

Practice Questions

[Practice Questions for CSIR-NET, GATE, SET, PSC, ...etc]



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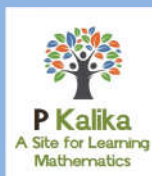
Practice Questions for:

- Calculus
- Number Theory
- ODE & PDE
- Integral Equation
- Calculus of Variation
- Runge-Kutta Method
- Linear Programming
- Probability & Markov Chain

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Que ① Find all the critical points following $f(x, y)$ and classify them as maxima, minima or saddle point.

(1). $f(x, y) = x^4 + y^4 - 2(x^2 + y^2) - 5$

(2). $f(x, y) = x^{36} + y^5 + 5$

(3). $f(x, y) = -x^{30} - y^4$

(4). $f(x, y) = x^3 + y^3$

②. Suppose that $z_1, z_2, \dots, z_n$ are vertices of a regular n -gon in order. Then the minimum value of $|z - z_1|^2 + |z - z_2|^2 + \dots + |z - z_n|^2$, $z \in \mathbb{C}$ is —

Ⓐ. ~~Ⓐ~~ obtained at —

(Ans: a)

Ⓐ. Centroid of n -gon

Ⓑ. $z = z_i$ for some $i \leq i \leq n$

Ⓒ. Outside n -gon

Ⓓ. On the boundary of n -gon.

(Hint: $z = x + iy$ & $z_k = x_k + iy_k$)

$\therefore f(x, y) = |z - z_1|^2 + |z - z_2|^2 + \dots + |z - z_n|^2$

$= (x - x_1)^2 + (y - y_1)^2 + (x - x_2)^2 + (y - y_2)^2 + \dots + (x - x_n)^2 + (y - y_n)^2$

then find (x, y) by $f_x = 0$ & $f_y = 0$

further, $f_{xx} = 2n$, $f_{xy} = 0$, $f_{yy} = 2n$

then

$D = f_{xx} \cdot f_{yy} - (f_{xy})^2 = 4n^2 > 0$ & $f_{xx} = 2n > 0$

Que (3) Find the min. value of $x^2 + 2y^2 + 3z^2$, subject to the constraint $x + 2y + 3z = 6$.

Hint $x = 6 - 2y - 3z$, put in $f(x, y, z) \rightarrow f(y, z)$

Que(4)

Find the point of the plane $x+2y+3z=12$, which is nearest to the point $(1,2,3)$.

Hint:

Minimize $(x-1)^2 + (y-2)^2 + (z-3)^2 = f(x,y,z)$

Subject to Constraint $x+2y+3z=12$

$$\Downarrow$$

$$x = 12 - 2y - 3z$$

thus put x in $f(x,y,z) \rightarrow f(y,z)$, further solve as usual.

Que.

(5)

Find the point on the plane $x+2y+3z=12$, which is nearest to the origin $(0,0,0)$

Hint:

Minimize $x^2 + y^2 + z^2 = f(x,y,z)$

Subject to $x+2y+3z=12 \Rightarrow x = 12 - 2y - 3z$

Putting x , $f(x,y,z) \rightarrow f(y,z) = (12 - 2y - 3z)^2 + y^2 + z^2$

Next $f_x = 0$, $f_y = 0$ gives $x = 12/7$, $z = 18/7$

thru

(P. Kalika Notes)

Ans

(3). $(1,1)$, min. value 6 (4). $(6/7)$ (5). $(6/7, 12/7, 18/7)$

(1) (a). $(0,0)$, $(1,0)$, $(-1,0)$ (b). $(0,0)$ (c). $(0,0)$ (d). $(0,0)$.

(2). $x = \frac{x_1 + x_2 + \dots + x_n}{n}$, $y = \frac{y_1 + y_2 + \dots + y_n}{n}$, $z = \frac{z_1 + z_2 + \dots + z_n}{n}$

Exercise (max. & min.)⁽⁴⁾

- Que (1). Find the minimum value of $2x^2 + 3y^2 + 5z^2$ subject to the constraint $2x + 3y + 5z = 10$.
- (2). Find the point on $4x^2 + 3y^2 + 4y - 6x + 11$, nearest to the origin.
- (3). Find the min. & max. value of $(x-11)^2 + (y-12)^2 + (z-13)^2$ subject to constraint $(x-6)^2 + (y-7)^2 + (z-8)^2 = 1$.
- (4). Find the min. value of $x^2 + y^2 + z^2$ subject to constraint $x + y + z = 1$.
- (5). Find the min. & max. value of $x + y + z$ subject to constraint $x^2 + y^2 + z^2 = 1$.
- (6). Find the maximum value of $2x^2 + 3y^2 + 5z^2$, when given that $3x + 7y + 11z = 1$.

(P. Kalika Notes)

(6). pt. $\left(\frac{45}{1351}, \frac{70}{1351}, \frac{66}{1351}\right)$, Value = $\frac{2(45)^2 + 3(70)^2 + 5(66)^2}{(1351)^2}$

Ans. (1). pt. $(1, 1, 1)$, value = 10 (2). pt. $\left(\frac{6d}{2+8d}, \frac{-4d}{2+6d}\right)$

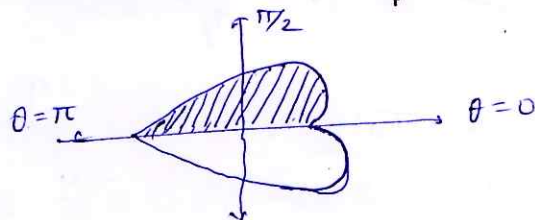
(3). min. $(5\sqrt{3}-1)^2$, max. $(5\sqrt{3}+1)^2$ (4). $\frac{1}{3}$ (5). min. $-\sqrt{3}$, max. $\sqrt{3}$

Que(1). Find the area b/w the curve $y^2 = 4x$ & $x^2 = 4y$.

Ans: $A = \int_0^4 \left(2\sqrt{x} - \frac{x^2}{4} \right) dx = \frac{16}{3}$

Que(2). Find the volume of solid generated by revolving the cardioid $r = a(1 - \cos \theta)$ about the line $\theta = 0$.

Hint: $V = \int_{\theta_1}^{\theta_2} \frac{2\pi}{3} r^3 \sin \theta d\theta$
 $= \frac{8\pi a^3}{3}$



Que(3). Find the circumference of a triangle whose vertices are $(0,0)$, $(1,0)$ and $(1,1)$.

Ans: $2 + \sqrt{2}$

Que(4). Find arc length of the curve which encloses region b/w parabola $y^2 = x$ & $x^2 = y$

Ans: $l = \int_0^1 \sqrt{1 + 4x^2} dx =$
 $= 2 \left[\frac{\sqrt{5}}{4} + \frac{1}{8} \log \left(1 + \frac{\sqrt{5}}{2} \right) - \frac{1}{8} \log 2 \right]$

Que(5). Find area enclosed b/w curve $y = \sin x$ and x -axis, when x -varies 0 to 2π .

Ans: , $A = 4$

Que(6). Find area of triangle whose sides are of length a , b and c

Ans: , Area = $\frac{\sqrt{((a+b)^2 - c^2)(c^2 - (a-b)^2)}}{4}$

Que(7). Find the area of the circle with $r^2 = a^2$

Ans: πa^2

Que(8). Find the sectoral Area of the curve $r = 7e^{3\theta}$ b/w $\theta = 0$ to π .

Ans: Area = $\int_0^\pi \frac{1}{2} \cdot 49 e^{6\theta} d\theta = \frac{49}{12} [e^{6\pi} - 1]$

Assignment (APPⁿ of double Integral)

- (1). Using application of double integral, Find the surface area of sphere of radius r .

①

(2)

Hint: $x^2 + y^2 + z^2 = r^2$, find z_x, z_y , then

$$S = 2 \iint_R \sqrt{1 + z_x^2 + z_y^2} \, dx \, dy$$

$$\text{Surface Area} = 4\pi r^2$$

②

- (2). Find the surface area of sphere $x^2 + y^2 + z^2 = 4$ lying inside cylinder $x^2 + y^2 = 1$.

Hint: $x^2 + y^2 + z^2 = 16$, $z_x = -x/2$, $z_y = -y/2$

$$\text{Surface area } S = 2 \iint_R \frac{4}{2} \, dx \, dy = 16(4 - \sqrt{15}) \cdot 2\pi$$

③

- (3) Find the volume of cone, whose radius is r and height is h .

Hint: $\frac{r}{\sqrt{x^2 + y^2}} = \frac{h - z}{z} = \frac{h}{z} - 1 \Rightarrow z = \frac{h\sqrt{x^2 + y^2}}{r + \sqrt{x^2 + y^2}}$

$$\begin{aligned} \text{Vol.} &= \iiint_R z \, dx \, dy = \iint_R f(x, y) \, dy \, dx \\ &= 2 \end{aligned}$$

- (4). Find the volume enclosed by plane $z = 0$ & $x^2 + y^2 = 4 - z$. Also find ^{curved} surface area

Hint: Vol. = 8π ($z = 4 - x^2 - y^2$)

Curved surface area = 1.

(7)
Exercise (Stokes & Gauss Theorem)

Q. (1) Evaluate $\oint_C (y dx + z dy + x dz)$ where C is the curve of intersection of $x^2 + y^2 + z^2 = a^2$ & $x + z = a$

Ans:
$$= \int_S (\vec{\nabla} \times \vec{F}) \cdot \vec{n} ds = -\frac{\pi a^2}{\sqrt{2}}$$

Q. (2) The value of $\int_C \vec{F} \cdot d\vec{r}$, where $\vec{F} = -y^3 \hat{i} + x^3 \hat{j}$ & C is the curve (Circular disc $x^2 + y^2 \leq 1$, $z = 0$) is _____

Ans:
$$\int_C \vec{F} \cdot d\vec{r} = \int_S (\vec{\nabla} \times \vec{F}) \cdot \vec{n} ds = \frac{3\pi}{2}$$

Q. (3) Evaluate $\int_S \vec{\nabla} \times \vec{F} \cdot \vec{n} ds$, where $\vec{F} = 4x^2z \hat{i} - (yz - 7) \hat{j} + xy^2z \hat{k}$ and S is the surface bdd by $y^2 + z^2 = 25$ & $x = 0$ to $x = 2$.

Ans:
$$\int_S (\vec{\nabla} \times \vec{F}) \cdot \vec{n} ds = \int_V \text{div}(\vec{\nabla} \times \vec{F}) dV = 0$$

Q. (4) Evaluate $\int_S \vec{F} \cdot \vec{n} ds$ where $\vec{F} = 4x \hat{i} + 2y^2 \hat{j} + z^2 \hat{k}$ taken over the region bdd by $x^2 + y^2 = 4$ & $z = 0$ to $z = 3$.

Ans:
$$\int_S \vec{F} \cdot \vec{n} ds = \int_V \vec{\nabla} \cdot \vec{F} dV = 84\pi$$

Q. (5) The value of $\int_S \vec{F} \cdot \vec{n} ds$, where $\vec{F} = 4x^2 \hat{i} - 3y \hat{j} + 8xz \hat{k}$ & S is a surface $0 \leq x \leq 1$, $0 \leq y \leq 2$ & $0 \leq z \leq 3$ is _____

Ans = 30

Q.(6). Evaluate $\int_S (x^2 + 2y^2 + 3z^2) ds$, where S is the surface $x^2 + y^2 + z^2 = 1$

Ans: $\int_S \vec{F} ds = \int_V \nabla \cdot \vec{F} dV = 8\pi$

Q.(7). The value of $\int_S (x dy dz + y dz dx + z dx dy)$, where S is a surface

(i). sphere $x^2 + y^2 + z^2 = 9$

Ans: $4 \cdot 3^3 \pi$

(ii). cylinder $x^2 + z^2 = 16$, $y = 0$ to 3

Ans: $4^2 \cdot 3^2 \pi$

Q.(8). Evaluate $\int_V \nabla \cdot \vec{F} dV$, where $\vec{F} = (2x^2 - 3z)\hat{i} - 2xy\hat{j} - 4xz\hat{k}$ and V is the region bdd by the planes $x=0, y=0, z=0$ & $2x+2y+z=4$,

Ans: $8/3$

Q.(9). The volume of the object expressed in spherical coordinate $V = \int_0^{2\pi} \int_0^{\pi/2} \int_0^1 r^2 \sin \theta dr d\phi d\theta$, then V is -

(a) $\pi/3$

(b) $\pi/6$

(c) $\pi/2$

(d) $\pi/4$

Ans: (a)

Q.(10) Find the volume of cylinder, whose radius is r and height is h .

Ans: $V = \pi r^2 h$

Q.(11). The value of $\int_S \vec{F} \cdot \vec{n} ds$, where $\vec{F} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$ & S is surface of the cube bdd by $x=0, x=1, y=0, y=1, z=0$ & $z=1$ is —

Ans: $3/2$

(1). Evaluate $\oint_C x^2 y dx + y^2 x dy$, where C is closed triangle formed with points $(0,0)$, $(1,0)$ & $(1,1)$.

(2). Evaluate $\oint_C (x^2 + y^2 + xy) dx + (2x^2 + 3y^2) dy$, $C: x^2 + y^2 = 1$

(3). If C is triangle with vertices $(0,0)$, $(2,0)$ & $(1,1)$, then evaluate $\oint_C (x^2 + xy) dx + (y^2 + 2xy) dy$.

(4). If C is contour of semi-circular arc along with its diameter with centre at $(0,0)$ & radius 4 unit in upper half plane. Then evaluate —

(a). $\oint_C x^2 dy + xy dx$

(b). $\oint_C (y^3 + x^3) dy + (3x^2 y + x^3) dx$

(5). Find $\oint_C (3x^2 - 8y^2) dx + (4y - 6xy) dy$, where C is a curve bounded by $y = \sqrt{x}$ & $y = x^2$.

(6). The value of integral $\oint x dy - y dx$ along closed path $x^2 + y^2 = 1$ is —

(a). π (b). 2π (c). 4π (d). 8π

Ans: (1). $-1/6$ (2). 0 (3). $-1/3$ (4) (a). 0 (b). 0 (5). $3/2$
(6). b (7).

Instruction:

Try to solve these Questions in
in less time as much as possible (max. 30 min.)

MCQ

①

W.O.T.f intervals contains an integers satisfying the following three congruences.

$$x \equiv 2 \pmod{5}$$

$$x \equiv 3 \pmod{7}$$

$$x \equiv 4 \pmod{11}$$

(a). [400, 600]

(c). [801, 1000]

(b). [601, 800]

(d). [1001, 1200]

(NET - 2015 Dec)

②

W.O.T.f primes satisfy the congruence

$$a^2 \equiv 6a + 2 \pmod{13}$$

(a). 41

(b). 47

(c). 67

(d). 83

(NET - 2015 June)

③

We are given a class of 4-Boys & 4-girls. A Committee that consists of a President, a Vice-President and a Secretary is to be chosen among the 8-students of the class. Let a denote the no. of ways of choosing the committee in such a way that the committee has at least one boy and at least one girl. Let b denote the no. of ways that no. of girls is greater than or equal to that of the boys. then —

(a) $a = 288$

(b) $b = 168$

(c) $a = 144$

(d) $b = 192$

(NET - 2014 June)

Hint: (Permutation & Combination)

MCQ

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(4) Let x & y are the values of remainders when 2^{100} is divided by 11 and 2^{105} is divided by 11, respectively. Then w.o.t.f is true?

- (a) $\phi(x+y) = 10$ (b) $\phi(xy) = 6$
 (c) $\phi(xy) = 4$ (d) $\phi(x+y) = \phi(x) + \phi(y)$

(5) Consider the congruence $x^n \equiv 2 \pmod{13}$. This congruence has a solution for x if —

- (a) $n=5$ (b) $n=6$ (c) $n=7$ (d) $n=8$

(NET-2013 June)

(6) Which of the following (w.o.t.f) is/are true?

- (a) $5^6 \equiv 1 \pmod{7}$
 (b) $2^{\phi(7)} \equiv 1 \pmod{7}$
 (c) $5^7 \equiv 5 \pmod{7}$
 (d) $5^{\phi(7)} \equiv 5 \pmod{7}$

(7) Let a, b be integers, not both zero, then w.o.t.f is/are true?

- (a) If $\gcd(a, b) = 1$, then $\gcd(a+b, a-b) = 1$ or 2 .
 (b) If $\gcd(a, 4) = 2 = \gcd(b, 4)$, then $\gcd(a+b, 4) = 4$.
 (c) $\gcd(a, b) = \gcd(a, b+ac)$ for any integers.
 (d) $\gcd(a^2, b^2) = \gcd(a, b)$.

(Number Theory Practice Test)

— P. Kalika

CSIR-NET, Number Theory
Practice Set-1

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(8). If d is a positive integer s.t. $d \mid (13n+6)$, $d \mid (12n+5)$ for some integer n , then possible value of d is/are —

- (a) 1 (b) 3 (c) 5 (d) 7 (e) None

(9). How many digits are there in $2^{17} \times 3^2 \times 5^{14} \times 7$?

- (a) 14 (b) 15 (c) 16 (d) 17

(NET - 2014 June)

(10). W.O.T. f is/are perfect square ?

- (a) 1022121 (b) 2042122
(c) 3063126 (d) 4083128

(NET - 2014 Dec)

(11). We define a function $f(N)$ = sum of digit of N , expressed as decimal number.
(eg. $f(137) = 1+3+7 = 11$. Then Evaluate $f(2^7 3^5 5^6) =$ —

- (a) 10 (b) 18 (c) 28 (d) 11

(NET 2014 Dec).

 P. Kalika

Answer key

- (1). a, b, d (2). a, c (3). a, b (4). a, c
(5). a, c (6). a, b, c (7). a, b, c (8). a, d
(9). d (10). a (11). b.

CSIR-NET ODE-PDE Practice Questions

ODE and PDE Practice Questions

1.

Consider a boundary value problem (BVP) $\frac{d^2y}{dx^2} = f(x)$ with boundary conditions $y(0) = y(1) = y'(1)$, where f is a real-valued continuous function on $[0, 1]$. Then which of the following are true?

1. the given BVP has a unique solution for every f
2. the given BVP does not have a unique solution for some f
3. $y(x) = \int_0^x x t f(t) dt + \int_x^1 (t - x + xt) f(t) dt$ is a solution of the given BVP
4. $y(x) = \int_0^x (x - t + xt) f(t) dt + \int_x^1 x t f(t) dt$ is a solution of the given BVP

2.

Consider the ODE on $\mathbb{R}$ $y'(x) = f(y(x))$. If f is an even function and y is an odd function, then

1. $-y(-x)$ is also a solution.
2. $y(-x)$ is also a solution.
3. $-y(x)$ is also a solution.
4. $y(x)y(-x)$ is also a solution.

3.

Consider the Lagrange equation $x^2 \frac{\partial z}{\partial x} + y^2 \frac{\partial z}{\partial y} = (x + y)z$. Then the general solution of the given equation is

1. $F\left(\frac{xy}{z}, \frac{x-y}{z}\right) = 0$ for an arbitrary differentiable function F
2. $F\left(\frac{x-y}{z}, \frac{1}{x} - \frac{1}{y}\right) = 0$ for an arbitrary differentiable function F
3. $z = f\left(\frac{1}{x} - \frac{1}{y}\right)$ for an arbitrary differentiable function f
4. $z = xy f\left(\frac{1}{x} - \frac{1}{y}\right)$ for an arbitrary differentiable function f

4.

Consider the system of ODE in $\mathbb{R}^2$, $\frac{dY}{dt} = AY$, $Y(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, $t > 0$ where $A = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}$ and $Y(t) = \begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix}$. Then

1. $y_1(t)$ and $y_2(t)$ are monotonically increasing for $t > 0$.
2. $y_1(t)$ and $y_2(t)$ are monotonically increasing for $t > 1$.
3. $y_1(t)$ and $y_2(t)$ are monotonically decreasing for $t > 0$.
4. $y_1(t)$ and $y_2(t)$ are monotonically decreasing for $t > 1$.

5.

The PDE

$$\frac{\partial^2 u}{\partial x^2} + 2 \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = x, \text{ has}$$

1. only one particular integral.
2. a particular integral which is linear in x and y .
3. a particular integral which is a quadratic polynomial in x and y .
4. more than one particular integral.

6.

Let $y : [0, \infty) \rightarrow [0, \infty)$ be a continuously differentiable function satisfying

$$y(t) = y(0) + \int_0^t y(s) ds \text{ for } t \geq 0.$$

Then

1. $y^2(t) = y^2(0) + \int_0^t y^2(s) ds.$
2. $y^2(t) = y^2(0) + 2 \int_0^t y^2(s) ds.$
3. $y^2(t) = y^2(0) + \int_0^t y(s) ds.$
4. $y^2(t) = y^2(0) + \left(\int_0^t y(s) ds\right)^2 + 2y(0) \int_0^t y(s) ds.$

CSIR-NET ODE-PDE Practice Questions

7.

Let $a, b \in \mathbb{R}$ be such that $a^2 + b^2 \neq 0$. Then the Cauchy problem

$$a \frac{\partial u}{\partial x} + b \frac{\partial u}{\partial y} = 1; \quad x, y \in \mathbb{R}$$

$$u(x, y) = x \text{ on } ax + by = 1$$

1. has more than one solution if either a or b is zero
2. has no solution
3. has a unique solution
4. has infinitely many solutions

8.

Let $y : [0, \infty) \rightarrow [0, \infty)$ be a continuously differentiable function satisfying

$$y(t) = y(0) + \int_0^t y(s) ds \text{ for } t \geq 0.$$

Then

1. $y^2(t) = y^2(0) + \int_0^t y^2(s) ds.$
2. $y^2(t) = y^2(0) + 2 \int_0^t y^2(s) ds.$
3. $y^2(t) = y^2(0) + \int_0^t y(s) ds.$
4. $y^2(t) = y^2(0) + \left(\int_0^t y(s) ds \right)^2 + 2y(0) \int_0^t y(s) ds.$

9.

Consider the boundary value problem

$$-u''(x) = \pi^2 u(x); \quad x \in (0, 1)$$

$$u(0) = u(1) = 0.$$

If u and u' are continuous on $[0, 1]$, then

1. $u'^2(x) + \pi^2 u^2(x) = u'^2(0)$
2. $\int_0^1 u'^2(x) dx - \pi^2 \int_0^1 u^2(x) dx = 0$
3. $u'^2(x) + \pi^2 u^2(x) = 0$
4. $\int_0^1 u'^2(x) dx - \pi^2 \int_0^1 u^2(x) dx = u'^2(0)$

10.

The solution of the initial value problem

$$(x - y) \frac{\partial u}{\partial x} + (y - x - u) \frac{\partial u}{\partial y} = u,$$

$u(x, 0) = 1$, satisfies

1. $u^2(x - y + u) + (y - x - u) = 0.$
2. $u^2(x + y + u) + (y - x - u) = 0.$
3. $u^2(x - y + u) - (x + y + u) = 0.$
4. $u^2(y - x + u) + (x + y - u) = 0.$

11.

Let $y(x)$ be a continuous solution of the initial value problem

$$y' + 2y = f(x), \quad y(0) = 0,$$

$$\text{where } f(x) = \begin{cases} 1, & 0 \leq x \leq 1 \\ 0, & x > 1 \end{cases}$$

Then $y\left(\frac{3}{2}\right)$ is equal to

- | | |
|---------------------------|---------------------------|
| 1. $\frac{\sinh(1)}{e^3}$ | 2. $\frac{\cosh(1)}{e^3}$ |
| 3. $\frac{\sinh(1)}{e^2}$ | 4. $\frac{\cosh(1)}{e^2}$ |

12.

Consider a system of first order differential equations

$$\frac{d}{dt} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} x(t) + y(t) \\ -y(t) \end{bmatrix}$$

The solution space is spanned by

1. $\begin{bmatrix} 0 \\ e^{-t} \end{bmatrix}$ and $\begin{bmatrix} e^t \\ 0 \end{bmatrix}$
2. $\begin{bmatrix} e^t \\ 0 \end{bmatrix}$ and $\begin{bmatrix} \cosh t \\ e^{-t} \end{bmatrix}$
3. $\begin{bmatrix} e^{-t} \\ -2e^{-t} \end{bmatrix}$ and $\begin{bmatrix} \sinh t \\ e^{-t} \end{bmatrix}$
4. $\begin{bmatrix} e^t \\ 0 \end{bmatrix}$ and $\begin{bmatrix} e^t - \frac{1}{2}e^{-t} \\ e^{-t} \end{bmatrix}$

CSIR-NET ODE-PDE Practice Questions

13.

The initial value problem $y' = 2\sqrt{y}$, $y(0) = a$, has

1. a unique solution if $a < 0$
2. no solution if $a > 0$
3. infinitely many solutions if $a = 0$
4. a unique solution if $a \geq 0$

14.

Consider the initial value problem

$$\frac{\partial u}{\partial x} + 2 \frac{\partial u}{\partial y} = 0, \quad u(0, y) = 4e^{-2y}.$$

Then the value of $u(1, 1)$ is

- | | |
|--------------|-----------|
| 1. $4e^{-2}$ | 2. $4e^2$ |
| 3. $2e^{-4}$ | 4. $4e^4$ |

15.

Let $u(x, t)$ satisfy the wave equation

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}; \quad x \in (0, 2\pi), t > 0$$

$$u(x, 0) = e^{i\omega x}$$

for some $\omega \in \mathbb{R}$. Then

1. $u(x, t) = e^{i\omega x} e^{i\omega t}$.
2. $u(x, t) = e^{i\omega x} e^{-i\omega t}$.
3. $u(x, t) = e^{i\omega x} \left(\frac{e^{i\omega t} + e^{-i\omega t}}{2} \right)$.
4. $u(x, t) = t + \frac{x^2}{2}$.

16.

Let $u(x, y)$ be the solution of the equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, which tends to zero as $y \rightarrow \infty$

and has the value $\sin x$ when $y = 0$. Then

1. $u = \sum_{n=1}^{\infty} a_n \sin(nx + b_n) e^{-ny}$, where a_n are arbitrary and b_n are non-zero constants.
2. $u = \sum_{n=1}^{\infty} a_n \sin(nx + b_n) e^{-n^2 y}$, where $a_1 = 1$ and a_n ($n > 1$), b_n are non-zero constants.
3. $u = \sum_{n=1}^{\infty} a_n \sin(nx + b_n) e^{-ny}$, where $a_1 = 1$, $a_n = 0$ for $n > 1$ and $b_n = 0$ for $n \geq 1$.
4. $u = \sum_{n=1}^{\infty} a_n \sin(nx + b_n) e^{-n^2 y}$, where $b_n = 0$ for $n \geq 0$ and a_n are all nonzero.

17.

Consider the differential equation

$$\frac{d^2 y}{dx^2} - 2 \tan x \frac{dy}{dx} - y = 0$$

defined on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Which among the following are true?

1. there is exactly one solution $y = y(x)$ with $y(0) = y'(0) = 1$ and $y\left(\frac{\pi}{3}\right) = 2\left(1 + \frac{\pi}{3}\right)$
2. there is exactly one solution $y = y(x)$ with $y(0) = 1, y'(0) = -1$ and $y\left(-\frac{\pi}{3}\right) = 2\left(1 + \frac{\pi}{3}\right)$
3. any solution $y = y(x)$ satisfies $y''(0) = y(0)$
4. if y_1 and y_2 are any two solutions then $(ax + b)y_1 = (cx + d)y_2$ for some $a, b, c, d \in \mathbb{R}$

CSIR-NET ODE-PDE Practice Questions

18.

For $f \in C[0,1]$ and $n > 1$,

$$\text{let } T(f) = \frac{1}{n} \left[\frac{1}{2} f(0) + \frac{1}{2} f(1) + \sum_{j=1}^{n-1} f\left(\frac{j}{n}\right) \right]$$

be an approximation of the integral

$I(f) = \int_0^1 f(x) dx$. For which of the following functions f is $T(f) = I(f)$?

1. $1 + \sin 2\pi nx$
2. $1 + \cos 2\pi nx$
3. $\sin^2 2\pi nx$
4. $\cos^2 2\pi(n+1)x$

19.

Let $B = \{(x_1, x_2) \in \mathbb{R}^2 \mid x_1^2 + x_2^2 < 1\}$, and let

$$C_{ld}^2(\bar{B}; \mathbb{R}^2) = \{u \in C^2(\bar{B}; \mathbb{R}^2) \mid u(x_1, x_2) = (x_1, x_2), \text{ for } (x_1, x_2) \in \partial B\}.$$

Let $u = (u_1, u_2)$ and define $J : C_{ld}^2(\bar{B}; \mathbb{R}^2) \rightarrow \mathbb{R}$ by

$$J(u) = \int_B \left(\frac{\partial u_1}{\partial x_1} \frac{\partial u_2}{\partial x_2} - \frac{\partial u_1}{\partial x_2} \frac{\partial u_2}{\partial x_1} \right) dx_1 dx_2$$

Then,

1. $\inf\{J(u) : u \in C_{ld}^2(\bar{B}; \mathbb{R}^2)\} = 0$
2. $J(u) > 0$, for all $u \in C_{ld}^2(\bar{B}; \mathbb{R}^2)$
3. $J(u) = 1$, for infinitely many $u \in C_{ld}^2(\bar{B}; \mathbb{R}^2)$
4. $J(u) = \pi$, for all $u \in C_{ld}^2(\bar{B}; \mathbb{R}^2)$

20.

A solution of the PDE

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + \left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial u}{\partial y} \right)^2 - u = 0$$

represents

1. an ellipse in the x - y plane.
2. an ellipsoid in the xyu space.
3. a parabola in the u - x plane.
4. a hyperbola in the u - y plane.

21.

Let $X = \{u \in C^1[0,1] \mid u(0) = 0\}$ and let $I : X \rightarrow \mathbb{R}$ be defined as

$$I(u) = \int_0^1 (u'(t)^2 - u(t)^2) dt$$

Which of the following are correct?

1. I is bounded below
2. I is not bounded below
3. I attains its infimum
4. I does not attain its infimum

Practice Questions of ODE & PDE for CSIR-NET Mathematics

-: P. Kalika & K. Munesh

Practice Problems

1. The critical point of the system

$$x'(t) = -4x - y$$

$$y'(t) = x - 2y$$

(NET-June 2015)

- (a) Asymptotically stable Node
- (b) Unstable node
- (c) Asymptotically stable spiral
- (d) Unstable spiral

Answer: (a)

2. Consider the system of differential equations

$$x'(t) = 2x - 7y$$

$$y'(t) = 3x - 8y$$

(NET-June 2018)

Then critical point $(0, 0)$ of the system is an

- (a) Asymptotically stable Node
- (b) Unstable node
- (c) Asymptotically stable spiral
- (d) Unstable spiral

Answer: (a)

3. Then critical point $(0, 0)$ for the system

$$x'(t) = x - 2y + y^2 \sin(x)$$

$$y'(t) = 2x - 2y - 3y \cos(y^2)$$

(NET-Dec 2018)

- (a) is a Stable spiral point
- (b) is a Unstable spiral point
- (c) is a Saddle point
- (d) is a Stable node

Answer: (c)

4. Consider the system of differential equations [1]

$$x'(t) = x + 4y - x^2$$

$$y'(t) = 6x - y + 2xy$$

(Practice Que.)

Then critical point $(0, 0)$ of the system is an

- (a) Asymptotically stable Node
- (b) Unstable saddle point
- (c) Asymptotically stable spiral
- (d) Unstable spiral

Answer: (b)

5. Then critical point $(0, 0)$ for the system

$$x'(t) = \sin(x) - 4y$$

$$y'(t) = \sin(2x) - 5y$$

(Practice Que.)

- (a) is a Stable spiral point
- (b) is a Asymptotically stable Node
- (c) is a Saddle point
- (d) is a Stable node

Answer: (b)

6. Consider the system of differential equations [1]

$$x'(t) = 8x - y^2$$

$$y'(t) = -6y + 6x^2$$

(Practice Que.)

Then critical point $(0, 0)$ of the system is an

- (a) Asymptotically stable Node
- (b) Asymptotically stable spiral
- (c) Unstable saddle point
- (d) Unstable spiral

Answer: (c)

Hint: There are critical points $(0, 0)$ & $(2, 4)$

At $(0, 0)$: Unstable saddle point

At $(2, 4)$: Unstable spiral point

7. Consider the systems of differential equations [1]

$$x'(t) = -y - x^2$$

$$y'(t) = x$$

and

$$x'(t) = -y - x^3$$

$$y'(t) = x$$

(Practice Que.)

Find all the critical point and nature of system on the each critical points.

Hint: Nature- Centre or Spiral Point

References

- [1] S. Ross, *DIFFERENTIAL EQUATIONS, 3RD ED.* Wiley India Pvt. Limited, 2007.
- [2] A. Jeffrey, *Advanced engineering mathematics.* Elsevier, 2001.
- [3] E. Kreyszig, “Advanced engineering mathematics, 8-th edition,” 1999.

Note: *Full Notes of dynamical system with non-linear will be available soon.*

Summary (Integrating factor)

Given: $M(x,y)dx + N(x,y)dy = 0$, Then —

Type	Form (of given DE)	I.F.
Type-I	When M and N are homogeneous f ⁿ of same deg.	$\frac{1}{Mx+Ny}$; if $Mx+Ny \neq 0$
Type-II	if $M = yf_1(xy)$ & $N = xf_2(xy)$	$\frac{1}{Mx-Ny}$; if $Mx-Ny \neq 0$
Type-III	if $\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) \cdot \frac{1}{N} = \phi(x)$	I.F. = $e^{\int \phi(x) dx}$
Type-IV	if $\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) \cdot \frac{1}{M} = \psi(y)$	I.F. = $e^{\int \psi(y) dy}$
Type-V	if $Mdx + Ndy = 0$ can be expressed as — $x^a y^b (mydx + nx dy) +$ $x^r y^s (u_1 y dx + v_1 x dy) = 0$ Where a, b, r, s, m, n, u ₁ , v ₁ are known constants	I.F. = $x^h y^k$, where h & k can be found by solving $\frac{a+h+1}{m} = \frac{b+k+1}{n}$ & $\frac{r+h+1}{u_1} = \frac{s+k+1}{v_1}$

Once eqⁿ $M(x,y)dx + N(x,y)dy = 0$ becomes exact diffⁿ eqⁿ (check by $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$)

It's general solution is given by —

$$\int M(x,y) dx + \int N dy = C$$

(treat y as constant)

(only those part of N, which doesn't contain terms of x)

P. Kalika Notes

Exercise (21) (Exact DE)

(P. Kalika Notes)

#. Find the general solⁿ of following: —

(1). $x dx + y dy + \frac{xdy - ydx}{(x^2 + y^2)} = 0$

(2). $e^{x/4} (1 + e^{-x/4}) dx + (e^{x/4} - \frac{x}{4} e^{x/4}) dy = 0$

(3). Value of a & b , for which diffⁿ eqⁿ is exact
 $(3a^2x^2 + by \cos x) dx + (2 \sin x - 4ay^3) dy = 0$

(a). $a=3, b=2$

(b). $a=2, b=3$

(c). $a=3, b=4$

(d). $a=2, b=5$

(4). Consider the D.E —

$2 \cos(y^2) dx - xy \sin(y^2) dy = 0$, then I.F. can be —

(a). e^x

(b). e^{-x}

(c). $3x$

(d). x^3

(5). The eqⁿ. $(\alpha x y^3 + y \cos x) dx + (x^2 y^2 + \beta \sin x) dy = 0$ is exact if —

(a). $\alpha = 3/2, \beta = 1$

(b). $\alpha = 1, \beta = 3/2$

(c). $\alpha = 2/3, \beta = 1$

(d). $\alpha = 1, \beta = 2/3$

----- Answer -----

(1). $x^2 + y^2 - 2 \tan^{-1}(x/y) = C_1$

(2). $x + y \cdot e^{x/4} = C$

(3). (a)

(4). (d)

(5).

[Integrating Factor] ⁽²²⁾ Assignment (Exercise)

P. Kalika

④ Solve the following differential Equations.

(1). $x dx + y dy + \frac{x dy - y dx}{x^2 + y^2} = 0$

(2). $(3x^2y^4 + 2xy) dx + x^2(2xy^3 - 1) dy = 0$

(3). $(2xy^4e^y + 2xy^3 + y) dx + (x^2y^4e^y - x^2y^2 - 3x) dy = 0$
(Hint: type-IV)

(4). $(x^2 + y^2 + 2x) dx + 2y dy = 0$

(5). $(6y + 2y^3 + 3x^2) dx + \frac{3}{2}(1 + y^2)x dy = 0$

(6). $(xy^2 + 2x^2y^3) dx + (x^2y - x^3y^2) dy = 0$

(7). $(2x^2y - 3y^4) dx + (3x^3 + 2xy^3) dy = 0$

(8). Given that I.F of $(x^7y^2 + 3y) dx + (3x^8y - x) dy = 0$ is $x^h y^k$ then —

(a) $h = -7, k = 1$ (c) $h = k = 0$

(b) $h = 1, k = -7$ (d) $h = k = 1$ (e) None

(9). Given that $\frac{dy}{dx} - \frac{y}{x} = 0$. Then w.o.t.f. is/are true for I.F —

(a) $\frac{1}{x^2}$

(c) $\frac{1}{xy}$

(b) $\frac{1}{y^2}$

(d) $\frac{1}{x+y}$

(e) None

(10). $(x^2y^3 + xy^2 + y) dx + (x^2y^2 - x^2y + x) dy = 0$

(11) If $x^3 y^2$ is an integrating factor of

$$(6y^2 + axy) dx + (6xy + bx^2) dy = 0 \quad \text{where } a, b \in \mathbb{R}$$

Then —

(A). $3a - 5b = 0$

(B). $2a - b = 0$

(C). $3a + 5b = 0$

(D). $2a + b = 0$

GATE 2017

(12). An Integrating factor of eqⁿ.

IIT-JAM-2018

$$\left(y + \frac{1}{3}y^3 + \frac{1}{2}x^2\right) dx + \frac{1}{4}(x + xy^2) dy = 0 \quad \text{is —}$$

- (a). x^2 (b). $3 \log_e x$ (c). x^3 (d). $2 \log_e x$ (e) None

(13). An integrating factor of

IIT-JAM 2005

$$x \frac{dy}{dx} + (3x+1)y = x e^{-2x} \quad \text{is —}$$

- (a). $x e^{3x}$ (b). $3x e^x$ (c). $x e^x$ (d). $x^3 e^x$

(14). The solⁿ of the DE $x \frac{dy}{dx} + (1+x)y = e^{-x}$ with the

B.C $y(x=1)=0$ is —

- (a). $\frac{x-1}{x} e^{-x}$ (b). $\frac{x-1}{x^2} e^{-x}$ (c). $\frac{1-x}{x^2} e^{-x}$ (d). $(x-1)^2 e^{-x}$

CSIR-NET 2019 June
Physics

Answer

(1) $x^2 + y^2 - 2 \tan^{-1}(x/y) = C_1$

(2) $(xy)^3 + x^2 = cy$

(3) $x^2 e^y + \frac{x^2}{y} + \frac{x}{y^3} = C$

(4) $x^2 e^x + y^2 e^x = C$

(5) $3x^4 y + x^4 y^3 + x^6 = C$

(6) $-\frac{1}{3xy} + \frac{2}{3} \log x - \frac{1}{3} \log y = C$

(7) $I.F = x^{-4/3} y^{-20/13}$

(8). (a).

(9). (a, b, c)

(10) $xy - \frac{1}{xy} + \log \frac{x}{y} = C$

(11). (A).

(12). (C).

(13). (a).

(14). (a)

P. Kalika Notes

Find the solutions of following:

(1). $(x^2 + y^2 + 2x)dx + 2ydy = 0$

(2). $\sec^2 y \frac{dy}{dx} + x \tan y = x^3$

(3). $\frac{\sin x}{y^2} \frac{dy}{dx} + \frac{3}{y} = \cos x$

W.O.T.f statements ~~are~~ is/are true?

(1). Solution of the problem $y' \cos x + y \sin x = 1, y(0) = 0$ is -

(a). Periodic fⁿ

(b). positive in $(0, \pi)$

(c). Monotonic $\uparrow$ in $(0, \pi/2)$

(d). Invertible in $(0, \pi)$

(2). The solⁿ $\phi(x)$ of $x^2 y' + 2xy = 1$ satisfies $\phi(x) = 2\phi(1)$ then —

(a). $\phi(x) \rightarrow \infty$ as $x \rightarrow \infty$

(b). $\phi(x) \rightarrow -2$ as $x \rightarrow +\infty$

(c). $\phi(x) \rightarrow 0$ as $x \rightarrow \infty$

(d). None.

(3). D.E $y' + P(x)y = Q(x)y^n$ is —

(a). linear D.E for $n=0$

(c). Bernoulli eqⁿ for $n \neq 0, 1$

(b). linear D.E for $n=1$

(d). Bernoulli eqⁿ for $n=1, 0$

(4). The value of 'a', for which y^a is an IF of D.E

$2xydx - (2x^2 - y^2)dy = 0$ is —

(a). -4

(b). 4

(c). -3

(d). 3

Answer

(1). $y^2 = -x^2 + Ce^{-x}$

(2). $y = \tan^{-1}(x^2 - 2 + Ce^{-1/2 x^2})$

(3). IF = $(\tan x/2)^{-3}$

Objective (Answer)

(1). a, b, c

(2). (c).

(3). A, I

(4). (c)

Assignment (Particular Integral)

☐ Solve the following differential equations

①. $(D^2 - 1)y = 2^x + e^x \cos x$

②. $(D^2 - 4D + 4)y = 8x^2 e^{2x} \sin 2x$

③. $(D^3 + D^2 - D - 1)y = \cos 2x$

④. $(D^2 - 4D + 4)y = e^{2x} \cos^2 x$ (H

⑤. $(D^2 + 4)y = x \cos x$ (Hint: $y_p = \frac{1}{4} [3x \cos x + 2 \sin x]$)

⑥. $(D^2 - 2D + 1)y = x e^x \sin 2x$ (Hint: $y_p = -\frac{e^x}{4} (x \sin 2x + \cos 2x)$)

⑦. $(D^2 + 1)y = \frac{1}{\sin x}$ (Hint: $y_p = \sin x \cdot \log \sin x - x \cos x$)

⑧. $(D^3 - 4D^2 - 3D + 18)y = 0$

(Hint: $y_{cf} = C_1 e^{-2x} + (C_2 + C_3 x) e^{3x}$)

⑨.

- Answer (Hint):

①. $y_p = e^x \left[\frac{-2 \sin x + \cos x}{5} \right] + \frac{2^x}{(\log 2)^2 - 1}$

②. $y_p = -e^{2x} [2x^2 \sin 2x + 4x \cos 2x - 3 \sin 2x]$

③. $y_p = -\frac{1}{25} [2 \sin 2x + \cos 2x]$

④. $y_p = \frac{1}{8} e^{2x} [2x^2 - \cos 2x]$

(26)
Practice Questions (Variation of Parameter)

①. Solve $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} - y = x^2 e^x$

(Ans: $y(x) = C_1 x + C_2/x + \frac{1}{2} x^2 + e^x - \frac{e^x}{x}$)

CSIR-NET

②. Using method of VOP, the particular solⁿ of the diffⁿ eqⁿ. $y'' + 4y = \frac{3}{\sin 2x}$, $0 < x < \pi/2$ is —

(a). $\frac{3}{4} \sin 2x \cdot \log(\sin 2x) - \frac{3}{2} \cos 2x$.

(b). $\frac{3}{2} \sin 2x \cdot \log(\cos 2x) - \frac{3}{4} \cos 2x$

(c). $\frac{3}{2} \sin 2x \log(\sin 2x) - \frac{3}{2} \cos 2x$

(d). $\frac{3}{4} \sin 2x \cdot \log(\sin 2x) - \frac{3}{2} x \cos 2x$

(Ans: d)

CSIR-NET

③. The general solⁿ of the diffⁿ eqⁿ.

$y'' + y = f(x)$, $x \in (-\infty, \infty)$, where $f(x)$ is rts.

real valued fⁿ on $(-\infty, \infty)$ is —

(a). $y = A \cos x + B \sin x + \int_0^x f(x-t) \sin t \, dt$.

(b). $y = A \cos x + B \sin x + \int_0^x f(x+t) \cos t \, dt$

(c). $y = A \cos x + B \sin x + \int_0^x f(t) \sin(x-t) \, dt$

(d). $y = \cos(x+k) + \int_0^x f(t) \sin(x-t) \, dt$

(Ans: c).

CSIR-NET

Que (4). The gen. solⁿ of DE $y'' = -f(x)$, $x \in \mathbb{R}$, $f \in C^1(\mathbb{R})$ then —

(a). $y(x) = A + Bx + \int_0^x (x-t) f(t) \, dt$.

(b). $y(x) = Ax - \int_0^x x f(x-t) \, dt$

(c). $y(x) = Ax + B + \int_0^x (x-t) f(x-t) \, dt$

(d). $y(x) = A + Bx + \int_0^x (x-t) f(t) \, dt$.

(Ans: d).

[P. Kalika Notes] (27) Practice Set (Existence, Uniqueness & Lipschitz) [ODE]

① Consider the IVP $y' = 2\sqrt{y}$, $y(0) = k$, then given problem has —

- (a) Unique solⁿ if $a > 0$ (b) Unique solⁿ if $a < 0$
 (c) Unique solⁿ if $a \geq 0$ (d) Unique solⁿ if $a = 0$

②. Find the largest interval predicted by Picard theorem for $\frac{dy}{dx} = 1 + y + y^2 \cos x$, $y(0) = 0$ —

- (a). $[-\frac{1}{3}, \frac{1}{3}]$ (b). $[0, \frac{1}{3}]$
 (c). $[-1, 1]$ (d). None

③. For $\lambda \in \mathbb{R}$, consider DE $y'(x) = \lambda \sin(x + y(x))$, $y(0) = 1$. Then this IVP has —

- (a) no solⁿ in any nb.d of 0.
 (b) a solⁿ in $\mathbb{R}$ if $|\lambda| < 1$ (NET-2016)
 (c) a solⁿ in a nb.d of 0.
 (d) a solⁿ in $\mathbb{R}$ only if $|\lambda| > 1$

④. Consider the IVP $y' = xy^{\frac{1}{3}}$, $y(0) = 0$, $(x, y) \in \mathbb{R} \times \mathbb{R}$
 Then w.o.t.f are correct?

- (a) The f^n $f(x, y) = xy^{\frac{1}{3}}$ doesn't satisfy Lipschitz Condⁿ w.r.t. y in any nb.d of $y = 0$.
 (b) $\exists$ a unique solⁿ for the IVP.
 (c) $\exists$ no solution for the IVP
 (d) $\exists$ more than one solⁿ for the IVP

(NET-2013)

(5) The diffⁿ eqⁿ. $y' = 60(y^2)^{1/5}$, $x > 0$

$y(0) = 0$ has —

(a). a unique solⁿ

(b). two solⁿ.

(c). No solution

(d). infinitely many solⁿ.

(NET-2012 Dec)

(6) Consider the IVP $y' = y^2$, $y(0) = 1$, $(x, y) \in \mathbb{R} \times \mathbb{R}$.
Then $\exists$ a unique solⁿ of the IVP on

(a). $\mathbb{R} = (-\infty, \infty)$

(b). $(-\infty, 1)$

(c). $(-2, 2)$

(d). $(-1, \infty)$

(NET-2013 June)

(7) Consider the IVP $y'(t) = f(t) \cdot y(t)$, $y(0) = 1$,
where $f: \mathbb{R} \rightarrow \mathbb{R}$ is cts. Then the IVP has —

(a). infinitely many solⁿ for some f .

(b). a unique solⁿ in $\mathbb{R}$.

(c). no solⁿ in $\mathbb{R}$ for some f .

(d). a solⁿ in an interval containing 0 but not on $\mathbb{R}$ for some f .

(NET 2012 June)

(8) Find h for — $\left(h = \min \left\{ a, \frac{b}{m} \right\} \right)$

(a). $y' = e^{-x^2+y^2}$, $y(0) = 1$

(b). $y' = (4y + e^{-x^2})e^y$, $y(0) = 0$

Ans:

(1). a

(6). b

(2). a

(7). b

(3). b, c

(8) (a) $h = \frac{\sqrt{2}}{1 + (\sqrt{2} + 1)^2}$

(4). a, d

(b) $h = \frac{1}{8\sqrt{e}}$

(5). d

Exercise⁽²⁰⁾ (EUT)

Find Solution

①. $\frac{dy}{dx} = 1 + y^2$, $y(1) = 0$

(Hint: Unique solⁿ)

②. $\frac{dy}{dx} = \frac{2y}{x}$, $y(0) = 0$

(Hint: IVP has infinite no. of solⁿ.)

③. $\frac{dy}{dx} = \frac{2y}{x}$, $y(0) = 1$ | $\frac{dy}{dx} = \frac{2y}{x}$, $y(1) = 0$

(Hint: No solⁿ.)

(Hint: Unique solⁿ.)

④. The IVP $\frac{dy}{dx} = y^2$, $y(0) = 1$ has unique solⁿ in —

(a) $(-2, 2)$

(b) $(-3, 3)$

(c) $\mathbb{R}$

(d) $(-1/2, 1/2)$

(Hint: d)

⑤. The IVP $2x \frac{dy}{dx} = 3(2y-1)$, $y(0) = 1/2$ has —

(a) unique solⁿ

(b) finitely many solⁿ.

(c) infinitely many solⁿ. (d) no. solⁿ.

⑥. If $y'(t) = f(t) \cdot y(t)$, $y(0) = 1$, $f: \mathbb{R} \rightarrow \mathbb{R}$ is cts. Then this IVP has — (Ans: c)

(a) infinitely many solⁿ for some f .

(b) Unique solⁿ in $\mathbb{R}$.

(Any: d).

(c) No solⁿ in $\mathbb{R}$ for some f .

(d) a solⁿ in an interval containing 0 but not in $\mathbb{R}$ for some f .

⑦. The IVP $\frac{dy}{dx} = xy^{1/3}$, $y(0)=1$ has —
 (Hint: Unique solⁿ by EUT).

⑧. If $y'(t) = f(t)$ s.t. $y(0)=0$ where $f: \mathbb{R} \rightarrow \mathbb{R}$ is Cts., then —

(Ans: IVP has unique solⁿ in an open interval containing 0 but not on $\mathbb{R}$)

⑨. If $\frac{dy}{dx} = y^{2/3}$ s.t. $y(0)=0$, the IVP has —

(a) Unique solⁿ

(b) NO solⁿ.

(c) More than one solⁿ

(d) $y = \frac{1}{27} x^3$ is a solⁿ.

(Ans: c, d)

Exercise (Self-Adjoint)

Que: Transform the following eqⁿ. into an equivalent Self-Adjoint equation.

(1). $x^2 \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} + 5y = 0$

(Hint: factor = $1/x^4$)

(2). $(x^4 + x^2) \frac{d^2 y}{dx^2} + 2x^3 \frac{dy}{dx} + 3x = 0$

[Ans: $\frac{d}{dx} \left[(x^2 + 1) \frac{dy}{dx} \right] + \frac{3}{x^2} y = 0$]

(3). $a(x) \frac{d^2 y}{dx^2} + b(x) y = 0$ [Ans: $\frac{d}{dx} \left[e^{\int \frac{b(x)}{a(x)} dx} \frac{dy}{dx} \right] = 0$]

Que
(2). Consider the ODE $x''(t) + e^{t^2} x(t) = 0$ for $t \in [0, 3\pi]$ where $x: [0, 3\pi] \rightarrow \mathbb{R}$. Then what is cardinality of the set $\{t \in [0, 3\pi] : x(t) = 0\}$. (Ans: ≥ 3)

(3). Check (Prove) that the following Boundary Value Problem (BVP) is Sturm-Liouville Problem.

(a). $\frac{d}{dt} \left[t \frac{dy}{dt} \right] + [2t^2 + 1t^3] y = 0$

with conditions, $3y(1) + 4y'(1) = 0$

& $5y(2) - 3y'(2) = 0$

(b). $\frac{d^2 y}{dx^2} + 1y = 0$ with conditions

$y(0) = 0$ & $y(\pi) = 0$

(4). Find the non-trivial solⁿ of Sturm-Liouville BVP $y''(x) + 1y = 0$ with $y(0) = 0 = y(\pi)$.

(4-75M)
NET-June 2013

Practice set (Green fⁿ)

①. Consider the BVP $u'' = -f$, $u(0) = u'(1) = 0$ on $[0, 1]$ where $u' \equiv \frac{du}{dx}$ & $u'' = \frac{d^2u}{dx^2}$. Assume $f(x)$ is real-valued cts. fⁿ. on $[0, 1]$. Then w.o.t.f is/are correct?

(a). The Green's fⁿ is $G(x, t) = \begin{cases} x & \text{for } 0 \leq x \leq t \\ t & \text{for } t \leq x \leq 1 \end{cases}$ (for the above BVP)

(b). Both G & $\frac{\partial G}{\partial x}$ are cts. on $[0, 1] \times [0, 1]$ with $\frac{\partial^2 G}{\partial x^2}$ having a discontinuity along $x = t$

(c). $G(x, t)$ satisfies the homo. eqⁿ. $u'' = 0$ and $0 \leq x < t < t \leq x \leq 1$

(d). The solⁿ of the given BVP is

$$u(x) = \int_0^x t f(t) dt + \int_x^1 x f(t) dt$$

(NET-Dec-2012)

②. The solⁿ of the diffⁿ eqⁿ $y'' = f(x)$, $x \in (0, 1)$ $y(0) = y(1) = 0$ is given by $y(x) = \int_0^1 G(x, t) f(t) dt$

where —

(a). $G(x, t) = \begin{cases} x(t-1) & : x \leq t \\ t(x-1) & : x \geq t \end{cases}$

(b). $G(x, t) = \begin{cases} x^2(t-1) & : x \leq t \\ t^2(x-1) & : x > t \end{cases}$

(c). $G(x, t) = \begin{cases} x(t^2-1) & : x \leq t \\ t(x^2-1) & : x > t \end{cases}$

(d). $G(x, t) = \begin{cases} \sin x(t-1) & : x \leq t \\ \sin t(x-1) & : x \geq t \end{cases}$

NET-2011

③. The green fn $G(x, t)$ of the BVP $y'' - \frac{1}{x} y' = 1$
 $y(0) = y(1) = 0$ is $G(x, t) = \begin{cases} f_1(x, t) & : x \leq t \\ f_2(x, t) & : t \geq x \end{cases}$ where

(a). $f_1(x, t) = -\frac{1}{2} t (1 - x^2)$, $f_2(x, t) = -\frac{1}{2t} x^2 (1 - t^2)$

(b). $f_1(x, t) = -\frac{1}{2x} t^2 (1 - x^2)$, $f_2(x, t) = -\frac{1}{2t} x^2 (1 - t^2)$

(c). $f_1(x, t) = -\frac{1}{2t} x^2 (1 - t^2)$, $f_2(x, t) = -\frac{1}{2} t (1 - x^2)$

(d). $f_1(x, t) = -\frac{1}{2t} x^2 (1 - t^2)$, $f_2(x, t) = -\frac{1}{2x} t^2 (1 - x^2)$

2011
NET-June

④. The Green's function $G(x, t)$, $0 \leq x, t \leq 1$
 of the BVP $y'' + \lambda y = 0$, $y(0) = 0 = y(1)$ is —

(a). Symmetric in x & t

(b). Continuous at $x = t$

(c). $\frac{\partial G(x, t)}{\partial x} \Big|_{x=t^-} - \frac{\partial G(x, t)}{\partial x} \Big|_{x=t^+} = -1$

(d). $\frac{\partial G(x, t)}{\partial x} \Big|_{x=t^-} - \frac{\partial G(x, t)}{\partial x} \Big|_{x=t^+} = 1$

GATE-2016

⑤. The difference b/w the least two
 eigen-values of the BVP

$$y'' + \lambda y = 0, \quad 0 < x < \pi, \quad y(0) = y'(\pi) = 0$$

is equal to _____

Assignment Set

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Q1) Que:

(b) Convert the following DE to I.E —

$$(1) \quad y'' + y = 0 \quad \text{when} \quad y(0) = 0 = y'(0)$$

$$\text{Ans: } y(x) = -\int_0^x (x-t) y(t) dt$$

$$(2) \quad y''(x) - 3y'(x) + 2y(x) = 4 \sin x \quad \text{with} \quad y(0) = 1, y'(0) = -2$$

$$\text{Ans: } y(x) = 1 - x - 4 \sin x + \int_0^x (3 - 2(x-t)) y(t) dt$$

(3) The IVP corresponding to the DE

GATE
(2001)

$$y(x) = 1 + \int_0^x y(t) dt \quad \text{is —}$$

$$\checkmark (a) \quad y' - y = 0, y(0) = 1$$

$$(c) \quad y' - y = 0, y(0) = 0$$

$$(b) \quad y' + y = 0, y(0) = 0$$

$$(d) \quad y' + y = 0, y(0) = 1$$

Hint: $\because y(0) = 1 \Rightarrow \text{op(b) \& op(c) discarded.}$

$$y'(x) = 0 + y(x) - y(0) \cdot 0 = y(x) \Rightarrow y'(0) = y(0) = 1$$

$$\Rightarrow y' - y = 0 \Rightarrow \text{op(a) TRUE}$$

(4) I.E corresponding to $y''(x) + y(x) = 0, x > 0$
(NET)

$$y(0) = 1 \quad \& \quad y'(0) = 0 \quad \text{is —}$$

$$\times (a) \quad y(x) = \int_0^x (t-x) y(t) dt \quad \Rightarrow y(0) \neq 1$$

$$\times (b) \quad y(x) = \int_0^x (t-x) y(t) dt - 1 \quad \Rightarrow y(0) \neq 1$$

$$\checkmark (c) \quad y(x) = \int_0^x (t-x) y(t) dt + 1$$

$$(d) \quad y(x) = 1 - \frac{1}{2} \int_0^x (t-x) y(t) dt$$

Hint: $y''(x) + y = 0 \Rightarrow y'(x) - y'(0) = -\int_0^x y(t) dt$

$$\Rightarrow y'(x) = -\int_0^x y(t) dt \Rightarrow y(x) - y(0) = -\int_0^x \int_0^t y(t) dt$$

$$\Rightarrow y(x) = 1 + \int_0^x (t-x) y(t) dt$$

Practice Questions

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- (5) Show that, solⁿ $y(x)$ of $y''(x) = F(x)$ with I.C $y(0) = y_0$ and $y'(0) = y'_0$, gives

$$y(x) = y_0 + xy'_0 + \int_0^x (x-t)F(t)dt$$

- (6) Convert $y''(x) - 2xy'(x) - 3y(x) = 0$ with I.C $y(0) = 1, y'(0) = 0$ to Volterra IE of 2nd kind.

Conversely, derive the Original DE with IC from the IE. (Ans: $y = 1 + \int_0^x (x+t)y(t)dt$)

- (7) Reduce the following IVP into IE —

$$\frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0, \quad y(0) = 1, \quad y'(0) = 1$$

Ans: $y(x) = 1 + x - \int_0^x ty(t)dt$

- (8) Show that, the solⁿ of Volterra IE $y(x) = 1 + \int_0^x (t-x)y(t)dt$ satisfies the DE

$$y''(x) + y(x) = 0 \quad \& \quad \text{B.C } y(0) = 1, y'(0) = 1$$

- (9) $y'' + xy = 1, \quad y(0) = y'(0) = 0$

Ans: $u(x) = 1 - \int_0^x x(x-t)u(t)dt, \quad u(x) = y''$

- (10) $y'' - 5y' + 6y = 0, \quad y(0) = 0, y'(0) = -1$

Ans: $u(x) = 6x - 5 + \int_0^x (5 - 6x + 6t)u(t)dt, \quad u(x) = y''$

- (11) $y'' + y = \cos x, \quad y(0) = 0, y'(0) = 1$

Ans: $u(x) = \cos x - x - \int_0^x (x-t)u(t)dt, \quad u(x) = y''$

P. KALKA

Practice Questions

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- (1). Find the eigen-values & Eigen-function of

$$y(x) = \lambda \int_0^1 e^x \cdot e^t y(t) dt.$$

(Ans: Already discussed)

- (2). Show that

$$y(x) = \lambda \int_0^1 (3x-2)t y(t) dt$$
 has no eigen value & eigen-function.

GATE

- (3). The values of λ for which the IE

$y(x) = \lambda \int_0^1 (6x-t)y(t) dt$ has non-trivial solⁿ is roots of —

(a) $(3\lambda-1)(2+\lambda)-\lambda^2=0$

(b) $(3\lambda-1)(2+\lambda)+2=0$

[GATE-2004]

(c) $(3\lambda-1)(2+\lambda)-4\lambda^2=0$

(d) $(3\lambda-1)(2+\lambda)+\lambda^3=0$

(Ans: C)

- (4). Find the eigen-values & eigen-function of homo. IE

$$y(x) = \lambda \int_0^1 \sin \pi x \cos \pi t y(t) dt$$

Ans: NO characteristic no. & eigen function.

GATE

- (5). The eigen-values of IE

$y(x) = \lambda \int_0^{2\pi} \sin(x+t) dt$ has —

- (a) $\pm \frac{1}{2}\pi$ (b) $\pm \frac{1}{\pi}$ (c) $\pm \pi$ (d) $\pm 2\pi$

Ans: (b)

[GATE-2005]

- (6). Solve the Fredholm IE (homo.) of 2nd kind

$$y(x) = \lambda \int_0^{2\pi} \sin(x+t) y(t) dt$$

Ans: $\lambda = \pm \frac{1}{\pi}$, $y(x) = \sin x \pm \cos x$.

GATE

- (7). The IE $y(x) = \lambda \int_0^{2\pi} \sin(x+t) y(t) dt$ has —

[GATE-2003]

Practice Questions

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- (a) two solⁿs for any value of d .
 (b) Unique solⁿ for every value of d .
 (c) Infinitely many solⁿ for only one values of d .
 (d) " " " " two values of d .

(Ans: d)

Ans: $\left(\begin{array}{l} d_1 = 4/\pi, y_1(x) = \cos^2 x \\ d_2 = 8/\pi, y_2(x) = \cos 3x \end{array} \right)$

GATE
(8)The eigenvalue λ of the Fredholm IE

$$y(x) = \lambda \int_0^1 x^2 + y(t) dt \quad \text{is}$$

- (a) 4 (b) 2 (c) -2 (d) -4 [GATE-2011]

- (Ans: a)

(9) Solve the following homo. I.E -

(a) $y(x) = \frac{1}{2} \int_0^\pi \sin x y(t) dt \quad (y=0)$

(b) $y(x) = \frac{1}{e^2 - 1} \int_0^1 2e^{x+t} y(t) dt \quad (y=0)$

(c) $y(x) = \lambda \int_0^\pi \cos(x+t) y(t) dt, \quad \lambda = \text{Find}$

(d) $y(x) = \lambda \int_{-\pi}^1 (5xt^3 + 4x^2t) y(t) dt, \quad \text{Find } \lambda.$

(e) $y(x) = \lambda \int_0^\pi \sin^2 x y(t) dt.$

GATE

(10) Prove that, the linear DE of 2nd order

$$y''(x) + a_1(x)y'(x) + a_2(x)y(x) = F(x).$$

with IC $y(0) = C_0$ & $y'(0) = C_1$ can be transformed into non-homo. Volterra's IE of 2nd kind.

[GATE-2006]

(Hint: look at Page-27)

(Download Notes at www.pkalika.wordpress.com)

Exercise Questions

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Que (40). Soln of I.E. $y(x) = x - \int_0^x x y(t) dt$,
 $y \in C^1[1, \infty)$ is —

- (a). $y = x(1 - \log x)$ (b). $y = x e^{(x-1/2)}$
 (c). $y = x e^{1/2(1-x^2)}$ (d). $y = x - x(e^{x^2} - e)$

Ans: (c)

Que (41) I.E. $\phi(x) = \lambda \int_0^1 [\sqrt{x+t} - \sqrt{t+x}] \phi(t) dt$ has —

- (a). One characteristic number.
 (b). Two characteristic numbers
 (c). Infinite no. of eigen-value
 (d). No eigen-value.

Que (42) Solution of I.E. $y(x) = 6 - 3x^2 + \int_0^x y(t) dt$
 $y(0) = 0$ is —

- (a). $y(x) = x^2$ (b). $y(x) = 6x$
 (c). $y(x) = \sin x$ (d). $y(x) = x \sin x$

Ans: (b)

Que (43). Eigen-values of following I.E are —

$$\phi(x) = \lambda \left[x \int_0^1 t \phi(t) dt + 2x^2 \int_0^1 \phi(t) dt \right]$$

- (a). $\lambda_1 = -36 - 12\sqrt{10}$, $\lambda_2 = -36 + 12\sqrt{10}$
 (b). $\lambda_1 = -9 - 6\sqrt{10}$, $\lambda_2 = -9 + 6\sqrt{10}$
 (c). $\lambda_1 = -18 - 6\sqrt{10}$, $\lambda_2 = -18 + 6\sqrt{10}$
 (d). No eigen-values.

Ans: (c)

Que (44). Solution of I.E. $y(x) = f(x) + \int_0^1 x^2 y(t) dt$
 is given by — $\phi(x) =$ —

- (a). $\phi(x) = f(x)$ (b). $\phi(x) = x^3$
 (c). $\phi(x) = f(x) + \frac{4}{3} x^2 \int_0^1 x^2 f(x) dx$
 (d). None

Ans: (c)

COV : Collection of Problem from NET

P. Kalika

(1). The variational problem of extremizing the functional

$$I(y(x)) = \int_0^{2\pi} \left[\left(\frac{dy}{dx} \right)^2 - y^2 \right] dx, \quad y(0) = 1 = y(2\pi)$$
 has —

- (a). Unique solⁿ (c). Infinite no. of solⁿ
 (b). exactly two solⁿ (d). No solution

JUNE
NET-2011

(2). Suppose any two pts. $P(x_0, y_0)$ & $Q(x_1, y_1)$ and $F(x, y, y')$ of 3 independent variables are given, where $y' = \frac{dy}{dx}$. In order to find among all curve $y = y(x)$ joining P & Q that one which furnished for the definite Integral $I(y) = \int_{x_0}^{x_1} F(x, y, y') dx$. The smallest value. w.o.t.f assumption suffices —

- (a). Function F is of class C^1
 (b). Functional F is of class C^2 for all system of values x, y, y' furnished by all of the admissible fⁿs.
 (c). — — — — — C^3 — — — — —
 (d). It is enough to treat y & F to be of class C^1 only with r.t. their arguments.

NET-DEC-2011

(3) The variational problem of extremizing the functional $I(y(x)) = \int_1^3 y(3x - y) dx$; $y(3) = 4\frac{1}{2}$, $y(1) = 1$ has —

- (a). Unique solⁿ (c). Infinite no. of solⁿ
 (b). exactly two solⁿ (d). No solution

NET-JUNE-2012

(4). Let $J(u) = \int_0^1 \left(u_x^2 + 4 \frac{u^2}{x^2} \right) x dx$ where $u(x)$ is smooth fⁿ on $[0, 1]$ satisfying $u(0) = 0$ & $u(1) = 1$. w.o.t.f minimizes J ?

- (a). $u(x) = x^2$ (c). $u(x) = \frac{x^2}{2}$
 (b). $u(x) = \frac{x^2}{\sqrt{2}}$ (d). $u(x) = \frac{x^2}{4}$

NET-DEC-2012

(5) Consider the functional $I(y) = \int_a^b F(x, y, y') dx$; $y(a) = y_1$, $y(b) = y_2$, $y \in C^1[a, b]$ & F has 2nd order P.D. w.r.t y & y' and y_1, y_2 are given real no.s. Let $y = y(x)$ be an extremizing fⁿ for the functional I . Then along the extremizing curve —

(a) F remains constant (c) $F - y' \frac{\partial F}{\partial y'} = \text{const.}$

(b) $\frac{\partial F}{\partial y} = 0$ (d) $F - y' \frac{\partial F}{\partial y'} = \text{const.}$

NET-June-2013

(6) Consider the functional $J(y) = \int_a^b F(x, y, y') dx$, $F = y' + y$ for admissible fⁿ, then J has —

(a) No extremals (c) $y(x) = e^x$ as an extremal

(b) Several extremals (d) $y = \text{const}$ " " " "

NET-DEC-2013

(7) The curve extremizing the fⁿ $I(y(x)) = \int_1^2 \frac{1+y'^2}{x} dx$, $y(1) = 0$, $y(2) = 1$

(a) a ellipse (c) a circle

(b) a parabola (d) a st. line

NET-JUNE-2014

(8) Consider the functional $J(y) = y^2(1) + \int_0^1 y'^2 dx$, $y(0) = 1$, $y \in C^2[0, 1]$. If y extremize J then —

(a) $y(x) = 1 - \frac{1}{2}x^2$

(c) $y(x) = 1 + \frac{1}{2}x$

(b) $y(x) = 1 - \frac{1}{2}x$

(d) $y(x) = 1 + \frac{1}{2}x^2$

NET-DEC-2014

(9) The functional $I(y(x)) = \int_a^b (y^2 + y'^2 - 2y \sin x) dx$ has the following extremal with C_1 and C_2 as arbitrary constants —

(a) $y = C_1 e^{2x} + C_2 e^{-2x} + \frac{1}{2} \sin x$ (c) $y = C_1 e^x + C_2 e^{-x} + \frac{1}{2} \sin x$

(b) $y = C_1 e^x + C_2 e^{-x} + \frac{1}{2} \sin x$ (d) $y = C_1 e^{2x} + C_2 e^{-2x} + \frac{1}{2} \cos x$

NET-DEC-2015

(10) The extremal of the functional $I = \int_0^{2\pi} y^2(y')^2 dx$ that passes through $(0, 0)$ & (π, y) is —

(a) a constant fⁿ (c) part of a parabola

(b) a linear a fⁿ of x (d) parts of an ellipse.

NET-JUNE-2015

Que (1). The 4th order R-K Method given by $u_{j+1} = u_j + \frac{h}{6} [K_1 + 2K_2 + 2K_3 + K_4]$, $j = 0, 1, 2, \dots$ is used to solve the IVP $u' = u$, $u(0) = \alpha$, if $u(1) = 1$ is obtained by taking step size $h=1$, then the value of K_4 is

Que (2). Using Euler's Method, taking step size 0.1, the approximate value of y obtained corresponding to $x=0.2$ for IVP $\frac{dy}{dx} = x^2 + y^2$ and $y(0) = 1$ is —

- (A). 1.322 (B). 1.122 (C). 1.222 (D). 1.110

Que (3). Consider the IVP $\frac{dy}{dx} = f(x, y)$, $y(x_0) = y_0$. The aim is to compute the value of $y_1 = y(x_1)$, where $x_1 = x_0 + h$ ($h > 0$). At $x = x_1$, if the value of y_1 is equal to the corresponding value of st. line passing through (x_0, y_0) and having the slope equal to the slope of the curve $y(x)$ at $x = x_0$, then the method is called —

- (A). Euler's Method (B). Improved Euler's Method
(C). Backward Euler's Method (D). Taylor Series method of order

Que (4). Consider the IVP $\frac{dy}{dx} = f(x, y)$, $y(x_0) = y_0$. Let $y_1 = y_0 + w_1 k_1 + w_2 k_2$ approximate the solution of the given IVP at $x_1 = x_0 + h$ with $k_1 = hf(x_0, y_0)$, $k_2 = hf(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2})$ and h being the step-size. If the formula for y_1 yields a second order method, then the value of w_1 is —

- (A). -1 (B). -2 (C). 3 (D). $\frac{1}{6}$

Answer: (1). 1.017 (2). C (3). A (4). B



Que(5). Consider the IVP $\frac{dy}{dx} = x+y$, $y(0)=1$.

(NET 2013D)

Then the approximate value of the solⁿ $y(x)$ at $x=0.2$, using improved Euler method with $h=0.2$ is —

- (a) 1.11 (b) 1.20 (c) 1.24 (d) 1.48

Que(6) Using R-K Method of order 4, Find an approximate value of y at $x=0.2$, if $\frac{dy}{dx} = x+y^2$, $y(0)=1$.

Que(7). Solve the DE $\frac{dy}{dx} = 1-y$ with I.C $y(0)=0$. Using Euler's method & find solⁿ at $x=0.1, 0.2, 0.3$.

Ans:

x	0	0.1	0.2	0.3
sol ⁿ	0	0.1	0.19	0.271

Que(8). Find the value of y at $x=0.1, 0.2$ by using R-K Method for $y' = -y$, $y(0)=1$.

(a) of 2nd order

(b) of 4th order R-K Method

Ans: (a)

x	0.1	0.2
y	0.905	0.819

(b)

x	0.1	0.2
y	0.965	0.887

Que(9). Given $\frac{dy}{dx} = x^2+y$, $y(0)=1$, Evaluate $y(0.02)$ & $y(0.04)$

By Euler's Method

(Ans: 1.0202, 1.0408, 1.0619)

Que(10). Using R-K method of 4th order, Solve $\frac{dy}{dx} = \frac{y^2-x^2}{y^2+x^2}$ with $y(0)=1$, at $x=0.2$ & 0.4 .

(Ans: 1.296, 1.3752)

Answers: (5). (c), (6). $y(0.1)=1.1165$, $y(0.2)=1.2736$ (7)



- (1). Consider the following Linear Programming Problem. Max $x_1 + \frac{5}{2}x_2$ subject to

$$5x_1 + 3x_2 \leq 15$$

$$-x_1 + x_2 \leq 1$$

$$2x_1 + 5x_2 \leq 10.$$

$$x_1, x_2 \geq 0.$$

The problem

1. has no feasible solution.
2. has infinitely many optimal solutions.
3. has a unique optimal solution.
4. has an unbounded solution.

- (2). Suppose ABC is a triangle on the xy -plane with centroid D. Which of the following points can NEVER be a minimizer of the function $7x - 10y + 1$ as (x, y) runs over the triangle ABC?

1. A
2. B
3. C
4. D

- (3). Consider the variables $x_1 \geq 0$ and $x_2 \geq 0$ satisfying the constraints $x_1 + x_2 \geq 15$, $4x_1 - x_2 \leq 15$ and $4x_2 - x_1 \leq 15$. Which of the following statements is/are correct?

- (1) The maximum value of $3x_1 + 2x_2$ is 25
- (2) The minimum value of $3x_1 + 2x_2$ is 11
- (3) $3x_1 + 2x_2$ has no finite maximum
- (4) $3x_1 + 2x_2$ has no finite minimum

- (4). Consider the LP problem maximize $x_1 + x_2$ subject to

$$x_1 - 2x_2 \leq 10$$

$$x_2 - 2x_1 \leq 10$$

$$x_1, x_2 \geq 0$$

Then

- (1) The LP problem admits an optimal solution
- (2) The LP problem is unbounded
- (3) The LP problem admits no feasible solution
- (4) The LP problem admits a unique feasible solution

- (5). Suppose that the LP problem

$$\text{maximise } c^T x$$

subject to

$$Ax \leq b$$

$$x \geq 0$$

admits a feasible solution and the dual

minimize $b^T y$

subject to

$$A^T y \geq c$$

$$y \geq 0$$

admits a feasible solution y_0 . Then

(1) the dual admits an optimal solution.

(2) any feasible solution x_0 of the primal and y_0

of the dual satisfies $b^T y_0 \leq c^T x_0$.

(3) the dual problem is unbounded

Practice Set by P. Kalika

Assam PSC-19

① If x is a discrete r.v. & $E(ax+b) = aE(x) + b$, where a & b are constants, then the variance $ax+b$ is —

(a). $a \text{Var}(x)$

(b). $a^2 \text{Var}(x)$

(c). $b \text{Var}(x)$

(d). $ab \text{Var}(x)$

Assam PSC-19

NET-17D

② Let X & Y be independent exp. r.v. If $E(X) = 1$ & $E(Y) = 1/2$, then $P(X > 2Y | X > Y)$ is —

(a). $1/2$

(b). $1/3$

(c). $2/3$

(d). $3/4$

Ans: (d)

NET-16D

③ A & B play a game of tossing a fair coin. A starts the game by tossing the coin once & B then tosses the coin twice followed by A tossing the coin once & B tossing the coin twice & hits untill a head turns up. whoever gets the first head wins the game. Then —

(1). $P(B \text{ wins}) > P(A \text{ wins})$

(2). $P(B \text{ wins}) = 2 P(A \text{ wins})$

(3). $P(A \text{ wins}) > P(B \text{ wins})$

(4). $P(A \text{ wins}) = 1 - P(B \text{ wins})$

Ans: 3, 4

NET-16D

④ For any two events A & B, w.o.t.f always holds?

(1). $P^2(A \cap B^c) + P^2(A \cap B) + P^2(A^c) \geq 1/3$

(2). $P^2(A \cap B^c) + P^2(A \cap B) + P^2(A^c) = 1/3$

(3). " " " " = 1

(4). " " " " $\leq 1/3$

Ans: 1

NET-2018 D

⑤ Suppose $X \sim \text{Cauchy}(0,1)$. Then the distⁿ of

$$\frac{1-X}{1+X} \text{ is } \underline{\hspace{2cm}}$$

(1). Uniform (0,1)

(2). Normal (0,1)

(3). Double exp. (0,1)

(4). Cauchy (0,1)

NET-2018 J

⑥ In a data set with mean 2.5 & std. deviation 0.5. The median must be

(1) bigger than 2.5

(2) smaller than 2.5

(3) smaller than 3

(4) bigger than 2

Ans: 3, 4

NET-2018 J

⑦ Let X & Y be two random variables satisfying for $x, y \geq 0$, $E(X) = 3$, $V(X) = 9$, $E(Y) = 2$ and $V(Y) = 4$. w.o.t.f statements are correct?

(1). $0 \leq \text{Cov}(X, Y) \leq 4$ (2). $E(XY) \leq 6$ (3). $V(X+Y) \leq 25$ (4). $E(X+Y)^2 \geq 25$

Ans: 3, 4

NET-2018 J

⑧ Let X & Y be two r.v with joint p.d.f -

$$f(x, y) = \begin{cases} \frac{1}{\pi} & : 0 \leq x^2 + y^2 \leq 1 \\ 0 & : \text{o.w} \end{cases}$$

Then

w.o.t.f statements are true?

(1). X & Y are independent(2). $P(X > 0) = 1/2$ (3). $E(Y) = 0$ (4). $\text{Cov}(X, Y) = 0$

Ans: 2, 3, 4

(By: P. Kalika (maths.whisperer@gmail.com))

NET-18J

(9) A std. fair die is rolled until some face other than 5 or 6 turns up. Let X denote the face value of the last roll and $A = \{X \text{ is even}\}$ & $B = \{X \text{ is at most 2}\}$. Then—

P. Kalika

(1). $P(A \cap B) = 0$

(2). $P(A \cap B) = 1/6$

(3). $P(A \cap B) = 1/4$

(4). $P(A \cap B) = 1/3$

Ans: 3

NET-18J

(10) A simple random sample of size n will be drawn from a class of 125 students, and the mean maths score of the sample will be computed. If the std. error of the sample mean for "with replacement sampling" is twice as much as the std. error of the sample mean for "without replacement" sampling the value of n is —

(1). 32

(2). 63

(3). 79

(4). 94

Ans:

NET
2011-June

(11) w.o.t.f is/are C.d.f of random variables —

(1). $F_1(x) = \begin{cases} 0 & : x \leq 0 \\ e^{-x} & : x > 0 \end{cases}$

(2). $F_2(x) = \begin{cases} 0 & : x \leq 0 \\ 1 - e^{-x} & : x > 0 \end{cases}$

(3). $F_3(x) = \begin{cases} 0 & : x \leq 0 \\ 1 & : x > 0 \end{cases}$

(4). $F_4(x) = \begin{cases} 0 & : x < 0 \\ x/2 & : 0 \leq x < 1 \\ 1 & : x \geq 1 \end{cases}$

NET-2011J

(12) Let X be a r.v taking values in set E . Let $P(X > a+b | X > a) = P(X > b) \forall a, b \in E$. Then w.o.t.f is a possible distⁿ of X ?

Ans: 1

(1). Poisson

(2). Geometric

(3). Log-normal

(4). Exponential.

Ans:

GATE 2019

GATE 2018

P. Kalik



- (1). Five numbers 10, 7, 5, 4 and 2 are to be arranged in a sequence from left to right following the directions given below:
1. No two odd or even numbers are next to each other.
 2. The second number from the left is exactly half of the left-most number.
 3. The middle number is exactly twice the right-most number.

Which is the second number from the right?

- (A) 2 (B) 4 (C) 7 (D) 10

- (2). Forty students watched films A, B and C over a week. Each student watched either only one film or all three. Thirteen students watched film A, sixteen students watched film B and nineteen students watched film C. How many students watched all three films?

- (A) 0 (B) 2 (C) 4 (D) 8

- (3). Let X and Y have joint probability density function given by

$$f_{X,Y}(x,y) = \begin{cases} 2, & 0 \leq x \leq 1-y, \quad 0 \leq y \leq 1 \\ 0, & \text{otherwise.} \end{cases}$$

If f_Y denotes the marginal probability density function of Y , then $f_Y(1/2) =$ _____.

- (4). Let the cumulative distribution function of the random variable X be given by

$$F_X(x) = \begin{cases} 0, & x < 0, \\ x, & 0 \leq x < 1/2, \\ (1+x)/2, & 1/2 \leq x < 1, \\ 1, & x \geq 1. \end{cases}$$

Then $\mathbb{P}(X = 1/2) =$ _____.

- (5). Let $\{X_j\}$ be a sequence of independent Bernoulli random variables with $\mathbb{P}(X_j = 1) = 1/4$ and let $Y_n = \frac{1}{n} \sum_{j=1}^n X_j^2$. Then Y_n converges, in probability, to _____.

(6).

Let X be the number of heads in 4 tosses of a fair coin by Person 1 and let Y be the number of heads in 4 tosses of a fair coin by Person 2. Assume that all the tosses are independent. Then the value of $\mathbb{P}(X = Y)$ correct up to three decimal places is 273.

(7).

Let X_1 and X_2 be independent geometric random variables with the same probability mass function given by $\mathbb{P}(X = k) = p(1 - p)^{k-1}$, $k = 1, 2, \dots$. Then the value of $\mathbb{P}(X_1 = 2 | X_1 + X_2 = 4)$ correct up to three decimal places is 13.

(8).

An urn contains four balls, each ball having equal probability of being white or black. Three black balls are added to the urn. The probability that five balls in the urn are black is

- (A) $2/7$ (B) $3/8$ (C) $1/2$ (D) $5/7$

GATE 2018

(9).

An unbiased coin is tossed six times in a row and four different such trials are conducted. One trial implies six tosses of the coin. If H stands for head and T stands for tail, the following are the observations from the four trials:

- (1) HTHTHT (2) TTHHHT (3) HTTHHT (4) HHHT__ __.

Which statement describing the last two coin tosses of the fourth trial has the highest probability of being correct?

- (A) Two T will occur.
(B) One H and one T will occur.
(C) Two H will occur.
(D) One H will be followed by one T.

GATE 2018



- (1). An urn has 3 red and 6 black balls. Balls are drawn at random one by one without replacement. The probability that second red ball appears at the fifth draw is
1. $\frac{1}{9!}$
 2. $\frac{4!}{9!}$
 3. $4 \left(\frac{6!4!}{9!} \right)$
 4. $\frac{6!4!}{9!}$
- (2). From the six letters A, B, C, D, E and F , three letters are chosen at random with replacement. What is the probability that either the word BAD or the word CAD can be formed from the chosen letters?
1. $\frac{1}{216}$
 2. $\frac{3}{216}$
 3. $\frac{6}{216}$
 4. $\frac{12}{216}$
- (3). Let X be a random variable which is symmetric about 0. Let F be the cumulative distribution function of X . Which of the following statements is always true?
1. $F(x) + F(-x) = 1$ for all $x \in \mathbb{R}$.
 2. $F(x) - F(-x) = 0$ for all $x \in \mathbb{R}$.
 3. $F(x) + F(-x) = 1 + P(X = x)$ for all $x \in \mathbb{R}$.
 4. $F(x) + F(-x) = 1 - P(X = -x)$ for all $x \in \mathbb{R}$.
- (4). Suppose X has density $f(x|\theta) = \frac{1}{\theta} e^{-x/\theta}, x > 0$ where $\theta > 0$ is unknown. Define Y as follows:
 $Y = k$ if $k \leq X < k + 1, k = 0, 1, 2, \dots$
 Then the distribution of Y is
1. normal.
 2. binomial.
 3. Poisson.
 4. geometric.
- (5). Let X and Y be independent exponential random variables. If $E[X] = 1$ and $E[Y] = \frac{1}{2}$ then $P(X > 2Y | X > Y)$ is
1. $\frac{1}{2}$
 2. $\frac{1}{3}$
 3. $\frac{2}{3}$
 4. $\frac{3}{4}$
- (6). Let S be the set of all 3×3 matrices having 3 entries equal to 1 and 6 entries equal to 0. A matrix M is picked uniformly at random from the set S . Then
1. $P\{M \text{ is nonsingular}\} = \frac{1}{8}$
 2. $P\{M \text{ has rank } 1\} = \frac{1}{14}$
 3. $P\{M \text{ is identity}\} = \frac{1}{14}$
 4. $P\{\text{trace}(M) = 0\} = \frac{1}{14}$

Practice / Asked Que - Set

Assam PSC-19

① Consider a Markov chain with state space $\{0, 1, 2\}$ and the transition prob. matrix

The the period of 0 is -

$$P = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

- (A) . 0 (B) . 1
(C) . 2 (D) . None

Ans: 2

NET-17 June

② Let $\{X_n\}$ be a Markov chain on $\{0, 1, 2, \dots\}$ with $[4.75]$ $P_{00} = \frac{2}{3}$, $P_{01} = \frac{1}{3}$, $P_{i,i+1} = \frac{2}{3}$, $P_{i,i-1} = \frac{1}{3}$, $i \geq 1$, $P_{ij} = 0$ otherwise. W.O.T.f statements are correct

- (a) $\{X_n\}$ is recurrent
(b) $\{X_n\}$ is transient.
(c) $P(\lim_{n \rightarrow \infty} X_n = 0) > 0$
(d) $P(\lim_{n \rightarrow \infty} X_n = +\infty) > 0$

Ans: 2, 4

NET-17 June

③. W.O.T.f statements are correct?

- (4.75) (a) For a finite state Markov chain there is at least one transient state.
(b) For a finite state Markov chain there is at least one stationary dist.
(c) For a countable state Markov chain, every state can be transient.
(d) For an aperiodic countable state Markov chain there is at least one stationary dist.

Ans: 2, 3

NET-11D

④. An aperiodic Markov chain with stationary distⁿ prob. on the state space $\{1, 2, 3, 4, 5\}$ must have at least one

- (a) null recurrent state (b) positive recurrent state
(c) true rec. & null rec. (d) transient state.

Ans: (a) (b)

Markov Chain Practice Set - 1

NET
2011-Dec

(5) Let $\pi = (\pi_1, \pi_2, \pi_3, \pi_4)$ be the stationary distⁿ for a Markov chain on the state space $\{1, 2, 3, 4\}$ with transition prob. matrix P . Suppose that states 1 & 2 are transient & states 3 & 4 form a communicating class - w.o.d.f is/are true?

(1) $\pi P^3 = \pi P^5$

(2) $\pi_1 = 0$ & $\pi_2 = 0$

(3) $\pi_3 + \pi_4 = 1$

(4) One of π_3 & π_4 is 0.

Ans: 2, 3

NET
2011-June

(6) What are c.d.f of random

Let $\{X_n\}$ be stationary ~~distⁿ~~ Markov Chain s.t. —
 $P(X_{i+1} = 1 | X_i = 1) = p_1 = 1 - P(X_{i+1} = 0 | X_i = 1)$,
 $P(X_{i+1} = 1 | X_i = 0) = p_0 = 1 - P(X_{i+1} = 0 | X_i = 0)$ & $P(X_i = 1) = \pi_1 = 1 - P(X = 0)$. Then —

(a) $\pi_1 = p_1$

(b) $\pi_1 = p_0$

(c) $\pi_1 = \frac{p_0}{1 - p_0 + p_1}$

(d) $\pi_1 = 1/2$

Ans: All

Markov Chain Practice Set - 2

- (1). Let $(X_n)_{n \geq 0}$ be a Markov chain on the state space $S := \{1, 2, \dots, 23\}$ with transition probability given by

$$p_{i,i+1} = p_{i,i-1} = \frac{1}{2} \quad \forall 2 \leq i \leq 22$$

$$p_{1,2} = p_{1,23} = \frac{1}{2}$$

$$p_{23,1} = p_{23,22} = \frac{1}{2}.$$

Then, which of the following statements are true?

1. $(X_n)_{n \geq 0}$ has a unique stationary distribution.
2. $(X_n)_{n \geq 0}$ is irreducible.
3. $\mathbb{P}(X_n = 1) \rightarrow \frac{1}{23}$.
4. $(X_n)_{n \geq 0}$ is recurrent.

- (4). Consider a Markov chain with five states $\{1, 2, 3, 4, 5\}$ and transition matrix

$$P = \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{7} & 0 & 0 & \frac{6}{7} \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \\ \frac{1}{3} & 0 & 0 & \frac{2}{3} & 0 \\ 0 & \frac{5}{8} & 0 & 0 & \frac{3}{8} \end{pmatrix}$$

Which of the following are true?

1. 3 and 1 are in the same communicating class
2. 1 and 4 are in the same communicating class
3. 4 and 2 are in the same communicating class
4. 2 and 5 are in the same communicating class

- (2). Consider a Markov chain $\{X_n \mid n \geq 0\}$ with state space $\{1, 2, 3\}$ and transition matrix

$$P = \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}. \text{ Then } P(X_3 = 1 \mid X_0 = 1)$$

equals

1. 0
2. $\frac{1}{4}$
3. $\frac{1}{2}$
4. $\frac{1}{8}$

- (3). Consider a Markov Chain with state space $S = \{0, 1, 2, 3\}$ and with transition probability matrix P given by

$$P = \begin{matrix} & \begin{matrix} 0 & 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \end{matrix} & \begin{pmatrix} 2/3 & 0 & 1/3 & 0 \\ 1 & 0 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 \\ 1/4 & 1/4 & 1/4 & 1/4 \end{pmatrix} \end{matrix}$$

Then

1. 1 is a recurrent state.
2. 0 is a recurrent state.
3. 3 is a recurrent state.
4. 2 is a recurrent state.