Given that $\dfrac{dy}{dx} - y/x = 0$. Then w.o.t.f. is/are true for I.F —

(a). $1/x^2$ (c). $1/xy$

(b). $1/y^2$ (d). $\dfrac{1}{x+y}$ (e). None

(10). $(x^2y^3 + xy^2 + y)\,dx + (x^3y^2 - x^2y + x)\,dy = 0$

(11) If x^3y^2 is an integrating factor of

$$(6y^2+axy)\,dx + (6xy+bx^2)\,dy = 0 \quad \text{where } a,b \in \mathbb{R},$$

Then —

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(A). $3a-5b=0$ (B). $2a-b=0$

(C). $3a+5b=0$ (D). $2a+b=0$

(12). An Integrating factor of eq^n.

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$$\left(y+\tfrac{1}{3}y^3+\tfrac{1}{2}x^2\right)dx + \tfrac{1}{4}(x+xy^2)\,dy = 0 \quad \text{is —}$$

(a). x^2 (b). $3\log_e x$ (c). x^3 (d). $2\log_e x$ (e) None

(13). An integrating factor of

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$$x\frac{dy}{dx} + (3x+1)y = xe^{-2x} \quad \text{is —}$$

(a) xe^{3x} (b). $3xe^{x}$ (c). xe^{x} (d). x^3e^{x}

(14). The soln. of the DE $x\frac{dy}{dx} + (1+x)y = e^{-x}$ with the B.C $y(x=1)=0$ is —

(a). $\frac{x-1}{x}e^{-x}$ (b). $\frac{x-1}{x^2}e^{-x}$ (c). $\frac{1-x}{x^2}e^{-x}$ (d). $(x-1)^2e^{-x}$

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Answer

(1) $x^2+y^2-2\tan^{-1}(x/y) = C_1$

(2). $(xy)^3 + x^2 = cy$

(3). $x^2e^y + x^2/y + x/y^3 = C$

(4). $x^2e^x + y^2e^x = C$

(5). $3x^4y + x^4y^3 + x^6 = C$

(6). $-\frac{1}{3xy} + \frac{2}{3}\log x - \frac{1}{3}\log y = C$

(7). I.F $= x^{-49/3}\cdot y^{-20/13}$

(8). (a).

(9). (a, b, c)

(10). $xy - \frac{1}{xy} + \log\frac{x}{y} = C$

(11). (A).

(12). (C).

(13). (a).

(14). (a)

P. Kalika Notes

 P. Kalika

Find the solutions of followings:

(1). $(x^2+y^2+2x)\,dx + 2y\,dy = 0$

(2). $\sec^2 y \dfrac{dy}{dx} + x\tan y = x^3$

(3). $\dfrac{\sin x}{y^2}\dfrac{dy}{dx} + \dfrac{3}{y} = \cos x$

W.o.t.f statements is/are true ?

(1). Solution of the problem $y'\cos x + y\sin x = 1,\ y(0)=0$ is –

(a). Periodic f^n (b). positive in $(0,\pi)$

(c). Monotonic ↑ in $(0,\pi/2)$ (d). Invertible in $(0,\pi)$

(2). The sol^n $\phi(x)$ of $x^2y' + 2xy = 1$ satisfies $\phi(2) = 2\,\phi(1)$ then —

(a). $\phi(x) \to \infty$ as $x \to \infty$ (b). $\phi(x) \to -2$ as $x \to +\infty$

(c). $\phi(x) \to 0$ as $x \to \infty$ (d). None.

(3). D.E $y' + P(x)y = Q(x)y^n$ is a —

(a). linear DE for $n=0$ (c). Bernoulli eq^n for $n \neq 0, 1$

(b). linear DE for $n=1$ (d). Bernoulli eq^n for $n = 10$

(4). The value of 'a', for which y^a is an IF of DE

$2xy\,dx - (2x^2 - y^2)\,dy = 0$ is —

(a). -4 (b) 4 (c) -3 (d) 3

Answer

(1). $y^2 = -x^2 + ce^{-x}$

(2). $y = \tan^{-1}(x^2 - 2 + c\,e^{-\frac{1}{2}x^2})$

(3). $IF = (\tan x/2)^{-3}$

Objective (Answer)

(1). a, b, c

(2). (c).

(3). All

(4). (c)

Assignment (Particular Integral)

Solve the following differential equations

1. $(D^2-1)y = 2^x + e^x \cos x$

2. $(D^2-4D+4)y = 8x^2 e^{2x} \sin 2x$

3. $(D^3+D^2-D-1)y = \cos 2x$

4. $(D^2-4D+4)y = e^{2x} \cos^2 x$ (H

5. $(D^2+4)y = x \cos x$ (Hint: $y_p = \frac{1}{9}[3x\cos x + 2\sin x]$

6. $(D^2-2D+1)y = x e^x \sin 2x$ (Hint: $y_p = -\frac{e^x}{4}(x\sin 2x + \cos 2x)$)

7. $(D^2+1)y = \frac{1}{\sin x}$ (Hint: $y_p = \sin x \cdot \log \sin x - x\cos x$)

8. $(D^3-4D^2-3D+18)y = 0$ (Hint: $y_{cf} = c_1 e^{-2x} + (c_2+c_3x)e^{3x}$)

9.

Answer (Hint):

1. $y_p = e^x\left[\frac{-2\sin x + \cos x}{5}\right] + \frac{2^x}{(\log 2)^2 - 1}$

2. $y_p = -e^{2x}[2x^2\sin 2x + 4x\cos 2x - 3\sin 2x]$

3. $y_p = \frac{-1}{25}[2\sin 2x + \cos 2x]$

4. $y_p = \frac{1}{8}e^{2x}[2x^2 - \cos 2x]$

Practice Questions (Variation of Parameters)

①. Solve $x^2\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} - y = x^2e^x$

(Ans: $y(x) = c_1x + c_2/x + \cancel{\tfrac{1}{2}e^x} + e^x - \dfrac{e^x}{x}$)

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②. Using Method of VOP, the particular solⁿ of the diffⁿ eqⁿ $y'' + 4y = \dfrac{3}{\sin 2x}$, $0 < x < \pi/2$ is —

ⓐ. $\frac{3}{4}\sin 2x \cdot \log(\sin 2x) - \frac{3}{2}\cos 2x$.

ⓑ. $\frac{3}{2}\sin 2x \cdot \log(\cos 2x) - \frac{3}{4}\cos 2x$

ⓒ. $\frac{3}{2}\sin 2x \log(\sin 2x) - \frac{3}{2}\cos 2x$

ⓓ. $\frac{3}{4}\sin 2x \cdot \log(\sin 2x) - \frac{3}{2}x\cos 2x$

(Ans: d)

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③. The general solⁿ of the diffⁿ eqⁿ $y'' + y = f(x)$, $x \in (-\infty, \infty)$, where $f(x)$ is cts. real valued fⁿ on $(-\infty, \infty)$ is —

ⓐ. $y = A\cos x + B\sin x + \int_0^x f(x-t)\sin t\,dt$.

ⓑ. $y = A\cos x + B\sin x + \int_0^x f(x+t)\cos t\,dt$

ⓒ. $y = A\cos x + B\sin x + \int_0^x f(t)\sin(x-t)\,dt$

ⓓ. $y = \cos(x+k) + c\int_0^x f(t)\sin(x-t)\,dt$

(Ans: c).

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Que(4). The gen. solⁿ of DE $y'' = -f(x)$, $x \in \mathbb{R}$, $f \in C^1[\mathbb{R}]$ then —

ⓐ. $y(x) = A + Bx + \int_0^x (x-t)f(t)\,dt$.

ⓑ $y(x) = Ax - \int_0^x x f(x-t)\,dt$

ⓒ. $y(x) = Ax + B + \int_0^x (x-t)f(x-t)\,dt$

ⓓ. $y(x) = A + Bx - \int_0^x (x-t)f(t)\,dt$.

(Ans: d).

[P. Kalika Notes] Practice Set (Existence, Uniqueness & Lipschitz) [ODE]

(1) Consider the IVP $y' = 2\sqrt{y}$, $y(0) = k$, then given problem has —

(a) Unique solⁿ if $a > 0$ (b) unique solⁿ if $a < 0$
(c) unique solⁿ if $a \geq 0$ (d) Unique solⁿ if $a = 0$

(2). Find the largest interval predicted by Picard theorem for $\frac{dy}{dx} = 1 + y + y^2 \cos x$, $y(0) = 0$ —

(a). $[-\frac{1}{3}, \frac{1}{3}]$ (b). $[0, \frac{1}{3}]$
(c). $[-1, 1]$ (d). None

(3). For $\lambda \in \mathbb{R}$, Consider DE $y'(x) = \lambda \sin(x + y(x))$, $y(0) = 1$. Then this IVP has —

(a) no solⁿ in any nb.d of 0.
(b). a solⁿ in $\mathbb{R}$ if $|\lambda| < 1$ (C NET-2016)
(c). a solⁿ in a nb.d of 0.
(d). a solⁿ in $\mathbb{R}$ only if $|\lambda| > 1$

(4). Consider the IVP $y' = x y^{1/3}$, $y(0) = 0$, $(x, y) \in \mathbb{R} \times \mathbb{R}$
Then w.o.t.f are correct.?

(a). The fⁿ $f(x,y) = xy^{1/3}$ doesn't satisfy Lipschitz Condⁿ w.r.t. y in any nb.d of $y = 0$.
(b). ∃ a unique solⁿ for the IVP.
(c). ∃ no solution for the IVP
(d). ∃ more than one solⁿ for the IVP

(NET- 2013)

(5). The diff^n eq^n. $y' = 60\,(y^2)^{1/5}$, $x > 0$.
$y(0) = 0$ has —

(a). a unique sol^n. (b). two sol^n.
(c). No solution (d). infinitely many sol^n.

(NET-2012 Dec)

(6). Consider the IVP $y' = y^2$, $y(0) = 1$, $(x,y) \in \mathbb{R}\times\mathbb{R}$.
Then $\exists$s a unique sol^n of the IVP on

(a). $\mathbb{R} = (-\infty, \infty)$ (b). $(-\infty, 1)$
(c). $(-2, 2)$ (d). $(-1, \infty)$

(NET-2013 June)

(7). Consider the IVP $y'(t) = f(t).y(t)$, $y(0) = 1$,
where $f : \mathbb{R} \to \mathbb{R}$ is cts. Then the IVP has —

(a). infinitely many sol^n for some f.
(b). a unique sol^n in $\mathbb{R}$.
(c). no sol^n in $\mathbb{R}$ for some f.
(d). a sol^n in an interval containg 0 but not on $\mathbb{R}$
for some f.

(NET 2012 June)

(8). Find h for — $\left(h = \min\{a, \frac{b}{M}\}\right)$

(a). $y' = e^{-x^2} + y^2$, $y(0) = 1$
(b). $y' = (4y + e^{-x^2})e^y$, $y(0) = 0$

Ans:

(1). a
(2). a
(3). b, c
(4). a, d
(5). d
(6). b
(7). b
(8) (a) $h = \sqrt{2}/1 + (\sqrt{2}+1)^2$
(b) $h = 1/8\sqrt{e}$

Exercise (EUT)

Find Solution

① $\frac{dy}{dx} = 1 + y^2$, $y(1) = 0$

(Hint: Unique sol^n)

② $\frac{dy}{dx} = 2y/x$, $y(0) = 0$

(Hint: IVP has infinite no. of sol^n.)

③ $\frac{dy}{dx} = \frac{2y}{x}$, $y(0) = 1$

(Hint: No sol^n.)

$\frac{dy}{dx} = \frac{2y}{x}$, $y(1) = 0$

(Hint: Unique sol^n.)

④ The IVP $\frac{dy}{dx} = y^2$, $y(0) = 1$ has unique sol^n. in —

ⓐ $(-2, 2)$ ⓑ $(-3, 3)$

ⓒ $\mathbb{R}$ ⓓ $(-1/2, 1/2)$

(Hint: d)

⑤ The IVP $2x\frac{dy}{dx} = 3(2y-1)$, $y(0) = 1/2$ has —

ⓐ unique sol^n ⓑ finitely many sol^n.

ⓒ Infinitely many sol^n. ⓓ No. sol^n.

(Ans: c)

⑥ If $y'(t) = f(t, y(t))$, $y(0) = 1$, $f: \mathbb{R} \to \mathbb{R}$ is cts. Then this IVP has —

ⓐ infinitely many sol^n for some f.

ⓑ Unique sol^n in $\mathbb{R}$.

ⓒ No sol^n in $\mathbb{R}$ for some f.

ⓓ a sol^n. in an interval containing 0 but not in $\mathbb{R}$ for some f.

(Ans: d).

7. The IVP $\frac{dy}{dx} = xy^{1/3}$, $y(0)=1$ has —

(Hint: Unique sol^n by EUT).

8. If $y'(t) = f(t)$ s.t $y(0)=0$ where $f : \mathbb{R} \to \mathbb{R}$ is cts., then —

(Ans: IVP has unique sol^n in an open interval containing 0 but not on $\mathbb{R}$)

9. If $\frac{dy}{dx} = y^{2/3}$ s.t $y(0)=0$, the IVP has —

(a) Unique sol^n (b). No sol^n.

(c). More than one sol^n (d). $y = \frac{1}{27}x^3$ is a sol^n.

(Ans: c, d)

Exercise (Self-Adjoint)

Que (1): Transform the following eqn into an equivalent self-Adjoint equation.

(1). $x^2 \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} + 5y = 0$

(Hint: factor $= 1/x^4$)

(2). $(x^4 + x^2) \frac{d^2y}{dx^2} + 2x^3 \frac{dy}{dx} + 3x = 0$

[Ans: $\frac{d}{dx}\left[(x^2+1) \frac{dy}{dx} \right] + \frac{3}{x^2}y = 0$]

(3). $a(x) \frac{d^2y}{dx^2} + b(x)y = 0$ [Ans: $\frac{d}{dx}\left[e^{\int \frac{b(x)}{a(x)}} \frac{dy}{dx} \right] = 0$]

Que (2). Consider the ODE $x''(t) + e^{t^2} x(t) = 0$ for $t \in [0, 3\pi]$ where $x : [0, 3\pi] \to \mathbb{R}$. Then what is Cardinality of the set $\{ t \in [0, 3\pi] : x(t) = 0 \}$. (Ans: ≥ 3)

(3). Check (Prove) that the following Boundary Value Problem (BVP) is Strum- Liouville Problem.

(a). $\frac{d}{dt}\left[x(t) \frac{dy}{dt} \right] + [2t^2 + \lambda t^3] y = 0$

with conditions, $3y(1) + 4y'(1) = 0$

& $5y(2) - 3y'(2) = 0$

(b). $\frac{d^2y}{dx^2} + \lambda y = 0$ with conditions

$y(0) = 0$ & $y(\pi) = 0$

(4). Find the non-trivial soln of Strum-Liouville BVP $y''(x) + \lambda y = 0$ with $y(0) = 0 = y(\pi)$.

(4.75M)
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Practice set (Green f^n)

1. Consider the BVP $u'' = -f$, $u(0) = u''(1) = 0$ on $[0,1]$ where $u' \equiv \frac{du}{dx}$ & $u'' = \frac{d^2u}{dx^2}$. Assume $f(x)$ is real-valued cts. f^n. on $[0,1]$. Then w.o.t.f is/are correct?

(a). The Green's f^n (for the above BVP) is $G(x,t) = \begin{cases} x & \text{for } 0 \le x \le t \\ t & \text{for } t \le x \le 1 \end{cases}$

(b). Both G & $\frac{\partial G}{\partial x}$ are cts. on $[0,1] \times [0,1]$ with $\frac{\partial^2 G}{\partial x^2}$ having a discontinuity along $x = t$

(c) $G(x,t)$ satisfies the homo. eq^n. $u'' = 0$ and $0 \le x < t$ & $t < x \le 1$

(d). The sol^n of the given BVP is

$$u(x) = \int_0^x t\, f(t)\, dt + \int_x^1 x\, f(t)\, dt$$

(NET-Dec-2012)

2. The sol^n. of the diff^n eq^n $y'' = f(x)$, $x \in [0,1)$ $Y(0) = Y(1) = 0$ is given by $y(x) = \int_0^1 G(x,t)\, f(t)\, dt$

where —

(a). $G(x,t) = \begin{cases} x(t-1) & : x \le t \\ t(x-1) & : x \ge t \end{cases}$

(b). $G(x,t) = \begin{cases} x^2(t-1) & : x \le t \\ t^2(x-1) & : x > t \end{cases}$

(c). $G(x,t) = \begin{cases} x(t^2-1) & : x \le t \\ t(x^2-1) & : x > t \end{cases}$

(d). $G(x,t) = \begin{cases} \sin x(t-1) & : x \le t \\ \sin t(x-1) & : \cancel{x \ge t} \end{cases}$

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(3). The green f^n $G(x,t)$ of the BVP $y'' - \frac{1}{x} y' = 1$

$y(0) = y(1) = 0$ is $G(x,t) = \begin{cases} f_1(x,t) & : x \le t \\ f_2(x,t) & : t \ge x \end{cases}$ where

(a). $f_1(x,t) = -\frac{1}{2} t (1-x^2)$, $f_2(x,t) = -\frac{1}{2t} x^2 (1-t^2)$

(b). $f_1(x,t) = \frac{-1}{2x} t^2 (1-x^2)$, $f_2(x,t) = -\frac{1}{2t} x^2 (1-t^2)$

(c). $f_1(x,t) = -\frac{1}{2t} x^2 (1-t^2)$, $f_2(x,t) = -\frac{1}{2} t (1-x^2)$

(d). $f_1(x,t) = -\frac{1}{2t} x^2 (1-t^2)$, $f_2(x,t) = -\frac{1}{2x} t^2 (1-x^2)$

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(4). The Green's function $G(x,t)$, $0 \le x, t \le 1$ of the BVP $y'' + \lambda y = 0$, $y(0) = 0 = y(1)$ is ____

(a). Symmetric in x & t

(b). Continuous at $x = t$

(c). $\left.\frac{\partial G(x,t)}{\partial x}\right|_{x=t^-} - \left.\frac{\partial G(x,t)}{\partial x}\right|_{x=t^+} = -1$

(d). $\left.\frac{\partial G(x,t)}{\partial x}\right|_{x=t^-} - \left.\frac{\partial G(x,t)}{\partial x}\right|_{x=t^+} = 1$

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(5). The difference b/w the least two eigen-values of the BVP

$$y'' + \lambda y = 0, \quad 0 < x < \pi, \quad y(0) = y'(\pi) = 0$$

is equal to ________