HYDERABAD CENTRAL UNIVERSITY (HCU)
M.Sc. Mathematics Entrance - 2016

Time : 2 Hours **Max. Marks: 100**

Instructions:

(*i*) There are a total of **50** questions in **Part-A** and **Part-B** together.

(*ii*) There is a negative marking in **Part-A**. Each correct answer carries **1 mark** and each wrong answer carries **–0.33 mark**. Each question in **Part-A** has only one correct option.

(*iii*) There is **no negative** marking in **Part-B**. Each correct answer carries **3 marks**. In **Part-B** some questions have **more than** one correct option. All the correct options have to be marked in OMR sheet other wise **zero marks** will be credited.

PART-A

1. Which of the following is an uncountable subset of $\mathbb{R}^2$?

(a) $\{(x,y)\in\mathbb{R}^2 : x\in\mathbb{Q} \text{ or } x+y\in\mathbb{Q}\}$
(b) $\{(x,y)\in\mathbb{R}^2 : x\in\mathbb{Q} \text{ and } y\in\mathbb{Q}\}$
(c) $\{(x,y)\in\mathbb{R}^2 : x\in\mathbb{Q} \text{ or } y\in\mathbb{Q}\}$
(d) $\{(x,y)\in\mathbb{R}^2 : x\in\mathbb{Q} \text{ or } y^2\in\mathbb{Q}\}$

2. Which of the following is an unbounded subset of $\mathbb{R}^2$?

(a) $\{(x,y)\in\mathbb{R}^2 : x^2+y^2\le 1\}$
(b) $\{(x,y)\in\mathbb{R}^2 : x+y\le 1\}$
(c) $\{(x,y)\in\mathbb{R}^2 : |x|+|y|\le 1\}$
(d) $\{(x,y)\in\mathbb{R}^2 : |x|+y^2\le 1\}$

3. Let $f:\mathbb{R}\to\mathbb{R}$ be a twice differentiable function. Then which of the following is true?

(a) If $f(0)=0=f''(0)$ then $f'(0)=0$
(b) f is a polynomial
(c) f' is continuous
(d) If $f''(x)>0$ for all x in $\mathbb{R}$ then $f(x)>0$ for all x in $\mathbb{R}$

4. The non-zero values for x_0 and x_1 such that the sequence defined by the recurrence relation $x_{n+2}=2x_n$, is convergent are

(a) $x_0=1$ and $x_2=1$
(b) $x_2=1/2$ and $x_1=1/4$
(c) $x_0=1/10$ and $x_1=1/20$
(d) none of the above

5. The set of all values of a for which the series $\sum_{n=1}^{\infty}\frac{a^n}{n!}$ converges is

(a) $(0,\infty)$ (b) $(-\infty,0]$ (c) $(-\infty,\infty)$ (d) $(-1,1)$

6. Consider $f(x)=\begin{cases}|x|, & \text{if } -1\le x\le 1,\\ x^2, & \text{otherwise}\end{cases}$. Then

(a) f is not continuous at 0 (b) f is not continuous at 1
(c) f is not continuous at –1 (d) f is continuous at all points

7. Consider the statement S: "Not all students in this class are tall". The statement S means
(a) All students in this class are short (b) All short students are in this class
(c) At least one student in this class is not tall (d) No short student is in this class

8. A subset S of $\mathbb{N}$ is infinite if and only if
(a) S is not bounded below (b) S is not bounded above
(c) $\exists n_0\in\mathbb{N}$ such that $\forall n\ge n_0, n\in S$ (d) $\forall a\in S, \exists x\in\mathbb{N}$ such that $x<a$

9. In a cyclic group of order 35, the number of elements of order 35 is
(a) 1 (b) 4 (c) 6 (d) 24

10. Let A be an $n\times n$ matrix. Consider the following statements
S_1 : If the rank of A is equal to n then the rank of the adjoint matrix of A is also equal to n.
S_2 : If the rank of A is equal to $n-2$ then the rank of the adjoint matrix of A is also equal to $n-2$.
Pick up a true statement from the following
(a) S_1 and S_2 are true (b) S_1 is true but not S_2
(c) S_2 is true but not S_1 (d) Neither S_1 nor S_2 is true

11. The locus of a point P which is at the same distance from two planes $x+y+z=1, z=0$ is
(a) an unbounded set (b) a sphere
(c) a pair of parallel planes (d) a pair of intersecting planes

12. The value of the integral $\oint_C (x^3+x)dx+(1+y^2+y^3)dy$, where

$C=\{(x(t),y(t))/x(t)=2+3\cos t, y(t)=5+4\sin t, 0\le t<2\pi\}$, is

(a) 0 (b) π (c) 10π (d) 12π

13. Let V be the region which is common to the solid sphere $x^2+y^2+z^2\le 1$ and the solid cylinder $x^2+y^2\le 0.5$.
Let ∂V be the boundary of V and $\hat{n}$ be the unit outward normal drawn at the boundary. Let
$\vec{F}=(y^2+z^2)\hat{i}+(z^2-2x^2)\hat{j}+(x^2+2y^2)\hat{k}$. Then the value of $\iint_{\partial V}\vec{F}\cdot\hat{n}dS$ is equal to

(a) 0 (b) 1 (c) –1 (d) π

14. Let $R([a,b])$ be the set of all Riemann integrable functions on $[a,b]$. Consider the following statements.

S_1 : $f \in R([a,b])$ whenever there exist, $g, h \in R([a,b])$ such that $g \le f \le h$

S_2 : $f \in R([a,b])$ whenever there exist two continuous functions g, h on $[a,b]$ such that $g \le f \le h$

Which of the following statements is true?

(a) S_1 and S_2 are true
(b) S_1 is true but not S_2
(c) S_2 is true but not S_1
(d) Neither S_1 nor S_2 is true

15. The shortest distance from the sphere $x^2+y^2+z^2-2x-4y-6z+11=0$ to the plane $x+y+z=3$ is equal to

(a) $\sqrt{3}$
(b) $2\sqrt{3}$
(c) $3\sqrt{3}$
(d) $4\sqrt{3}$

16. A solution of $(x^2y^2+y^4+2x)dx+2y(x^3+xy^2+1)dy=0$ is

(a) $x^2+\log|x^2-y^2|=\text{constant}$
(b) $x^2y+\log|x^2-y^2|=\text{constant}$
(c) $x^2y+\log(x^2+y^2)=\text{constant}$
(d) $xy^2+\log(x^2+y^2)=\text{constant}$

17. Let $f, g : \mathbb{R} \to \mathbb{R}$, $f(x)=x^2$, $g(x)=\sin x$. Which of the following statements is true?

(a) f and g are uniformly continuous
(b) f and g are not uniformly continuous
(c) f is uniformly continuous but g is not
(d) f is not uniformly continuous but g is uniformly continuous

18. If A and B are square matrices satisfying $AB=BA$, $\det(A)=1$ and $\det(B)=0$ then $\det(A^3B^2+A^2B^3)$ is equal to

(a) –1
(b) 0
(c) 1
(d) 2

19. Which of the following subsets are subspaces of $\mathbb{R}^3$?

(a) $\{(x,y,z)\in\mathbb{R}^3 / 5x-y+z=0\}$
(b) $\{(x,y,z)\in\mathbb{R}^3 / 5x-y+z=-1\}$
(c) $\{(x,y,z)\in\mathbb{R}^3 / x,y,z \text{ are rationals}\}$
(d) $\{(x,y,z)\in\mathbb{R}^3 / x^2+y^2+z^2=1\}$

20. A palindrome is a word which reads the same backward or forward (eg. MADAM, ANNA). The number of palindromes of length 11 (eleven) can be formed from an alphabet of K letters is equal to

(a) K^6
(b) K^5
(c) $\binom{K}{6}$
(d) $\binom{K}{5}$

21. The center of the ring of 2 × 2 matrices over $\mathbb{R}$ is

(a) $\left\{\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} / a,b\in\mathbb{R}\right\}$
(b) $\left\{\begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} / a\in\mathbb{R}\right\}$
(c) $\left\{\begin{pmatrix} 0 & b \\ a & 0 \end{pmatrix} / a,b\in\mathbb{R}\right\}$
(d) $\left\{\begin{pmatrix} 0 & a \\ a & 0 \end{pmatrix} / a\in\mathbb{R}\right\}$

22. The set of units of the Gaussian ring $\{a+ib / a,b \in \mathbb{Z}\}$ is

(a) $\{\pm 1, \pm i\}$ (b) $\mathbb{Z} \cup i\mathbb{Z}$

(c) $\{a+ib / a,b \in \{\pm 1, 0\}\}$ (d) $\mathbb{Z}$

23. Group of automorphisms of $(\mathbb{Z}/10\mathbb{Z}, +)$ is isomorphic to

(a) $(\mathbb{Z}/2\mathbb{Z}) \times (\mathbb{Z}/2\mathbb{Z})$ (b) $\mathbb{Z}/2\mathbb{Z}$

(c) $\mathbb{Z}/4\mathbb{Z}$ (d) $\mathbb{Z}/10\mathbb{Z}$

24. The system of equations $6x_1 - 2x_2 + 2\alpha x_3 = 1$ and $3x_1 - x_2 + x_3 = 5$ has no solution if α is equal to

(a) –5 (b) –1 (c) 1 (d) 5

25. Suppose every collection of three distinct numbers from 1, 2, ..., 9 is equally likely to be selected, then

(a) the probability that the sum of the selected numbers is even is more than the probability that the sum of the selected numbers is odd

(b) the probability that the sum of the selected number is even is equal to the probability that the sum of the selected numbers is odd

(c) the probability that the product of the selected numbers is even is less than 1/2

(d) the probability that the product of the selected numbers is odd is equal to 2/3

PART-B

26. Which of the following are true?

(a) There is a surjective function $f : \mathbb{N} \to \mathbb{N} \times \mathbb{N}$

(b) There is an injective function $f : \mathbb{N} \times \mathbb{N} \times \mathbb{N} \to \mathbb{N}$

(c) There is a bijective function $f : \mathbb{N} \times \mathbb{N} \to \mathbb{N} \times \mathbb{N} \times \mathbb{N}$

(d) There is a bijective function $f : \{0,1\} \times \mathbb{N} \to \mathbb{N} \times \mathbb{N}$

27. Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function such that $f(1) = f(0) + 1$. Then which of the following are true?

(a) f' is constant (b) $f(2) = f(1) + 1$

(c) $f'(x) = 1$ for some x in (0, 1) (d) $|f'(x)| \le 1$ for all x in (0, 1)

28. Let $P(\mathbb{N}) = \{\text{All subsets of } \mathbb{N}\}$. Then which of the following are equivalence relations on $P(\mathbb{N})$?

(a) $A \sim B$ if and only if $|A| = |B|$ (b) $A \sim B$ if and only if $A \cup B = B$

(c) $A \sim B$ if and only if $A \cup B = \mathbb{N}$ (d) $A \sim B$ if and only if $A \cap B \ne \phi$

29. Let $f(x) = x^3 + ax^2 + bx + c$ where a, b, c are real numbers. Suppose $c < 0, a + b + c > -1$ and $a - b + c > 1$. Then which of the following are true?

(a) All roots of $f(x)$ are real

(b) $f(x)$ has one real root and two complex roots

(c) $f(x)$ has two roots in (–1, 1)

(d) $f(x)$ has at least one negative root

30. Suppose $T:\mathbb{R}^3 \to \mathbb{R}^3$ is the map $T((x_1,x_2,x_3)) = (2x_1, x_2, 2x_1)$. Then which of the following is true?

(a) T has only two distinct eigenvalues

(b) $\ker(T) \neq \{(0,0,0)\}$

(c) T has three distinct eigenvalues

(d) Range of T is isomorphic to $\mathbb{R}^2$

31. Consider the following statements

$S_1 : \sum_{n=3}^{\infty} \frac{1}{(\log\log n)^{\log n}}$ is a convergent series

$S_2 : \sum_{n=3}^{\infty} \frac{1}{n^{\log\log\log n}}$ is a convergent series

Which of the following statements are true?

(a) S_1 and S_2 are true

(b) S_1 is true but not S_2

(c) S_2 is true but not S_1

(d) Neither S_1 nor S_2 is true

32. The functions $f(z)$ and $g(z)$ are such that the function

$\vec{F} = (2x + yf(z))\hat{i} + (2y + xf(z))\hat{j} + xyg(z)\hat{k}$

can be written as a gradient of some scalar function

Pick up the possible choices for f and g from the following

(a) $f(z) = z^3$ and $g(z) = 3z^2$

(b) $f(z) = 0$ and $g(z) = 0$

(c) $f(z) = 1$ and $g(z) = 0$

(d) $f = 1$ and $g = z$

33. If β is the radius of the circle of intersection of the sphere $x^2 + y^2 + z^2 - 2x - 4y - 6z + \alpha = 0$ and the plane $x + y + z = 1$, then a relation between α and β is

(a) $3\alpha + 3\beta^2 = 17$

(b) $3\alpha^2 - 3\beta^2 = 17$

(c) $3\alpha^2 - 3\beta^2 = 67$

(d) $3\alpha + 3\beta^2 = 67$

34. Which of the following sequences converge to e?

(a) $\left(1 + \frac{1}{2n}\right)^n$

(b) $\left(2 + \frac{1}{2!} + \frac{1}{3!} + ... + \frac{1}{n!}\right)$

(c) $\left(1 + \frac{1}{n}\right)^n$

(d) $\left(\frac{2n+1}{2n-2}\right)^n$

35. For every pair of continuous functions $f, g : \mathbb{R} \to \mathbb{R}$, which of the following statements are "always" true?

(a) If $f(x) = g(x), \forall x \in \mathbb{Q}$, then $f(x) = g(x), \forall x \in \mathbb{Q}$

(b) $\{x \in \mathbb{R} / f(x) = g(x)\}$ is an open subset of $\mathbb{R}$

(c) The product of f and g is continuous

(d) If $h(x) = \begin{cases} \frac{f(x)}{g(x)}, & \text{for } g(x) \neq 0 \\ 0, & \text{otherwise,} \end{cases}$ then h is continuous

36. Let A and B be two $n\times n$ matrices such that $\text{rank}(A)=n$, $\text{rank}(B)=n-1$. Then which of the following are true?

(a) $\det(A^3)=0$ (b) $\det(B)=0$

(c) $\text{rank}(AB)=n-1$ (d) $\text{rank}(BA)=n-1$

37. Which of the following statements are true?

(a) Every finite group of even order contains at least one element of order 2

(b) If every subgroup of a group is normal then the group is abelian

(c) If G is an abelian group of odd order, then $x\to x^2$ is an automorphism of G

(d) If the elements a, b in a group have finite order then the element ab is also of finite order.

38. For $A\subset\mathbb{R}$, define $\chi_A(x)=\begin{cases}1 & \text{for } x\in A,\\ 0 & \text{for } x\notin A.\end{cases}$ Then χ_A is Riemann integrable over [–1, 1] if

(a) $A=\mathbb{Q}\cap[-1,1]$ (b) $A=[-1,1]\setminus\mathbb{Q}$

(c) $A=\left\{1,\frac{1}{2},\frac{1}{3},\ldots\right\}$ (d) $A=\{\pm 10^{-n}/n\in\mathbb{N}\}$

39. A solution of $(D^2+1)^3y=\sin x$ is

(a) $\sum_{n=1}^{3}(c_nx^{n-1}\sin x+d_nx^{n-1}\cos x)$ for some $c_n,d_n\in\mathbb{R},1\le n\le 3$

(b) $\sum_{n=1}^{3}(c_n\sin^n x+d_n\cos^n x)$ for some $c_n,d_n\in\mathbb{R},1\le n\le 3$

(c) $\sum_{n=1}^{3}(c_n\sin nx+d_n\cos nx)$ for some $c_n,d_n\in\mathbb{R},1\le n\le 3$

(d) none of the above

40. Let $\sum_{n=0}^{\infty}a_n$ be a divergent series of positive terms. Then it follows that

(a) $\sum_{n=0}^{\infty}a_n^2$ is also divergent (b) the sequence (a_n) does not converge to 0

(c) the sequence (a_n) is not bounded (d) $\sum_{n=0}^{\infty}\sqrt{a_n}$ is also divergent

41. Let (a_n) be a sequence where all rational numbers are terms (and all terms are rational). Then

(a) no subsequence of (a_n) converges

(b) there are uncountable many convergent subsequence of (a_n)

(c) every limit point of (a_n) is a rational number

(d) no limit point of (a_n) is a rational number

42. Let $y_1(x)=\sum_{n=1}^{3}c_n\phi_n(x)$ and $y_2(x)=\sum_{n=1}^{3}d_n\psi_n(x)$ be complementary solutions to $P(D)y=0$ and $Q(D)y=0$ respectively, where $P(D)y=(D^3+a_1D^2+a_2D+a_3)y$ and $Q(D)y=(D^3+b_1D^2+b_2D+b_3)y$. Then the general solution of $P(D)Q(D)y=0$ is equal to

(a) $y=\sum_{n=1}^{3}(c_n\phi_n+d_n\psi_n)$

(b) $y=\left(\sum_{n=1}^{3}c_n\phi_n\right)\left(\sum_{n=1}^{3}d_n\psi_n\right)$

(c) $y=\sum_{n=1}^{3}(c_n\phi_n^2+d_n\psi_n^2)$

(d) None of these

43. The number of elements in S_5 whose order is 2 is

(a) 10 (b) 12 (c) 25 (d) 40

44. Consider the following statements

S_1 : There is no polynomial $P(x)$ with integer coefficients such that $P(5)=5$ and $P(9)=7$.

S_2 : If α and β are two old integers then $\alpha^2+\beta^2$ is not a perfect square

Which of the following statements are true?

(a) S_1 and S_2 are true

(b) S_1 is true but not S_2

(c) S_2 is true but not S_1

(d) Neither S_1 nor S_2 is true

45. Let A be an $n\times n$ matrix. If $A^m=0$ for some integer m then which of the following statements are true?

(a) If A is nilpotent then $\det(I+A)=1$

(b) If A is nilpotent then $A^n=0$

(c) If every eigenvalue of A is 0, then A is nilpotent

(d) If A is nilpotent then every eigenvalue of A is 0

46. The number of automorphisms of $\mathbb{Z}[\sqrt{2}]$ is

(a) 1 (b) 2 (c) 4 (d) infinity

47. The kernel of a ring homomorphism from $\mathbb{R}[X]$ to $\mathbb{C}$ defined by $f(X)\to f(3+2i)$ is

(a) $\langle X^2-6X+13\rangle$

(b) $\langle X^2+6X+5\rangle$

(c) $\mathbb{R}[X]$

(d) $\{0\}$

48. Pick up prime elements of the ring of Gaussian integers $G=\{x+iy / x,y\in\mathbb{Z}\}$ from the following

(a) 2 (b) 3 (c) 7 (d) 13

49. A subset S of $\mathbb{N}$ is said to be thick if among any 2016 consecutive positive integers, at least one should belong to S. Which of these subsets are thick?

(a) The set of the geometric progression $\{2,2^2,2^3,\ldots\}$

(b) The set of the arithmetic progression $\{1000,2000,3000,\ldots\}$

(c) $\{n\in\mathbb{N} / n>2016\}$

(d) The set of all composite numbers

50. Three students are selected at random from a class of 10 students among which 4 students know C programming of whom 2 students are experts. If every such selection is equally likely, then the probability of selecting three students such that at least two of them know C programming with at least one out of the two selected being an expert in C programming is

(a) less than 1/4
(b) greater than 1/4 but less than 1/2
(c) greater than 1/2 but less than 3/4
(d) greater than 3/4

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ANSWER KEY

PART-A

1.	(a, c, d)	**2.**	(b)	**3.**	(c)	**4.**	(d)	**5.**	(c)
6.	(d)	**7.**	(c)	**8.**	(b)	**9.**	(d)	**10.**	(b)
11.	(d)	**12.**	(a)	**13.**	(a)	**14.**	(d)	**15.**	(a)
16.	(d)	**17.**	(d)	**18.**	(b)	**19.**	(a)	**20.**	(a)
21.	(b)	**22.**	(a)	**23.**	(c)	**24.**	(c)	**25.**	(b)

PART-B

26.	(a, b, c, d)	**27.**	(c)	**28.**	(a)	**29.**	(a, c, d)	**30.**	(b, c, d)
31.	(c)	**32.**	(a, b, c)	**33.**	(a)	**34.**	(b, c)	**35.**	(a, c)
36.	(b, c, d)	**37.**	(a, c)	**38.**	(c, d)	**39.**	(d)	**40.**	(d)
41.	(b)	**42.**	(a)	**43.**	(c)	**44.**	(a)	**45.**	(a, b, c, d)
46.	(b)	**47.**	(a)	**48.**	(b, c)	**49.**	(b, d)	**50.**	(b)

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(*iii*) There is **no negative** marking in **Part-B**. Each correct answer carries **3 marks**. In **Part-B** some questions have **more than** one correct option. All the correct options have to be marked in OMR sheet other wise **zero marks** will be credited.

PART-A

1. Let A be an $m\times n$ matrix and B be an $n\times m$ matrix. Then which of the following statements is true?

(a) $\text{rank}(AB) > \min(\text{rank}(A), \text{rank}(B))$

(b) $\text{rank}(AB) \le \min(\text{rank}(A), \text{rank}(B))$

(c) $\text{rank}(AB) \le \max(\text{rank}(A), \text{rank}(B)) - \min(\text{rank}(A), \text{rank}(B))$

(d) $\text{rank}(AB) > \max(\text{rank}(A), \text{rank}(B)) - \min(\text{rank}(A), \text{rank}(B))$

2. Let A be an $n\times n$ non-zero matrix where A is not an identity matrix. If $A^2 = A,$ then the eigenvalues of A are given by

(a) 1 and –1 (b) 0 and 1 (c) –1 and 0 (d) 0 and n

3. Let A be a 7 × 5 matrix over $\mathbb{R}$ having at least 5 linearly independent rows. Then the dimension of the null space of A is

(a) 0 (b) 1 (c) 2 (d) at least 2

4. The dimension of the vector subspace W of $M_2(\mathbb{C})$ given by

$W = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a,b,c,d \in \mathbb{C}, a+b=c, b+c=d, c+a=d \right\}$ is equal to

(a) 4 (b) 3 (c) 2 (d) 1

5. If $|a-b| = |c-d|,$ then

(a) $a = b+c-d$ (b) $a = b-c+d$

(c) $a = b+c-d$ and $a = b-c+d$ (d) $a = b+c-d$ or $a = b-c+d$

6. The set of all real numbers x for which there is some positive real number y such that $x < y$ is equal to

(a) $\mathbb{R}$ (b) the set of all negative real numbers

(c) $\{0\}$ (d) the empty set

7. Let $\hat{n}$ be the unit outward normal to the sphere of radius α in $\mathbb{R}^3$. Then the value of the integral $\int \vec{r} \cdot \hat{n} dS$ evaluated on the sphere is equal to

(a) $\frac{4}{3}\pi\alpha^3$ (b) $4\pi\alpha^2$ (c) $\frac{4}{3}\pi\alpha^2$ (d) $4\pi\alpha^3$

8. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, $r = \sqrt{x^2 + y^2 + z^2}$ and $n \in \mathbb{N}$ then ∇r^n is equal to

(a) $nr^{n-1}\vec{r}$ (b) $(n-1)r^{n-2}\vec{r}$ (c) $nr^{n-2}\vec{r}$ (d) $(n-1)r^n\vec{r}$

9. The value of the integral $\int_C \left(\frac{-y}{x^2 + y^2} dx + \frac{x}{x^2 + y^2} dy \right)$ where C is the circle with radius α centered at the origin is equal to

(a) 0 (b) $\frac{\pi}{2}$ (c) 2π (d) $2\pi\alpha$

10. The volume of the cube whose two faces lie on the planes $6x - 3y + 2z + 1 = 0$ and $6x - 3y + 2z + 4 = 0$ is equal to

(a) 27 (b) $\frac{27}{343}$ (c) $\frac{3}{7}$ (d) 10

11. The number of common tangent planes to the spheres $(x+2)^2 + y^2 + z^2 = 1$, $(x-2)^2 + y^2 + z^2 = 1$ passing through the origin is equal to

(a) 0 (b) 1 (c) 2 (d) none of these

12. Let c be an arbitrary non-zero constant. Then the orthogonal family of curves to the family $y(1-cx) = 1 + cx$ is

(a) $3y - y^3 + 3x^2 =$ constant (b) $3y + y^3 - 3x^2 =$ constant

(c) $3y - y^3 - 3x^2 =$ constant (d) $3y + y^3 + 3x^2 =$ constant

13. Consider the following two statements.

S_1 : If (a_n) is any real sequence, then $\left(\frac{a_n}{1 + |a_n|} \right)$ has a convergent subsequence

S_2 : If every subsequence of (a_n) has a convergent subsequence, then (a_n) is bounded.

Which of the following statements is true?

(a) Both S_1 and S_2 are true (b) Both S_1 and S_2 are false

(c) S_1 is false but S_2 is true (d) S_1 is true but S_2 is false

14. The largest interval I such that the series $\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$ converges whenever $x \in I$ is equal to

(a) $[-1, 1]$ (b) $[-1, 1)$ (c) $(-1, 1]$ (d) $(-1, 1)$

15. Let $\sum a_n$ be a convergent series. Let $b_n = a_{n+1} - a_n$ for all $n \in \mathbb{N}$. Then

(a) $\sum b_n$ should also be convergent and $(b_n) \to 0$ as $n \to \infty$

(b) $\sum b_n$ need not be convergent but $(b_n) \to 0$ as $n \to \infty$

(c) $\sum b_n$ is convergent but (b_n) need not tend to zero as $n \to \infty$

(d) none of the above statements is true

16. Consider the real sequences (a_n) and (b_n) such that $\sum a_n b_n$ converges. Which of the following statements is true?

(a) If $\sum a_n$ converges, then (b_n) is bounded (b) If $\sum b_n$ converges, then (a_n) is bounded

(c) If (a_n) is bounded, then (b_n) is converges (d) If (a_n) is unbounded, then (b_n) bounded

17. If $f : \mathbb{R} \to \mathbb{R}$ and $\lim_{h \to 0}(f(x+h) - f(x-h)) = 0$ for all $x \in \mathbb{R}$, then

(a) f need not be continuous (b) f is continuous but not differentiable

(c) f is differentiable but f' need not be continuous (d) f is differentiable and f' is continuous

18. If $f : [0,1] \to \mathbb{R}$ is continuous and $f(1) < f(0)$, then

(a) $f([0,1]) \subseteq [f(1), f(0)]$ (b) $f([0,1]) \supseteq [f(1), f(0)]$

(c) $f([0,1]) = [f(1), f(0)]$ (d) $f([0,1])$ need not be a closed interval

19. Consider $f : [-1, 2] \to \mathbb{R}$ defined by $f(x) = \begin{cases} -x, & \text{if } -1 \le x \le 0 \\ 2x^3 - 4x^2 + 2x, & \text{if } 0 < x \le 2 \end{cases}$. Then the maximum value of $f(x)$ is equal to

(a) 0 (b) 2 (c) 4 (d) 10

20. The function e^x from $\mathbb{R}$ to $\mathbb{R}$ is

(a) both one-one and onto (b) one-one but not onto

(c) onto but not one-one (d) neither one-one nor onto

21. The number of elements of order 6 in a cyclic group of order 36 is equal to

(a) 2 (b) 3 (c) 4 (d) 6

22. Consider the following two statements

S_1 : There cannot exist an infinite group in which every element has a finite order.

S_2 : In a group G if $a \in G,\ a^7 = e$ and $a^9 = e,$ then $a = e$

Which of the following statements is true?

(a) Both S_1 and S_2 are true
(b) Both S_1 and S_2 are false
(c) S_1 is false but S_2 are true
(d) S_1 is true but S_2 are false

23. Let R be a commutative ring with unity and $1 \neq 0$. Let a be a nilpotent element, x be a unit. Then

(a) $1+a$ is not a unit
(b) $a-x$ is a nilpotent element
(c) $x+a$ is a unit
(d) none of the above statements is true

24. Let R be a commutative ring with unity. Consider the following two statements.

S_1 : If for any $a \in R, a^2 = 0$ implies $a = 0$ then R does not have non zero nilpotent elements.

S_2 : If A and B are two ideals of R with $A+B=R$ then $A \cap B = AB$

Then which of the following statements is true?

(a) Both S_1 and S_2 are true
(b) Both S_1 and S_2 are false
(c) S_1 is false but S_2 are true
(d) S_1 is true but S_2 are false

25. In how many ways can one place 8 identical balls in 3 different boxes so that no box is empty?

(a) 8 (b) 28 (c) 36 (d) 21

PART-B

26. The projection of the point (11, –1, 6) onto the plane $3x+2y-7z-51=0$ is equal to

(a) (14, 1, –1) (b) (4, 2, –5) (c) (18, 2, 1) (d) none of these

27. The projection of the straight line $x-y-z=0$ and $2x+3y+z=5$ onto the yz-plane is

(a) $5y=-3z+5$ and $x=0$
(b) $y=3z+5$ and $x=0$
(c) $y=z+5$ and $x=0$
(d) $y=-z+5$ and $x=0$

28. The number of spheres of radii $\sqrt{2}$ such that the area of each circle of intersection with the three coordinate planes is π is equal to

(a) 1 (b) 3 (c) 4 (d) 8

29. If all blind horses are white then it follows that

(a) no blind horse is black
(b) no brown horse is blind
(c) all white horses are blind
(d) all horses are blind and white

30. The set of all real roots of the polynomial $P(x)=x^4-x$ is

(a) {0, 1}
(b) the set of roots of (x^2-x)
(c) a set having four elements
(d) an infinite set

31. Let $f, g : \mathbb{R} \to \mathbb{R}$ be polynomials. Then which of the following is false?

(a) If $f(x) = g(x)$ for all $x \in [0,1]$ then $f = g$

(b) If $f\left(\frac{1}{n}\right) = g\left(\frac{1}{n}\right)$ for all $n \in \mathbb{N}$ then $f = g$

(c) If $f(x) \le g(x)$ for all $x \in \mathbb{R}$ then $\text{degree}(f) \le \text{degree}(g)$

(d) If $\{x \in \mathbb{R} : f(x) = 0\} = \{x \in \mathbb{R} : g(x) = 0\}$ then $f = g$

32. If the graph of the function $y = f(x)$ is symmetrical about the line $x = a$, then

(a) $f(x) = f(-x)$
(b) $f(x+a) = f(-x-a)$
(c) $f(x+a) = f(a-x)$
(d) $f(2a-x) = f(x)$

33. Consider the following two statements

S_1 : There exists a linear transformation $T : \mathbb{R}^5 \to \mathbb{R}^2$ such that T is onto and $\text{Ker}(T) = \{(x_1, x_2, x_3, x_4, x_5) : x_1 + x_2 + x_3 = 0\}$

S_2 : For every linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ there exists $\mu \in \mathbb{R}$ such that $T - \mu I$ is invertible

Which of the following statements are true?

(a) Both S_1 and S_2 are true
(b) Both S_1 and S_2 are false
(c) S_1 is false but S_2 is true
(d) S_1 is true but S_2 is false

34. The set $S = \{-1, 1\}$ is the set of eigenvalues of the square matrix A, if

(a) $A \pm I \ne 0$, A is a real, orthogonal and symmetric matrix

(b) $A \pm I \ne 0$, A is a symmetric matrix

(c) $A \pm I \ne 0$, $A^2 = I$

(d) $A \pm I \ne 0$, A is a Hermitian matrix

35. If $A \ne 0$ is a 2 × 2 real matrix and suppose $A^2\vec{v} = -\vec{v}$ for all vectors $\vec{v} \in \mathbb{R}^2$, then

(a) –1 is an eigenvalue of A

(b) the characteristic polynomial of A is $\lambda^2 + 1$

(c) the map from $\mathbb{R}^2 \to \mathbb{R}^2$ given by $\vec{v} \to A\vec{v}$ is surjective

(d) det A = 1

36. Consider a linear system of equations $A\vec{x} = \vec{b}$ where A is a 3 × 3 matrix and $\vec{b} \ne 0$. Suppose the rank of the matrix of coefficients $A = (a_{ij})$ is equal to 2 then

(a) there definitely exists a solution to the system of equations

(b) there exists a non-zero column vector $\vec{v}$ in $\mathbb{R}^3$ such that $A\vec{v} = \vec{0}$

(c) if there exists a solution to the system of equations $A\vec{x} = \vec{b}$ then at least one equation is a linear combination of the other two equations

(d) $\det A = 0$

37. Which of the following sets are closed and bounded?

(a) $\{(x,y)\in\mathbb{R}^2 : x^2+y^2=3\}$ (b) $\{(x,y)\in\mathbb{R}^2 : x+y=3\}$

(c) $\{(x,y)\in\mathbb{R}^2 :|x|+|y|\le 3\}$ (d) $\{(x,y)\in\mathbb{R}^2 : \max\{|x|,|y|\}\le 3\}$

38. Let $\ell\in\mathbb{R}$, and (a_n) be a real sequence. Then which of the following conditions is equivalent to '$(a_n)\to\ell$ as $n\to\infty$' ?

(a) $\forall_{\in}>0, \exists n_0\in\mathbb{N}$ such that $|a_n-\ell|<2\in$ whenever $n\ge n_0$

(b) $\forall_{\in}>0, \exists n_0\in\mathbb{N}$ such that $|a_n-\ell|<\in$ whenever $n\ge 2n_0$

(c) $\forall_{\in}>0, \exists n_0\in 3\mathbb{N}$ such that $|a_n-a_m|<2\in$ whenever $m,n\ge n_0$

(d) $\forall_{\in}>0, \exists n_0\in\mathbb{N}$ such that $|a_n-a_m|<2\in$ whenever $m,n\ge n_0$

39. Which of the following series converge?

(a) $\sum_{n=1}^{\infty}\left(\frac{\log n}{n^{1+2\in}}\right)$ (b) $\sum_{n=1}^{\infty}\left(\frac{(\log n)^2}{n^{1+2\in}}\right)$ (c) $\sum_{n=1}^{\infty}\left(\frac{n^2+1}{n^3+n}\right)$ (d) $\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^n$

40. Let $f(x)=\begin{cases} x^{3/2}(1-x)^{5/4}, & x\in(0,1) \\ 0, & x\in\mathbb{R}\setminus(0,1)\end{cases}$. Then

(a) f is discontinuous at 0 and 1

(b) f is continuous but not differentiable at 0 and 1

(c) f is differentiable at 0 and 1 but f' is not continuous at 0 and 1

(d) none of the above

41. The value of the integral $\int_0^2 (x-[x^2])dx$ is equal to

(a) $\sqrt{2}+\sqrt{3}+3$ (b) $\sqrt{2}+\sqrt{3}-3$ (c) $\sqrt{2}-\sqrt{3}+3$ (d) $\sqrt{2}-\sqrt{3}-3$

42. Let f and g be real valued functions on [0, 1] which are Riemann integrable. Let $f(x)\le g(x)$ for all $x\in[0,1]$ and $f\left(\frac{1}{2}\right)<g\left(\frac{1}{2}\right)$. The inequality $\int\int dx<\int gdx$ holds if

(a) f and g are continuous in [0, 1]

(b) f is continuous

(c) g is continuous

(d) f and g are continuous in a neighbourhood containing $\frac{1}{2}$

43. The general solution of $y'''-4y''+y'=0$ is

(a) $c_1\sinh^2 x+c_2\cosh^2 x+c_3$ (b) $c_1\sinh 2x+c_2\cosh 2x+c_3$

(c) $c_1\sin 2x+c_2\cos 2x+c_3$ (d) $c_1e^{2x}+c_2e^{-2x}+c_3$

44. Which of the following are solutions of the differential equation $yy'' - (y')^2 + 1 = 0$?

(a) x

(b) $\sin(x+c)$ where c is an arbitrary constant

(c) $\sinh(x+c)$ where c is an arbitrary constant

(d) none of the above

45. Which of the following statements are true?

(a) In a cyclic group of order n, if m divides n, then there exists a unique subgroup of order m.

(b) A cyclic group of order n will have $(n-1)$ elements of order n

(c) In a cyclic group of order 24 there is a unique element of order 2

(d) In the group $(\mathbb{Z}_{12}, +)$ of integers modulo 12 the order of $\overline{5}$ is 12

46. Let G be a finite group with no nontrivial proper subgroups. Then which of the following statements are true?

(a) G is cyclic
(b) G is abelian
(c) G is of prime order
(d) G is non-abelian

47. The equation 5X = 7 (mod 12) has

(a) a unique solution in $\mathbb{Z}$

(b) a unique solution in the set {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11}

(c) a unique solution in the set $\{n, n+1, n+2, n+3, n+4, n+5, n+6, n+7, n+8, n+9, n+10, n+11\}$

(d) no solution in $\mathbb{Z}$

48. Which of the following maps are ring homormorphisms?

(a) $f : \mathbb{Z}_4 \to \mathbb{Z}_{10}, f(x) = 5x$

(b) $f : \mathbb{Z}_5 \to \mathbb{Z}_{10}, f(x) = 5x$

(c) $f : \mathbb{Z}_4 \to \mathbb{Z}_{12}, f(x) = 3x$

(d) $f : \mathbb{Z}_4 \to R, f(x) = xe$ where R is a ring with unity e

49. Let R be a finite commutative ring with no zero divisors then

(a) R is a field
(b) R has a unity
(c) characteristic of R is a prime number
(d) none of the above

50. Each question in a text has 4 options of which only one is correct. Ashok does not know which of the options are correct or wrong in 3 questions. He decides to select randomly the options for these 3 questions independently. The probability that he will choose at least 2 correctly is

(a) more than 0.25
(b) in the interval (0.2, 0.25)
(c) in the interval (1/6, 0.2]
(d) less than 1/6

HYDERABAD CENTRAL UNIVERSITY (HCU)
M.Sc. Mathematics Entrance - 2015

ANSWER KEY

PART-A

1.	(b)	**2.**	(b)	**3.**	(a)	**4.**	(d)	**5.**	(c)
6.	(a)	**7.**	(d)	**8.**	(c)	**9.**	(c)	**10.**	(__)
11.	(__)	**12.**	(c)	**13.**	(a)	**14.**	(b)	**15.**	(a)
16.	(d)	**17.**	(a)	**18.**	(b)	**19.**	(c)	**20.**	(b)
21.	(a)	**22.**	(c)	**23.**	(c)	**24.**	(a)	**25.**	(d)

PART-B

26.	(a)	**27.**	(__)	**28.**	(__)	**29.**	(a, b)	**30.**	(a, b)
31.	(__)	**32.**	(c, d)	**33.**	(c)	**34.**	(a, c)	**35.**	(b, c, d)
36.	(b, c, d)	**37.**	(a, c, d)	**38.**	(a, b, d)	**39.**	(__)	**40.**	(d)
41.	(b)	**42.**	(__)	**43.**	(__)	**44.**	(a, b, c)	**45.**	(a, c, d)
46.	(a, c, b)	**47.**	(b, c)	**48.**	(a, d)	**49.**	(a, b, c)	**50.**	(d)

HYDERABAD CENTRAL UNIVERSITY (HCU)
M.Sc. Mathematics Entrance - 2014

Time : 2 Hours **Max. Marks: 100**

Instructions:

(*i*) There are a total of **50** questions in **Part-A** and **Part-B** together.

(*ii*) There is a negative marking in **Part-A**. Each correct answer carries **1 mark** and each wrong answer carries **–0.33 mark**. Each question in **Part-A** has only one correct option.

(*iii*) There is **no negative** marking in **Part-B**. Each correct answer carries **3 marks**. In **Part-B** some questions have **more than** one correct option. All the correct options have to be marked in OMR sheet other wise **zero marks** will be credited.

PART-A

1. Let $0 \neq \overline{v} \in \mathbb{R}^2$. For $0 \leq \theta < \pi$, let $A = \begin{pmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{pmatrix}$. Then the angle between $\overline{v}$ and $A\overline{v}$ is

(a) $\pi - \theta$ (b) θ (c) $\frac{\pi}{2} - \theta$ (d) 0

2. Let $A \in M_n(\mathbb{R})$. If $A^2 = -I$ (where I is the identity matrix), then

(a) n is even (b) $A = \pm I$

(c) all the eigen values of A are in $\mathbb{R}$ (d) A is a diagonal matrix

3. Let $\overline{f} = (u, v, w)$ be a vector field which is solinoidal. If $\text{curl}(\text{curl}\,\overline{f}) = 0,$ then

(a) $\text{curl}(\overline{f}) = 0$ (b) $\text{grad}(\overline{f} \cdot \overline{f}) = 0$

(c) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 1$ (d) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$

4. Let $\overline{v}, \overline{w} \in \mathbb{R}^3$. Then a sufficient condition for $\overline{v} \times \overline{w} \neq 0,$ is

(a) both $\overline{v}$ and $\overline{w}$ are non-zero

(b) dimension of the linear span of $\{\overline{v}, \overline{w}, \overline{v} \times \overline{w}\}$ is ≥ 2

(c) either $\overline{v}$ or $\overline{w}$ is non-zero

(d) none of the above

5. Let $f:\mathbb{R}\to\mathbb{R}$ be a function such that $|f(x)|\le|x^2|$ for all $x\in\mathbb{R}$. Then

(a) f is continuous but not differentiable at $x=0$

(b) f is differentiable at $x=0$

(c) f is an increasing function

(d) f is a decreasing function

6. The number of points of continuity of the function $f=\begin{cases}|x^2-1| & \text{if } x \text{ is irrational,}\\ 0 & \text{if } x \text{ is rational}\end{cases}$

(a) 0 (b) 1 (c) 2 (d) infinite

7. The number of words formed by permuting the letters L, O, C, K, U, P such that neither 'LOCK' nor 'UP' appears in any such arrangement is

(a) $6!-4!-2!+1$ (b) $6!-5!-3!+2$ (c) $6!-5!-3!+1$ (d) $6!-2!+1$

8. The domain of the real-valued function $f(x)=\dfrac{1}{\sqrt{x+1}}+\dfrac{1}{\sqrt{6-x}}$ is

(a) $(-\infty,\infty)\setminus\{-1,6\}$ (b) $\mathbb{R}$

(c) $(-\infty,6)\cap(-1,\infty)$ (d) $(-1,\ 6)$

9. Let G be a group with identity element e, and N be a normal subgroup. Let the index of N is G be 12, i.e., $[G:N]=12$. Then

(a) $x^{12}=e$ for all $x\in N$

(b) $x^{12}=e$, the identity element in G, for all $x\in G$

(c) $x^{24}\in N$ for all $x\in G$

(d) none of the above

10. If $\{a_n\}$ is a sequence converging to l. Let $b_n=\begin{cases}a_{2n}, & \text{if } n \text{ is odd,}\\ a_{3n}, & \text{if } n \text{ is even}\end{cases}$. Then the sequence $\{b_n\}$

(a) need not converge (b) should converge to 0

(c) should converge to $2l$ or to $3l$ (d) should converge to l

11. Two fair dice 1 red and 1 blue are rolled. The probability that the sum of the numbers that show up on the two dice is a prime number is

(a) 7/18 (b) 7/36 (c) 15/36 (d) 29/72

12. Let V be a vector space of dimension n over $\mathbb{R}$ and $\{v_1,\ldots,v_n\}$ be a basis of V. Let σ be a permutation of the numbers $\{1,\ldots,n\}$, i.e., $\sigma:\{1,\ldots,n\}\to\{1,\ldots,n\}$ is a bijective map. Then the linear transformation defined by $T(v_i)=v_{\sigma(i)}$ is

(a) 1-1 but not onto (b) onto but not 1-1

(c) neither 1-1 nor onto (d) an isomorphism on V

13. Let $\{x_n\}$ and $\{y_n\}$ be two sequences in $\mathbb{R}$ such that $\lim_{n\to\infty} x_n = 2$ and $\lim_{n\to\infty} y_n = -2$. Then

(a) $x_n \geq y_n$ for all $n \in \mathbb{N}$

(b) $x_n^2 \geq y_n$ for all $n \in \mathbb{N}$

(c) there exists an $m \in \mathbb{N}$ such that $|x_n| \leq y_n^2$ for all $n > m$

(d) there exists an $m \in \mathbb{N}$ such that $|x_n| = |y_n|$ for all $n > m$

14. Let $\{x_n\}$ be an increasing sequence of irrational numbers in [0, 2]. Then

(a) $\{x_n\}$ converges to 2

(b) $\{x_n\}$ converges to $\sqrt{2}$

(c) $\{x_n\}$ converges to some number in [0, 2]

(d) $\{x_n\}$ may not converge

15. Let X be a set. For $A \subset X$, let $A^c = X \setminus A$. The correct statement for $A, B \subset X$ is

(a) $A \setminus B = B^c \setminus A^c$, always

(b) If $A \setminus B = B^c \setminus A^c$ then $A \subset B$ or $B \subset A$

(c) If $A \setminus B = B^c \setminus A^c$ then $A \cap B = \phi$

(d) If $A \setminus B = B^c \setminus A^c$ then $A = X$ or $B = X$

16. The value of $\int_0^{2\sqrt{\pi}} |\pi - x^2|\, dx$ is

(a) $2\pi\sqrt{\pi}$

(b) $2\sqrt{\pi}$

(c) 2π

(d) none of the above

17. Let $\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and ϕ be a homogeneous function with degree 3. Then $\text{div}(\bar{r}\phi)$ is

(a) 3ϕ

(b) 6ϕ

(c) 9ϕ

(d) 27ϕ

18. Let $f : [-2, 5] \to \mathbb{R}$ be the function given by $f(x) = x^6 + 3x^2 + 60$. Then

(a) f is a bounded function

(b) there exists a $c \in [-2, 5]$ such that $f(c) = 0$

(c) f is increasing

(d) f is decreasing

19. Write the logical negation of the following statement about a sequence $\{a_n\}$ of real numbers:

"For all $n \in \mathbb{N}$ there exists an $m \in \mathbb{N}$ such that $m > n$ and $a_m \neq a_n$

(a) There exists an $n \in \mathbb{N}$ such that $a_m = a_n$ for all $m > n$

(b) For all $n \in \mathbb{N}$ there exists $m \in \mathbb{N}$ such that $m > n$ and $a_m = a_n$

(c) There exists $n \in \mathbb{N}$ such that $a_m \neq a_n$, for all $m > n$

(d) There exists $n \in \mathbb{N}$ such that $a_m = a_n$, for all $m \leq n$

20. Let G be a group of order 6. Then
    (a) G has 2 possibilities (upto isomorphism)
    (b) G is cyclic
    (c) G is abelian but not cyclic
    (d) there is not sufficient information to determine G

21. The value of $\int_0^1 (1-x+\frac{x^2}{2!}-\frac{x^3}{3!}+...+\frac{(-1)^n x^n}{n!}+...)e^x dx$ is

    (a) 0 (b) e (c) 1 (d) not defined

22. The general solution of the differential equation $y'''-5y''+8y-4=0$ is

    (a) $ae^{2x}-bxe^{2x}-ce^{x}$ (b) $e^{2x}+xe^{2x}-e^{x}$

    (c) $ae^{2x}+be^{x}$ (d) $ae^{x}+b(e^{2x}+xe^{2x})$

23. The number of common tangents to the spheres $x^2+y^2+z^2-2x-4y+6z+13=0$ and $x^2+y^2+z^2-6x-2y+2z-5=0$ is

    (a) 0 (b) 1 (c) 3 (d) 1

24. Let $f(x)=\begin{cases} \frac{\sin x}{x}, & \text{if } x>0 \\ \cos x, & \text{if } x\le 0 \end{cases}$. Then,

    (a) f is continuous but not differentiable at 0
    (b) f is differentiable at 0
    (c) f is not continuous at 0
    (d) f is neither integrable nor continuous 0

25. The least positive integer n such that every integer is greater than n is of the form $2a+11b$ for some positive integers a and b is

    (a) 13 (b) 23 (c) 35 (d) 44

PART-B

26. Let $f(x)=\max\{\sin x,\cos x\}$, for $x\in\mathbb{R}$. Then
    (a) f is discontinuous at $(2n+1)\pi/4, n\in\mathbb{Z}$
    (b) f is continuous everywhere
    (c) f is differentiable everywhere
    (d) f is differentiable everywhere except at $(4n+1)\pi/4, n\in\mathbb{Z}$

27. $\lim_{n\to\infty}\left(\frac{1}{n}+\frac{1}{n+1}+...+\frac{1}{3n}\right)=$

    (a) 0 (b) log 2 (c) log 3 (d) ∞

28. Let $f:\mathbb{R}\to\mathbb{R}$ such that $(f(x))^2 = 1+\int_0^x f(t)dt$. Then $f(x)=$

(a) $\frac{x}{2}-1$ (b) $\frac{x}{2}+1$ (c) $\frac{x}{2}$ (d) $\frac{x}{2}\pm 1$

29. Let $A\neq \pm I$ be a 2 × 2 matrix over $\mathbb{R}$ whose square is *I*. Then which of the following statements are correct

(a) A is a diagonal matrix (b) sum of diagonal elements of A is 0

(c) there are infinitely many such matrices (d) 1 must be an eigen value

30. Let $f:(0,\pi/2)\to\mathbb{R}$ given by $f(x)=\sin x+\cos 2x$ is

(a) increasing in $(0,\pi/4)$ (b) decreasing in $(0,\pi/4)$

(c) has a minimum in $(0,\pi/4)$ (d) has a maximum in $(0,\pi/4)$

31. The numbers 0, 1, 2, ..., 9 are arranged randomly (without repetitions) in a row to get a 10-digit number greater than 10^9. What is the probability that the number so obtained is a multiple of 5?

(a) $\frac{8\times 8!}{9\times 9!}$ (b) $\frac{2\times 9!}{9\times 9!}$ (c) $\frac{2\times 9!}{10!}$ (d) $\frac{8\times 8!}{9\times 9!}+\frac{9!}{9\times 9!}$

32. Let *f*, *g* be Riemann integrable on $[a,b]$. Define, $h(x):=\min(f(x),g(x))$ and $l(x):=\int_a^x f(t)dt$ for $x\in[a,b]$. Then

(a) *h* need not be Riemann integrable but *l* always is

(b) *l* need not be Riemann integrable but *h* always is

(c) *h*, *l* are Riemann integrable, always.

(d) whenever *h* and *l* are Riemann integrable, $\int_a^x h(t)dt\le l(x)$ for all $x\in(a,b)$

33. Let $f:\mathbb{R}\to\mathbb{R}$ be a continuous function. Which of the following are sufficient conditions for *f* to have a fixed point in [0, 1]?

(a) $f(0)=f(1)$ (b) $f(1)<0<f(0)$

(c) $0<f(1)<f(0)$ (d) $f(0)<0<1<f(1)$

34. Let $x_n\in\mathbb{R}$ such that $\sum_{n=1}^{\infty}x_n=-5$. Then

(a) $\lim_{n\to\infty}x_n=0$

(b) there exists an $m\in\mathbb{N}$ such that $x_n\le 0$ for all $n>m$

(c) $\sum_{n=1}^{\infty}|x_n|=5$

(d) $|x_n|\le 5$ for all $n\in\mathbb{N}$

35. Which of the following series are convergent?

(a) $\sum_{n=1}^{\infty} \frac{(-1)^n + \frac{1}{2}}{n}$ (b) $\sum_{n=1}^{\infty} e^{-n} n^2$

(c) $\sum_{n=1}^{\infty} \frac{1+2+...+n}{1^2+2^2+...+n^2}$ (d) $\sum_{n=1}^{\infty} \frac{1.2.3}{4.5.6} + \frac{7.8.9}{10.11.12} + ...$

36. In a class with 200 students, all the students know either Hindi, English or Telugu (and no other language). Of them, 100 know English, 150 know Telugu, 80 know Hindi, 50 know both Telugu and Hindi, 40 know only Telugu and no other language, 10 know all the three languages. Which of the following statements are correct

(a) 30 students know only English and no other language

(b) 110 know atleast two languages

(c) 60 know Hindi but not English

(d) 110 know exactly two languages

37. For a group G, which of the following statements are true?

(a) If $x, y \in G$ such that order of x is 3, order of y is 2 then order of xy is 6.

(b) If every element is of finite order in G then G is a finite group

(c) If all subgroups are normal in G then G is abelian

(d) If G is abelian then all subgroups of G are normal

38. Let V be the vector space of all polynomials with coefficients in $\mathbb{R}$, i.e., $V = \mathbb{R}[X]$. Then which one of the following $T: V \to V$ are not linear transformations: for $f(X)$ in V, define $T(f(x))$ as

(a) $f(X^2)$ (b) $f(X)^2$ (c) $X^2 f(x)$ (d) $f(X^2+1)$

39. Let S, T and U be three sets of horses. Let U be the set of all white horses. If all the horses in the set S are white and if no horse in the set T is black, then it necessarily follows that

(a) S and T are disjoint (b) $S \subset U$

(c) $U \subset T$ (d) $S \cap T = U$

40. For the real number system, which of the following statements are true:

(a) let $x, y \in \mathbb{R}$ such that $0 < x < y$ then there exists an $n \in \mathbb{N}$ such that $y < nx$

(b) let $x, y \in \mathbb{R}$ such that $x < y$ then there exists an $r \in \mathbb{Q}$ such that $x < r < y$

(c) let $x, y \in \mathbb{R}$ such that $x < y$ then there exists an $\alpha \in \mathbb{R} \setminus \mathbb{Q}$ such that $x < \alpha < y$

(d) For $y \in \mathbb{R}$ such that $y > 0$ there exists an $n \in \mathbb{N}$ such that $n \le y < n+1$

41. The value of α such that the sum of the squares of the roots of $x^2 - (\alpha - 2)x - \alpha - 1$ is minimum is

(a) 0 (b) 1 (c) $1/\sqrt{2}$ (d) 4

42. An integrating factor for $ydx + (x - 2x^2y^3)dy = 0$ is

(a) $\frac{1}{x^2+y^2}$ (b) $\frac{1}{x+y}$ (c) $e^{\frac{1}{x^2y^2}}$ (d) $\frac{1}{x^2y^2}$

43. Let $\overline{r} = x\hat{i} + y\hat{j} + z\hat{k}$, which of the following statements are true:

(a) There exists a non constant vector valued function $\overline{f}$ such that $\overline{f}$ is both irrotational and solinoidal

(b) $\text{div}(\text{curl}(\overline{r}\,|\overline{r}|^2) = 0$

(c) $\text{curl}(\text{grad}\,(|\overline{r}|^6)) = 0$

(d) If $\overline{f}$ is solinoidal then $\text{div}(|\overline{r}|^2\ \overline{f}) = 2\overline{r} \cdot \overline{f}$

44. Let B be the unit sphere in $\mathbb{R}^3$. The value of $\iint\limits_B (x^2 + 2y^2 - 3z^2)dS$ is

(a) 4π (b) $\frac{4}{3}\pi$ (c) 6π (d) none of the above

45. Let $A, B \subset [0,1]$ be two uncountable sets. Which of the following are false statements

(a) If $A \cap B = \phi$, then either $\sup(A) \le \inf B$ or $\sup B \le \inf A$

(b) If $A \cap B = \phi$, then $[0,1] \setminus (A \cap B)$ is countable

(c) If $\inf A = \inf B$ and $\sup A = \sup B$, then $A \cap B \ne \phi$

(d) If $A \subset B$, then B\A is countable

46. The value of λ such that the plane $2x - y + \lambda z = 0$ is a tangent plane to the sphere

$x^2 + y^2 + z^2 - 2x - 2y - 2z + 2 = 0$ is

(a) 4 (b) 1 (c) 2 (d) -2

47. The distance between the straight lines

$\frac{x}{2} = \frac{y-1}{1} = \frac{z+1}{1}$ and $\frac{x+1}{2} = \frac{y+1}{1} = \frac{z+1}{1}$ is

(a) $\frac{\sqrt{21}}{3}$ (b) $\frac{21}{9}$

(c) 0 (d) impossible to find from the given data

48. Let V be a 3-dimensional vector space over $\mathbb{C}$. Let $T : V \to V$ be a linear transformation whose characteristic polynomials is $(X-2)(X-1)(X+1)$. Let B be a basis of V. Then which of the following are correct?

(a) The matrix of T w.r.t. B is conjugate to $\begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(b) The matrix of T^{-1} w.r.t. B is conjugate to $\begin{pmatrix} 1/2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

(c) The matrix of T w.r.t. B is conjugate to $\begin{pmatrix} 2 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$

(d) The matrix of T w.r.t. B is conjugate to $\begin{pmatrix} 2 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

49. Let S be the group of all permutation of the letters U, N, I, V, E, R, S, I, T, Y such that the letter ‘I’ is fixed. Then

(a) There exists elements of order 21 and of order 11

(b) There exists an element of order 21 and no element of order 11

(c) There is an element of order 11 and no element of order 21

(d) There are no elements of order 11 or or order 21

50. The least positive integer r such that $\begin{pmatrix} 2014 \\ r \end{pmatrix}$ is a multiple of 10 is

(a) 5 (b) 10 (c) 11 (d) 14

HYDERABAD CENTRAL UNIVERSITY (HCU)
M.Sc. Mathematics Entrance - 2014

ANSWER KEY

PART-A

1.	(c)	**2.**	(a)	**3.**	(d)	**4.**	(b)	**5.**	(b)
6.	(c)	**7.**	(b)	**8.**	(d)	**9.**	(__)	**10.**	(a)
11.	(c)	**12.**	(d)	**13.**	(c)	**14.**	(a)	**15.**	(a)
16.	(b)	**17.**	(b)	**18.**	(a)	**19.**	(a)	**20.**	(a)
21.	(c)	**22.**	(a)	**23.**	(__)	**24.**	(a)	**25.**	(b)

PART-B

26.	(b, d)	**27.**	(c)	**28.**	(a, b, d)	**29.**	(b, c, d)	**30.**	(d)
31.	(d)	**32.**	(c)	**33.**	(b, d)	**34.**	(a)	**35.**	(b)
36.	(c, d)	**37.**	(d)	**38.**	(b)	**39.**	(b)	**40.**	(a, b, c)
41.	(b)	**42.**	(d)	**43.**	(a, b, c, d)	**44.**	(d)	**45.**	(a, b, c, d)
46.	(__)	**47.**	(__)	**48.**	(a, b)	**49.**	(d)	**50.**	(__)

HYDERABAD CENTRAL UNIVERSITY (HCU)
M.Sc. Mathematics Entrance - 2013

Time : 2 Hours **Max. Marks: 100**

Instructions:

(*i*) There are a total of **50** questions in **Part-A** and **Part-B** together.

(*ii*) There is a negative marking in **Part-A**. Each correct answer carries **1 mark** and each wrong answer carries **–0.33 mark**. Each question in **Part-A** has only one correct option.

(*iii*) There is **no negative** marking in **Part-B**. Each correct answer carries **3 marks**. In **Part-B** some questions have **more than** one correct option. All the correct options have to be marked in OMR sheet other wise **zero marks** will be credited.

PART-A

1. We say that a sequence (a_n) does NOT converge to l if

(a) $\forall \in > 0, \forall n_0 \in \mathbb{N}, \forall n \geq n_0$ we have $|a_n - l| > \in$

(b) $\forall \in > 0, \forall n_0 \in \mathbb{N}, \exists n \geq n_0$ such that $|a_n - l| > \in$

(c) $\exists \in > 0, \forall n_0 \in \mathbb{N}, \exists n \geq n_0$ such that $|a_n - l| > \in$

(d) $\exists \in > 0, \forall n_0 \in \mathbb{N}, \forall n \geq n_0$ we have $|a_n - l| > \in$

2. Consider a sequence (a_n) of positive numbers satisfying the condition $a_n a_{n+2} \leq a_{n+1}^2, \forall n \in \mathbb{N}$ then (a_n) is a

(a) convergent sequence if $a_1 \neq 2a_2$

(b) monotonically increasing sequence if $a_1 \neq 2a_2$

(c) convergent sequence if $a_1 = 2a_2$

(d) monotonically increasing sequence if $a_1 = 2a_2$

3. The sum of the series $\sum_{n=1}^{\infty}[(n+1)^{\frac{1}{5}} - n^{\frac{1}{5}}]$ is

(a) less than –1 (b) equal to –1

(c) greater than –1 but less than 2 (d) none of the above

4. Let $S=\{x\in\mathbb{R}/x^2\le 5\}\cap\mathbb{Q}$. Which of the following statements is true about S'?

(a) S is bounded above the $\sup S\in\mathbb{Q}$

(b) S is bounded above and $\sup S\in\mathbb{R}-\mathbb{Q}$

(c) S is a closed interval

(d) S is an open interval

5. The value of $\lim_{x\to 0}\frac{e^{(1/x)}-e^{(-1/x)}}{e^{(1/x)}+e^{(-1/x)}}$ is

(a) 0 (b) 1 (c) –1 (d) none of the above

6. Consider the function $f:\mathbb{R}\to\mathbb{R}$ given by $f(x)=\begin{cases}-x+3, & x\in\mathbb{Q},\\ x^2-6x+9 & x\notin\mathbb{Q}\end{cases}$. The set of all points at which f is continuous is

(a) $\{2, 3\}$ (b) $\{3\}$ (c) $\mathbb{R}-\{2,3\}$ (d) $\mathbb{R}-\{3\}$

7. Consider the function $f:\mathbb{R}\to\mathbb{R}$ given by $f(x)=\begin{cases}\sin x, & x\ge 0,\\ 1-\cos x, & x<0\end{cases}$. Which of the following statements is true about f?

(a) f is differentiable (b) f is continuous but NOT differentiable

(c) f is discontinuous (d) none of the above statements is true

8. Let $f:[0,1]\to\mathbb{R}, g:[0,1]\to\mathbb{R}$ given by $f(x)=\begin{cases}1, & x\in\mathbb{Q}\cap[0,1]\\ 0, & x\notin\mathbb{Q}\cap[0,1]\end{cases}$ and $g(x)=\begin{cases}0, & x\in\mathbb{Q}\cap[0,1]\\ 1, & x\notin\mathbb{Q}\cap[0,1]\end{cases}$, then

(a) both f and g are Riemann integrable

(b) f is Riemann integrable but g is NOT Riemann integrable

(c) g is Riemann integrable but f is NOT Riemann integrable

(d) both f and g are NOT Riemann integrable

9. $\lim_{n\to\infty}\sum_{k=0}^{n}\frac{2k}{k^2+n^2}=$

(a) 0 (b) log 2 (c) 2 (d) ∞

10. A solution of $xdy-ydx+(x^2+y^2)dx+(x^2+y^2)dy=0$ is

(a) $\arctan(y/x)+x+y=C$ (b) $\frac{y}{x}+x^2+y^2=C$

(c) $\arctan(y/x)+x^2+y^2=C$ (d) $\frac{y}{x}+x+y=C$

11. The general solution of $(D^4+I)^2 y=0$ is

(a) $C_1 \sin x + C_2 \cos x + C_3 e^x + C_4 e^{-x}$

(b) $C_1 x \sin x + C_2 x \cos x + C_3 e^x + C_4 e^{-x}$

(c) $(C_1 + C_2 x)\sin x + (C_3 + C_4 x)\cos x + C_5 e^x + C_6 e^{-x}$

(d) $(C_1 + C_2 x)\sin x + (C_3 + C_4 x)\cos x + (C_5 + C_6 x)e^x + (C_7 + C_8 x)e^{-x}$

12. Consider three different planes $a_{11}x + a_{12}y + a_{13}z = d_1, a_{21}x + a_{22}y + a_{23}z = d_2$ and $a_{31}x + a_{32}y + a_{33}z = d_3$. Let $A = (a_{ij}),\ 1 \le i, j \le 3$. Which of the following conditions necessarily implies that there exists a unique point of intersection of all three planes?

(a) $\det(A) = 0$

(b) $\det(A) \ne 0$

(c) $\text{Trace}(A) = 0$

(d) $\text{Trace}(A) \ne 0$

13. The number of planes containing both the lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x-2}{-1} = \frac{y-4}{-5} = \frac{z-6}{-1}$ is

(a) 0

(b) 1

(c) more than 1 but finitely many

(d) infinite

14. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 22 \\ 0 & 1/7 & \pi \end{bmatrix}$, then $\det(A)$ is

(a) zero

(b) a non-zero rational number

(c) an irrational number less than 1

(d) an irrational number greater than 1

15. Consider the vector space $\mathbb{R}^3$ over $\mathbb{R}$ and $A, B \subset \mathbb{R}^3$ such that $0 \notin A \cup B$. Let the number of elements in A and B are 4 and 2 respectively, then

(a) both A and B are linearly dependent sets

(b) A is linearly dependent set but B is linearly independent set

(c) both A and B are linearly independent sets

(d) none of the above is a true statement

16. The number of group homomorphisms from $\mathbb{Z}_{12}$ to $\mathbb{Z}_{13}$ is

(a) 0

(b) 1

(c) more than 1 but finitely many

(d) infinite

17. The centre of $\mathbb{Z}_{33}$ is

(a) $\{0\}$

(b) $\mathbb{Z}_3$

(c) $\mathbb{Z}_{11}$

(d) $\mathbb{Z}_{33}$

18. Let G be a group and H be a subgroup of G. Which of the following statements is true?

(a) If H is a normal subgroup of G then $gH = Hg, \forall g \in G$

(b) If H is a normal subgroup of G then $gH \neq Hg$, for some $g \in G$

(c) If $gH = Hg$, for some $g \in G$ then H is a normal subgroup of G

(d) If $gH \neq Hg$, for some $g \in G$ then H is a normal subgroup of G

19. The number of elements of order 8 in a cyclic group of order 16 is

(a) 1 (b) 2 (c) 3 (d) 4

20. If $x \neq e, y \neq e$ are elements in a group G such that the order of x is 2 and $x^{-1}yx = y^2$ then the order of y is

(a) 1 (b) 2 (c) 3 (d) 4

21. In the ring $(\mathbb{Z}, +, .)$ the set $\{12u + 30v \mid u, v \in \mathbb{Z}\}$ is the same as $n\mathbb{Z}$ for $n =$

(a) 6 (b) 4 (c) 3 (d) 2

22. Let S be the sphere with center at the origin and radius 1. Let $\overline{f}$ is a vector field given by $\overline{f}(x, y, z) = (z - 2xyz)\hat{i} + 9x^2yz^2\hat{j} + (yz^2 - 3x^2z^3)\hat{k}$. If $\hat{n}$ is the outward normal then, the value of $\iint_S \overline{f} \cdot \hat{n}\, dS =$

(a) 0 (b) $\frac{4}{3}\pi$ (c) π (d) $\frac{4}{3}\pi^3$

23. If ϕ is a real valued smooth function and $\overline{f}$ is a vector valued smooth function on $\mathbb{R}^3$. then $\text{div}(\phi \,\text{Curl}\, \overline{f}) =$

(a) $\nabla\phi \cdot \text{Curl}\, \overline{f}$ (b) $\nabla(\overline{f} \cdot \nabla\phi)$

(c) $\nabla\phi \cdot \text{Curl}\, \overline{f} + \nabla(\overline{f} \cdot \nabla\phi)$ (d) none of the above

24. What is the probability of that girls out number boys in a family with 5 children. Assume that births are independent trials and probability of a boy is equal to 1/2.

(a) 0 (b) $\frac{1}{2}$ (c) $\frac{15}{32}$ (d) $\frac{17}{32}$

25. Consider two boxes numbered Box 1 and Box 2. Let Box 1 contains 5 red balls and 4 black balls Box 2 contains 10 red balls and 17 black balls. Consider a random experiment of choosing a box, picking a ball from it. What is the probability that the color of the ball is red?

(a) $\frac{25}{54}$ (b) $\frac{50}{54}$ (c) $\frac{15}{36}$ (d) $\frac{15}{17}$

PART-B

26. Consider the statement "There is a train in which every compartment has at least one passenger without the ticket". Negation of this statement is
(a) There is a train in which every compartment has at least one passenger with the ticket
(b) There is a train in which every passenger of every compartment has the ticket
(c) Every train has a compartment in which every passenger has the ticket
(d) In every train every passenger in every compartment has the ticket

27. Consider a sequence (a_n) of real numbers. Which of the following conditions imply that (a_n) is convergent?
(a) $|a_{n+1} - a_n| < \frac{1}{n}, \forall n \in \mathbb{N}$
(b) $|a_{n+1} - a_n| < \frac{1}{3^n}, \forall n \in \mathbb{N}$
(c) $a_n > 0, \forall n \in \mathbb{N}$ and a_n is monotonically increasing
(d) $a_n > 0, \forall n \in \mathbb{N}$ and a_n is monotonically decreasing

28. Which of the following series are convergent?
(a) $\sum_{n=0}^{\infty} \frac{\log n}{n^{3/2}}$ (b) $\sum_{n=0}^{\infty} \frac{n^2}{n!}$ (c) $\sum_{n=0}^{\infty} \frac{1}{n \log n}$ (d) $\sum_{n=0}^{\infty} \frac{e^n}{n^{100}}$

29. Which of the following statements are true?
(a) If $A \subset \mathbb{Q}$ such that $\mathbb{Q} - A$ is finite then A is dense in $\mathbb{R}$
(b) There exists $A \subset \mathbb{Q}$ such that $\mathbb{Q} - A$ is finite and A is dense in $\mathbb{R}$
(c) There exists a pair of disjoint subsets of Q such that both of them are dense in $\mathbb{R}$
(d) None of the above is a true statement

30. Consider $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = \sin^3(|x|)$, then $f'(0)$
(a) is equal to –1
(b) is equal to 0
(c) is equal to 1
(d) does not exist

31. Consider the following two statements
S_1 : If $f : [0,1] \to [0,1]$ is continuous then $\exists x_0 \in [0,1]$ such that $f(x_0) = x_0$
S_2 : There exists a continuous function $f : [0,1] \to [0,1] - \left\{\frac{1}{2}\right\}$ such that *f* is on to
(a) Both S_1 and S_2 are true
(b) S_1 is true but S_2 is false
(c) S_2 is true but S_1 is false
(d) Both S_1 and S_2 are false

32. Consider the following two statements.

$S_1 : \int_0^{\pi/2} \frac{\sin x}{x} dx$ exists

$S_2 : \int_0^{1} \frac{x}{\log x} dx$ exists

(a) Both S_1 and S_2 are true
(b) S_1 is true but S_2 is false
(c) S_2 is true but S_1 is false
(d) Both S_1 and S_2 are false

33. Solution of $(x^2+y^2)xdx+(x^2+y^2)ydy+2xy(xdy-ydx)=0$ is

(a) $\log(\sqrt{x^2+y^2})-\frac{x^2}{x^2+y^2}=C$
(b) $\log(x^2+y^2)-\frac{x^2}{x^2+y^2}=C$
(c) $\log(\sqrt{x^2+y^2})-\tan^{-1}\frac{y}{x}=C$
(d) $\log(x^2+y^2)-\tan^{-1}\frac{y}{x}=C$

34. The general solution of $(D^2-1)y=x^2+e^{-x}$ is

(a) $C_1e^x+C_2e^{-x}-\left[\frac{1}{4}(2x+1)e^{-x}+x^2+2\right]$
(b) $C_1\sin x+C_2\cos x-\left[\frac{1}{4}(2x+1)e^{-x}+x^2+2\right]$
(c) $C_1e^x+C_2e^{-x}-\left[\frac{1}{2}e^{-x}+x^2+2\right]$
(d) $C_1\sin x+C_2\cos x-\left[\frac{1}{2}e^{-x}+x^2+2\right]$

35. The value of k such that the lines $\frac{x-1}{k}=\frac{y-1}{4}=\frac{z-2}{3}$ and $\frac{x+1}{2}=\frac{y-1}{1}=\frac{z}{3}$ are coplanar is

(a) –1
(b) 1
(c) –2
(d) 2

36. Consider a plane which is at a distance p from the origin O = (0, 0, 0). Let A, B, C be the points of intersection of that plane with the co-ordinate axis. The locus of the centre of the sphere passing through O, A, B and C is

(a) $\frac{2}{x^2}+\frac{2}{y^2}+\frac{2}{z^2}=\frac{1}{p^2}$
(b) $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{4}{p^2}$
(c) $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{2}{p^2}$
(d) $\frac{4}{x^2}+\frac{4}{y^2}+\frac{4}{z^2}=\frac{1}{p^2}$

37. Consider the circle C which is the intersection of the sphere $x^2+y^2+z^2-x-y-z=0$ and the plane $x+y+z=1$. The radius of the sphere with centre at the origin, containing the circle C is

(a) 1
(b) 2
(c) 3
(d) 4

38. Which of the following statements are true?

(a) All groups of order 4 are abelian
(b) All groups of order 6 are abelian
(c) 73^{12} – 1 is divisible by 7
(d) A subgroup of a cyclic group must be cyclic

39. Consider the quotient group $G = \frac{\mathbb{Q}}{\mathbb{Z}}$ under addition. Which of the following statements about G are true?

(a) G is a finite group
(b) In G every element has a finite order
(c) G has no non-trivial proper subgroups
(d) G is NOT a cyclic group

40. Let $G = \left\{ \begin{pmatrix} a & b \\ 0 & 1 \end{pmatrix} \middle| a \in \mathbb{Q} - \{0\}, b \in \mathbb{Q} \right\}, U = \left\{ \begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix} \middle| b \in \mathbb{Q} \right\}, \; D = \left\{ \begin{pmatrix} a & 0 \\ 0 & 1 \end{pmatrix} \middle| a \in \mathbb{Q} - \{0\} \right\}$

Which of the following statements are true?

(a) G, U, D are all groups under multiplication
(b) D is a normal subgroup of G
(c) U is a normal subgroup of G
(d) For every matrix $A \in U$, $ADA^{-1} \subseteq D$

41. Let $X = \{1, 2, 3, 4, 5\}$, $P(X)$ be the power set of X. Consider the ring $R = (P(X), \Delta, \cap)$, for subsets A and B of X, $A \Delta B = (A \cup B) - (A \cap B)$. Which of the following statements are true about R?

(a) R is a commutative ring with unity
(b) R is field
(c) Every element in R has 'additive' order 2.
(d) Every element in R has 'multiplicative' order 2

42. Consider the ring $R = (\mathbb{Z}_{60}, +, .)$. Which of the following statements are true about R?

(a) There are no maximal ideals in R
(b) There are three maximal ideals in R
(c) There are ten non-zero proper ideals in R
(d) All non-zero ideals in R are maximal

43. Consider the group $\mathbb{Z}$ under addition +. Define the binary operation * on $\mathbb{Z}$ by $a * b = 0, \forall a \cdot b \in \mathbb{Z}$. Which of the following statements are true about R?

(a) $(\mathbb{Z}, +, .)$ is a commutative ring with unity
(b) $(\mathbb{Z}, +, .)$ is a ring
(c) Every additive subgroup of $\mathbb{Z}$ is an ideal
(d) The only ideals in $\mathbb{Z}$ are of the form $n\mathbb{Z} = \{nx \mid x \in \mathbb{Z}\}$

44. Let A be a non-singular 3 × 3 matrix with real entries. For every non-zero eigenvalue λ of A,

(a) λ is an eigenvalue of both $P^{-1}AP, PAP^{-1}$ where $\det(P) \neq 0$
(b) $1 + \lambda$ is an eigenvalue of $I + A$
(c) if $\det(A) < 1$ then $|\lambda| < 1$
(d) if μ is an eigen value of A^{-1} then $\mu\lambda = 1$

45. Let A be a 2 × 2 real matrix. Let the sum of the entries in each row of A be qual to 2. Which of the following statements is true?

(a) 0 is always an eigen value of A
(b) 0 and 2 are always eigenvalues of A
(c) 2 is always an eigenvalues of A
(d) None of the above

46. Let A, B be a 4 × 4 matrices. Denote rank of a matrix A, B by $\rho(A), \rho(B)$ and adjoint of A by adj(A). Which of the following statements are true?

(a) $\rho(A+B) \le \rho(A)+\rho(B)$
(b) $\rho(A-B) \le \rho(A)-\rho(B)$
(c) $\rho(AB) \le \rho(A)\rho(B)$
(d) If $\rho(A)=2$ then adj (A) = $O_{4\times 4}$

47. Consider the vector space $V=\mathbb{R}^3(\mathbb{R})$ and $B=\{v_1, v_2\} \subset V, 0 \notin B$. Which of the following statements are true?

(a) If B is a linearly dependent set then $\exists(\alpha_1, \alpha_2) \ne (0,0)$ such that $\alpha_1 v_1 + \alpha_2 v_2 = 0$

(b) If B is a linearly dependent set then $\exists(\alpha_1 \cdot \alpha_2)$ such that $\alpha_1 \ne 0,\ \alpha_2 \ne 0$ and $\alpha_1 v_1 + \alpha_2 v_2 = 0$

(c) If B is linearly independent then $\exists$ no nonzero 2-tuple (α_1, α_2) such that $\alpha_1 v_1 + \alpha_2 v_2 \ne 0$

(d) If B is linearly independent then $\exists$ no nonzero 2-tuple (α_1, α_2) such that $\alpha_1 v_1 + \alpha_2 v_2 = 0$

48. Let $\overline{r} = x\hat{i} + y\hat{j} + z\hat{k},\ |\overline{r}| = \sqrt{x^2+y^2+z^2}$ and $\overline{f}:\mathbb{R}^3 - \{0\} \to \mathbb{R}^3$ be given by $f(x,y,z) = \dfrac{\overline{r}}{|\overline{r}|^n}$. The value of n for which $div(\overline{f}) = 0$ is

(a) 1
(b) 2
(c) 3
(d) 4

49. Let R be a region in the *xy*-plane. The boundary of R is a smooth simple closed curved C which is parametrized by $C=(x(t), y(t)), t \in [0,1]$. The area of R is NOT equal to

(a) $\int_0^1 x(t)y'(t)dt$
(b) $-\int_0^1 y(t)x'(t)dt$
(c) $\frac{1}{2}\int_0^1 (x(t)y'(t) + y(t)x'(t))dt$
(d) $\frac{3}{4}\int_0^1 x(t)y'(t)dt - \frac{1}{4}\int_0^1 y(t)x'(t)dt$

50. A storage depot contains 10 machines 4 of which are defective. If a company selects 5 of these machines randomly, then what is the probability that at least 4 of the machines are NON DEFECTIVE?

(a) $\frac{11}{42}$
(b) $\frac{5}{21}$
(c) $\frac{1}{252}$
(d) None of the above

HYDERABAD CENTRAL UNIVERSITY (HCU)
M.Sc. Mathematics Entrance - 2013

ANSWER KEY

PART-A

1.	(c)	**2.**	(c)	**3.**	(d)	**4.**	(b)	**5.**	(d)
6.	(a)	**7.**	(b)	**8.**	(d)	**9.**	(b)	**10.**	(a)
11.	(d)	**12.**	(b)	**13.**	(__)	**14.**	(c)	**15.**	(d)
16.	(b)	**17.**	(d)	**18.**	(a)	**19.**	(d)	**20.**	(c)
21.	(a)	**22.**	(a)	**23.**	(a)	**24.**	(b)	**25.**	(a)

PART-B

26.	(c)	**27.**	(b, d)	**28.**	(a, b)	**29.**	(a, b, c)	**30.**	(b)
31.	(b)	**32.**	(a)	**33.**	(__)	**34.**	(a)	**35.**	(__)
36.	(__)	**37.**	(__)	**38.**	(a, c, d)	**39.**	(__)	**40.**	(__)
41.	(a, c)	**42.**	(b, c)	**43.**	(__)	**44.**	(a, b, d)	**45.**	(c)
46.	(a, c, d)	**47.**	(a, b, d)	**48.**	(c)	**49.**	(b, d)	**50.**	(a)

HYDERABAD CENTRAL UNIVERSITY (HCU)
M.Sc. Mathematics Entrance - 2012

Time : 2 Hours **Max. Marks: 100**

Instructions:

(*i*) There are a total of **50** questions in **Part-A** and **Part-B** together.

(*ii*) There is a negative marking in **Part-A**. Each correct answer carries **1 mark** and each wrong answer carries **–0.33 mark**. Each question in **Part-A** has only one correct option.

(*iii*) There is **no negative** marking in **Part-B**. Each correct answer carries **3 marks**. In **Part-B** some questions have **more than** one correct option. All the correct options have to be marked in OMR sheet other wise **zero marks** will be credited.

PART-A

1. If α, β and γ are the roots of $x^3 + ax^2 + bx + c = 0$ then the value of $\alpha^2 + \beta^2 + \gamma^2$ is

(a) $a^2 - 2b$ (b) $b^2 - 2c$ (c) $c^2 + 2a$ (d) $b^2 + 2c$

2. Let $f : \mathbb{R} \to \mathbb{R}$ and $f(x) = |x-1| + |x-2|$. Let $S_1 = \{x \mid f \text{ is continuous at } x\}$ and $S_2 = \{x \mid f \text{ is differentiable at } x\}$. Then

(a) $S_1 = \mathbb{R}, S_2 = \mathbb{R}$
(b) $S_1 = \mathbb{R}, S_2 = \mathbb{R} \setminus \{1, 2\}$
(c) $S_1 = \mathbb{R} \setminus \{1, 2\}, S_2 = \mathbb{R}$
(d) $S_1 = \mathbb{R} \setminus \{1, 2\}, S_2 = \mathbb{R} \setminus \{1, 2\}$

3. Consider the following statements

S_1 : If f is Riemann integrable in [0, 1] then f^2 is Riemann integrable in [0, 1]

S_2 : If f^2 is Riemann integrable in [0, 1] then f is Riemann integrable in [0, 1]

Then

(a) S_1 is true but S_2 is false
(b) S_1 is false but S_2 is true
(c) both S_1 and S_2 are false
(d) both S_1 and S_2 are true

4. The function $f(x) = \sin(x) + \cos(x)$ is

(a) increasing in $[0, \pi/2]$
(b) decreasing in $[0, \pi/2]$
(c) increasing in $[0, \pi/4]$ and decreasing in $[\pi/4, \pi/2]$
(d) decreasing in $[0, \pi/4]$ and increasing in $[\pi/4, \pi/2]$

5. Let G_1 and G_2 be two finite groups with $|G_1| = 100$ and $|G_2| = 25$. If $f : G_1 \longrightarrow G_2$ is a surjective group homomorphism, then

(a) $|Ker(f)| = 2$ (b) $|Ker(f)| = 4$ (c) $|Ker(f)| = 5$ (d) $|Ker(f)| = 10$

6. Let $\{p_n\}$ be a strictly increasing sequence of prime numbers and let $x_n = (-1)^{p_n+1}\left(1+\frac{1}{p_n}\right)$ then

(a) $\lim_{n\to\infty} x_n = -1/2$ (b) $\lim_{n\to\infty} x_n = -1$

(c) $\lim_{n\to\infty} x_n = 1$ (d) $\lim_{n\to\infty} x_n$ does not exist

7. Let V be a vector space of dimension n and $\{v_1, v_2, ..., v_n\}$ be a basis of V. Let $\sigma \in S_n$ and $T : V \to V$ be a linear transformation defined by $T(v_i) = v_{\sigma(i)}$. Then

(a) T is nilpotent (b) T is one-one but not onto

(c) T is onto but not one-one (d) T is an isomorphism

8. Let G be a group and $a \in G$ be a unique element of order n where $n > 1$. Let $Z(G)$ denote the centre of the group G. Then

(a) $O(G) = n$ (b) $O(Z(G)) > 1$ (c) $Z(G) = G$ (d) $G = S_2$

9. If the series $\sum_{n=0}^{\infty} (\sin x)^n$ converges to the value $(4+2\sqrt{3})$ for some value of x is $(0, \pi/2)$, then the value of x is

(a) $\pi/3$ (b) $\pi/4$ (c) $\pi/5$ (d) $\pi/6$

10. If m and M are respectively the greatest lower bound and the least upper bound of the set $S = \left\{\frac{2x+3}{x+2}, x \geq 0\right\}$ then

(a) $m \in S, M \notin S$ (b) $m \notin S, M \notin S$ (c) $m \notin S, M \in S$ (d) $m \in S, M \in S$

11. The value of $\lim_{x\to 0} (\cos x)^{(1/\sin^2 x)}$ is

(a) exp(–1) (b) exp(1) (c) exp(–1/2) (d) exp(1/2)

12. The graphs of the real valued functions $f(x) = 2\log(x)$ and $g(x) = \log(2x)$

(a) do not intersect (b) intersect at one point only

(c) intersect at two points (d) intersect at more than two points

13. The points of continuity of the function $f : \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = \begin{cases} |x^2 - 1|, & \text{if } x \text{ is irrational} \\ 0, & \text{if } x \text{ is rational} \end{cases}$$ are

(a) $x = -1, x = 0, x = 1$ (b) $x = -1, x = 1$ (c) $x = -1, x = 0$ (d) $x = 0, x = 1$

14. The smallest positive integer n such that $5^n - 1$ is divisible by 36 is

(a) 2 (b) 3 (c) 5 (d) 6

15. Let $f(x) = x^5 + a_1x^4 + a_2x^3 + a_3x^2$. Suppose $f(-1) > 0$ and $f(1) < 0$ then

(a) f has at least 3 real roots (b) f has at most 3 real roots

(c) f has at most 1 real root (d) all roots of f are real

16. Let $\{u, v\}$ be a linearly independent subset of a real vector space V. Then which of the following is **not** a linearly independent set?

(a) $\{u, u-v\}$ (b) $\{u+\sqrt{2}v, u-\sqrt{2}v\}$

(c) $\{v, 2v-u/2\}$ (d) $\{2u+v, -4u-2v\}$

17. Let V be a vector space of 2 × 2 real matrices. Let $A = \begin{bmatrix} 1 & 2 \\ 1 & -1 \end{bmatrix}$ then the dimension of the subspace spanned by $\{A, A^2, A^3, A^4\}$ is

(a) 2 (b) 3 (c) 4 (d) 5

18. Let $A \in M_3(\mathbb{Q})$. Consider the statements

P : Matrix A is nilpotent

Q : $A^3 = 0$

Pick up true statements from the following

(a) $P \Rightarrow Q$ (b) $Q \Rightarrow P$ and $P \not\Rightarrow Q$

(c) $P \not\Rightarrow Q$ and $Q \Rightarrow P$ (d) None of (a), (b), (c) is true

19. Consider the statements

$S_1 : 1-1+1-1+1-1+.... = \pm 1$

$S_2 : \dfrac{1}{1+2} = 1-2+2^2-2^3+...$ then

(a) S_1 is true but S_2 is false (b) S_1 is false but S_2 is true

(c) both S_1 and S_2 are true (d) both S_1 and S_2 are false

20. Let $x_0 < x_1 < ... < x_n$ and $y_1, y_2, ..., y_n \in \mathbb{R}$. Then

(a) there exists a unique continuous function f such that $F(x_i) = y_i$ for all i.

(b) there exists a unique differentiable function f such that $F(x_i) = y_i$ for all i.

(c) there exists a unique n times differentiable function f such that $F(x_i) = y_i$ for all i.

(d) there exists a unique polynomial function f of degree n such that $F(x_i) = y_i$ for all i

21. Solution of the differential equation $y'' - x(y')^2 = 0,$ subject to the boundary conditions $y(0) = 0,\ y'(0) = -1$ is

(a) $y = \sqrt{\frac{-2}{a}} \tan^{-1}\left(\frac{x}{\sqrt{2a}}\right) + b,$ where a and b are arbitrary constants

(b) $y = -\sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$

(c) $y = \sqrt{\frac{2}{a}} \tan^{-1}\left(\frac{x}{\sqrt{2a}}\right) + b,$ where a and b are arbitrary constants

(d) $y = \frac{-1}{\sqrt{2}} \tan^{-1}(\sqrt{2}x)$

22. Let V be the vector space of all continuous functions on $\mathbb{R}$ over the field $\mathbb{R}$.

Let $S = \{|x|, |x-1|, |x-2|\}$

(a) S is linearly independent and does not span V
(b) S is linearly independent and spans V
(c) S is linearly dependent and does not span V
(d) S is linearly dependent and spans V

23. 10 red balls (all alike) and 10 blue balls (all alike) are to be arranged in a row. If every arrangement is equally likely, then the probability that the balls at two ends of the arrangement are of the same colour is

(a) equal to $\frac{1}{4}$
(b) equal to $\frac{1}{2}$
(c) less than $\frac{1}{2}$
(d) greater than $\frac{1}{2}$

24. 3 students are to be selected to form a committee from a class of 100 students. The chances that the tallest student is one among them is

(a) less than 5%
(b) 6 to 10%
(c) 15%
(d) 50%

25. Let $\vec{f}$ be a smooth vector valued function of a real variable. Consider the two statements

$S_1 : \text{div}\,\text{curl}\,\vec{f} = 0$

$S_1 : \text{grad}\,\text{div}\,\vec{f} = 0$. Then

(a) both S_1 and S_2 are true
(b) both S_1 and S_2 are false
(c) S_1 is true but S_2 is false
(d) S_1 is false but S_2 is true

PART-B

- The following questions may have more than one correct answer
- Find the correct answers and mark them on the OMR sheet. Correct answers (marked in OMR sheet) to a question get 3 marks and zero otherwise.
- For the answer to be right **all the correct options** have to be marked on the OMR sheet. **No credit** will be given for partially correct answers.

26. A sphere passing through the points (1, 0, 0), (0, 1, 0), (0, 0, 2) that has the least radius is

(a) $18(x^2+y^2+z^2)-16(x+y)-35z=2$ (b) $9(x^2+y^2+z^2)-5(x+y)-16z=4$

(c) $9(x^2+y^2+z^2)-7(x+y)-17z=2$ (d) None of the above

27. Let f be a function from $\mathbb{R}\to\mathbb{R}$. Consider the statement

P : There exists M in $\mathbb{R}$ such that $|f(x)|\le M$ for all x in $\mathbb{R}$. Which of the following statements are equivalent to P.

(a) The range of f is a bounded set of $\mathbb{R}$ (b) $|f|$ is a bounded function

(c) f is taking all values between –M and M (d) $|f|$ is taking all values between 0 and M/2

28. Let $\{x_n\}$ be a sequence of positive real numbers. Then which of the following is false?

(a) If $\sum_{n=1}^{\infty} x_n$ is convergent then $\sum_{n=1}^{\infty}\sqrt{x_n}$ is convergent

(b) If $\sum_{n=1}^{\infty} x_n$ is convergent then $\sum_{n=1}^{\infty} x_n^2$ is convergent

(c) If $\sum_{n=1}^{\infty} x_n^2$ is convergent then $\lim_{n\to\infty} x_n=0$

(d) If $\sum_{n=1}^{\infty}\sqrt{x_n}$ is convergent then $\lim_{n\to\infty} x_n=0$

29. Given S_1 and S_2, where

S_1 : A series $\sum_{n=0}^{\infty} a_n$ converges if for given $\in>0$ there exists $N_0\in\mathbb{N}$ such that $|a_{n+1}-a_n|<\in$ for all $n\ge N_0$.

S_2 : A series $\sum_{n=0}^{\infty} a_n$ converges if $|a_{n+1}-a_n|<\alpha^n$ where α is a fixed real number in (0, 1)

which of the following statements are true?

(a) S_1 is true but S_2 is false (b) S_1 is false but S_2 is true

(c) Both S_1 and S_2 are true (d) Both S_1 and S_2 are false

30. Let $x,y\in\mathbb{R}$. If $|x+y|=|x|+|y|$ then

(a) $x-y|=|x|-|y|$ (b) $|xy|=xy$

(c) $|x^2+y|=|x^2|+|y|$ (d) $|x+y|=x+y$

31. Let $f:\mathbb{R}\to\mathbb{R}$ be a quadratic polynomial. Then which of the following is impossible?

(a) $f(x)<f'(0)$, for all $x\in\mathbb{R}$ (b) $f'(x)>f(x)$, for all $x\ge 0$

(c) $f'(0)=0$ and $f(1)=f(4)$ (d) $f'(0)=0$ and $f(x)\ne 0$ for all $x\in\mathbb{R}$

32. If α, β and γ are the roots of the polynomial $x^3 + x^2 + x + 1$, then the value of $\frac{1}{\alpha - 1} + \frac{1}{\beta - 1} + \frac{1}{\gamma - 1}$ is

(a) 1/2 (b) –1/2 (c) 3/2 (d) –3/2

33. Let V be the vector space of polynomials of degree less than or equal to 2.

Let $S = \{x^2 + x + 1, x^2 + 2x + 2, x^2 + 3\}$. Then

(a) S is a linearly independent set
(b) S does not span V
(c) neither [A] nor [B] is false
(d) None of [A], [B], [C] is false

34. Let $f : \mathbb{R} \to \mathbb{R}$ be a function. Which of these four statements mean that *f* is a constant function?

(a) For all $x, y \in \mathbb{R}, f(x) = f(y)$

(b) There exists $x \in \mathbb{R}$ such that for all $y \in \mathbb{R}, f(x) = f(y)$

(c) There exists $x \in \mathbb{R}$ and there exists $y \in \mathbb{R}$ such that $f(x) = f(y)$

(d) For each $x \in \mathbb{R}$ there exists $y \in \mathbb{R}$ such that $f(x) = f(y)$

35. Let $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ be a 2 × 2 real matrix. Then

(a) 1 is the only eigenvalue of A
(b) A has two linearly independent eigenvectors
(c) A satisfies a polynomial equation with real coefficients of degree 2
(d) A is not invertible under multiplication

36. Let M and N be two smooth functions from $\mathbb{R}^2$ to $\mathbb{R}$. The form $(M\,dx + N\,dy)$ is exact if and only if

(a) there exists a smooth function *f* such that $M\,dx + N\,dy = df$

(b) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ for all x and y

(c) $\text{Curl}\,(M\hat{i} + N\hat{j}) = \hat{0}$

(d) all the above statements are true

37. The general solution of the differential equation $(D^2 - I)^2 y = 0$ is

(a) $(c_1 - c_2 x)\exp(x) + (c_3 - c_4 x)\exp(-x)$

(b) $(c_1 + c_2 x)\exp(ix) + (c_3 + c_4 x)\exp(-ix)$

(c) $(c_1 - c_2 x)\sin(x) + (c_3 - c_4 x)\cos(-x)$

(d) $c_1 \sin h(x) + c_2 x \sin h(-x) + c_3 \cosh(x) + c_4 x \cosh(-x)$

38. Let P be a polynomial of degree 5 having 5 distinct real roots. Then

(a) the roots of P and P' occur alternately

(b) the roots of P' and P'' occur alternately

(c) all the roots of P, P', P'', P''', P'''' are real

(d) it is possible to have a repeated root of P''

39. If each term of a 3 × 3 matrix A is constructed by selecting a number from the set {–1, 0, 1} with the same probability 1/3, then
    (a) The probability that the trace of A is greater than 0 is more than 1/3
    (b) The probability that A is a diagonal matrix is less than 1/81
    (c) The probability that A is a non-singular lower triangle matrix is more than 1/81
    (d) The probability that A is symmetric is less than 1/81

40. By revolving the curve $y = \sin(x)$ about the x-axis in the interval $[0, \pi]$, the surface area of the surface generated is
    (a) $6\pi + 2\pi \log(1+\sqrt{2})$ (b) $2\sqrt{2}\pi + 2\pi \log(1+\sqrt{2})$
    (c) $2\pi \log(1+\sqrt{2})$ (d) $2\pi(1+\log(1+\sqrt{2}))$

41. Let $A_i = \begin{bmatrix} \cos^2\theta_i & \cos\theta_i \sin\theta_i \\ \cos\theta_i \sin\theta_i & \sin^2\theta_i \end{bmatrix}, i = 1, 2.$ Then $A_1 A_2 = 0$ if
    (a) $\theta_1 = \theta_2 + (2k+1)\pi/2,\ k = 0,1,2,...$ (b) $\theta_1 = \theta_2 + k\pi,\ k = 0,1,2,...$
    (c) $\theta_1 = \theta_2 + 2k\pi,\ k = 0,1,2,...$ (d) $\theta_1 = \theta_2 + k\pi/2,\ k = 0,1,2,...$

42. Let $f : X \to Y$ and let A and B be subsets of X. Then
    (a) $f(A \cup B) \subseteq f[A] \cup f[B]$ (b) $f[A] \cup f[B] \subseteq f(A \cup B)$
    (c) $f(A \cap B) \subseteq f[A] \cap f[B]$ (d) $f[A] \cap f[B] \subseteq f(A \cap B)$

43. The value of the integral $\int_0^{10} (x - [x])dx$ is
    (a) 2 (b) 3 (c) 4 (d) 5

44. Let $f, g : (0,1) \to \mathbb{R}$. Let $f(x) = x\sin(1/x^2)$ and $g(x) = x^2$. Then
    (a) both f and g are uniformly continuous
    (b) f is uniformly continuous but g is not uniformly continuous
    (c) f is not uniformly continuous but g is uniformly continuous
    (d) both f and g are not uniformly continuous

45. Consider a linear transformation from $\mathbb{R}^4$ to $\mathbb{R}^4$ given by a matrix $A = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}$. Then the number of linearly independent vectors whose direction is invariant under this transformation is
    (a) 0 (b) 1 (c) 2 (d) 4

46. Let V be the vector space of polynomials of degree less than or equal to 2. Let $D:V \to V$ be defined as $Df = f'$. If $B_1 = \{1, x, x^2\}$, $B_2 = \{1, 1+x^2, 1+x+x^2\}$ be two ordered bases, then the matrix of linear transformation $[D]_{B_1,B_2}$ is

(a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -2 \\ 0 & 0 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & -2 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -2 \\ 0 & 0 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 1 & 0 \\ -2 & 0 & 0 \\ 2 & 0 & 0 \end{bmatrix}$

47. If α and β are the roots of $(7+4\sqrt{3})x^2 + (2+\sqrt{3})x - 2 = 0$ then the value of $|\alpha - \beta|$ is

(a) $2-\sqrt{3}$ (b) $2+\sqrt{3}$ (c) $6+3\sqrt{3}$ (d) $6-3\sqrt{3}$

48. Consider the following system of linear equations

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \dots + a_{15}x_5 &= b_1, \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{25}x_5 &= b_b, \\ \vdots \qquad & \quad \vdots \\ a_{81}x_1 + a_{82}x_2 + \dots + a_{85}x_5 &= b_8 \end{aligned}$$

A vector $(\lambda_1, \lambda_2, \dots, \lambda_5) \in \mathbb{R}^5$ is said to be a solution of the system if $x_i = \lambda_i, i = 1, 2 \dots, 5$ satisfies all the equations. Then

(a) If the system of equations has only finitely many solutions then it has exactly one solution
(b) If all the b_i's are zero then the set of solutions of the system is a subspace of $\mathbb{R}^5$.
(c) A system of 8 equations in 5 unknowns is always consistent
(d) If the system of equations has a unique solution then the rank of the matrix $[a_{ij}]$ must be 5.

49. What is the negation of the statement "There is a town in which all horses are white"
(a) In every town some horse in non-white
(b) There is a town in which no horse is white
(c) There is a town in which some horse is non-white
(d) There is no town without a non-white horse

50. Let S be the surface of the cylinder $x^2 + y^2 = 4$ bounded by the planes $z = 0$ and $z = 1$. Then the surface integral $\int\int_S ((x^2 - x)\hat{i} - 2xy\hat{j} + z\hat{k}) \cdot \hat{n} dS$

(a) –1 (b) 0 (c) 1 (d) None of (a), (b), (c)

HYDERABAD CENTRAL UNIVERSITY (HCU)
M.Sc. Mathematics Entrance - 2012

ANSWER KEY

PART-A

1.	(a)	**2.**	(b)	**3.**	(a)	**4.**	(c)	**5.**	(b)
6.	(c)	**7.**	(d)	**8.**	(b)	**9.**	(a)	**10.**	(a)
11.	(c)	**12.**	(b)	**13.**	(b)	**14.**	(d)	**15.**	(a, d)
16.	(d)	**17.**	(a)	**18.**	(d)	**19.**	(d)	**20.**	(d)
21.	(b)	**22.**	(__)	**23.**	(c)	**24.**	(a)	**25.**	(c)

PART-B

26.	(__)	**27.**	(a, b)	**28.**	(a)	**29.**	(b)	**30.**	(b, c)
31.	(c)	**32.**	(d)	**33.**	(a)	**34.**	(a, b)	**35.**	(a, c)
36.	(d)	**37.**	(a)	**38.**	(a, b, c)	**39.**	(__)	**40.**	(__)
41.	(a)	**42.**	(a, b, c)	**43.**	(d)	**44.**	(a)	**45.**	(c)
46.	(c)	**47.**	(__)	**48.**	(a, b, d)	**49.**	(a, d)	**50.**	(b)

HYDERABAD CENTRAL UNIVERSITY (HCU)
M.Sc. Mathematics Entrance - 2010

Time : 2 Hours **Max. Marks: 75**

Instructions:

(*i*) There are a total of **50** questions in **Part-A** and **Part-B** together.

(*ii*) **Part-A :** Each question carry **1 Mark**. **0.33 marks** will be deducted for each wrong answer. There will be no penalty if the questions if left unanswered.

(*iii*) **Part-B :** Each question carries **2 Marks**. **0.66 marks** will be deducted for a wrong answer. There will be no penalty if the questions if left unanswered.

PART-A

The set of real numbers is denoted by $\mathbb{R}$, the set of complex numbers by $\mathbb{C}$, the set of rational numbers by $\mathbb{Q}$, the set of integers by $\mathbb{Z}$, and the set of natural numbers by $\mathbb{N}$.

1. Let $f(x) = \cos|x|$ and $g(x) = \sin|x|$ then

(a) both *f* and *g* are even functions
(b) both *f* and *g* are odd functions
(c) *f* is an even function and *g* is an odd function
(d) *f* is an odd function and *g* is an even function

2. The sequence $\left\{(-1)^n\left(1+\frac{1}{n}\right)\right\}$ is

(a) bounded below but not bounded above
(b) bounded above but not bounded below
(c) bounded
(d) not bounded

3. If $f(x) = \begin{cases} \exp(x) - 1 - x, & x \neq 0 \\ 0, & x = 0, \end{cases}$ then $f'(0)$ is

(a) 0 (b) 1 (c) 1/2 (d) none of these

4. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be a matrix with integer entries such that $b \neq 0$. If $A^2 + A + I_2 = 0$ then

(a) $a^2 - a - bc = 1$
(b) $a^2 - a - bd = 1$
(c) $a^2 + a + bc = -1$
(d) $a^2 + a - bc = -1$

5. The number of points at which the function

$f(x) = |(|x|-3)\sin(\pi x)| + |(x^2-1)(x^3-27)|$ takes zero value is

(a) 1 (b) 2 (c) 3 (d) 4

6. Let $f(x) = \begin{cases} 2x, & \text{if } x \text{ is irrational,} \\ x+3, & \text{if } x \text{ is rational,} \end{cases}$ be a function defined from $\mathbb{R}$ to $\mathbb{R}$. Then the discontinuities of f are

(a) all rational numbers (b) all irrational numbers

(c) $\mathbb{R}\setminus\{2\}$ (d) $\mathbb{R}\setminus\{3\}$

7. Consider the system of equations $AX = 0,\ BX = 0$ where A and B are $n\times n$ matrices and X is a $n\times 1$ matrix. Which of the following statements are true.

(i) $\det(A) = \det(B)$ implies that the two systems have the same solutions

(ii) The two systems have the same solutions implies $\det(A) = \det(B)$

(iii) $\det(A) = 0 \neq \det(B)$ implies that the two systems can have different solutions

(a) All are true (b) (i) is true

(c) (iii) is true (d) (i) and (ii) are true

8. $\int \frac{(x+1)\exp(x)}{\cos^2(x\exp(x))}dx$ is equal to

(a) $-\cot(x\exp(x)) + C$ (b) $\tan(x\exp(x)) + C$

(c) $\log(\sec(x\exp(x)) + C$ (d) $\cos(x\exp(x)) + C$

9. If $f(x) = x^3 - 2x^2$ in (0, 5) then the value of c to satisfy the Mean Value theorem is

(a) 2 (b) 3 (c) 4 (d) None of these

10. A random variable X takes the values –1, 0 and 1 with probabilities 1/3 each. Then the mean value of X is

(a) 0 (b) 1 (c) 0.5 (d) 0.52

11. Two numbers are drawn without replacement from 1, 2, ..., 10. The probability that their sum is an even number strictly lies in

(a) (0, 1/3] (b) (1/3, 1/2] (c) (1/2, 3/4] (d) (3/4, 1]

12. $\lim_{x\to -1} \frac{\sqrt{2x+3}-1}{\sqrt{5+x}-2}$ is equal to

(a) 4 (b) 3 (c) 2 (d) None of these

13. For $X, Y \subset \mathbb{R}$, define $X + Y = \{x + y / x \in Y, y \in Y\}$. An example where $X + Y \neq \mathbb{R}$ is

(a) $X = \mathbb{Q},\ Y = \mathbb{R}\setminus\mathbb{Q}$ (b) $X = \mathbb{Z}, Y = [1/2, 1/2]$

(c) $X = (-\infty, 100],\ Y = \{p \in \mathbb{N} / p \text{ is prime}\}$ (d) $X = (-\infty, 100],\ Y = \mathbb{Z}$

14. Let $f:[0,5] \to \mathbb{R}$ be continuous function with a maximum at $x=2$ then
(a) the derivative of f at 2 may not exist
(b) the derivative of f at 2 must not exist and be nonzero
(c) the derivative of f at 2 must not exist and be zero
(d) the derivative of f at 2 can not exist

15. The perimeter of the Cardiod $r = a(1+\cos\theta)$ is
(a) $2a$ (b) $4a$ (c) $8a$ (d) none of these

16. If $P(x) = x^3 + 7x^2 + 6x + 5$ then
(a) P has no real root
(b) P has three real roots
(c) P has exactly one negative real root
(d) P has exactly two complex roots

17. The number of diagonal 3 × 3 complex matrices A such that $A^3 = I$ is
(a) 1 (b) 3 (c) 9 (d) 27

18. The number of subgroups of order 4 in a cyclic group of order 12 is
(a) 0 (b) 1 (c) 2 (d) 3

19. Let G be an abelian group and let $f(x) = x^2$ be an automorphism of G if G is
(a) finite
(b) finite cyclic
(c) prime order
(d) prime order ≥ 7

20. The series $\sum_{n=1}^{\infty}\left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}}\right)$
(a) converges to 1
(b) converges to 1/2
(c) converges to 3/4
(d) does not converge

21. The sequence $\left\{1 + \sum_{j=1}^{n} \frac{(-1)j}{2j+1}\right\}$ is
(a) unbounded and divergent
(b) bounded and divergent
(c) unbounded and convergent
(d) bounded and convergent

22. The function $f:\mathbb{R} \to \mathbb{R}$ defined by $f(x) = \begin{cases} \frac{1-x}{|1-x|}, & |x| < 1, \\ x^2, & |x| \geq 1 \end{cases}$ is
(a) continuous at all points
(b) not continuous at $x = \pm 1$
(c) differentiable at all points
(d) none of these

23. Let $T:\mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation defined by $T((x_1, x_2, x_3)) = (x_1 + x_2, x_2 + x_3, x_3 + x_1)$. Then an eigenvalue for T is
(a) 0 (b) 1 (c) 2 (d) 3

24. The solutions of $x^2y'' + xy' + 4y = 0,\ x \neq 0$ are

(a) $\cos(\log x)$, and $\sin(\log x)$
(b) $\cos(\log x)$, and $\sin(\log x^2)$
(c) $\cos(\log x^2)$, and $\sin(\log x)$
(d) $\cos(2\log x)$, and $\sin(2\log x)$

25. The series $\sum_{n=1}^{\infty} \frac{x^{2n}}{n}$

(a) converges in (–1, 1)
(b) converges in [–1, 1]
(c) converges in [–1, 1)
(d) converges in (–1, 1]

PART-B

26. The integrating factor of the differential equation $(y^2 - x^2y)dx + x^3dy = 0$ is

(a) $(xy)^{-1}$
(b) $(xy)^{-2}$
(c) xy
(d) x^3y^3

27. An example of an infinite group in which every element has finite order is

(a) non-singular 2 × 2 matrices with integer entries
(b) $(\mathbb{Q}/\mathbb{Z}, +)$
(c) the invertible elements in $\mathbb{Z}$ under multiplication
(d) the Quarternion group

28. The value of the determinant $\begin{vmatrix} 1^2 & 2^2 & 3^2 & 4^2 \\ 2^2 & 3^2 & 4^2 & 5^2 \\ 3^2 & 4^2 & 5^2 & 6^2 \\ 4^2 & 5^2 & 6^2 & 7^2 \end{vmatrix}$ is

(a) 0
(b) 1
(c) 2
(d) none of these

29. Three girls G_1, G_2, G_3 and 3 boys B_1, B_2, B_3 are made to sit in a row randomly. The probability that at lest two girls are next to each other is

(a) 0
(b) 1/10
(c) 1/20
(d) 9/10

30. 3 red balls (all alike), 4 blue balls (all alike) and 3 green balls (all alike) are arranged in a row. Then the probability that all 3 red balls are together is

(a) 1/15
(b) 1/10!
(c) 8!/10!
(d) 3/10!

31. The equation $|x-1| + |x| + |x+1| = x+2, x \in \mathbb{R}$ has

(a) no solution
(b) only one solution
(c) only two solutions
(d) infinitely many solutions

32. A natural number 'n' is said to be "petty" if all its prime divisors are $< \sqrt{n}$. A natural number is square free if the square of a prime can not divide it. Then
    (a) Every square free number is petty
    (b) All even numbers are petty
    (c) There exists an infinite numbers which are petty
    (d) Square of a prime number is petty

33. For the sequence $\left\{\sqrt{n}+\frac{(-1)^n}{\sqrt{n}}\right\}$ of real numbers
    (a) the greatest lower bound and least upper bound exist
    (b) the greatest lower bound exists but not least upper bound
    (c) the least upper bound exists but not the greatest lower bound
    (d) neither the greatest lower bound nor the least upper bound exist

34. Let $f:\mathbb{R}\to\mathbb{R}$ be a differentiable function and consider the following
    (i) $|f(x)-f(y)|\le 1, \forall x, y\in\mathbb{R}$ with $|x-y|\le 1$
    (ii) $|f'(x)|\le 1, \forall x\in\mathbb{R}$
    Then we have
    (a) (i) implies (ii) but (ii) does not imply (i)
    (b) (ii) implies (i) but (i) does not imply (ii)
    (c) (i) implies (ii) and (ii) implies (i)
    (d) (i) does not imply (ii) and (ii) does not imply (i)

35. Let $U=\{(a,b,c,d)/a+b=c+d\}, V=\{(a,b,c,d)/a=b, c=d\}$ be subspaces of $\mathbb{R}^4$. Then the dimensions U and V are
    (a) 1 and 2 respectively
    (b) 2 and 3 respectively
    (c) 3 and 2 respectively
    (d) 3 and 4 respectively

36. Let $f:[0,1]\to\mathbb{R}$ be continuous function and define $g:[0,1]\to\mathbb{R}$ as $g(x)=(f(x))^2$. Then
    (a) $\int_0^1 fdx=0 \Rightarrow \int_0^1 gdx=0$
    (b) $\int_0^1 gdx=0 \Rightarrow \int_0^1 fdx=0$
    (c) $\int_0^1 gdx=\left(\int_0^1 fdx\right)^2$
    (d) $\int_0^1 fdx \le \int_0^1 gdx$

37. Let X be a set, $\{A_\alpha / \alpha\in I\}$ be a collection of subsets of X and $f:X\to X$ be a function. Then we have

    $f\left(\bigcap_{\alpha\in I} A_\alpha\right)=\bigcap_{\alpha\in I} f(A_\alpha)$ if
    (a) X is finite
    (b) I is finite
    (c) f is one-one
    (d) f is onto

38. The value of the integral $\int_0^1 \log(\sqrt{1+x}+\sqrt{1-x})\,dx$ is

(a) $\log\sqrt{2}-1$ (b) $1-\log\sqrt{2}$

(c) $\log\sqrt{2}+1/2+\pi/4$ (d) $\log\sqrt{2}-1/2+\pi/4$

39. The derivative of the function $y=\sin^{-1}\left(\sqrt{\dfrac{x-1}{x+1}}\right)+\sec^{-1}\left(\sqrt{\dfrac{x+1}{x-1}}\right)$ is

(a) –1 (b) 0 (c) 1 (d) none of these

40. Let $T:\mathbb{R}^4 \to \mathbb{R}^3$ be the linear transformation defined by

$T((x_1,x_2,x_3,x_4)) = c(x_1-x_2, x_2-x_3, x_3-x_4)$. Then, which of the following statements are true?

(i) $\dim \operatorname{Ker}(T)=1$ if $c\neq 0$

(ii) $\dim \operatorname{Ker}(T)=4$ if $c=0$

(iii) $\dim \operatorname{Ker}(T)=1$ if T is onto

(a) (i) and (ii) (b) (ii) alone

(c) (ii) and (iii) (d) (i), (ii) and (iii)

41. Let S_1 and S_2 be two series defined for $x\in(-1,1)$ as $S_1=\sum_{n=0}^{\infty}(\sin n)x^n$ and $S_2=\sum_{n=0}^{\infty}(\sin n+\cos n)x^n$ then

(a) S_1 and S_2 are convergent

(b) S_1 and S_2 are bounded but are not convergent

(c) S_1 is convergent, S_1 but S_2 is only bounded

(d) S_1 and S_2 are divergent

42. If $P=\begin{bmatrix}3 & -3 & 3\\ 2 & -3 & 4\\ 0 & -1 & 1\end{bmatrix}$ then P is invertible and P^{-1} is equal to

(a) $(P^2+P+I)/3$ (b) $(P^2+P-I)/3$

(c) $(P^2-P+I)/3$ (d) $(P^2-P-I)/3$

43. Let $\{x_n\},\{y_n\}$ be two convergent real sequences and let $z_n=\max\{x_n,y_n\}$ for each $n\in\mathbb{N}$. Then

(a) $\{z_n\}$ is convergent

(b) $\{z_n\}$ is bounded but may not be convergent

(c) $\{z_n\}$ may not be convergent but $\{z_n\}$ has a convergent sub-sequence

(d) $\{z_n\}$ is convergent if and only if $\exists n_0\in\mathbb{N} \ni x_n=y_n \forall_n \geq n_0$

44. The solution of the differential equation $y' - y = xy^5$ is

(a) $y = (-x + c\exp(-4x) + 1/4)^4$
(b) $y = (-x + c\exp(-4x) + 1/4)^{-4}$
(c) $y = (-x + c\exp(-4x) + 1/4)^{-1/4}$
(d) $y = (-x + c\exp(-4x) + 1/4)^{1/4}$

45. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function such that $f(n) = n,\ \forall n \in \mathbb{Z}$. Then

(a) f is identity
(b) $|f(x)| \le x, \forall x \in \mathbb{R}$
(c) $f(x) > 0,\ \forall x \in (0, \infty)$
(d) none of these

46. Let $\{u, v, w\}$ be a linearly independent set in the vector space $\mathbb{R}^3$ and let $X = \text{span}\{u, v + w\}$ and $Y = \text{span}\{w, u + v\}$. Then the dimension of $X \cap Y$ is

(a) 0
(b) 1
(c) 2
(d) can not be found from the information

47. Let $f(x) = x|x|$ and $g(x) = \sin|x|$ then

(a) both f and g are differentiable functions
(b) f is differentiable function but g is not
(c) g is differentiable function but f is not
(d) both f and g are not differentiable functions

48. Let $u = x + ct,\ \ v = x - ct$ and $z = \log u + \sin v^2$ then $\dfrac{\partial^2 z}{\partial t^2} - c^2 \dfrac{\partial^2 z}{\partial x^2}$ is equal to

(a) $-c$
(b) -1
(c) $-2c$
(d) 0

49. If α, β and γ are the roots of the equation $15x^3 + 7x - 11 = 0$ then the value of $\alpha^3 + \beta^3 + \gamma^3$ is

(a) 3/5
(b) 7/5
(c) 9/5
(d) 11/5

50. Area of the region enclosed by the curves $y = x^2 - x - 2$ and $y = 0$ is

(a) 7/2
(b) –7/2
(c) 9/2
(d) –9/2

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M.Sc. Mathematics Entrance - 2010

ANSWER KEY

PART-A

1.	(a)	**2.**	(c)	**3.**	(a)	**4.**	(c)	**5.**	(c)
6.	(d)	**7.**	(c)	**8.**	(b)	**9.**	(b)	**10.**	(a)
11.	(b)	**12.**	(a)	**13.**	(a)	**14.**	(a)	**15.**	(c)
16.	(c, d)	**17.**	(d)	**18.**	(b)	**19.**	(d)	**20.**	(a)
21.	(d)	**22.**	(a)	**23.**	(c)	**24.**	(d)	**25.**	(a)

PART-B

26.	(b)	**27.**	(b)	**28.**	(a)	**29.**	(d)	**30.**	(a)
31.	(d)	**32.**	(c)	**33.**	(b)	**34.**	(b)	**35.**	(c)
36.	(b)	**37.**	(c)	**38.**	(d)	**39.**	(b)	**40.**	(d)
41.	(a)	**42.**	(c)	**43.**	(a)	**44.**	(c)	**45.**	(d)
46.	(b)	**47.**	(b)	**48.**	(d)	**49.**	(d)	**50.**	(c)