JAM Mathematics Que. Paper & Ans.

Contents:

No. of Pages: 271

JAM Mathematics (MA) Syllabus

Sequences and Series of Real Numbers: Sequence of real numbers, convergence of sequences, bounded and monotone sequences, convergence criteria for sequences of real numbers, Cauchy sequences, subsequences, Bolzano-Weierstrass theorem. Series of real numbers, absolute convergence, tests of convergence for series of positive terms – comparison test, ratio test, root test; Leibniz test for convergence of alternating series.

Functions of One Real Variable: Limit, continuity, intermediate value property, differentiation, Rolle's Theorem, mean value theorem, L'Hospital rule, Taylor's theorem, maxima and minima.

Functions of Two or Three Real Variables: Limit, continuity, partial derivatives, differentiability, maxima and minima.

Integral Calculus: Integration as the inverse process of differentiation, definite integrals and their properties, fundamental theorem of calculus. Double and triple integrals, change of order of integration, calculating surface areas and volumes using double integrals, calculating volumes using triple integrals.

Differential Equations: Ordinary differential equations of the first order of the form y'=f(x,y), Bernoulli's equation, exact differential equations, integrating factor, orthogonal trajectories, homogeneous differential equations, variable separable equations, linear differential equations of second order with constant coefficients, method of variation of parameters, Cauchy-Euler equation.

Vector Calculus: Scalar and vector fields, gradient, divergence, curl, line integrals, surface integrals, Green, Stokes and Gauss theorems.

Group Theory: Groups, subgroups, Abelian groups, non-Abelian groups, cyclic groups, permutation groups, normal subgroups, Lagrange's Theorem for finite groups, group homomorphisms and basic concepts of quotient groups.

Linear Algebra: Finite dimensional vector spaces, linear independence of vectors, basis, dimension, linear transformations, matrix representation, range space, null space, rank-nullity theorem. Rank and inverse of a matrix, determinant, solutions of systems of linear equations, consistency conditions, eigenvalues and eigenvectors for matrices, Cayley-Hamilton theorem.

Real Analysis: Interior points, limit points, open sets, closed sets, bounded sets, connected sets, compact sets, completeness of R. Power series (of real variable), Taylor's series, radius and interval of convergence, term-wise differentiation and integration of power series.

Paper Specific Instructions

1. The examination is of 3 hours duration. There are a total of 60 questions carrying 100 marks. The entire paper is divided into three sections, **A**, **B** and **C**. All sections are compulsory. Questions in each section are of different types.

2. **Section – A** contains a total of **30 Multiple Choice Questions (MCQ)**. Each MCQ type question has four choices out of which only **one** choice is the correct answer. Questions Q.1 – Q.30 belong to this section and carry a total of 50 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.30 carry 2 marks each.

3. **Section – B** contains a total of 10 **Multiple Select Questions (MSQ)**. Each MSQ type question is similar to MCQ but with a difference that there may be **one or more than one** choice(s) that are correct out of the four given choices. The candidate gets full credit if he/she selects all the correct answers only and no wrong answers. Questions Q.31 – Q.40 belong to this section and carry 2 marks each with a total of 20 marks.

4. **Section – C** contains a total of 20 **Numerical Answer Type (NAT)** questions. For these NAT type questions, the answer is a real number which needs to be entered using the virtual keyboard on the monitor. No choices will be shown for this type of questions. Questions Q.41 – Q.60 belong to this section and carry a total of 30 marks. Q.41 – Q.50 carry 1 mark each and Questions Q.51 – Q.60 carry 2 marks each.

5. In all sections, questions not attempted will result in zero mark. In **Section – A** (MCQ), wrong answer will result in **NEGATIVE** marks. For all 1 mark questions, 1/3 marks will be deducted for each wrong answer. For all 2 marks questions, 2/3 marks will be deducted for each wrong answer. In **Section – B** (MSQ), there is **NO NEGATIVE** and **NO PARTIAL** marking provisions. There is **NO NEGATIVE** marking in **Section – C** (NAT) as well.

6. Only Virtual Scientific Calculator is allowed. Charts, graph sheets, tables, cellular phone or other electronic gadgets are **NOT** allowed in the examination hall.

7. The Scribble Pad will be provided for rough work.

NOTATION

1. $\mathbb{N} = \{1, 2, 3, \cdots\}$
2. $\mathbb{R}$ - the set of all real numbers
3. $\mathbb{R} \setminus \{0\}$ - the set of all non-zero real numbers
4. $\mathbb{C}$ - the set of all complex numbers
5. $f \circ g$ - composition of the functions f and g
6. f' and f'' - first and second derivatives of the function f, respectively
7. $f^{(n)}$ - n^{th} derivative of f
8. $\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$
9. $\oint_C$ - the line integral over an oriented closed curve C
10. $\hat{i}, \hat{j}, \hat{k}$ - unit vectors along the Cartisean right handed rectangular co-ordinate system
11. $\hat{n}$ - unit outward normal vector
12. I - identity matrix of appropriate order
13. $\det(M)$ - determinant of the matrix M
14. M^{-1} - inverse of the matrix M
15. M^T - transpose of the matrix M
16. id - identity map
17. $\langle a \rangle$ - cyclic subgroup generated by an element a of a group
18. S_n - permutation group on n symbols
19. $S^1 = \{z \in \mathbb{C} : |z| = 1\}$
20. $o(g)$ - order of the element g in a group

SECTION – A

MULTIPLE CHOICE QUESTIONS (MCQ)

Q. 1 – Q. 10 carry one mark each.

Q. 1 Let $s_n = 1 + \dfrac{(-1)^n}{n}, n \in \mathbb{N}$. Then the sequence $\{s_n\}$ is

(A) monotonically increasing and is convergent to 1

(B) monotonically decreasing and is convergent to 1

(C) neither monotonically increasing nor monotonically decreasing but is convergent to 1

(D) divergent

Q. 2 Let $f(x) = 2x^3 - 9x^2 + 7$. Which of the following is true?

(A) f is one-one in the interval $[-1, 1]$

(B) f is one-one in the interval $[2, 4]$

(C) f is NOT one-one in the interval $[-4, 0]$

(D) f is NOT one-one in the interval $[0, 4]$

Q. 3 Which of the following is FALSE?

(A) $\lim\limits_{x\to\infty} \dfrac{x}{e^x} = 0$

(B) $\lim\limits_{x\to 0^+} \dfrac{1}{xe^{1/x}} = 0$

(C) $\lim\limits_{x\to 0^+} \dfrac{\sin x}{1+2x} = 0$

(D) $\lim\limits_{x\to 0^+} \dfrac{\cos x}{1+2x} = 0$

Q. 4 Let $g : \mathbb{R} \to \mathbb{R}$ be a twice differentiable function. If $f(x, y) = g(y) + xg'(y)$, then

(A) $\dfrac{\partial f}{\partial x} + y\dfrac{\partial^2 f}{\partial x \partial y} = \dfrac{\partial f}{\partial y}$

(B) $\dfrac{\partial f}{\partial y} + y\dfrac{\partial^2 f}{\partial x \partial y} = \dfrac{\partial f}{\partial x}$

(C) $\dfrac{\partial f}{\partial x} + x\dfrac{\partial^2 f}{\partial x \partial y} = \dfrac{\partial f}{\partial y}$

(D) $\dfrac{\partial f}{\partial y} + x\dfrac{\partial^2 f}{\partial x \partial y} = \dfrac{\partial f}{\partial x}$

Q. 5 If the equation of the tangent plane to the surface $z = 16 - x^2 - y^2$ at the point $P(1, 3, 6)$ is $ax + by + cz + d = 0$, then the value of $|d|$ is

(A) 16 (B) 26 (C) 36 (D) 46

Q. 6 If the directional derivative of the function $z = y^2 e^{2x}$ at $(2, -1)$ along the unit vector $\vec{b} = \alpha\hat{i} + \beta\hat{j}$ is zero, then $|\alpha + \beta|$ equals

(A) $\frac{1}{2\sqrt{2}}$ (B) $\frac{1}{\sqrt{2}}$ (C) $\sqrt{2}$ (D) $2\sqrt{2}$

Q. 7 If $u = x^3$ and $v = y^2$ transform the differential equation $3x^5 dx - y(y^2 - x^3)dy = 0$ to $\frac{dv}{du} = \frac{\alpha u}{2(u - v)}$, then α is

(A) 4 (B) 2 (C) -2 (D) -4

Q. 8 Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation given by $T(x, y) = (-x, y)$. Then

(A) $T^{2k} = T$ for all $k \geq 1$

(B) $T^{2k+1} = -T$ for all $k \geq 1$

(C) the range of T^2 is a proper subspace of the range of T

(D) the range of T^2 is equal to the range of T

Q. 9 The radius of convergence of the power series

$$\sum_{n=1}^{\infty} \left(\frac{n+2}{n}\right)^{n^2} x^n$$

is

(A) e^2 (B) $\frac{1}{\sqrt{e}}$ (C) $\frac{1}{e}$ (D) $\frac{1}{e^2}$

Q. 10 Consider the following group under matrix multiplication:

$$H = \left\{ \begin{bmatrix} 1 & p & q \\ 0 & 1 & r \\ 0 & 0 & 1 \end{bmatrix} : p, q, r \in \mathbb{R} \right\}.$$

Then the center of the group is isomorphic to

(A) $(\mathbb{R} \setminus \{0\}, \times)$

(B) $(\mathbb{R}, +)$

(C) $(\mathbb{R}^2, +)$

(D) $(\mathbb{R}, +) \times (\mathbb{R} \setminus \{0\}, \times)$

Q. 11 – Q. 30 carry two marks each.

Q. 11 Let $\{a_n\}$ be a sequence of positive real numbers. Suppose that $l = \lim_{n\to\infty} \frac{a_{n+1}}{a_n}$. Which of the following is true?

(A) If $l = 1$, then $\lim_{n\to\infty} a_n = 1$

(B) If $l = 1$, then $\lim_{n\to\infty} a_n = 0$

(C) If $l < 1$, then $\lim_{n\to\infty} a_n = 1$

(D) If $l < 1$, then $\lim_{n\to\infty} a_n = 0$

Q. 12 Define $s_1 = \alpha > 0$ and $s_{n+1} = \sqrt{\frac{1+s_n^2}{1+\alpha}}$, $n \geq 1$. Which of the following is true?

(A) If $s_n^2 < \frac{1}{\alpha}$, then $\{s_n\}$ is monotonically increasing and $\lim_{n\to\infty} s_n = \frac{1}{\sqrt{\alpha}}$

(B) If $s_n^2 < \frac{1}{\alpha}$, then $\{s_n\}$ is monotonically decreasing and $\lim_{n\to\infty} s_n = \frac{1}{\alpha}$

(C) If $s_n^2 > \frac{1}{\alpha}$, then $\{s_n\}$ is monotonically increasing and $\lim_{n\to\infty} s_n = \frac{1}{\sqrt{\alpha}}$

(D) If $s_n^2 > \frac{1}{\alpha}$, then $\{s_n\}$ is monotonically decreasing and $\lim_{n\to\infty} s_n = \frac{1}{\alpha}$

Q. 13 Suppose that S is the sum of a convergent series $\sum_{n=1}^{\infty} a_n$. Define $t_n = a_n + a_{n+1} + a_{n+2}$. Then the series $\sum_{n=1}^{\infty} t_n$

(A) diverges

(B) converges to $3S - a_1 - a_2$

(C) converges to $3S - a_1 - 2a_2$

(D) converges to $3S - 2a_1 - a_2$

Q. 14 Let $a \in \mathbb{R}$. If $f(x) = \begin{cases} (x+a)^2, & x \leq 0 \\ (x+a)^3, & x > 0, \end{cases}$

then

(A) $\frac{d^2 f}{dx^2}$ does not exist at $x = 0$ for any value of a

(B) $\frac{d^2 f}{dx^2}$ exists at $x = 0$ for exactly one value of a

(C) $\frac{d^2 f}{dx^2}$ exists at $x = 0$ for exactly two values of a

(D) $\frac{d^2 f}{dx^2}$ exists at $x = 0$ for infinitely many values of a

Q. 15 Let $f(x, y) = \begin{cases} x^2 \sin \frac{1}{x} + y^2 \sin \frac{1}{y}, & xy \neq 0 \\ x^2 \sin \frac{1}{x}, & x \neq 0, y = 0 \\ y^2 \sin \frac{1}{y}, & y \neq 0, x = 0 \\ 0, & x = y = 0. \end{cases}$

Which of the following is true at $(0, 0)$?

(A) f is not continuous

(B) $\frac{\partial f}{\partial x}$ is continuous but $\frac{\partial f}{\partial y}$ is not continuous

(C) f is not differentiable

(D) f is differentiable but both $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ are not continuous

Q. 16 Let S be the surface of the portion of the sphere with centre at the origin and radius 4, above the xy-plane. Let $\vec{F} = y\hat{i} - x\hat{j} + yx^3\hat{k}$. If $\hat{n}$ is the unit outward normal to S, then

$$\iint_S (\nabla \times \vec{F}) \cdot \hat{n}\, dS$$

equals

(A) -32π (B) -16π (C) 16π (D) 32π

Q. 17 Let $f(x, y, z) = x^3 + y^3 + z^3 - 3xyz$. A point at which the gradient of the function f is equal to zero is

(A) $(-1, 1, -1)$ (B) $(-1, -1, -1)$ (C) $(-1, 1, 1)$ (D) $(1, -1, 1)$

Q. 18 The area bounded by the curves $x^2 + y^2 = 2x$ and $x^2 + y^2 = 4x$, and the straight lines $y = x$ and $y = 0$ is

(A) $3\left(\frac{\pi}{2} + \frac{1}{4}\right)$ (B) $3\left(\frac{\pi}{4} + \frac{1}{2}\right)$ (C) $2\left(\frac{\pi}{4} + \frac{1}{3}\right)$ (D) $2\left(\frac{\pi}{3} + \frac{1}{4}\right)$

Q. 19 Let M be a real 6×6 matrix. Let 2 and -1 be two eigenvalues of M. If $M^5 = aI + bM$, where $a, b \in \mathbb{R}$, then

(A) $a = 10, b = 11$ (B) $a = -11, b = 10$

(C) $a = -10, b = 11$ (D) $a = 10, b = -11$

Q. 20 Let M be an $n \times n$ $(n \geq 2)$ non-zero real matrix with $M^2 = 0$ and let $\alpha \in \mathbb{R} \setminus \{0\}$. Then

(A) α is the only eigenvalue of $(M + \alpha I)$ and $(M - \alpha I)$

(B) α is the only eigenvalue of $(M + \alpha I)$ and $(\alpha I - M)$

(C) $-\alpha$ is the only eigenvalue of $(M + \alpha I)$ and $(M - \alpha I)$

(D) $-\alpha$ is the only eigenvalue of $(M + \alpha I)$ and $(\alpha I - M)$

Q. 21 Consider the differential equation $L[y] = (y - y^2)dx + xdy = 0$. The function $f(x, y)$ is said to be an integrating factor of the equation if $f(x, y)L[y] = 0$ becomes exact.

If $f(x, y) = \dfrac{1}{x^2y^2}$, then

(A) f is an integrating factor and $y = 1 - kxy,\ k \in \mathbb{R}$ is NOT its general solution

(B) f is an integrating factor and $y = -1 + kxy,\ k \in \mathbb{R}$ is its general solution

(C) f is an integrating factor and $y = -1 + kxy,\ k \in \mathbb{R}$ is NOT its general solution

(D) f is NOT an integrating factor and $y = 1 + kxy,\ k \in \mathbb{R}$ is its general solution

Q. 22 A solution of the differential equation $2x^2\dfrac{d^2y}{dx^2} + 3x\dfrac{dy}{dx} - y = 0, x > 0$ that passes through the point $(1, 1)$ is

(A) $y = \dfrac{1}{x}$ (B) $y = \dfrac{1}{x^2}$ (C) $y = \dfrac{1}{\sqrt{x}}$ (D) $y = \dfrac{1}{x^{3/2}}$

Q. 23 Let M be a 4×3 real matrix and let $\{e_1, e_2, e_3\}$ be the standard basis of $\mathbb{R}^3$. Which of the following is true?

(A) If rank$(M) = 1$, then $\{Me_1, Me_2\}$ is a linearly independent set

(B) If rank$(M) = 2$, then $\{Me_1, Me_2\}$ is a linearly independent set

(C) If rank$(M) = 2$, then $\{Me_1, Me_3\}$ is a linearly independent set

(D) If rank$(M) = 3$, then $\{Me_1, Me_3\}$ is a linearly independent set

Q. 24 The value of the triple integral $\iiint_V (x^2y + 1)\, dxdydz$, where V is the region given by $x^2 + y^2 \leq 1, 0 \leq z \leq 2$ is

(A) π (B) 2π (C) 3π (D) 4π

Q. 25 Let S be the part of the cone $z^2 = x^2 + y^2$ between the planes $z = 0$ and $z = 1$. Then the value of the surface integral $\iint_S (x^2 + y^2)\, dS$ is

(A) π (B) $\dfrac{\pi}{\sqrt{2}}$ (C) $\dfrac{\pi}{\sqrt{3}}$ (D) $\dfrac{\pi}{2}$

Q. 26 Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}, x, y, z \in \mathbb{R}$. Which of the following is FALSE?

(A) $\nabla(\vec{a} \cdot \vec{r}) = \vec{a}$

(B) $\nabla \cdot (\vec{a} \times \vec{r}) = 0$

(C) $\nabla \times (\vec{a} \times \vec{r}) = \vec{a}$

(D) $\nabla \cdot ((\vec{a} \cdot \vec{r})\vec{r}) = 4(\vec{a} \cdot \vec{r})$

Q. 27 Let $D = \{(x, y) \in \mathbb{R}^2 : |x| + |y| \leq 1\}$ and $f : D \to \mathbb{R}$ be a non-constant continuous function. Which of the following is TRUE?

(A) The range of f is unbounded

(B) The range of f is a union of open intervals

(C) The range of f is a closed interval

(D) The range of f is a union of at least two disjoint closed intervals

Q. 28 Let $f : [0, 1] \to \mathbb{R}$ be a continuous function such that $f\left(\frac{1}{2}\right) = -\frac{1}{2}$ and

$$|f(x) - f(y) - (x - y)| \leq \sin\left(|x - y|^2\right)$$

for all $x, y \in [0, 1]$. Then $\int_0^1 f(x)\, dx$ is

(A) $-\frac{1}{2}$

(B) $-\frac{1}{4}$

(C) $\frac{1}{4}$

(D) $\frac{1}{2}$

Q. 29 Let $S^1 = \{z \in \mathbb{C} : |z| = 1\}$ be the circle group under multiplication and $i = \sqrt{-1}$. Then the set $\{\theta \in \mathbb{R} : \langle e^{i2\pi\theta} \rangle$ is infinite$\}$ is

(A) empty

(B) non-empty and finite

(C) countably infinite

(D) uncountable

Q. 30 Let $F = \{\omega \in \mathbb{C} : \omega^{2020} = 1\}$. Consider the groups

$$G = \left\{ \begin{pmatrix} \omega & z \\ 0 & 1 \end{pmatrix} : \omega \in F, z \in \mathbb{C} \right\}$$

and

$$H = \left\{ \begin{pmatrix} 1 & z \\ 0 & 1 \end{pmatrix} : z \in \mathbb{C} \right\}$$

under matrix multiplication. Then the number of cosets of H in G is

(A) 1010 (B) 2019 (C) 2020 (D) infinite

SECTION – B

MULTIPLE SELECT QUESTIONS (MSQ)

Q. 31 – Q. 40 carry two marks each.

Q. 31 Let $a, b, c \in \mathbb{R}$ such that $a < b < c$. Which of the following is/are true for any continuous function $f : \mathbb{R} \to \mathbb{R}$ satisfying $f(a) = b, f(b) = c$ and $f(c) = a$?

(A) There exists $\alpha \in (a, c)$ such that $f(\alpha) = \alpha$

(B) There exists $\beta \in (a, b)$ such that $f(\beta) = \beta$

(C) There exists $\gamma \in (a, b)$ such that $(f \circ f)(\gamma) = \gamma$

(D) There exists $\delta \in (a, c)$ such that $(f \circ f \circ f)(\delta) = \delta$

Q. 32 If $s_n = \dfrac{(-1)^n}{2^n + 3}$ and $t_n = \dfrac{(-1)^n}{4n - 1}, n = 0, 1, 2, ...,$ then

(A) $\sum\limits_{n=0}^{\infty} s_n$ is absolutely convergent

(B) $\sum\limits_{n=0}^{\infty} t_n$ is absolutely convergent

(C) $\sum\limits_{n=0}^{\infty} s_n$ is conditionally convergent

(D) $\sum\limits_{n=0}^{\infty} t_n$ is conditionally convergent

Q. 33 Let $a, b \in \mathbb{R}$ and $a < b$. Which of the following statement(s) is/are true?

(A) There exists a continuous function $f : [a, b] \to (a, b)$ such that f is one-one

(B) There exists a continuous function $f : [a, b] \to (a, b)$ such that f is onto

(C) There exists a continuous function $f : (a, b) \to [a, b]$ such that f is one-one

(D) There exists a continuous function $f : (a, b) \to [a, b]$ such that f is onto

Q. 34 Let V be a non-zero vector space over a field F. Let $S \subset V$ be a non-empty set. Consider the following properties of S:

(I) For any vector space W over F, any map $f : S \to W$ extends to a linear map from V to W.

(II) For any vector space W over F and any two linear maps $f, g : V \to W$ satisfying $f(s) = g(s)$ for all $s \in S$, we have $f(v) = g(v)$ for all $v \in V$.

(III) S is linearly independent.

(IV) The span of S is V.

Which of the following statement(s) is /are true?

(A) (I) implies (IV) (B) (I) implies (III)

(C) (II) implies (III) (D) (II) implies (IV)

Q. 35 Let $L[y] = x^2 \dfrac{d^2y}{dx^2} + px\dfrac{dy}{dx} + qy$, where p, q are real constants. Let $y_1(x)$ and $y_2(x)$ be two solutions of $L[y] = 0, x > 0$, that satisfy $y_1(x_0) = 1, y_1'(x_0) = 0, y_2(x_0) = 0$ and $y_2'(x_0) = 1$ for some $x_0 > 0$. Then,

(A) $y_1(x)$ is not a constant multiple of $y_2(x)$

(B) $y_1(x)$ is a constant multiple of $y_2(x)$

(C) $1, \ln x$ are solutions of $L[y] = 0$ when $p = 1, q = 0$

(D) $x, \ln x$ are solutions of $L[y] = 0$ when $p + q \neq 0$

Q. 36 Consider the following system of linear equations

$$x + y + 5z = 3, \quad x + 2y + mz = 5 \quad \text{and} \quad x + 2y + 4z = k.$$

The system is consistent if

(A) $m \neq 4$ (B) $k \neq 5$ (C) $m = 4$ (D) $k = 5$

Q. 37 Let $a = \lim_{n\to\infty}\left(\frac{1}{n^2} + \frac{2}{n^2} + \cdots + \frac{(n-1)}{n^2}\right)$ and $b = \lim_{n\to\infty}\left(\frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{n+n}\right)$.
Which of the following is/are true?

(A) $a > b$ (B) $a < b$ (C) $ab = \ln\sqrt{2}$ (D) $\frac{a}{b} = \ln\sqrt{2}$

Q. 38 Let S be that part of the surface of the paraboloid $z = 16 - x^2 - y^2$ which is above the plane $z = 0$ and D be its projection on the xy-plane. Then the area of S equals

(A) $\iint_D \sqrt{1 + 4(x^2 + y^2)}\, dxdy$

(B) $\iint_D \sqrt{1 + 2(x^2 + y^2)}\, dxdy$

(C) $\int_0^{2\pi}\int_0^{4} \sqrt{1 + 4r^2}\, drd\theta$

(D) $\int_0^{2\pi}\int_0^{4} \sqrt{1 + 4r^2}\, rdrd\theta$

Q. 39 Let f be a real valued function of a real variable, such that $|f^{(n)}(0)| \leq K$ for all $n \in \mathbb{N}$, where $K > 0$. Which of the following is/are true?

(A) $\left|\frac{f^{(n)}(0)}{n!}\right|^{\frac{1}{n}} \to 0$ as $n \to \infty$

(B) $\left|\frac{f^{(n)}(0)}{n!}\right|^{\frac{1}{n}} \to \infty$ as $n \to \infty$

(C) $f^{(n)}(x)$ exists for all $x \in \mathbb{R}$ and for all $n \in \mathbb{N}$

(D) The series $\sum_{n=1}^{\infty} \frac{f^{(n)}(0)}{(n-1)!}$ is absolutely convergent

Q. 40 Let G be a group with identity e. Let H be an abelian non-trivial proper subgroup of G with the property that $H \cap gHg^{-1} = \{e\}$ for all $g \notin H$.

If $K = \{g \in G : gh = hg \text{ for all } h \in H\}$, then

(A) K is a proper subgroup of H

(B) H is a proper subgroup of K

(C) $K = H$

(D) there exists no abelian subgroup $L \subseteq G$ such that K is a proper subgroup of L

SECTION – C

NUMERICAL ANSWER TYPE (NAT)

Q. 41 – Q. 50 carry one mark each.

Q. 41 Let $x_n = n^{\frac{1}{n}}$ and $y_n = e^{1-x_n}, n \in \mathbb{N}$. Then the value of $\lim_{n\to\infty} y_n$ is ________.

Q. 42 Let $\vec{F} = x\hat{i} + y\hat{j} + z\hat{k}$ and S be the sphere given by $(x-2)^2 + (y-2)^2 + (z-2)^2 = 4$. If $\hat{n}$ is the unit outward normal to S, then

$$\frac{1}{\pi} \iint_S \vec{F} \cdot \hat{n}\, dS$$

is ________.

Q. 43 Let $f : \mathbb{R} \to \mathbb{R}$ be such that f, f', f'' are continuous functions with $f > 0, f' > 0$ and $f'' > 0$. Then

$$\lim_{x\to-\infty} \frac{f(x) + f'(x)}{2}$$

is ________.

Q. 44 Let $S = \left\{\frac{1}{n} : n \in \mathbb{N}\right\}$ and $f : S \to \mathbb{R}$ be defined by $f(x) = \frac{1}{x}$. Then

$$\max\left\{\delta : \left|x - \frac{1}{3}\right| < \delta \implies \left|f(x) - f\left(\frac{1}{3}\right)\right| < 1\right\}$$

is ________. (rounded off to two decimal places)

Q. 45 Let $f(x, y) = e^x \sin y, x = t^3 + 1$ and $y = t^4 + t$. Then $\frac{df}{dt}$ at $t = 0$ is ________. (rounded off to two decimal places)

Q. 46 Consider the differential equation

$$\frac{dy}{dx} + 10y = f(x),\ x > 0,$$

where $f(x)$ is a continuous function such that $\lim_{x\to\infty} f(x) = 1$. Then the value of

$$\lim_{x\to\infty} y(x)$$

is ________.

Q. 47 If $\int_0^1 \int_{2y}^2 e^{x^2} dxdy = k(e^4 - 1)$, then k equals ________.

Q. 48 Let $f(x, y) = 0$ be a solution of the homogeneous differential equation

$$(2x + 5y)dx - (x + 3y)dy = 0.$$

If $f(x + \alpha, y - 3) = 0$ is a solution of the differential equation

$$(2x + 5y - 1)dx + (2 - x - 3y)dy = 0,$$

then the value of α is ________.

Q. 49 Consider the real vector space $P_{2020} = \{\sum_{i=0}^{n} a_i x^i : a_i \in \mathbb{R} \text{ and } 0 \le n \le 2020\}$. Let W be the subspace given by

$$W = \left\{ \sum_{i=0}^{n} a_i x^i \in P_{2020} : a_i = 0 \text{ for all odd } i \right\}.$$

Then, the dimension of W is ________.

Q. 50 Let $\phi : S_3 \to S^1$ be a non-trivial non-injective group homomorphism. Then, the number of elements in the kernel of ϕ is ________.

Q. 51 – Q. 60 carry two marks each.

Q. 51 The sum of the series $\dfrac{1}{2(2^2-1)} + \dfrac{1}{3(3^2-1)} + \dfrac{1}{4(4^2-1)} + \cdots$ is ________.

Q. 52 Consider the expansion of the function $f(x) = \dfrac{3}{(1-x)(1+2x)}$ in powers of x, that is valid in $|x| < \dfrac{1}{2}$. Then the coefficient of x^4 is ________.

Q. 53 The minimum value of the function $f(x,y) = x^2 + xy + y^2 - 3x - 6y + 11$ is ________.

Q. 54 Let $f(x) = \sqrt{x} + \alpha x,\ x > 0$ and

$$g(x) = a_0 + a_1(x-1) + a_2(x-1)^2$$

be the sum of the first three terms of the Taylor series of $f(x)$ around $x = 1$. If $g(3) = 3$, then α is ________.

Q. 55 Let C be the boundary of the square with vertices $(0,0)$, $(1,0)$, $(1,1)$ and $(0,1)$ oriented in the counter clockwise sense. Then, the value of the line integral

$$\oint_C x^2y^2dx + (x^2 - y^2)dy$$

is ________. (rounded off to two decimal places)

Q. 56 Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function with $f'(x) = f(x)$ for all x. Suppose that $f(\alpha x)$ and $f(\beta x)$ are two non-zero solutions of the differential equation

$$4\frac{d^2y}{dx^2} - p\frac{dy}{dx} + 3y = 0$$

satisfying

$$f(\alpha x)f(\beta x) = f(2x) \text{ and } f(\alpha x)f(-\beta x) = f(x).$$

Then, the value of p is ________.

Q. 57 If $x^2 + xy^2 = c$, where $c \in \mathbb{R}$, is the general solution of the exact differential equation

$$M(x, y)\, dx + 2xy\, dy = 0,$$

then $M(1, 1)$ is ________.

Q. 58 Let $M = \begin{bmatrix} 9 & 2 & 7 & 1 \\ 0 & 7 & 2 & 1 \\ 0 & 0 & 11 & 6 \\ 0 & 0 & -5 & 0 \end{bmatrix}$. Then, the value of $\det\Big((8I - M)^3\Big)$ is ________.

Q. 59 Let $T : \mathbb{R}^7 \to \mathbb{R}^7$ be a linear transformation with Nullity$(T) = 2$. Then, the minimum possible value for Rank(T^2) is ________.

Q. 60 Suppose that G is a group of order 57 which is NOT cyclic. If G contains a unique subgroup H of order 19, then for any $g \notin H$, $o(g)$ is ________.

END OF THE QUESTION PAPER

Mathematics Answer Key-2020

Mathematics (MA)					
Q. No.	Session	QT	Section	Key	Marks
1	2	MCQ	A	C	1
2	2	MCQ	A	D	1
3	2	MCQ	A	D	1
4	2	MCQ	A	C	1
5	2	MCQ	A	Marks To All	1
6	2	MCQ	A	C	1
7	2	MCQ	A	D	1
8	2	MCQ	A	D	1
9	2	MCQ	A	D	1
10	2	MCQ	A	B	1
11	2	MCQ	A	D	2
12	2	MCQ	A	A	2
13	2	MCQ	A	D	2
14	2	MCQ	A	A	2
15	2	MCQ	A	D	2
16	2	MCQ	A	A	2
17	2	MCQ	A	B	2
18	2	MCQ	A	B	2
19	2	MCQ	A	A	2
20	2	MCQ	A	B	2
21	2	MCQ	A	C	2
22	2	MCQ	A	A	2
23	2	MCQ	A	D	2
24	2	MCQ	A	B	2
25	2	MCQ	A	B	2
26	2	MCQ	A	C	2
27	2	MCQ	A	C	2
28	2	MCQ	A	A	2
29	2	MCQ	A	D	2
30	2	MCQ	A	C	2
31	2	MSQ	B	A;C;D	2
32	2	MSQ	B	A;D	2
33	2	MSQ	B	A;C;D	2
34	2	MSQ	B	B;D	2
35	2	MSQ	B	A;C	2
36	2	MSQ	B	A;D	2
37	2	MSQ	B	B;C	2
38	2	MSQ	B	A;D	2
39	2	MSQ	B	A;D	2
40	2	MSQ	B	C;D	2
41	2	NAT	C	1 to 1	1
42	2	NAT	C	32 to 32	1
43	2	NAT	C	0 to INF	1
44	2	NAT	C	0.08 to 0.09	1
45	2	NAT	C	2.70 to 2.72	1
46	2	NAT	C	0.1 to 0.1	1
47	2	NAT	C	0.25 to 0.25	1
48	2	NAT	C	7 to 7	1

49	2	NAT	C	1011 to 1011	1
50	2	NAT	C	3 to 3	1
51	2	NAT	C	0.25 to 0.25	2
52	2	NAT	C	33 to 33	2
53	2	NAT	C	2 to 2	2
54	2	NAT	C	0.5 to 0.5	2
55	2	NAT	C	0.65 to 0.67	2
56	2	NAT	C	8 to 8	2
57	2	NAT	C	3 to 3	2
58	2	NAT	C	-216 to -216	2
59	2	NAT	C	3 to 3	2
60	2	NAT	C	3 to 3	2

Paper Specific Instructions

1. The examination is of 3 hours duration. There are a total of 60 questions carrying 100 marks. The entire paper is divided into three sections, **A**, **B** and **C**. All sections are compulsory. Questions in each section are of different types.

2. **Section – A** contains a total of 30 **Multiple Choice Questions (MCQ).** Each MCQ type question has four choices out of which only **one** choice is the correct answer. Questions Q.1 – Q.30 belong to this section and carry a total of 50 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.30 carry 2 marks each.

3. **Section – B** contains a total of 10 **Multiple Select Questions (MSQ).** Each MSQ type question is similar to MCQ but with a difference that there may be **one or more than one** choice(s) that are correct out of the four given choices. The candidate gets full credit if he/she selects all the correct answers only and no wrong answers. Questions Q.31 – Q.40 belong to this section and carry 2 marks each with a total of 20 marks.

4. **Section – C** contains a total of 20 **Numerical Answer Type (NAT)** questions. For these NAT type questions, the answer is a real number which needs to be entered using the virtual keyboard on the monitor. No choices will be shown for these type of questions. Questions Q.41 – Q.60 belong to this section and carry a total of 30 marks. Q.41 – Q.50 carry 1 mark each and Questions Q.51 – Q.60 carry 2 marks each.

5. In all sections, questions not attempted will result in zero mark. In **Section – A** (MCQ), wrong answer will result in **NEGATIVE** marks. For all 1 mark questions, 1/3 marks will be deducted for each wrong answer. For all 2 marks questions, 2/3 marks will be deducted for each wrong answer. In **Section – B** (MSQ), there is **NO NEGATIVE** and **NO PARTIAL** marking provisions. There is **NO NEGATIVE** marking in **Section – C** (NAT) as well.

6. Only Virtual Scientific Calculator is allowed. Charts, graph sheets, tables, cellular phone or other electronic gadgets are **NOT** allowed in the examination hall.

7. The Scribble Pad will be provided for rough work.

Notation

$\mathbb{N}$	set of all natural numbers $1, 2, 3, \cdots$
$\mathbb{R}$	set of all real numbers
$M_{m\times n}(\mathbb{R})$	real vector space of all matrices of size $m \times n$ with entries in $\mathbb{R}$
$\emptyset$	empty set
$X \setminus Y$	set of all elements from the set X which are not in the set Y
$\mathbb{Z}_n$	group of all congruence classes of integers modulo n
$\hat{\imath}, \hat{\jmath}, \hat{k}$	unit vectors having the directions of the positive x, y and z axes of a three dimensional rectangular coordinate system, respectively
S_n	group of all permutations of the set $\{1, 2, 3, \cdots, n\}$
ln	logarithm to the base e
log	logarithm to the base 10
∇	$\hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$
$\det(M)$	determinant of a square matrix M

SECTION – A

MULTIPLE CHOICE QUESTIONS (MCQ)

Q. 1 – Q.10 carry one mark each.

Q.1 Let $a_1 = b_1 = 0$, and for each $n \geq 2$, let a_n and b_n be real numbers given by

$$a_n = \sum_{m=2}^{n} \frac{(-1)^m m}{(\log(m))^m} \text{ and } b_n = \sum_{m=2}^{n} \frac{1}{(\log(m))^m} .$$

Then which one of the following is TRUE about the sequences $\{a_n\}$ and $\{b_n\}$?

(A) Both $\{a_n\}$ and $\{b_n\}$ are divergent
(B) $\{a_n\}$ is convergent and $\{b_n\}$ is divergent
(C) $\{a_n\}$ is divergent and $\{b_n\}$ is convergent
(D) Both $\{a_n\}$ and $\{b_n\}$ are convergent

Q.2 Let $T \in M_{m\times n}(\mathbb{R})$. Let V be the subspace of $M_{n\times p}(\mathbb{R})$ defined by

$$V = \{X \in M_{n\times p}(\mathbb{R}) : TX = 0\}.$$

Then the dimension of V is

(A) $pn -$ rank(T)
(B) $mn - p$ rank(T)
(C) $p(m -$ rank$(T))$
(D) $p(n -$ rank$(T))$

Q.3 Let $g: \mathbb{R} \to \mathbb{R}$ be a twice differentiable function. Define $f: \mathbb{R}^3 \to \mathbb{R}$ by

$$f(x, y, z) = g(x^2 + y^2 - 2z^2).$$

Then $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$ is equal to

(A) $4(x^2 + y^2 - 4z^2)\, g''(x^2 + y^2 - 2z^2)$
(B) $4(x^2 + y^2 + 4z^2)\, g''(x^2 + y^2 - 2z^2)$
(C) $4(x^2 + y^2 - 2z^2)\, g''(x^2 + y^2 - 2z^2)$
(D) $4(x^2 + y^2 + 4z^2)\, g''(x^2 + y^2 - 2z^2) + 8g'(x^2 + y^2 - 2z^2)$

Q.4 Let $\{a_n\}_{n=0}^{\infty}$ and $\{b_n\}_{n=0}^{\infty}$ be sequences of positive real numbers such that $na_n < b_n < n^2 a_n$ for all $n \geq 2$. If the radius of convergence of the power series $\sum_{n=0}^{\infty} a_n x^n$ is 4, then the power series $\sum_{n=0}^{\infty} b_n x^n$

(A) converges for all x with $|x| < 2$
(B) converges for all x with $|x| > 2$
(C) does not converge for any x with $|x| > 2$
(D) does not converge for any x with $|x| < 2$

Q.5 Let S be the set of all limit points of the set $\left\{\frac{n}{\sqrt{2}} + \frac{\sqrt{2}}{n} : n \in \mathbb{N}\right\}$. Let $\mathbb{Q}_+$ be the set of all positive rational numbers. Then

(A) $\mathbb{Q}_+ \subseteq S$
(B) $S \subseteq \mathbb{Q}_+$
(C) $S \cap (\mathbb{R} \setminus \mathbb{Q}_+) \neq \emptyset$
(D) $S \cap \mathbb{Q}_+ \neq \emptyset$

Q.6 If $x^h y^k$ is an integrating factor of the differential equation

$$y(1+xy)\,dx + x(1-xy)\,dy = 0,$$

then the ordered pair (h,k) is equal to

(A) $(-2,-2)$ (B) $(-2,-1)$ (C) $(-1,-2)$ (D) $(-1,-1)$

Q.7 If $y(x) = \lambda e^{2x} + e^{\beta x}$, $\beta \neq 2$, is a solution of the differential equation

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$$

satisfying $\frac{dy}{dx}(0) = 5$, then $y(0)$ is equal to

(A) 1 (B) 4 (C) 5 (D) 9

Q.8 The equation of the tangent plane to the surface $x^2 z + \sqrt{8 - x^2 - y^4} = 6$ at the point $(2, 0, 1)$ is

(A) $2x + z = 5$ (B) $3x + 4z = 10$
(C) $3x - z = 10$ (D) $7x - 4z = 10$

Q.9 The value of the integral

$$\int_{y=0}^{1} \int_{x=0}^{1-y^2} y \sin\left(\pi(1-x)^2\right) dx\, dy$$

is

(A) $\frac{1}{2\pi}$ (B) 2π (C) $\frac{\pi}{2}$ (D) $\frac{2}{\pi}$

Q.10 The area of the surface generated by rotating the curve $x = y^3$, $0 \le y \le 1$, about the y-axis, is

(A) $\frac{\pi}{27} 10^{3/2}$ (B) $\frac{4\pi}{3}(10^{3/2} - 1)$ (C) $\frac{\pi}{27}(10^{3/2} - 1)$ (D) $\frac{4\pi}{3} 10^{3/2}$

Q. 11 – Q. 30 carry two marks each.

Q.11 Let H and K be subgroups of $\mathbb{Z}_{144}$. If the order of H is 24 and the order of K is 36, then the order of the subgroup $H \cap K$ is

(A) 3 (B) 4 (C) 6 (D) 12

Q.12 Let P be a 4×4 matrix with entries from the set of rational numbers. If $\sqrt{2} + i$, with $i = \sqrt{-1}$, is a root of the characteristic polynomial of P and I is the 4×4 identity matrix, then

(A) $P^4 = 4P^2 + 9I$ (B) $P^4 = 4P^2 - 9I$ (C) $P^4 = 2P^2 - 9I$ (D) $P^4 = 2P^2 + 9I$

Q.13 The set $\left\{\frac{x}{1+x} : -1 < x < 1\right\}$, as a subset of $\mathbb{R}$, is

(A) connected and compact
(B) connected but not compact
(C) not connected but compact
(D) neither connected nor compact

Q.14 The set $\left\{\frac{1}{m}+\frac{1}{n} : m,n \in \mathbb{N}\right\} \cup \{0\}$, as a subset of $\mathbb{R}$, is

(A) compact and open
(B) compact but not open
(C) not compact but open
(D) neither compact nor open

Q.15 For $-1 < x < 1$, the sum of the power series $1 + \sum_{n=2}^{\infty} (-1)^{n-1}\, n^2 x^{n-1}$ is

(A) $\frac{1-x}{(1+x)^3}$
(B) $\frac{1+x^2}{(1+x)^4}$
(C) $\frac{1-x}{(1+x)^2}$
(D) $\frac{1+x^2}{(1+x)^3}$

Q.16 Let $f(x) = (\ln x)^2$, $x > 0$. Then

(A) $\lim_{x\to\infty} \frac{f(x)}{x}$ does not exist
(B) $\lim_{x\to\infty} f'(x) = 2$
(C) $\lim_{x\to\infty} \big(f(x+1) - f(x)\big) = 0$
(D) $\lim_{x\to\infty} \big(f(x+1) - f(x)\big)$ does not exist

Q.17 Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function such that $f'(x) > f(x)$ for all $x \in \mathbb{R}$, and $f(0) = 1$. Then $f(1)$ lies in the interval

(A) $(0,\ e^{-1})$
(B) $\left(e^{-1}, \sqrt{e}\right)$
(C) $(\sqrt{e},\ e)$
(D) (e, ∞)

Q.18 For which one of the following values of k, the equation

$$2x^3 + 3x^2 - 12x - k = 0$$

has three distinct real roots?

(A) 16
(B) 20
(C) 26
(D) 31

Q.19 Which one of the following series is divergent?

(A) $\sum_{n=1}^{\infty} \frac{1}{n} \sin^2 \frac{1}{n}$
(B) $\sum_{n=1}^{\infty} \frac{1}{n} \log n$
(C) $\sum_{n=1}^{\infty} \frac{1}{n^2} \sin \frac{1}{n}$
(D) $\sum_{n=1}^{\infty} \frac{1}{n} \tan \frac{1}{n}$

Q.20 Let S be the family of orthogonal trajectories of the family of curves

$$2x^2 + y^2 = k, \text{ for } k \in \mathbb{R} \text{ and } k > 0.$$

If $\mathcal{C} \in S$ and $\mathcal{C}$ passes through the point $(1, 2)$, then $\mathcal{C}$ also passes through

(A) $(4, -\sqrt{2})$ (B) $(2, -4)$ (C) $(2, 2\sqrt{2})$ (D) $(4, 2\sqrt{2})$

Q.21 Let x, $x + e^x$ and $1 + x + e^x$ be solutions of a linear second order ordinary differential equation with constant coefficients. If $y(x)$ is the solution of the same equation satisfying $y(0) = 3$ and $y'(0) = 4$, then $y(1)$ is equal to

(A) $e + 1$ (B) $2e + 3$ (C) $3e + 2$ (D) $3e + 1$

Q.22 The function

$$f(x, y) = x^3 + 2xy + y^3$$

has a saddle point at

(A) $(0, 0)$ (B) $\left(-\frac{2}{3}, -\frac{2}{3}\right)$ (C) $\left(-\frac{3}{2}, -\frac{3}{2}\right)$ (D) $(-1, -1)$

Q.23 The area of the part of the surface of the paraboloid $x^2 + y^2 + z = 8$ lying inside the cylinder $x^2 + y^2 = 4$ is

(A) $\frac{\pi}{2}(17^{3/2} - 1)$ (B) $\pi(17^{3/2} - 1)$ (C) $\frac{\pi}{6}(17^{3/2} - 1)$ (D) $\frac{\pi}{3}(17^{3/2} - 1)$

Q.24 Let $\mathcal{C}$ be the circle $(x - 1)^2 + y^2 = 1$, oriented counter clockwise. Then the value of the line integral

$$\oint_{\mathcal{C}} -\frac{4}{3}xy^3\, dx + x^4\, dy$$

is

(A) 6π (B) 8π (C) 12π (D) 14π

Q.25 Let $\vec{F}(x, y, z) = 2y\,\hat{\imath} + x^2\,\hat{\jmath} + xy\,\hat{k}$ and let $\mathcal{C}$ be the curve of intersection of the plane $x + y + z = 1$ and the cylinder $x^2 + y^2 = 1$. Then the value of

$$\left|\oint_{\mathcal{C}} \vec{F} \cdot d\vec{r}\,\right|$$

is

(A) π (B) $\frac{3\pi}{2}$ (C) 2π (D) 3π

Q.26 The tangent line to the curve of intersection of the surface $x^2 + y^2 - z = 0$ and the plane $x + z = 3$ at the point $(1, 1, 2)$ passes through

(A) $(-1, -2, 4)$ (B) $(-1, 4, 4)$ (C) $(3, 4, 4)$ (D) $(-1, 4, 0)$

Q.27 The set of eigenvalues of which one of the following matrices is NOT equal to the set of eigenvalues of $\begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$?

(A) $\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$ (B) $\begin{pmatrix} 3 & 2 \\ 4 & 1 \end{pmatrix}$ (C) $\begin{pmatrix} 3 & 4 \\ 2 & 1 \end{pmatrix}$ (D) $\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$

Q.28 Let $\{a_n\}$ be a sequence of positive real numbers. The series $\sum_{n=1}^{\infty} a_n$ converges if the series

(A) $\sum_{n=1}^{\infty} a_n^2$ converges
(B) $\sum_{n=1}^{\infty} \frac{a_n}{2^n}$ converges
(C) $\sum_{n=1}^{\infty} \frac{a_{n+1}}{a_n}$ converges
(D) $\sum_{n=1}^{\infty} \frac{a_n}{a_{n+1}}$ converges

Q.29 For $\beta \in \mathbb{R}$, define

$$f(x, y) = \begin{cases} \dfrac{x^2|x|^\beta y}{x^4 + y^2}, & x \neq 0, \\ 0, & x = 0. \end{cases}$$

Then, at $(0, 0)$, the function f is

(A) continuous for $\beta = 0$
(B) continuous for $\beta > 0$
(C) not differentiable for any β
(D) continuous for $\beta < 0$

Q.30 Let $\{a_n\}$ be a sequence of positive real numbers such that

$$a_1 = 1, \; a_{n+1}^2 - 2a_n a_{n+1} - a_n = 0 \text{ for all } n \geq 1.$$

Then the sum of the series $\sum_{n=1}^{\infty} \frac{a_n}{3^n}$ lies in the interval

(A) $(1, 2]$ (B) $(2, 3]$ (C) $(3, 4]$ (D) $(4, 5]$

SECTION - B

MULTIPLE SELECT QUESTIONS (MSQ)

Q. 31 – Q. 40 carry two marks each.

Q.31 Let G be a noncyclic group of order 4. Consider the statements I and II:

I. There is NO injective (one-one) homomorphism from G to $\mathbb{Z}_8$
II. There is NO surjective (onto) homomorphism from $\mathbb{Z}_8$ to G

Then

(A) I is true (B) I is false
(C) II is true (D) II is false

Q.32 Let G be a nonabelian group, $y \in G$, and let the maps f, g, h from G to itself be defined by

$$f(x) = yxy^{-1},\ g(x) = x^{-1} \text{ and } h = g \circ g.$$

Then

(A) g and h are homomorphisms and f is not a homomorphism
(B) h is a homomorphism and g is not a homomorphism
(C) f is a homomorphism and g is not a homomorphism
(D) f, g and h are homomorphisms

Q.33 Let S and T be linear transformations from a finite dimensional vector space V to itself such that $S(T(v)) = 0$ for all $v \in V$. Then

(A) $\text{rank}(T) \geq \text{nullity}(S)$ (B) $\text{rank}(S) \geq \text{nullity}(T)$
(C) $\text{rank}(T) \leq \text{nullity}(S)$ (D) $\text{rank}(S) \leq \text{nullity}(T)$

Q.34 Let $\vec{F}$ and $\vec{G}$ be differentiable vector fields and let g be a differentiable scalar function. Then

(A) $\nabla \cdot (\vec{F} \times \vec{G}) = \vec{G} \cdot \nabla \times \vec{F} - \vec{F} \cdot \nabla \times \vec{G}$ (B) $\nabla \cdot (\vec{F} \times \vec{G}) = \vec{G} \cdot \nabla \times \vec{F} + \vec{F} \cdot \nabla \times \vec{G}$
(C) $\nabla \cdot (g\vec{F}) = g\nabla \cdot \vec{F} - \nabla g \cdot \vec{F}$ (D) $\nabla \cdot (g\vec{F}) = g\nabla \cdot \vec{F} + \nabla g \cdot \vec{F}$

Q.35 Consider the intervals $S = (0, 2]$ and $T = [1, 3)$. Let S° and T° be the sets of interior points of S and T, respectively. Then the set of interior points of $S \setminus T$ is equal to

(A) $S \setminus T^\circ$ (B) $S \setminus T$ (C) $S^\circ \setminus T^\circ$ (D) $S^\circ \setminus T$

Q.36 Let $\{a_n\}$ be the sequence given by

$$a_n = \max\left\{\sin\left(\frac{n\pi}{3}\right), \cos\left(\frac{n\pi}{3}\right)\right\}, \quad n \geq 1.$$

Then which of the following statements is/are TRUE about the subsequences $\{a_{6n-1}\}$ and $\{a_{6n+4}\}$?

(A) Both the subsequences are convergent
(B) Only one of the subsequences is convergent
(C) $\{a_{6n-1}\}$ converges to $-\frac{1}{2}$
(D) $\{a_{6n+4}\}$ converges to $\frac{1}{2}$

Q.37 Let

$$f(x) = \cos(|\pi - x|) + (x - \pi)\sin|x| \text{ and } g(x) = x^2 \text{ for } x \in \mathbb{R}.$$

If $h(x) = f(g(x))$, then

(A) h is not differentiable at $x = 0$
(B) $h'(\sqrt{\pi}) = 0$
(C) $h''(x) = 0$ has a solution in $(-\pi, \pi)$
(D) there exists $x_0 \in (-\pi, \pi)$ such that $h(x_0) = x_0$

Q.38 Let $f: \left(0, \frac{\pi}{2}\right) \to \mathbb{R}$ be given by

$$f(x) = (\sin x)^{\pi} - \pi \sin x + \pi.$$

Then which of the following statements is/are TRUE?

(A) f is an increasing function
(B) f is a decreasing function
(C) $f(x) > 0$ for all $x \in \left(0, \frac{\pi}{2}\right)$
(D) $f(x) < 0$ for some $x \in \left(0, \frac{\pi}{2}\right)$

Q.39 Let

$$f(x, y) = \begin{cases} \frac{|x|}{|x| + |y|}\sqrt{x^4 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0). \end{cases}$$

Then at $(0, 0)$,

(A) f is continuous
(B) $\frac{\partial f}{\partial x} = 0$ and $\frac{\partial f}{\partial y}$ does not exist
(C) $\frac{\partial f}{\partial x}$ does not exist and $\frac{\partial f}{\partial y} = 0$
(D) $\frac{\partial f}{\partial x} = 0$ and $\frac{\partial f}{\partial y} = 0$

Q.40 Let $\{a_n\}$ be the sequence of real numbers such that

$$a_1 = 1 \text{ and } a_{n+1} = a_n + a_n^2 \text{ for all } n \geq 1.$$

Then

(A) $a_4 = a_1(1 + a_1)(1 + a_2)(1 + a_3)$
(B) $\lim_{n\to\infty} \frac{1}{a_n} = 0$
(C) $\lim_{n\to\infty} \frac{1}{a_n} = 1$
(D) $\lim_{n\to\infty} a_n = 0$

SECTION – C

NUMERICAL ANSWER TYPE (NAT)

Q. 41 – Q. 50 carry one mark each.

Q.41 Let x be the 100-cycle $(1\ \ 2\ \ \ 3\ \cdots\ 100)$ and let y be the transposition $(49\ \ 50)$ in the permutation group S_{100}. Then the order of xy is ______

Q.42 Let W_1 and W_2 be subspaces of the real vector space $\mathbb{R}^{100}$ defined by

$$W_1 = \{\ (x_1, x_2, \dots, x_{100}) : x_i = 0 \text{ if } i \text{ is divisible by } 4\},$$

$$W_2 = \{\ (x_1, x_2, \dots, x_{100}) : x_i = 0 \text{ if } i \text{ is divisible by } 5\}.$$

Then the dimension of $W_1 \cap W_2$ is _____

Q.43 Consider the following system of three linear equations in four unknowns x_1, x_2, x_3 and x_4

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 &= 4, \\ x_1 + 2x_2 + 3x_3 + 4x_4 &= 5, \\ x_1 + 3x_2 + 5x_3 + kx_4 &= 5. \end{aligned}$$

If the system has no solutions, then $k =$ ________

Q.44 Let $\vec{F}(x, y) = -y\,\hat{\imath} + x\,\hat{\jmath}$ and let C be the ellipse

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

oriented counter clockwise. Then the value of $\oint_C \vec{F} \cdot d\vec{r}$ (round off to 2 decimal places) is________________

Q.45 The coefficient of $\left(x - \frac{\pi}{2}\right)$ in the Taylor series expansion of the function

$$f(x) = \begin{cases} \dfrac{4(1 - \sin x)}{2x - \pi}, & x \neq \dfrac{\pi}{2} \\ 0, & x = \dfrac{\pi}{2} \end{cases}$$

about $x = \frac{\pi}{2}$, is _____________

Q.46 Let $f: [0, 1] \to \mathbb{R}$ be given by

$$f(x) = \frac{\left(1+x^{\frac{1}{3}}\right)^3 + \left(1-x^{\frac{1}{3}}\right)^3}{8(1+x)}.$$

Then

$$\max\{f(x): x \in [0,1]\} - \min\{f(x): x \in [0,1]\}$$

is ___________

Q.47 If

$$g(x) = \int_{x(x-2)}^{4x-5} f(t)\, dt, \text{ where } f(x) = \sqrt{1+3x^4} \text{ for } x \in \mathbb{R}$$

then $g'(1) =$ _______

Q.48 Let

$$f(x,y) = \begin{cases} \dfrac{x^3+y^3}{x^2-y^2}, & x^2 - y^2 \neq 0 \\ 0, & x^2 - y^2 = 0. \end{cases}$$

Then the directional derivative of f at $(0, 0)$ in the direction of $\frac{4}{5}\hat{\imath} + \frac{3}{5}\hat{\jmath}$ is __________

Q.49 The value of the integral

$$\int_{-1}^{1}\int_{-1}^{1} |x+y|\, dx\, dy$$

(round off to 2 decimal places) is ___________

Q.50 The volume of the solid bounded by the surfaces $x = 1 - y^2$ and $x = y^2 - 1$, and the planes $z = 0$ and $z = 2$ (round off to 2 decimal places) is _____________

Q. 51 – Q. 60 carry two marks each.

Q.51 The volume of the solid of revolution of the loop of the curve $y^2 = x^4(x+2)$ about the x-axis (round off to 2 decimal places) is ___________

Q.52 The greatest lower bound of the set

$$\{(e^n + 2^n)^{\frac{1}{n}} : n \in \mathbb{N}\},$$

(round off to 2 decimal places) is _______________

Q.53 Let $G = \{n \in \mathbb{N} : n \leq 55, \ \gcd(n, 55) = 1\}$ be the group under multiplication modulo 55. Let $x \in G$ be such that $x^2 = 26$ and $x > 30$. Then x is equal to_____________

Q.54 The number of critical points of the function

$$f(x, y) = (x^2 + 3y^2)e^{-(x^2+y^2)}$$

is ____________

Q.55 The number of elements in the set $\{x \in S_3 : x^4 = e\}$, where e is the identity element of the permutation group S_3, is ____________

Q.56 If $\begin{pmatrix} 2 \\ y \\ z \end{pmatrix}$, $y, z \in \mathbb{R}$, is an eigenvector corresponding to a real eigenvalue of the matrix $\begin{pmatrix} 0 & 0 & 2 \\ 1 & 0 & -4 \\ 0 & 1 & 3 \end{pmatrix}$ then $z - y$ is equal to___________

Q.57 Let M and N be any two 4×4 matrices with integer entries satisfying

$$MN = 2\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

Then the maximum value of $\det(M) + \det(N)$ is ____________

Q.58 Let M be a 3×3 matrix with real entries such that $M^2 = M + 2I$, where I denotes the 3×3 identity matrix. If α, β and γ are eigenvalues of M such that $\alpha\beta\gamma = -4$, then $\alpha + \beta + \gamma$ is equal to__________

Q.59 Let $y(x) = xv(x)$ be a solution of the differential equation

$$x^2 \frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 3y = 0.$$

If $v(0) = 0$ and $v(1) = 1$, then $v(-2)$ is equal to_____________

Q.60 If $y(x)$ is the solution of the initial value problem

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0,\ y(0) = 2,\ \frac{dy}{dx}(0) = 0,$$

then $y(\ln 2)$ is (round off to 2 decimal places) equal to ____________

END OF THE QUESTION PAPER

JAM 2019 ANSWER KEY

Model Answer Key for MA Paper

Paper : **MATHEMATICS**	Code : **MA**

SECTION – A (MCQ)				SECTION – B (MSQ)		SECTION – C (NAT Type)			
Q. No.	**KEY**	**Q. No.**	**KEY**	**Q. No.**	**KEYS**	**Q. No.**	**KEY RANGE**	**Q. No.**	**KEY RANGE**
01	D	**16**	C	**31**	A, C	**41**	99 TO 99	**56**	3 TO 3
02	D	**17**	D	**32**	B, C	**42**	60 TO 60	**57**	17 TO 17
03	B	**18**	A	**33**	C, D	**43**	7 TO 7	**58**	3 TO 3
04	A	**19**	B	**34**	A, D	**44**	75.35 TO 75.45	**59**	4 TO 4
05	B	**20**	C	**35**	B, D	**45**	1 TO 1	**60**	1.12 TO 1.25
06	A	**21**	D	**36**	A	**46**	0.25 TO 0.25		
07	C	**22**	A	**37**	B, C, D	**47**	8 TO 8		
08	B	**23**	C	**38**	B, C	**48**	2.6 TO 2.6		
09	A	**24**	B	**39**	A, D	**49**	2.60 TO 2.70		
10	C	**25**	C	**40**	A, B	**50**	5.30 TO 5.50		
11	D	**26**	B			**51**	6.60 TO 6.80		
12	C	**27**	D			**52**	2.69 TO 2.74		
13	B	**28**	C			**53**	31 to 31 or 46 to 46		
14	B	**29**	B			**54**	5 TO 5		
15	A	**30**	A			**55**	4 TO 4		

Paper Specific Instructions

1. The examination is of 3 hours duration. There are a total of 60 questions carrying 100 marks. The entire paper is divided into three sections, **A**, **B** and **C**. All sections are compulsory. Questions in each section are of different types.

2. **Section – A** contains a total of 30 **Multiple Choice Questions (MCQ).** Each MCQ type question has four choices out of which only **one** choice is the correct answer. Questions Q.1 – Q.30 belong to this section and carry a total of 50 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.30 carry 2 marks each.

3. **Section – B** contains a total of 10 **Multiple Select Questions (MSQ).** Each MSQ type question is similar to MCQ but with a difference that there may be **one or more than one** choice(s) that are correct out of the four given choices. The candidate gets full credit if he/she selects all the correct answers only and no wrong answers. Questions Q.31 – Q.40 belong to this section and carry 2 marks each with a total of 20 marks.

4. **Section – C** contains a total of 20 **Numerical Answer Type (NAT)** questions. For these NAT type questions, the answer is a real number which needs to be entered using the virtual keyboard on the monitor. No choices will be shown for these type of questions. Questions Q.41 – Q.60 belong to this section and carry a total of 30 marks. Q.41 – Q.50 carry 1 mark each and Questions Q.51 – Q.60 carry 2 marks each.

5. In all sections, questions not attempted will result in zero mark. In **Section – A** (MCQ), wrong answer will result in **NEGATIVE** marks. For all 1 mark questions, 1/3 marks will be deducted for each wrong answer. For all 2 marks questions, 2/3 marks will be deducted for each wrong answer. In **Section – B** (MSQ), there is **NO NEGATIVE** and **NO PARTIAL** marking provisions. There is **NO NEGATIVE** marking in **Section – C** (NAT) as well.

6. Only Virtual Scientific Calculator is allowed. Charts, graph sheets, tables, cellular phone or other electronic gadgets are **NOT** allowed in the examination hall.

7. The Scribble Pad will be provided for rough work.

Useful information

$\mathbb{N}$	set of all natural numbers $\{1, 2, 3, \ldots\}$
$\mathbb{Z}$	set of all integers $\{0, \pm 1, \pm 2, \ldots\}$
$\mathbb{Q}$	set of all rational numbers
$\mathbb{R}$	set of all real numbers
$\mathbb{C}$	set of all complex numbers
$\mathbb{R}^n$	n-dimensional Euclidean space $\{(x_1, x_2, \ldots, x_n) \mid x_j \in \mathbb{R}, 1 \leq j \leq n\}$
S_n	group of all permutations of n distinct symbols
$\mathbb{Z}_n$	group of congruence classes of integers modulo n
$\hat{\imath}, \hat{\jmath}, \hat{k}$	unit vectors having the directions of the positive x, y and z axes of a three dimensional rectangular coordinate system
∇	$\hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$
$M_{m\times n}(\mathbb{R})$	real vector space of all matrices of order $m \times n$ with entries in $\mathbb{R}$
sup	supremum
inf	infimum

SECTION – A

MULTIPLE CHOICE QUESTIONS (MCQ)

Q. 1 – Q.10 carry one mark each.

Q.1 Which one of the following is TRUE?

(A) $\mathbb{Z}_n$ is cyclic if and only if n is prime
(B) Every proper subgroup of $\mathbb{Z}_n$ is cyclic
(C) Every proper subgroup of S_4 is cyclic
(D) If every proper subgroup of a group is cyclic, then the group is cyclic

Q.2 Let $a_n = \frac{b_{n+1}}{b_n}$, where $b_1 = 1$, $b_2 = 1$ and $b_{n+2} = b_n + b_{n+1}$, $n \in \mathbb{N}$. Then $\lim_{n\to\infty} a_n$ is

(A) $\frac{1-\sqrt{5}}{2}$ (B) $\frac{1-\sqrt{3}}{2}$ (C) $\frac{1+\sqrt{3}}{2}$ (D) $\frac{1+\sqrt{5}}{2}$

Q.3 If $\{v_1, v_2, v_3\}$ is a linearly independent set of vectors in a vector space over $\mathbb{R}$, then which one of the following sets is also linearly independent?

(A) $\{v_1 + v_2 - v_3,\ 2v_1 + v_2 + 3v_3,\ 5v_1 + 4v_2\}$
(B) $\{v_1 - v_2,\ v_2 - v_3,\ v_3 - v_1\}$
(C) $\{v_1 + v_2 - v_3,\ v_2 + v_3 - v_1,\ v_3 + v_1 - v_2,\ v_1 + v_2 + v_3\}$
(D) $\{v_1 + v_2,\ v_2 + 2v_3,\ v_3 + 3v_1\}$

Q.4 Let a be a positive real number. If f is a continuous and even function defined on the interval $[-a, a]$, then $\int_{-a}^{a} \frac{f(x)}{1+e^x}\,dx$ is equal to

(A) $\int_0^a f(x)\,dx$ (B) $2\int_0^a \frac{f(x)}{1+e^x}\,dx$

(C) $2\int_0^a f(x)\,dx$ (D) $2a\int_0^a \frac{f(x)}{1+e^x}\,dx$

Q.5 The tangent plane to the surface $z = \sqrt{x^2 + 3y^2}$ at $(1, 1, 2)$ is given by

(A) $x - 3y + z = 0$ (B) $x + 3y - 2z = 0$
(C) $2x + 4y - 3z = 0$ (D) $3x - 7y + 2z = 0$

Q.6 In $\mathbb{R}^3$, the cosine of the acute angle between the surfaces $x^2 + y^2 + z^2 - 9 = 0$ and $z - x^2 - y^2 + 3 = 0$ at the point $(2, 1, 2)$ is

(A) $\frac{8}{5\sqrt{21}}$ (B) $\frac{10}{5\sqrt{21}}$ (C) $\frac{8}{3\sqrt{21}}$ (D) $\frac{10}{3\sqrt{21}}$

Q.7 Let $f: \mathbb{R}^3 \to \mathbb{R}$ be a scalar field, $\vec{v}: \mathbb{R}^3 \to \mathbb{R}^3$ be a vector field and let $\vec{a} \in \mathbb{R}^3$ be a constant vector. If $\vec{r}$ represents the position vector $x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, then which one of the following is FALSE?

(A) $curl(f\,\vec{v}) = grad(f) \times \vec{v} + f\,curl(\vec{v})$

(B) $div(grad(f)) = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) f$

(C) $curl(\vec{a} \times \vec{r}) = 2\,|\vec{a}|\,\vec{r}$

(D) $div\left(\frac{\vec{r}}{|\vec{r}|^3}\right) = 0$, for $\vec{r} \neq \vec{0}$

Q.8 In $\mathbb{R}^2$, the family of trajectories orthogonal to the family of asteroids $x^{2/3} + y^{2/3} = a^{2/3}$ is given by

(A) $x^{4/3} + y^{4/3} = c^{4/3}$ (B) $x^{4/3} - y^{4/3} = c^{4/3}$

(C) $x^{5/3} - y^{5/3} = c^{5/3}$ (D) $x^{2/3} - y^{2/3} = c^{2/3}$

Q.9 Consider the vector space V over $\mathbb{R}$ of polynomial functions of degree less than or equal to 3 defined on $\mathbb{R}$. Let $T : V \to V$ be defined by $(Tf)(x) = f(x) - xf'(x)$. Then the rank of T is

(A) 1 (B) 2 (C) 3 (D) 4

Q.10 Let $s_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \cdots + \frac{1}{n!}$ for $n \in \mathbb{N}$. Then which one of the following is TRUE for the sequence $\{s_n\}_{n=1}^{\infty}$

(A) $\{s_n\}_{n=1}^{\infty}$ converges in $\mathbb{Q}$

(B) $\{s_n\}_{n=1}^{\infty}$ is a Cauchy sequence but does not converge in $\mathbb{Q}$

(C) the subsequence $\{s_{k^n}\}_{n=1}^{\infty}$ is convergent in $\mathbb{R}$, only when k is even natural number

(D) $\{s_n\}_{n=1}^{\infty}$ is not a Cauchy sequence

Q. 11 – Q. 30 carry two marks each.

Q.11 Let $a_n = \begin{cases} 2 + \frac{(-1)^{\frac{n-1}{2}}}{n} & , \quad if \ \ n \ is \ odd \\ 1 + \frac{1}{2^n} & , \quad if \ n \ is \ even \end{cases}$, $n \in \mathbb{N}$.

Then which one of the following is TRUE?

(A) $\sup \{a_n \mid n \in \mathbb{N}\} = 3$ and $\inf \{a_n \mid n \in \mathbb{N}\} = 1$

(B) $\liminf (a_n) = \limsup (a_n) = \frac{3}{2}$

(C) $\sup \{a_n \mid n \in \mathbb{N}\} = 2$ and $\inf \{a_n \mid n \in \mathbb{N}\} = 1$

(D) $\liminf (a_n) = 1$ and $\limsup (a_n) = 3$

Q.12 Let $a, b, c \in \mathbb{R}$. Which of the following values of a, b, c do NOT result in the convergence of the series

$$\sum_{n=3}^{\infty} \frac{a^n}{n^b \, (\log_e n)^c} \quad ?$$

(A) $|a| < 1, \ b \in \mathbb{R}, c \in \mathbb{R}$

(B) $a = 1, \ b > 1, \ c \in \mathbb{R}$

(C) $a = 1, b \geq 0, c < 1$

(D) $a = -1, \ b \geq 0, c > 0$

Q.13 Let $a_n = n + \frac{1}{n}$, $n \in \mathbb{N}$. Then the sum of the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{a_{n+1}}{n!}$ is

(A) $e^{-1} - 1$

(B) e^{-1}

(C) $1 - e^{-1}$

(D) $1 + e^{-1}$

Q.14 Let $a_n = \frac{(-1)^n}{\sqrt{1+n}}$ and let $c_n = \sum_{k=0}^{n} a_{n-k} a_k$, where $n \in \mathbb{N} \cup \{0\}$. Then which one of the following is TRUE?

(A) Both $\sum_{n=0}^{\infty} a_n$ and $\sum_{n=1}^{\infty} c_n$ are convergent

(B) $\sum_{n=0}^{\infty} a_n$ is convergent but $\sum_{n=1}^{\infty} c_n$ is not convergent

(C) $\sum_{n=1}^{\infty} c_n$ is convergent but $\sum_{n=0}^{\infty} a_n$ is not convergent

(D) Neither $\sum_{n=0}^{\infty} a_n$ nor $\sum_{n=1}^{\infty} c_n$ is convergent

Q.15 Suppose that $f, g : \mathbb{R} \to \mathbb{R}$ are differentiable functions such that f is strictly increasing and g is strictly decreasing. Define $p(x) = f(g(x))$ and $q(x) = g\big(f(x)\big)$, $\forall x \in \mathbb{R}$. Then, for $t > 0$, the sign of $\int_0^t p'(x)\,(\,q'(x) - 3)dx$ is

(A) positive (B) negative (C) dependent on t (D) dependent on f and g

Q.16 For $x \in \mathbb{R}$, let $f(x) = \begin{cases} x^3 \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$. Then which one of the following is FALSE?

(A) $\lim_{x\to 0} \frac{f(x)}{x} = 0$

(B) $\lim_{x\to 0} \frac{f(x)}{x^2} = 0$

(C) $\frac{f(x)}{x^2}$ has infinitely many maxima and minima on the interval $(0,1)$

(D) $\frac{f(x)}{x^4}$ is continuous at $x = 0$ but not differentiable at $x = 0$

Q.17 Let $f(x,y) = \begin{cases} \frac{xy}{(x^2+y^2)^\alpha}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$

Then which one of the following is TRUE for f at the point $(0,0)$?

(A) For $\alpha = 1$, f is continuous but not differentiable

(B) For $\alpha = \frac{1}{2}$, f is continuous and differentiable

(C) For $\alpha = \frac{1}{4}$, f is continuous and differentiable

(D) For $\alpha = \frac{3}{4}$, f is neither continuous nor differentiable

Q.18 Let $a, b \in \mathbb{R}$ and let $f: \mathbb{R} \to \mathbb{R}$ be a thrice differentiable function. If $z = e^u f(v)$, where $u = ax + by$ and $v = ax - by$, then which one of the following is TRUE?

(A) $b^2 z_{xx} - a^2 z_{yy} = 4a^2b^2 e^u f'(v)$ (B) $b^2 z_{xx} - a^2 z_{yy} = -4e^u f'(v)$

(C) $bz_x + az_y = abz$ (D) $bz_x + az_y = -abz$

Q.19 Consider the region D in the yz plane bounded by the line $y = \frac{1}{2}$ and the curve $y^2 + z^2 = 1$, where $y \geq 0$. If the region D is revolved about the z-axis in $\mathbb{R}^3$, then the volume of the resulting solid is

(A) $\frac{\pi}{\sqrt{3}}$ (B) $\frac{2\pi}{\sqrt{3}}$ (C) $\frac{\pi\sqrt{3}}{2}$ (D) $\pi\sqrt{3}$

Q.20 If $\vec{F}(x,y) = (3x - 8y)\hat{\imath} + (4y - 6xy)\hat{\jmath}$ for $(x,y) \in \mathbb{R}^2$, then $\oint_C \vec{F} \cdot d\,\vec{r}$, where C is the boundary of the triangular region bounded by the lines $x = 0, y = 0$ and $x + y = 1$ oriented in the anti-clockwise direction, is

(A) $\frac{5}{2}$ (B) 3 (C) 4 (D) 5

Q.21 Let U, V and W be finite dimensional real vector spaces, $T: U \to V$, $S: V \to W$ and $P: W \to U$ be linear transformations. If range (ST) = nullspace (P), nullspace (ST) = range (P) and rank (T) = rank (S), then which one of the following is TRUE?

(A) nullity of T = nullity of S

(B) dimension of $U \neq$ dimension of W

(C) If dimension of $V = 3$, dimension of $U = 4$, then P is not identically zero

(D) If dimension of $V = 4$, dimension of $U = 3$ and T is one-one, then P is identically zero

Q.22 Let $y(x)$ be the solution of the differential equation $\frac{dy}{dx} + y = f(x)$, for $x \geq 0$, $y(0) = 0$, where $f(x) = \begin{cases} 2, & 0 \leq x < 1 \\ 0, & x \geq 1 \end{cases}$. Then $y(x) =$

(A) $2(1 - e^{-x})$ when $0 \leq x < 1$ and $2(e - 1)e^{-x}$ when $x \geq 1$

(B) $2(1 - e^{-x})$ when $0 \leq x < 1$ and 0 when $x \geq 1$

(C) $2(1 - e^{-x})$ when $0 \leq x < 1$ and $2(1 - e^{-1})e^{-x}$ when $x \geq 1$

(D) $2(1 - e^{-x})$ when $0 \leq x < 1$ and $2\,e^{1-x}$ when $x \geq 1$

Q.23 An integrating factor of the differential equation $\left(y + \frac{1}{3}\,y^3 + \frac{1}{2}\,x^2\right) dx + \frac{1}{4}(x + x\,y^2)\,dy = 0$ is

(A) x^2 (B) $3\log_e x$ (C) x^3 (D) $2\log_e x$

Q.24 A particular integral of the differential equation $y'' + 3y' + 2y = e^{e^x}$ is

(A) $e^{e^x}e^{-x}$ (B) $e^{e^x}e^{-2x}$ (C) $e^{e^x}e^{2x}$ (D) $e^{e^x}e^{x}$

Q.25 Let G be a group satisfying the property that $f: G \to \mathbb{Z}_{221}$ is a homomorphism implies $f(g) = 0$, $\forall g \in G$. Then a possible group G is

(A) $\mathbb{Z}_{21}$ (B) $\mathbb{Z}_{51}$ (C) $\mathbb{Z}_{91}$ (D) $\mathbb{Z}_{119}$

Q.26 Let H be the quotient group $\mathbb{Q}/\mathbb{Z}$. Consider the following statements.

I. Every cyclic subgroup of H is finite.

II. Every finite cyclic group is isomorphic to a subgroup of H.

Which one of the following holds?

(A) I is TRUE but II is FALSE (B) II is TRUE but I is FALSE

(C) both I and II are TRUE (D) neither I nor II is TRUE

Q.27 Let I denote the 4×4 identity matrix. If the roots of the characteristic polynomial of a 4×4 matrix M are $\pm\sqrt{\frac{1\pm\sqrt{5}}{2}}$, then $M^8 =$

(A) $I + M^2$ (B) $2I + M^2$ (C) $2I + 3M^2$ (D) $3I + 2M^2$

Q.28 Consider the group $\mathbb{Z}^2 = \{(a, b) \mid a, b \in \mathbb{Z}\}$ under component-wise addition. Then which of the following is a subgroup of $\mathbb{Z}^2$?

(A) $\{(a, b) \in \mathbb{Z}^2 \mid ab = 0\}$

(B) $\{(a, b) \in \mathbb{Z}^2 \mid 3a + 2b = 15\}$

(C) $\{(a, b) \in \mathbb{Z}^2 \mid 7 \text{ divides } ab\}$

(D) $\{(a, b) \in \mathbb{Z}^2 \mid 2 \text{ divides } a \text{ and } 3 \text{ divides } b\}$

Q.29 Let $f: \mathbb{R} \to \mathbb{R}$ be a function and let J be a bounded open interval in $\mathbb{R}$. Define

$$W(f, J) = \sup \{f(x) \mid x \in J\} - \inf \{f(x) \mid x \in J\} .$$

Which one of the following is FALSE?

(A) $W(f, J_1) \le W(f, J_2)$ if $J_1 \subset J_2$

(B) If f is a bounded function in J and $J \supset J_1 \supset J_2 \cdots \supset J_n \supset \cdots$ such that the length of the interval J_n tends to 0 as $n \to \infty$, then $\lim_{n\to\infty} W(f, J_n) = 0$

(C) If f is discontinuous at a point $a \in J$, then $W(f, J) \neq 0$

(D) If f is continuous at a point $a \in J$, then for any given $\epsilon > 0$ there exists an interval $I \subset J$ such that $W(f, I) < \epsilon$

Q.30 For $x > \frac{-1}{2}$, let $f_1(x) = \frac{2x}{1+2x}$, $f_2(x) = \log_e(1+2x)$ and $f_3(x) = 2x$. Then which one of the following is TRUE?

(A) $f_3(x) < f_2(x) < f_1(x)$ for $0 < x < \frac{\sqrt{3}}{2}$

(B) $f_1(x) < f_3(x) < f_2(x)$ for $x > 0$

(C) $f_1(x) + f_2(x) < \frac{f_3(x)}{2}$ for $x > \frac{\sqrt{3}}{2}$

(D) $f_2(x) < f_1(x) < f_3(x)$ for $x > 0$

SECTION - B

MULTIPLE SELECT QUESTIONS (MSQ)

Q. 31 – Q. 40 carry two marks each.

Q.31 Let $f: \mathbb{R}\setminus\{0\} \to \mathbb{R}$ be defined by $f(x) = x + \frac{1}{x^3}$. On which of the following interval(s) is f one-one?

(A) $(-\infty, -1)$ (B) $(0, 1)$ (C) $(0, 2)$ (D) $(0, \infty)$

Q.32 The solution(s) of the differential equation $\frac{dy}{dx} = (\sin 2x)\, y^{1/3}$ satisfying $y(0) = 0$ is (are)

(A) $y(x) = 0$

(B) $y(x) = -\sqrt{\frac{8}{27}}\, \sin^3 x$

(C) $y(x) = \sqrt{\frac{8}{27}}\, \sin^3 x$

(D) $y(x) = \sqrt{\frac{8}{27}}\, \cos^3 x$

Q.33 Suppose f, g, h are permutations of the set $\{\alpha, \beta, \gamma, \delta\}$, where

f interchanges α and β but fixes γ and δ,

g interchanges β and γ but fixes α and δ,

h interchanges γ and δ but fixes α and β.

Which of the following permutations interchange(s) α and δ but fix(es) β and γ?

(A) $f \circ g \circ h \circ g \circ f$ (B) $g \circ h \circ f \circ h \circ g$ (C) $g \circ f \circ h \circ f \circ g$ (D) $h \circ g \circ f \circ g \circ h$

Q.34 Let P and Q be two non-empty disjoint subsets of $\mathbb{R}$. Which of the following is (are) FALSE?

(A) If P and Q are compact, then $P \cup Q$ is also compact

(B) If P and Q are not connected, then $P \cup Q$ is also not connected

(C) If $P \cup Q$ and P are closed, then Q is closed

(D) If $P \cup Q$ and P are open, then Q is open

Q.35 Let $\mathbb{C}^* = \mathbb{C} \setminus \{0\}$ denote the group of non-zero complex numbers under multiplication. Suppose $Y_n = \{ z \in \mathbb{C} \mid z^n = 1\},\ n \in \mathbb{N}$. Which of the following is (are) subgroup(s) of $\mathbb{C}^*$?

(A) $\cup_{n=1}^{100} Y_n$ (B) $\cup_{n=1}^{\infty} Y_{2^n}$ (C) $\cup_{n=100}^{\infty} Y_n$ (D) $\cup_{n=1}^{\infty} Y_n$

Q.36 Suppose $\alpha, \beta, \gamma \in \mathbb{R}$. Consider the following system of linear equations.
$x + y + z = \alpha,\ x + \beta y + z = \gamma,\ x + y + \alpha z = \beta$. If this system has at least one solution, then which of the following statements is (are) TRUE?

(A) If $\alpha = 1$ then $\gamma = 1$ (B) If $\beta = 1$ then $\gamma = \alpha$
(C) If $\beta \neq 1$ then $\alpha = 1$ (D) If $\gamma = 1$ then $\alpha = 1$

Q.37 Let $m, n \in \mathbb{N},\ m < n,\ P \in M_{n\times m}(\mathbb{R}),\ Q \in M_{m\times n}(\mathbb{R})$. Then which of the following is (are) NOT possible?

(A) $rank\ (PQ) = n$
(B) $rank\ (QP) = m$
(C) $rank\ (PQ) = m$
(D) $rank\ (QP) = \left\lceil \frac{m+n}{2} \right\rceil$, the smallest integer larger than or equal to $\frac{m+n}{2}$

Q.38 If $\vec{F}(x,y,z) = (2x + 3yz)\hat{\imath} + (3xz + 2y)\hat{\jmath} + (3xy + 2z)\hat{k}$ for $(x,y,z) \in \mathbb{R}^3$, then which among the following is (are) TRUE?

(A) $\nabla \times \vec{F} = \vec{0}$
(B) $\oint_C \vec{F} \cdot d\vec{r} = 0$ along any simple closed curve C
(C) There exists a scalar function $\phi: \mathbb{R}^3 \to \mathbb{R}$ such that $\nabla \cdot \vec{F} = \phi_{xx} + \phi_{yy} + \phi_{zz}$
(D) $\nabla \cdot \vec{F} = 0$

Q.39 Which of the following subsets of $\mathbb{R}$ is (are) connected?

(A) $\{ x \in \mathbb{R} \mid x^2 + x > 4\}$ (B) $\{ x \in \mathbb{R} \mid x^2 + x < 4\}$
(C) $\{ x \in \mathbb{R} \mid |x| < |x - 4|\}$ (D) $\{ x \in \mathbb{R} \mid |x| > |x - 4|\}$

Q.40 Let S be a subset of $\mathbb{R}$ such that 2018 is an interior point of S. Which of the following is (are) TRUE?

(A) S contains an interval

(B) There is a sequence in S which does not converge to 2018

(C) There is an element $y \in S$, $y \neq 2018$ such that y is also an interior point of S

(D) There is a point $z \in S$, such that $|z - 2018| = 0.002018$

SECTION – C

NUMERICAL ANSWER TYPE (NAT)

Q. 41 – Q. 50 carry one mark each.

Q.41 The order of the element $(1\ 2\ 3)\ (2\ 4\ 5)\ (4\ 5\ 6)$ in the group S_6 is ________________

Q.42 Let $\phi(x, y, z) = 3y^2 + 3yz$ for $(x, y, z) \in \mathbb{R}^3$. Then the absolute value of the directional derivative of ϕ in the direction of the line $\frac{x-1}{2} = \frac{y-2}{-1} = \frac{z}{-2}$, at the point $(1, -2, 1)$ is _______________

Q.43 Let $f(x) = \sum_{n=0}^{\infty} (-1)^n x(x-1)^n$ for $0 < x < 2$. Then the value of $f\left(\frac{\pi}{4}\right)$ is _____________

Q.44 Let $f: \mathbb{R}^2 \to \mathbb{R}$ be given by

$$f(x, y) = \begin{cases} \dfrac{x^2 y\,(x - y)}{x^2 + y^2}, & (x, y) \neq (0,0) \\ 0, & (x, y) = (0,0) \end{cases}$$

Then $\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) - \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right)$ at the point $(0,0)$ is _____________

Q.45 Let $f(x, y) = \sqrt{x^3 y}\ \sin\left(\frac{\pi}{2}\, e^{\left(\frac{y}{x}-1\right)}\right) + xy\ \cos\left(\frac{\pi}{3}\, e^{\left(\frac{x}{y}-1\right)}\right)$ for $(x, y) \in \mathbb{R}^2$, $x > 0$, $y > 0$.

Then $f_x(1,1) + f_y(1,1) =$ ________________

Q.46 Let $f: [0, \infty) \to [0, \infty)$ be continuous on $[0, \infty)$ and differentiable on $(0, \infty)$. If $f(x) = \int_0^x \sqrt{f(t)}\, dt$, then $f(6) =$ ___________

Q.47 Let $a_n = \frac{(1+(-1)^n)}{2^n} + \frac{(1+(-1)^{n-1})}{3^n}$. Then the radius of convergence of the power series $\sum_{n=1}^{\infty} a_n x^n$ about $x = 0$ is __________

Q.48 Let A_6 be the group of even permutations of 6 distinct symbols. Then the number of elements of order 6 in A_6 is __________________

Q.49 Let W_1 be the real vector space of all 5×2 matrices such that the sum of the entries in each row is zero. Let W_2 be the real vector space of all 5×2 matrices such that the sum of the entries in each column is zero. Then the dimension of the space $W_1 \cap W_2$ is ______________

Q.50 The coefficient of x^4 in the power series expansion of $e^{\sin x}$ about $x = 0$ is ____________ (correct up to three decimal places).

Q. 51 – Q. 60 carry two marks each.

Q.51 Let $a_k = (-1)^{k-1}$, $s_n = a_1 + a_2 + \cdots + a_n$ and $\sigma_n = (s_1 + s_2 + \cdots + s_n)/n$, where $k, n \in \mathbb{N}$. Then $\lim_{n\to\infty} \sigma_n$ is _______________ (correct up to one decimal place).

Q.52 Let $f: \mathbb{R} \to \mathbb{R}$ be such that f'' is continuous on $\mathbb{R}$ and $f(0) = 1$, $f'(0) = 0$ and $f''(0) = -1$. Then $\lim_{x\to\infty} \left(f\left(\sqrt{\frac{2}{x}} \right) \right)^x$ is ________________ (correct up to three decimal places).

Q.53 Suppose x, y, z are positive real numbers such that $x + 2y + 3z = 1$. If M is the maximum value of xyz^2 , then the value of $\frac{1}{M}$ is ________________

Q.54 If the volume of the solid in $\mathbb{R}^3$ bounded by the surfaces

$$x = -1, \quad x = 1, \quad y = -1, \quad y = 1, \quad z = 2, \quad y^2 + z^2 = 2$$

is $\alpha - \pi$, then $\alpha =$ ________________

Q.55 If $\alpha = \int_{\pi/6}^{\pi/3} \frac{\sin t + \cos t}{\sqrt{\sin 2t}}\, dt$, then the value of $\left(2\sin\frac{\alpha}{2} + 1\right)^2$ is ______________

Q.56 The value of the integral

$$\int_0^1 \int_x^1 y^4 e^{xy^2}\, dy\, dx$$

is ______________ (correct up to three decimal places).

Q.57 Suppose $Q \in M_{3\times3}(\mathbb{R})$ is a matrix of rank 2. Let $T: M_{3\times3}(\mathbb{R}) \to M_{3\times3}(\mathbb{R})$ be the linear transformation defined by $T(P) = QP$. Then the rank of T is _______________

Q.58 The area of the parametrized surface

$S = \{((2 + \cos u)\cos v,\ (2 + \cos u)\sin v,\ \sin u) \in \mathbb{R}^3 \mid 0 \le u \le \frac{\pi}{2},\ 0 \le v \le \frac{\pi}{2}\}$

is ____________ (correct up to two decimal places).

Q.59 If $x(t)$ is the solution to the differential equation $\frac{dx}{dt} = x^2 t^3 + xt$, for $t > 0$, satisfying $x(0) = 1$, then the value of $x(\sqrt{2})$ is _____________ (correct up to two decimal places).

Q.60 If $y(x) = v(x)\sec x$ is the solution of $y'' - (2\tan x)\, y' + 5y = 0$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$, satisfying $y(0) = 0$ and $y'(0) = \sqrt{6}$, then $v\left(\frac{\pi}{6\sqrt{6}}\right)$ is _______________ (correct up to two decimal places).

END OF THE QUESTION PAPER

Paper Code : MA			
Q No.	**Question Type (QT)**	**Section**	**Key/Range (KY)**
1	MCQ	A	B
2	MCQ	A	D
3	MCQ	A	D
4	MCQ	A	A
5	MCQ	A	B
6	MCQ	A	C
7	MCQ	A	C
8	MCQ	A	B
9	MCQ	A	C
10	MCQ	A	B
11	MCQ	A	A
12	MCQ	A	C
13	MCQ	A	D
14	MCQ	A	B
15	MCQ	A	A
16	MCQ	A	D
17	MCQ	A	C
18	MCQ	A	A
19	MCQ	A	C
20	MCQ	A	B
21	MCQ	A	C
22	MCQ	A	A
23	MCQ	A	C

Paper Code : MA			
Q No.	**Question Type (QT)**	**Section**	**Key/Range (KY)**
24	MCQ	A	B
25	MCQ	A	A
26	MCQ	A	C
27	MCQ	A	C
28	MCQ	A	D
29	MCQ	A	B
30	MCQ	A	C
31	MSQ	B	B
32	MSQ	B	A,B,C
33	MSQ	B	A,D
34	MSQ	B	B,C,D
35	MSQ	B	B,C,D
36	MSQ	B	A,B
37	MSQ	B	A,D
38	MSQ	B	A,B,C
39	MSQ	B	B,C,D
40	MSQ	B	A,B,C
41	NAT	C	4 to 4
42	NAT	C	6.5 to 7.5
43	NAT	C	1 to 1
44	NAT	C	1 to 1
45	NAT	C	3 to 3
46	NAT	C	9 to 9

Paper Code : MA			
Q No.	Question Type (QT)	Section	Key/Range (KY)
47	NAT	C	2 to 2
48	NAT	C	0 to 0
49	NAT	C	4 to 4
50	NAT	C	-0.130 to -0.120
51	NAT	C	0.4 to 0.6
52	NAT	C	0.350 to 0.380
53	NAT	C	1140 to 1160
54	NAT	C	5.99 to 6.01
55	NAT	C	2.9 to 3.1
56	NAT	C	0.230 to 0.250
57	NAT	C	6 to 6
58	NAT	C	6.30 to 6.70
59	NAT	C	-2.80 to -2.70
60	NAT	C	0.5 to 0.5

Notation

$\mathbb{Z}_n$	Set of all residue classes modulo n
$X \setminus Y$	The set of elements from X which are not in Y
$\mathbb{N}$	The set of all natural numbers 1,2,3, ...
$\mathbb{R}$	The set of all real numbers
S_n	Set of all permutations of the set $\{1,2, \dots, n\}$
$GL_n(\mathbb{R})$	Set of all $n \times n$ invertible matrices with real entries
$\hat{\imath}, \hat{\jmath}, \hat{k}$	unit vectors having the directions of the positive x, y and z axes in a three dimensional rectangular coordinate system, respectively
M^T	Transpose of a matrix M

SECTION – A

MULTIPLE CHOICE QUESTIONS (MCQ)

Q. 1 – Q.10 carry one mark each.

Q.1 Consider the function $f(x, y) = 5 - 4\sin x + y^2$ for $0 < x < 2\pi$ and $y \in \mathbb{R}$. The set of critical points of $f(x, y)$ consists of

(A) a point of local maximum and a point of local minimum
(B) a point of local maximum and a saddle point
(C) a point of local maximum, a point of local minimum and a saddle point
(D) a point of local minimum and a saddle point

Q.2 Let $\varphi: \mathbb{R} \to \mathbb{R}$ be a differentiable function such that φ' is strictly increasing with $\varphi'(1) = 0$. Let α and β denote the minimum and maximum values of $\varphi(x)$ on the interval $[2, 3]$, respectively. Then which one of the following is TRUE?

(A) $\beta = \varphi(3)$ (B) $\alpha = \varphi(2.5)$ (C) $\beta = \varphi(2.5)$ (D) $\alpha = \varphi(3)$

Q.3 The number of generators of the additive group $\mathbb{Z}_{36}$ is equal to

(A) 6 (B) 12 (C) 18 (D) 36

Q.4

$$\lim_{n\to\infty} \frac{\pi}{n} \sum_{k=1}^{n} \sin\left(\frac{\pi}{2} + \frac{5\pi}{2} \cdot \frac{k}{n}\right) =$$

(A) $\frac{2\pi}{5}$ (B) $\frac{5}{2}$ (C) $\frac{2}{5}$ (D) $\frac{5\pi}{2}$

Q.5 Let $f: \mathbb{R} \to \mathbb{R}$ be a twice differentiable function. If $g(u, v) = f(u^2 - v^2)$, then

$$\frac{\partial^2 g}{\partial u^2} + \frac{\partial^2 g}{\partial v^2} =$$

(A) $4(u^2 - v^2)f''(u^2 - v^2)$
(B) $4(u^2 + v^2)f''(u^2 - v^2)$
(C) $2f'(u^2 - v^2) + 4(u^2 - v^2)f''(u^2 - v^2)$
(D) $2(u - v)^2 f''(u^2 - v^2)$

Q.6

$$\int_0^1 \int_x^1 \sin(y^2)\,dy\,dx =$$

(A) $\frac{1+\cos 1}{2}$ (B) $1 - \cos 1$ (C) $1 + \cos 1$ (D) $\frac{1-\cos 1}{2}$

Q.7 Let $f_1(x), f_2(x), g_1(x), g_2(x)$ be differentiable functions on $\mathbb{R}$. Let $F(x) = \begin{vmatrix} f_1(x) & f_2(x) \\ g_1(x) & g_2(x) \end{vmatrix}$ be the determinant of the matrix $\begin{bmatrix} f_1(x) & f_2(x) \\ g_1(x) & g_2(x) \end{bmatrix}$. Then $F'(x)$ is equal to

(A) $\begin{vmatrix} f_1'(x) & f_2'(x) \\ g_1(x) & g_2(x) \end{vmatrix} + \begin{vmatrix} f_1(x) & g_1'(x) \\ f_2'(x) & g_2(x) \end{vmatrix}$

(B) $\begin{vmatrix} f_1'(x) & f_2'(x) \\ g_1(x) & g_2(x) \end{vmatrix} + \begin{vmatrix} f_1(x) & g_1'(x) \\ f_2(x) & g_2'(x) \end{vmatrix}$

(C) $\begin{vmatrix} f_1'(x) & f_2'(x) \\ g_1(x) & g_2(x) \end{vmatrix} - \begin{vmatrix} f_1(x) & g_1'(x) \\ f_2(x) & g_2'(x) \end{vmatrix}$

(D) $\begin{vmatrix} f_1'(x) & f_2'(x) \\ g_1'(x) & g_2'(x) \end{vmatrix}$

Q.8 Let

$$f(x) = \frac{x + |x|(1+x)}{x} \sin\left(\frac{1}{x}\right), \qquad x \neq 0.$$

Write $L = \lim_{x\to 0^-} f(x)$ and $R = \lim_{x\to 0^+} f(x)$. Then which one of the following is TRUE?

(A) L exists but R does not exist

(B) L does not exist but R exists

(C) Both L and R exist

(D) Neither L nor R exists

Q.9 If $\lim_{T\to\infty} \int_0^T e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$, then

$$\lim_{T\to\infty} \int_0^T x^2 e^{-x^2}\, dx =$$

(A) $\frac{\sqrt{\pi}}{4}$ (B) $\frac{\sqrt{\pi}}{2}$ (C) $\sqrt{2\pi}$ (D) $2\sqrt{\pi}$

Q.10 If

$$f(x) = \begin{cases} 1 + x & \text{if } x < 0 \\ (1-x)(px+q) & \text{if } x \geq 0 \end{cases}$$

satisfies the assumptions of Rolle's theorem in the interval $[-1, 1]$, then the ordered pair (p, q) is

(A) $(2, -1)$ (B) $(-2, -1)$ (C) $(-2,1)$ (D) $(2,1)$

Q. 11 – Q. 30 carry two marks each.

Q.11 The flux of the vector field

$$\vec{F} = \left(2\pi x + \frac{2x^2y^2}{\pi}\right)\hat{\imath} + \left(2\pi xy - \frac{4y}{\pi}\right)\hat{\jmath}$$

along the outward normal, across the ellipse $x^2 + 16y^2 = 4$ is equal to

(A) $4\pi^2 - 2$ (B) $2\pi^2 - 4$ (C) $\pi^2 - 2$ (D) 2π

Q.12 Let $\mathcal{M}$ be the set of all invertible 5×5 matrices with entries 0 and 1. For each $M \in \mathcal{M}$, let $n_1(M)$ and $n_0(M)$ denote the number of 1's and 0's in M, respectively. Then

$$\min_{M \in \mathcal{M}} |n_1(M) - n_0(M)| =$$

(A) 1 (B) 3 (C) 5 (D) 15

Q.13 Let $M = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} \\ 0 & 1 \end{bmatrix}$ and $x = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$. Then

$$\lim_{n \to \infty} M^n x$$

(A) does not exist (B) is $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

(C) is $\begin{bmatrix} 2 \\ 4 \end{bmatrix}$ (D) is $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$

Q.14 Let $\vec{F} = (3 + 2xy)\hat{\imath} + (x^2 - 3y^2)\hat{\jmath}$ and let L be the curve

$$\vec{r}(t) = e^t \sin t\, \hat{\imath} + e^t \cos t\, \hat{\jmath}, \qquad 0 \le t \le \pi.$$

Then

$$\int_L \vec{F} \cdot d\vec{r} =$$

(A) $e^{-3\pi} + 1$ (B) $e^{-6\pi} + 2$ (C) $e^{6\pi} + 2$ (D) $e^{3\pi} + 1$

Q.15 The line integral of the vector field

$$\vec{F} = zx\,\hat{\imath} + xy\,\hat{\jmath} + yz\,\hat{k}$$

along the boundary of the triangle with vertices (1,0,0), (0,1,0) and (0,0,1), oriented anti-clockwise, when viewed from the point (2,2,2), is

(A) $\frac{-1}{2}$ (B) -2 (C) $\frac{1}{2}$ (D) 2

Q.16 The area of the surface $z = \frac{xy}{3}$ intercepted by the cylinder $x^2 + y^2 \le 16$ lies in the interval

(A) $(20\pi, 22\pi]$ (B) $(22\pi, 24\pi]$ (C) $(24\pi, 26\pi]$ (D) $(26\pi, 28\pi]$

Q.17 For $a > 0, b > 0$, let $\vec{F} = \frac{x\hat{j} - y\,\hat{\imath}}{b^2x^2+a^2y^2}$ be a planar vector field. Let

$$C = \{\,(x,y) \in \mathbb{R}^2 \mid x^2 + y^2 = a^2 + b^2\}$$

be the circle oriented anti-clockwise. Then

$$\oint_C \vec{F} \cdot d\vec{r} =$$

(A) $\frac{2\pi}{ab}$ (B) 2π (C) $2\pi ab$ (D) 0

Q.18 The flux of $\vec{F} = y\,\hat{\imath} - x\hat{j} + z^2\hat{k}$ along the outward normal, across the surface of the solid

$$\left\{ (x,y,z) \in \mathbb{R}^3 \,\middle|\, 0 \le x \le 1,\ 0 \le y \le 1,\ 0 \le z \le \sqrt{2 - x^2 - y^2} \right\}$$

is equal to

(A) $\frac{2}{3}$ (B) $\frac{5}{3}$ (C) $\frac{8}{3}$ (D) $\frac{4}{3}$

Q.19 Let $f: \mathbb{R} \to \mathbb{R}$ be a differentiable function such that $f(2) = 2$ and

$$|f(x) - f(y)| \le 5(|x - y|)^{3/2}$$

for all $x \in \mathbb{R}, y \in \mathbb{R}$. Let $g(x) = x^3 f(x)$. Then $g'(2) =$

(A) 5 (B) $\frac{15}{2}$ (C) 12 (D) 24

Q.20 Let $f: \mathbb{R} \to [0, \infty)$ be a continuous function. Then which one of the following is NOT TRUE?

(A) There exists $x \in \mathbb{R}$ such that $f(x) = \frac{f(0)+f(1)}{2}$

(B) There exists $x \in \mathbb{R}$ such that $f(x) = \sqrt{f(-1)f(1)}$

(C) There exists $x \in \mathbb{R}$ such that $f(x) = \int_{-1}^{1} f(t)dt$

(D) There exists $x \in \mathbb{R}$ such that $f(x) = \int_{0}^{1} f(t)dt$

Q.21 The interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{1}{(-3)^{n+2}} \frac{(4x-12)^n}{n^2+1}$$

is

(A) $\frac{10}{4} \le x < \frac{14}{4}$

(B) $\frac{9}{4} \le x < \frac{15}{4}$

(C) $\frac{10}{4} \le x \le \frac{14}{4}$

(D) $\frac{9}{4} \le x \le \frac{15}{4}$

Q.22 Let $\mathcal{P}_3$ denote the real vector space of all polynomials with real coefficients of degree at most 3. Consider the map $T: \mathcal{P}_3 \to \mathcal{P}_3$ given by $T(p(x)) = p''(x) + p(x)$. Then

(A) T is neither one-one nor onto

(B) T is both one-one and onto

(C) T is one-one but not onto

(D) T is onto but not one-one

Q.23 Let $f(x,y) = \frac{x^2y}{x^2+y^2}$ for $(x,y) \neq (0,0)$. Then

(A) $\frac{\partial f}{\partial x}$ and f are bounded

(B) $\frac{\partial f}{\partial x}$ is bounded and f is unbounded

(C) $\frac{\partial f}{\partial x}$ is unbounded and f is bounded

(D) $\frac{\partial f}{\partial x}$ and f are unbounded

Q.24 Let S be an infinite subset of $\mathbb{R}$ such that $S \setminus \{\alpha\}$ is compact for some $\alpha \in S$. Then which one of the following is TRUE?

(A) S is a connected set

(B) S contains no limit points

(C) S is a union of open intervals

(D) Every sequence in S has a subsequence converging to an element in S

Q.25

$$\sum_{n=1}^{\infty} \tan^{-1} \frac{2}{n^2} =$$

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ (C) $\frac{3\pi}{4}$ (D) π

Q.26 Let $0 < a_1 < b_1$. For $n \geq 1$, define

$$a_{n+1} = \sqrt{a_n b_n} \text{ and } b_{n+1} = \frac{a_n + b_n}{2}.$$

Then which one of the following is NOT TRUE?

(A) Both $\{a_n\}$ and $\{b_n\}$ converge, but the limits are not equal

(B) Both $\{a_n\}$ and $\{b_n\}$ converge and the limits are equal

(C) $\{b_n\}$ is a decreasing sequence

(D) $\{a_n\}$ is an increasing sequence

Q.27

$$\lim_{n\to\infty} \frac{1}{\sqrt{n}}\left(\frac{1}{\sqrt{3}+\sqrt{6}} + \frac{1}{\sqrt{6}+\sqrt{9}} + \cdots + \frac{1}{\sqrt{3n}+\sqrt{3n+3}}\right) =$$

(A) $1+\sqrt{3}$ (B) $\sqrt{3}$ (C) $\frac{1}{\sqrt{3}}$ (D) $\frac{1}{1+\sqrt{3}}$

Q.28 Which one of the following is TRUE?

(A) Every sequence that has a convergent subsequence is a Cauchy sequence

(B) Every sequence that has a convergent subsequence is a bounded sequence

(C) The sequence $\{\sin n\}$ has a convergent subsequence

(D) The sequence $\left\{n \cos \frac{1}{n}\right\}$ has a convergent subsequence

Q.29 A particular integral of the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = e^{2x} \sin x$$

is

(A) $\frac{e^{2x}}{10}(3\cos x - 2\sin x)$

(B) $-\frac{e^{2x}}{10}(3\cos x - 2\sin x)$

(C) $-\frac{e^{2x}}{5}(2\cos x + \sin x)$

(D) $\frac{e^{2x}}{5}(2\cos x - \sin x)$

Q.30 Let $y(x)$ be the solution of the differential equation

$$(xy + y + e^{-x})dx + (x + e^{-x})dy = 0$$

satisfying $y(0) = 1$. Then $y(-1)$ is equal to

(A) $\frac{e}{e-1}$ (B) $\frac{2e}{e-1}$ (C) $\frac{e}{1-e}$ (D) 0

SECTION - B

MULTIPLE SELECT QUESTIONS (MSQ)

Q. 31 – Q. 40 carry two marks each.

Q.31 For $\alpha, \beta \in \mathbb{R}$, define the map $\varphi_{\alpha,\beta}: \mathbb{R} \to \mathbb{R}$ by $\varphi_{\alpha,\beta}(x) = \alpha x + \beta$. Let

$$G = \{\varphi_{\alpha,\beta} \mid (\alpha, \beta) \in \mathbb{R}^2\}$$

For $f, g \in G$, define $g \circ f \in G$ by $(g \circ f)(x) = g(f(x))$. Then which of the following statements is/are TRUE?

(A) The binary operation $\circ$ is associative

(B) The binary operation $\circ$ is commutative

(C) For every $(\alpha, \beta) \in \mathbb{R}^2$, $\alpha \neq 0$ there exists $(a, b) \in \mathbb{R}^2$ such that $\varphi_{\alpha,\beta} \circ \varphi_{a,b} = \varphi_{1,0}$

(D) $(G, \circ)$ is a group

Q.32 The volume of the solid

$$\left\{(x, y, z) \in \mathbb{R}^3 \,\middle|\, 1 \le x \le 2, \quad 0 \le y \le \frac{2}{x}, \quad 0 \le z \le x\right\}$$

is expressible as

(A) $\int_1^2 \int_0^{2/x} \int_0^x dz\, dy\, dx$

(B) $\int_1^2 \int_0^x \int_0^{2/x} dy\, dz\, dx$

(C) $\int_0^2 \int_1^z \int_0^{2/x} dy\, dx\, dz$

(D) $\int_0^2 \int_{\max\{z,1\}}^2 \int_0^{2/x} dy\, dx\, dz$

Q.33 Let $f: \mathbb{R}^2 \to \mathbb{R}$ be a function. Then which of the following statements is/are TRUE?

(A) If f is differentiable at (0,0), then all directional derivatives of f exist at (0,0)

(B) If all directional derivatives of f exist at (0,0), then f is differentiable at (0,0)

(C) If all directional derivatives of f exist at (0,0), then f is continuous at (0,0)

(D) If the partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ exist and are continuous in a disc centered at (0,0), then f is differentiable at (0,0)

Q.34 If X and Y are $n \times n$ matrices with real entries, then which of the following is/are TRUE?

(A) If $P^{-1}XP$ is diagonal for some real invertible matrix P, then there exists a basis for $\mathbb{R}^n$ consisting of eigenvectors of X

(B) If X is diagonal with distinct diagonal entries and $XY = YX$, then Y is also diagonal

(C) If X^2 is diagonal, then X is diagonal

(D) If X is diagonal and $XY = YX$ for all Y, then $X = \lambda I$ for some $\lambda \in \mathbb{R}$

Q.35 Let G be a group of order 20 in which the conjugacy classes have sizes $1, 4, 5, 5, 5$. Then which of the following is/are TRUE?

(A) G contains a normal subgroup of order 5

(B) G contains a non-normal subgroup of order 5

(C) G contains a subgroup of order 10

(D) G contains a normal subgroup of order 4

Q.36 Let $\{x_n\}$ be a real sequence such that $7x_{n+1} = x_n^3 + 6$ for $n \geq 1$. Then which of the following statements is/are TRUE?

(A) If $x_1 = \frac{1}{2}$, then $\{x_n\}$ converges to 1

(B) If $x_1 = \frac{1}{2}$, then $\{x_n\}$ converges to 2

(C) If $x_1 = \frac{3}{2}$, then $\{x_n\}$ converges to 1

(D) If $x_1 = \frac{3}{2}$, then $\{x_n\}$ converges to -3

Q.37 Let S be the set of all rational numbers in $(0,1)$. Then which of the following statements is / are TRUE?

(A) S is a closed subset of $\mathbb{R}$

(B) S is not a closed subset of $\mathbb{R}$

(C) S is an open subset of $\mathbb{R}$

(D) Every $x \in (0,1)\setminus S$ is a limit point of S

Q.38 Let M be an $n \times n$ matrix with real entries such that $M^3 = I$. Suppose that $Mv \neq v$ for any non-zero vector v. Then which of the following statements is / are TRUE?

(A) M has real eigenvalues

(B) $M + M^{-1}$ has real eigenvalues

(C) n is divisible by 2

(D) n is divisible by 3

Q.39 Let $y(x)$ be the solution of the differential equation

$$\frac{dy}{dx} = (y-1)(y-3)$$

satisfying the condition $y(0) = 2$. Then which of the following is/are TRUE?

(A) The function $y(x)$ is not bounded above

(B) The function $y(x)$ is bounded

(C) $\lim_{x\to+\infty} y(x) = 1$

(D) $\lim_{x\to-\infty} y(x) = 3$

Q.40 Let $k, \ell \in \mathbb{R}$ be such that every solution of

$$\frac{d^2y}{dx^2} + 2k\frac{dy}{dx} + \ell y = 0$$

satisfies $\lim_{x\to\infty} y(x) = 0$. Then

(A) $3k^2 + \ell < 0$ and $k > 0$

(B) $k^2 + \ell > 0$ and $k < 0$

(C) $k^2 - \ell \leq 0$ and $k > 0$

(D) $k^2 - \ell > 0, k > 0$ and $\ell > 0$

SECTION – C

NUMERICAL ANSWER TYPE (NAT)

Q. 41 – Q. 50 carry one mark each.

Q.41 If the orthogonal trajectories of the family of ellipses $x^2 + 2y^2 = c_1,\ c_1 > 0$, are given by $y = c_2 x^\alpha,\ c_2 \in \mathbb{R}$, then $\alpha =$ ____________

Q.42 Let G be a subgroup of $GL_2(\mathbb{R})$ generated by $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $\begin{bmatrix} 0 & -1 \\ 1 & -1 \end{bmatrix}$. Then the order of G is ____________

Q.43 Consider the permutations $\sigma = \begin{pmatrix} 1\,2\,3\,4\,5\,6\,7\,8 \\ 4\,5\,3\,7\,8\,6\,1\,2 \end{pmatrix}$ and $\tau = \begin{pmatrix} 1\,2\,3\,4\,5\,6\,7\,8 \\ 4\,5\,3\,1\,7\,6\,8\,2 \end{pmatrix}$ in S_8. The number of $\eta \in S_8$ such that $\eta^{-1}\sigma\eta = \tau$ is equal to ____________

Q.44 Let P be the point on the surface $z = \sqrt{x^2 + y^2}$ closest to the point $(4,2,0)$. Then the square of the distance between the origin and P is ____________

Q.45 $$\left(\int_0^1 x^4(1-x)^5\,dx\right)^{-1} = \_\_\_\_\_\_\_\_$$

Q.46 Let $v_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ and $v_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$. Let M be the matrix whose columns are $v_1, v_2, 2v_1 - v_2, v_1 + 2v_2$ in that order. Then the number of linearly independent solutions of the homogeneous system of linear equations $Mx = 0$ is ____________

Q.47 $$\frac{1}{2\pi}\left(\frac{\pi^3}{1!\,3}-\frac{\pi^5}{3!\,5}+\frac{\pi^7}{5!\,7}-\cdots+\frac{(-1)^{n-1}\pi^{2n+1}}{(2n-1)!\,(2n+1)}+\cdots\right)= \_\_\_\_\_\_\_\_$$

Q.48 Let P be a 7×7 matrix of rank 4 with real entries. Let $\boldsymbol{a}\in\mathbb{R}^7$ be a column vector. Then the rank of $P+\boldsymbol{a}\boldsymbol{a}^T$ is at least ____________

Q.49 For $x>0$, let $\lfloor x\rfloor$ denote the greatest integer less than or equal to x. Then

$$\lim_{x\to 0^+} x\left(\left\lfloor\frac{1}{x}\right\rfloor+\left\lfloor\frac{2}{x}\right\rfloor+\cdots+\left\lfloor\frac{10}{x}\right\rfloor\right)= \_\_\_\_\_\_\_\_$$

Q.50 The number of subgroups of $\mathbb{Z}_7\times\mathbb{Z}_7$ of order 7 is ____________

Q. 51 – Q. 60 carry two marks each.

Q.51 Let $y(x),\ x>0$ be the solution of the differential equation

$$x^2\frac{d^2y}{dx^2}+5x\frac{dy}{dx}+4y=0$$

satisfying the conditions $y(1)=1$ and $y'(1)=0$. Then the value of $e^2y(e)$ is ____________

Q.52 Let T be the smallest positive real number such that the tangent to the helix

$$\cos t\ \hat{\imath}+\sin t\ \hat{\jmath}+\frac{t}{\sqrt{2}}\hat{k}$$

at $t=T$ is orthogonal to the tangent at $t=0$. Then the line integral of $\vec{F}=x\hat{\jmath}-y\,\hat{\imath}$ along the section of the helix from $t=0$ to $t=T$ is ____________

Q.53 Let $f(x)=\frac{\sin\pi x}{\pi\sin x}$, $x\in(0,\pi)$, and let $x_0\in(0,\pi)$ be such that $f'(x_0)=0$. Then

$$\left(f(x_0)\right)^2\left(1+(\pi^2-1)\sin^2 x_0\right)= \_\_\_\_\_\_\_\_$$

Q.54 The maximum order of a permutation σ in the symmetric group S_{10} is ____________

Q.55 Let $a_n=\sqrt{n}$, $n\geq 1$, and let $s_n=a_1+a_2+\cdots+a_n$. Then

$$\lim_{n\to\infty}\left(\frac{a_n/s_n}{-\ln(1-a_n/s_n)}\right)= \_\_\_\_\_\_\_\_$$

Q.56 For a real number x, define $\lceil x \rceil$ to be the smallest integer greater than or equal to x. Then

$$\int_0^1 \int_0^1 \int_0^1 (\lceil x \rceil + \lceil y \rceil + \lceil z \rceil)\, dx\, dy\, dz = \_\_\_\_\_\_\_\_$$

Q.57 For $x > 1$, let

$$f(x) = \int_1^x \left(\sqrt{\log t} - \frac{1}{2}\log\sqrt{t}\right) dt$$

The number of tangents to the curve $y = f(x)$ parallel to the line $x + y = 0$ is __________

Q.58 Let $\alpha, \beta, \gamma, \delta$ be the eigenvalues of the matrix

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

Then $\alpha^2 + \beta^2 + \gamma^2 + \delta^2 =$ __________

Q.59 The radius of convergence of the power series

$$\sum_0^\infty n!\, x^{n^2}$$

is __________

Q.60 If

$$y(x) = \int_{\sqrt{x}}^{x} \frac{e^t}{t}\, dt, \quad x > 0$$

then $y'(1) =$ __________

END OF THE QUESTION PAPER

JAM 2017 ANSWER KEY

Model Answer Key for MA Paper

Paper: **MATHEMATICS**	Code:**MA**

SECTION – A (MCQ)				SECTION – B (MSQ)		SECTION – C (NAT Type)			
Q. No.	**KEY**	**Q. No.**	**KEY**	**Q. No.**	**KEYS**	**Q. No.**	**KEY RANGE**	**Q. No.**	**KEY RANGE**
01	D	16	A	31	A, C	41	1.9 – 2.1	56	2.9 – 3.1
02	A	17	A	32	A, B, D	42	5.9 – 6.1	57	0.9 – 1.1
03	B	18	D	33	A, D	43	-0.01 – +0.01	58	5.9 – 6.1
04	C	19	D	34	A, B, D	44	9.9 – 10.1	59	0.9 – 1.1
05	B	20	C	35	A, C	45	1259.9 – 1260.1	60	1.34 – 1.36
06	D	21	D	36	A, C	46	1.9 – 2.1		
07	B	22	B	37	B, D	47	0.49 – 0.51		
08	A	23	B	38	B, C	48	2.9 – 3.1		
09	A	24	D	39	B, C, D	49	54.9 – 55.1		
10	D	25	C	40	C, D	50	7.9 – 8.1		
11	B	26	A			51	2.9 – 3.1		
12	A	27	C			52	2.0 – 2.2		
13	C	28	C			53	0.9 – 1.1		
14	D	29	C			54	29.9 – 30.1		
15	C	30	B			55	0.9 – 1.1		

Notation

$\mathbb{N}$	The set of all natural numbers $\{1,2,3,\ldots\}$
$\mathbb{Z}$	The set of all integers
$\mathbb{Q}$	The set of all rational numbers
$\mathbb{R}$	The set of all real numbers
S_n	The group of permutations of n distinct symbols
$\mathbb{Z}_n$	$\{0, 1, 2, \ldots, n-1\}$ with addition and multiplication modulo n
ϕ	empty set
A^T	Transpose of A
i	$\sqrt{-1}$
$\hat{\imath}, \hat{\jmath}, \hat{k}$	unit vectors having the directions of the positive x, y and z axes of a three dimensional rectangular coordinate system
∇	$\hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$
I_n	Identity matrix of order n
ln	logarithm with base e

SECTION – A

MULTIPLE CHOICE QUESTIONS (MCQ)

Q. 1 – Q.10 carry one mark each.

Q.1 The sequence $\{s_n\}$ of real numbers given by

$$s_n = \frac{\sin\frac{\pi}{2}}{1\cdot 2} + \frac{\sin\frac{\pi}{2^2}}{2\cdot 3} + \cdots + \frac{\sin\frac{\pi}{2^n}}{n\cdot(n+1)}$$

is

(A) a divergent sequence
(B) an oscillatory sequence
(C) not a Cauchy sequence
(D) a Cauchy sequence

Q.2 Let P be the vector space (over $\mathbb{R}$) of all polynomials of degree ≤ 3 with real coefficients. Consider the linear transformation $T: P \to P$ defined by

$$T(a_0 + a_1x + a_2x^2 + a_3x^3) = a_3 + a_2x + a_1x^2 + a_0x^3.$$

Then the matrix representation M of T with respect to the ordered basis $\{1, x, x^2, x^3\}$ satisfies

(A) $M^2 + I_4 = 0$ (B) $M^2 - I_4 = 0$
(C) $M - I_4 = 0$ (D) $M + I_4 = 0$

Q.3 Let $f: [-1, 1] \to \mathbb{R}$ be a continuous function. Then the integral

$$\int_0^{\pi} x\, f(\sin x)\, dx$$

is equivalent to

(A) $\frac{\pi}{2}\int_0^{\pi} f(\sin x)\, dx$ (B) $\frac{\pi}{2}\int_0^{\pi} f(\cos x)\, dx$

(C) $\pi\int_0^{\pi} f(\cos x)\, dx$ (D) $\pi\int_0^{\pi} f(\sin x)\, dx$

Q.4 Let σ be an element of the permutation group S_5. Then the maximum possible order of σ is

(A) 5 (B) 6 (C) 10 (D) 15

Q.5 Let f be a strictly monotonic continuous real valued function defined on $[a, b]$ such that $f(a) < a$ and $f(b) > b$. Then which one of the following is TRUE?

(A) There exists exactly one $c \in (a, b)$ such that $f(c) = c$
(B) There exist exactly two points $c_1, c_2 \in (a, b)$ such that $f(c_i) = c_i,\ i = 1,2$
(C) There exists no $c \in (a, b)$ such that $f(c) = c$
(D) There exist infinitely many points $c \in (a, b)$ such that $f(c) = c$

Q.6 The value of $\lim_{(x,\ y)\to(2,-2)} \frac{\sqrt{(x-y)}-2}{x-y-4}$ is

(A) 0 (B) $\frac{1}{4}$ (C) $\frac{1}{3}$ (D) $\frac{1}{2}$

Q.7 Let $\vec{r} = \left(x\,\hat{\imath} + y\,\hat{\jmath} + z\,\hat{k} \right)$ and $r = |\vec{r}|$. If $f(r) = \ln r$ and $g(r) = \frac{1}{r}$, $r \neq 0$, satisfy $2\,\nabla f + h(r)\nabla g = \vec{0}$, then $h(r)$ is

(A) r (B) $\frac{1}{r}$ (C) $2r$ (D) $\frac{2}{r}$

Q.8 The nonzero value of n for which the differential equation

$$(3xy^2 + n^2x^2y)dx + (nx^3 + 3x^2y)\,dy = 0, \quad x \neq 0,$$

becomes exact is

(A) -3 (B) -2 (C) 2 (D) 3

Q.9 One of the points which lies on the solution curve of the differential equation

$$(y - x)dx + (x + y)dy = 0,$$

with the given condition $y(0) = 1$, is

(A) $(1,\ -2)$ (B) $(2,\ -1)$ (C) $(2,\ 1)$ (D) $(-1,\ 2)$

Q.10 Let S be a closed subset of $\mathbb{R}$, T a compact subset of $\mathbb{R}$ such that $S \cap T \neq \phi$. Then $S \cap T$ is

(A) closed but not compact
(B) not closed
(C) compact
(D) neither closed nor compact

Q. 11 – Q. 30 carry two marks each.

Q.11 Let S be the series

$$\sum_{k=1}^{\infty} \frac{1}{(2k-1)\,2^{(2k-1)}}$$

and T be the series

$$\sum_{k=2}^{\infty} \left(\frac{3k-4}{3k+2}\right)^{\frac{(k+1)}{3}}$$

of real numbers. Then which one of the following is TRUE?

(A) Both the series S and T are convergent
(B) S is convergent and T is divergent
(C) S is divergent and T is convergent
(D) Both the series S and T are divergent

Q.12 Let $\{a_n\}$ be a sequence of positive real numbers satisfying

$$\frac{4}{a_{n+1}} = \frac{3}{a_n} + \frac{a_n^3}{81}, \qquad n \geq 1, \; a_1 = 1.$$

Then all the terms of the sequence lie in

(A) $\left[\frac{1}{2}, \frac{3}{2}\right]$ (B) $[0, 1]$ (C) $[1, 2]$ (D) $[1, 3]$

Q.13 The largest eigenvalue of the matrix $\begin{bmatrix} 1 & 4 & 16 \\ 4 & 16 & 1 \\ 16 & 1 & 4 \end{bmatrix}$ is

(A) 16 (B) 21

(C) 48 (D) 64

Q.14 The value of the integral

$$\frac{(2n)!}{2^{2n}\,(n!)} \int_{-1}^{1} (1-x^2)^n \, dx, \qquad n \in \mathbb{N}$$

is

(A) $\frac{2}{(2n+1)!}$ (B) $\frac{2n}{(2n+1)!}$

(C) $\frac{2\,(n!)}{2n+1}$ (D) $\frac{(n+1)!}{2n+1}$

Q.15 If the triple integral over the region bounded by the planes

$$2x + y + z = 4, \qquad x = 0, \qquad y = 0, \qquad z = 0$$

is given by

$$\int_0^2 \int_0^{\lambda(x)} \int_0^{\mu(x,y)} dz\, dy\, dx\,,$$

then the function $\lambda(x) - \mu(x, y)$ is

(A) $x + y$ (B) $x - y$ (C) x (D) y

Q.16 The surface area of the portion of the plane $y + 2z = 2$ within the cylinder $x^2 + y^2 = 3$ is

(A) $\frac{3\sqrt{5}}{2}\pi$ (B) $\frac{5\sqrt{5}}{2}\pi$ (C) $\frac{7\sqrt{5}}{2}\pi$ (D) $\frac{9\sqrt{5}}{2}\pi$

Q.17 Let $f: \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x,y) = \begin{cases} \dfrac{xy^2}{x+y} & \text{if } x+y \neq 0 \\ 0 & \text{if } x+y = 0 \end{cases}.$$

Then the value of $\left(\frac{\partial^2 f}{\partial x\,\partial y} + \frac{\partial^2 f}{\partial y\,\partial x}\right)$ at the point $(0,0)$ is

(A) 0 (B) 1 (C) 2 (D) 4

Q.18 The function $f(x,y) = 3x^2y + 4y^3 - 3x^2 - 12y^2 + 1$ has a saddle point at

(A) $(0,\ 0)$ (B) $(0,\ 2)$ (C) $(1,\ 1)$ (D) $(-2,\ 1)$

Q.19 Consider the vector field $\vec{F} = r^\beta(y\,\hat{\imath} - x\hat{\jmath}\,)$, where $\beta \in \mathbb{R}$, $\vec{r} = x\hat{\imath} + y\hat{\jmath}$ and $r = |\vec{r}|$. If the absolute value of the line integral $\oint_c \vec{F} \cdot d\vec{r}$ along the closed curve $C: x^2 + y^2 = a^2$ (oriented counter clockwise) is 2π, then β is

(A) -2 (B) -1 (C) 1 (D) 2

Q.20 Let S be the surface of the cone $z = \sqrt{x^2 + y^2}$ bounded by the planes $z = 0$ and $z = 3$. Further, let C be the closed curve forming the boundary of the surface S. A vector field $\vec{F}$ is such that $\nabla \times \vec{F} = -x\hat{\imath} - y\hat{\jmath}$. The absolute value of the line integral $\oint_c \vec{F} \cdot d\vec{r}$, where $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ and $r = |\vec{r}|$, is

(A) 0 (B) 9π (C) 15π (D) 18π

Q.21 Let $y(x)$ be the solution of the differential equation

$$\frac{d}{dx}\left(x\frac{dy}{dx}\right) = x; \quad y(1) = 0, \quad \left.\frac{dy}{dx}\right|_{x=1} = 0\,.$$

Then $y(2)$ is

(A) $\frac{3}{4} + \frac{1}{2}\ln 2$ (B) $\frac{3}{4} - \frac{1}{2}\ln 2$

(C) $\frac{3}{4} + \ln 2$ (D) $\frac{3}{4} - \ln 2$

Q.22 The general solution of the differential equation with constant coefficients

$$\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$$

approaches zero as $x \to \infty$, if

(A) b is negative and c is positive
(B) b is positive and c is negative
(C) both b and c are positive
(D) both b and c are negative

Q.23 Let $S \subset \mathbb{R}$ and ∂S denote the set of points x in $\mathbb{R}$ such that every neighbourhood of x contains some points of S as well as some points of complement of S. Further, let $\overline{S}$ denote the closure of S. Then which one of the following is FALSE?

(A) $\partial\mathbb{Q} = \mathbb{R}$
(B) $\partial(\mathbb{R} \setminus T) = \partial T, \quad T \subset \mathbb{R}$
(C) $\partial(T \cup V) = \partial T \cup \partial V, \quad T, V \subset \mathbb{R},\ T \cap V \neq \phi$
(D) $\partial T = \overline{T} \cap (\overline{\mathbb{R} \setminus T}), \quad T \subset \mathbb{R}$

Q.24 The sum of the series

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{n^2 + n - 2}$$

is

(A) $\frac{1}{3}\ln 2 - \frac{5}{18}$ (B) $\frac{1}{3}\ln 2 - \frac{5}{6}$ (C) $\frac{2}{3}\ln 2 - \frac{5}{18}$ (D) $\frac{2}{3}\ln 2 - \frac{5}{6}$

Q.25 Let $f(x) = \frac{1}{1+|x|} + \frac{1}{1+|x-1|}$ for all $x \in [-1, 1]$. Then which one of the following is TRUE?

(A) Maximum value of $f(x)$ is $\frac{3}{2}$
(B) Minimum value of $f(x)$ is $\frac{1}{3}$
(C) Maximum of $f(x)$ occurs at $x = \frac{1}{2}$
(D) Minimum of $f(x)$ occurs at $x = 1$

Q.26 The matrix $M = \begin{bmatrix} \cos\alpha & \sin\alpha \\ i\sin\alpha & i\cos\alpha \end{bmatrix}$ is a unitary matrix when α is

(A) $(2n+1)\frac{\pi}{2}, n \in \mathbb{Z}$ (B) $(3n+1)\frac{\pi}{3}, n \in \mathbb{Z}$

(C) $(4n+1)\frac{\pi}{4}, n \in \mathbb{Z}$ (D) $(5n+1)\frac{\pi}{5}, n \in \mathbb{Z}$

Q.27 Let $M = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & \alpha \\ 2 & -\alpha & 0 \end{bmatrix}$, $\alpha \in \mathbb{R} \setminus \{0\}$ and $\boldsymbol{b}$ a non-zero vector such that $M\boldsymbol{x} = \boldsymbol{b}$ for some $\boldsymbol{x} \in \mathbb{R}^3$. Then the value of $\boldsymbol{x}^T\boldsymbol{b}$ is

(A) $-\alpha$ (B) α (C) 0 (D) 1

Q.28 The number of group homomorphisms from the cyclic group $\mathbb{Z}_4$ to the cyclic group $\mathbb{Z}_7$ is

(A) 7 (B) 3 (C) 2 (D) 1

Q.29 In the permutation group S_n $(n \geq 5)$, if H is the smallest subgroup containing all the 3-cycles, then which one of the following is TRUE?

(A) Order of H is 2
(B) Index of H in S_n is 2
(C) H is abelian
(D) $H = S_n$

Q.30 Let $f: \mathbb{R} \to \mathbb{R}$ be defined as

$$f(x) = \begin{cases} x(1 + x^{\alpha} \sin(\ln x^2)) & \text{if} \quad x \neq 0 \\ 0 & \text{if} \quad x = 0 \end{cases}.$$

Then, at $x = 0$, the function f is

(A) continuous and differentiable when $\alpha = 0$
(B) continuous and differentiable when $\alpha > 0$
(C) continuous and differentiable when $-1 < \alpha < 0$
(D) continuous and differentiable when $\alpha < -1$

SECTION - B

MULTIPLE SELECT QUESTIONS (MSQ)

Q. 31 – Q. 40 carry two marks each.

Q.31 Let $\{s_n\}$ be a sequence of positive real numbers satisfying

$$2\, s_{n+1} = s_n^2 + \frac{3}{4}, \qquad n \geq 1.$$

If α and β are the roots of the equation $x^2 - 2x + \frac{3}{4} = 0$ and $\alpha < s_1 < \beta$, then which of the following statement(s) is(are) TRUE ?

(A) $\{s_n\}$ is monotonically decreasing
(B) $\{s_n\}$ is monotonically increasing
(C) $\lim_{n\to\infty} s_n = \alpha$
(D) $\lim_{n\to\infty} s_n = \beta$

Q.32 The value(s) of the integral

$$\int_{-\pi}^{\pi} |x| \cos nx \, dx \ , \ \ n \geq 1$$

is (are)

(A) 0 when n is even
(B) 0 when n is odd
(C) $-\frac{4}{n^2}$ when n is even
(D) $-\frac{4}{n^2}$ when n is odd

Q.33 Let $f: \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x,y) = \begin{cases} \frac{xy}{|x|} & \text{if } x \neq 0 \\ 0 & \text{elsewhere} \end{cases}.$$

Then at the point $(0,0)$, which of the following statement(s) is(are) TRUE ?

(A) f is not continuous
(B) f is continuous
(C) f is differentiable
(D) Both first order partial derivatives of f exist

Q.34 Consider the vector field $\vec{F} = x\hat{\imath} + y\hat{\jmath}$ on an open connected set $S \subset \mathbb{R}^2$. Then which of the following statement(s) is(are) TRUE ?

(A) Divergence of $\vec{F}$ is zero on S
(B) The line integral of $\vec{F}$ is independent of path in S
(C) $\vec{F}$ can be expressed as a gradient of a scalar function on S
(D) The line integral of $\vec{F}$ is zero around any piecewise smooth closed path in S

Q.35 Consider the differential equation

$$\sin 2x \ \frac{dy}{dx} = 2y + 2\cos x \ , \ \ y\left(\frac{\pi}{4}\right) = 1 - \sqrt{2}.$$

Then which of the following statement(s) is(are) TRUE?

(A) The solution is unbounded when $x \to 0$
(B) The solution is unbounded when $x \to \frac{\pi}{2}$
(C) The solution is bounded when $x \to 0$
(D) The solution is bounded when $x \to \frac{\pi}{2}$

Q.36 Which of the following statement(s) is(are) TRUE?

(A) There exists a connected set in $\mathbb{R}$ which is not compact
(B) Arbitrary union of closed intervals in $\mathbb{R}$ need not be compact
(C) Arbitrary union of closed intervals in $\mathbb{R}$ is always closed
(D) Every bounded infinite subset V of $\mathbb{R}$ has a limit point in V itself

Q.37 Let $P(x) = \left(\frac{5}{13}\right)^x + \left(\frac{12}{13}\right)^x - 1$ for all $x \in \mathbb{R}$. Then which of the following statement(s) is(are) TRUE?

(A) The equation $P(x) = 0$ has exactly one solution in $\mathbb{R}$
(B) $P(x)$ is strictly increasing for all $x \in \mathbb{R}$
(C) The equation $P(x) = 0$ has exactly two solutions in $\mathbb{R}$
(D) $P(x)$ is strictly decreasing for all $x \in \mathbb{R}$

Q.38 Let G be a finite group and $o(G)$ denotes its order. Then which of the following statement(s) is(are) TRUE?

(A) G is abelian if $o(G) = pq$ where p and q are distinct primes
(B) G is abelian if every non identity element of G is of order 2
(C) G is abelian if the quotient group $\frac{G}{Z(G)}$ is cyclic, where $Z(G)$ is the center of G
(D) G is abelian if $o(G) = p^3$, where p is prime

Q.39 Consider the set $V = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 \;\middle|\; \alpha x + \beta y + z = \gamma,\ \alpha, \beta, \gamma \in \mathbb{R} \right\}$. For which of the following choice(s) the set V becomes a two dimensional subspace of $\mathbb{R}^3$ over $\mathbb{R}$?

(A) $\alpha = 0,\ \beta = 1,\ \gamma = 0$
(B) $\alpha = 0,\ \beta = 1,\ \gamma = 1$
(C) $\alpha = 1,\ \beta = 0,\ \gamma = 0$
(D) $\alpha = 1,\ \beta = 1,\ \gamma = 0$

Q.40 Let $S = \left\{ \frac{1}{3^n} + \frac{1}{7^m} \;\middle|\; n, m \in \mathbb{N} \right\}$. Then which of the following statement(s) is(are) TRUE?

(A) S is closed
(B) S is not open
(C) S is connected
(D) 0 is a limit point of S

SECTION – C

NUMERICAL ANSWER TYPE (NAT)

Q. 41 – Q. 50 carry one mark each.

Q.41 Let $\{s_n\}$ be a sequence of real numbers given by

$$s_n = 2^{(-1)^n}\left(1 - \frac{1}{n}\right)\sin\frac{n\pi}{2}, \qquad n \in \mathbb{N}.$$

Then the least upper bound of the sequence $\{s_n\}$ is ____________

Q.42 Let $\{s_k\}$ be a sequence of real numbers, where

$$s_k = k^{\alpha/k}, \qquad k \geq 1, \qquad \alpha > 0.$$

Then

$$\lim_{n \to \infty} \left(s_1\ s_2\ \ldots\ s_n\right)^{1/n}$$

is ____________

Q.43 Let $\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3$ be a non-zero vector and $A = \dfrac{\boldsymbol{x}\,\boldsymbol{x}^T}{\boldsymbol{x}^T\boldsymbol{x}}$. Then the dimension of the vector space $\left\{ \boldsymbol{y} \in \mathbb{R}^3 \;\middle|\; A\boldsymbol{y} = \boldsymbol{0} \right\}$ over $\mathbb{R}$ is ____________

Q.44 Let f be a real valued function defined by

$$f(x, y) = 2 \ln\left(x^2 y^2 e^{\frac{y}{x}}\right), \qquad x > 0, y > 0.$$

Then the value of $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}$ at any point (x, y), where $x > 0$, $y > 0$, is ____________

Q.45 Let $\vec{F} = \sqrt{x}\,\hat{\imath} + (x + y^3)\hat{\jmath}$ be a vector field for all (x, y) with $x \geq 0$ and $\vec{r} = x\hat{\imath} + y\hat{\jmath}$. Then the value of the line integral $\int_C \vec{F} \cdot d\vec{r}$ from $(0, 0)$ to $(1, 1)$ along the path
C: $x = t^2$, $y = t^3$, $0 \leq t \leq 1$ is ____________

Q.46 If $f: (-1, \infty) \to \mathbb{R}$ defined by $f(x) = \frac{x}{1+x}$ is expressed as

$$f(x) = \frac{2}{3} + \frac{1}{9}(x - 2) + \frac{c(x - 2)^2}{(1 + \xi)^3},$$

where ξ lies between 2 and x, then the value of c is ____________

Q.47 Let $y_1(x)$, $y_2(x)$ and $y_3(x)$ be linearly independent solutions of the differential equation

$$\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} - 6y = 0.$$

If the Wronskian $W(y_1, y_2, y_3)$ is of the form ke^{bx} for some constant k, then the value of b is____________

Q.48 The radius of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{(-4)^n}{n(n+1)} (x + 2)^{2n} \quad \text{is } \_\_\_\_\_\_\_\_\_\_$$

Q.49 Let $f: (0, \infty) \to \mathbb{R}$ be a continuous function such that

$$\int_0^x f(t)dt = -2 + \frac{x^2}{2} + 4x \sin 2x + 2\cos 2x.$$

Then the value of $\frac{1}{\pi} f\left(\frac{\pi}{4}\right)$ is ____________

Q.50 Let G be a cyclic group of order 12. Then the number of non-isomorphic subgroups of G is ____________

Q. 51 – Q. 60 carry two marks each.

Q.51 The value of $\lim_{n\to\infty}\left(8n - \frac{1}{n}\right)^{\frac{(-1)^n}{n^2}}$ is equal to _________

Q.52 Let R be the region enclosed by $x^2 + 4y^2 \geq 1$ and $x^2 + y^2 \leq 1$. Then the value of

$$\iint_R |xy|\, dx\, dy \quad \text{is } \_\_\_\_\_\_\_\_\_$$

Q.53 Let

$$M = \begin{bmatrix} \alpha & 1 & 1 \\ 1 & \beta & 1 \\ 1 & 1 & \gamma \end{bmatrix},\ \alpha\beta\gamma = 1,\ \alpha, \beta, \gamma \in \mathbb{R} \text{ and } \boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3.$$

Then $M\boldsymbol{x} = \boldsymbol{0}$ has infinitely many solutions if $trace(M)$ is ____________

Q.54 Let C be the boundary of the region enclosed by $y = x^2$, $y = x + 2$, and $x = 0$. Then the value of the line integral

$$\oint_C (xy - y^2)dx - x^3 dy,$$

where C is traversed in the counter clockwise direction, is ____________

Q.55 Let S be the closed surface forming the boundary of the region V bounded by $x^2 + y^2 = 3$, $z = 0$, $z = 6$. A vector field $\vec{F}$ is defined over V with $\nabla \cdot \vec{F} = 2y + z + 1$. Then the value of

$$\frac{1}{\pi} \iint_S \vec{F} \cdot \hat{n} \, ds,$$

where $\hat{n}$ is the unit outward drawn normal to the surface S, is ____________,

Q.56 Let $y(x)$ be the solution of the differential equation

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 0, \qquad y(0) = 1, \qquad \left.\frac{dy}{dx}\right|_{x=0} = -1.$$

Then $y(x)$ attains its maximum value at $x =$ ____________

Q.57 The value of the double integral

$$\int_0^{\pi} \int_0^{x} \frac{\sin y}{\pi - y} \, dy \, dx$$

is ____________

Q.58 Let H denote the group of all 2×2 invertible matrices over $\mathbb{Z}_5$ under usual matrix multiplication. Then the order of the matrix $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ in H is ____________

Q.59 Let $A = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 5 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ -1 & 0 \\ 3 & 1 \end{bmatrix}$, $N(A)$ the null space of A and $R(B)$ the range space of B. Then the dimension of $N(A) \cap R(B)$ over $\mathbb{R}$ is ____________

Q.60 The maximum value of $f(x, y) = x^2 + 2y^2$ subject to the constraint $y - x^2 + 1 = 0$ is ____________

END OF THE QUESTION PAPER

JAM 2016: Mathematics			
Qn. No.	Qn. Type	Key(s)	Mark(s)
1	MCQ	D	1
2	MCQ	B	1
3	MCQ	A	1
4	MCQ	B	1
5	MCQ	A	1
6	MCQ	B	1
7	MCQ	C	1
8	MCQ	D	1
9	MCQ	C	1
10	MCQ	C	1
11	MCQ	B	2
12	MCQ	D	2
13	MCQ	B	2
14	MCQ	C	2
15	MCQ	D	2
16	MCQ	A	2
17	MCQ	B	2
18	MCQ	D	2
19	MCQ	A	2
20	MCQ	MTA	2
21	MCQ	B	2
22	MCQ	C	2
23	MCQ	C	2
24	MCQ	C	2
25	MCQ	A	2
26	MCQ	A	2
27	MCQ	C	2
28	MCQ	D	2
29	MCQ	B	2
30	MCQ	MTA	2

JAM 2016: Mathematics			
Qn. No.	Qn. Type	Key(s)	Mark(s)
31	MSQ	A;C	2
32	MSQ	A;D	2
33	MSQ	B;D	2
34	MSQ	B;C;D	2
35	MSQ	C;D	2
36	MSQ	A;B	2
37	MSQ	A;D	2
38	MSQ	B;C	2
39	MSQ	A;C;D	2
40	MSQ	B;D	2

JAM 2016: Mathematics			
Qn. No.	Qn. Type	Key(s)	Mark(s)
41	NAT	0.5:0.5	1
42	NAT	1.0:1.0	1
43	NAT	2.0:2.0	1
44	NAT	8.0:8.0	1
45	NAT	1.49:1.55	1
46	NAT	-1:-1	1
47	NAT	6.0:6.0	1
48	NAT	0.5:0.5	1
49	NAT	0.25:0.25	1
50	NAT	6.0:6.0	1
51	NAT	1.0:1.0	2
52	NAT	0.35:0.4	2
53	NAT	3.0:3.0	2
54	NAT	0.8:1.9	2
55	NAT	72.0:72.0	2
56	NAT	-0.3:-0.25	2
57	NAT	2.0:2.0	2
58	NAT	3.0:3.0	2
59	NAT	1.0:1.0	2
60	NAT	2.0:2.0	2

JAM 2015: General Instructions during Examination

1. Total duration of the JAM 2015 examination is **180** minutes.

2. The clock will be set at the server. The countdown timer at the top right corner of screen will display the remaining time available for you to complete the examination. When the timer reaches zero, the examination will end by itself. You need not terminate the examination or submit your paper.

3. Any useful data required for your paper can be viewed by clicking on the **Useful Data** button that appears on the screen.

4. Use the scribble pad provided to you for any rough work. Submit the scribble pad at the end of the examination.

5. You are allowed to use only your own **non-programmable calculator**.

6. The Question Palette displayed on the right side of screen will show the status of each question using one of the following symbols:

   - 1 You have not visited the question yet.
   - 3 You have not answered the question.
   - 5 You have answered the question.
   - 7 You have NOT answered the question, but have marked the question for review.
   - 9 You have answered the question, but marked it for review.

7. The **Marked for Review** status for a question simply indicates that you would like to look at that question again. *If a question is 'answered, but marked for review', then the answer will be considered for evaluation unless the status is modified by the candidate.*

Navigating to a Question :

8. To answer a question, do the following:
   - **a.** Click on the question number in the Question Palette to go to that question directly.
   - **b.** Select the answer for a multiple choice type question and for the multiple select type question. Use the virtual numeric keypad to enter the answer for a numerical type question.
   - **c.** Click on **Save & Next** to save your answer for the current question and then go to the next question.
   - **d.** Click on **Mark for Review & Next** to save and to mark for review your answer for the current question, and then go to the next question.

   Caution: Note that your answer for the current question will not be saved, if you navigate to another question directly by clicking on a question number without saving the answer to the previous question.

9. You can view all the questions by clicking on the **Question Paper** button. This feature is provided, so that if you want you can just see the entire question paper at a glance.

Answering a Question :

10. Procedure for answering a multiple choice question (MCQ):
    - **a.** Choose the answer by selecting only one out of the 4 choices (A,B,C,D) given below the question and click on the bubble placed before the selected choice.

b. To deselect your chosen answer, click on the bubble of the selected choice again or click on the **Clear Response** button.

c. To change your chosen answer, click on the bubble of another choice.

d. To save your answer, you MUST click on the **Save & Next** button.

11. Procedure for answering a multiple select question (MSQ):

a. Choose the answer by selecting one or more than one out of the 4 choices (A,B,C,D) given below the question and click on the checkbox(es) placed before each of the selected choice (s).

b. To deselect one or more of your selected choice(s), click on the checkbox(es) of the choice(s) again. To deselect all the selected choices, click on the **Clear Response** button.

c. To change a particular selected choice, deselect this choice that you want to change and click on the checkbox of another choice.

d. To save your answer, you MUST click on the **Save & Next** button.

12. Procedure for answering a numerical answer type (NAT) question:

a. To enter a number as your answer, use the virtual numerical keypad.

b. A fraction (e.g. -0.3 or -.3) can be entered as an answer with or without '0' before the decimal point. As many as four decimal points, e.g. 12.5435 or 0.003 or -932.6711 or 12.82 can be entered.

c. To clear your answer, click on the **Clear Response** button.

d. To save your answer, you MUST click on the **Save & Next** button.

13. To mark a question for review, click on the **Mark for Review & Next** button. *If an answer is selected (for MCQ and MSQ types) or entered (for NAT) for a question that is* **Marked for Review**, *that answer will be considered in the evaluation unless the status is modified by the candidate.*

14. To change your answer to a question that has already been answered, first select that question and then follow the procedure for answering that type of question as described above.

15. Note that ONLY those questions for which answers are ***saved*** or ***marked for review after answering*** will be considered for evaluation.

Choosing a Section :

16. Sections in this question paper are displayed on the top bar of the screen. All sections are compulsory.

17. Questions in a section can be viewed by clicking on the name of that section. The section you are currently viewing will be highlighted.

18. To select another section, simply click the name of the section on the top bar. You can shuffle between different sections any number of times.

19. When you select a section, you will only be able to see questions in this Section, and you can answer questions in the Section.

20. After clicking the **Save & Next** button for the last question in a section, you will automatically be taken to the first question of the next section in sequence.

21. You can move the mouse cursor over the name of a section to view the answering status for that section.

JAM 2015 Examination

MA: Mathematics

Duration: **180 minutes** *Maximum Marks*: **100**

Read the following instructions carefully.

1. To login, enter your Registration Number and Password provided to you. Kindly go through the various coloured symbols used in the test and understand their meaning before you start the examination.
2. Once you login and after the start of the examination, you can view all the questions in the question paper, by clicking on the **Question Paper** button in the screen.
3. This test paper has a total of 60 questions carrying 100 marks. The entire question paper is divided into three **sections, A, B** and **C**. All sections are compulsory. Questions in each section are of different types.
4. **Section – A** contains **Multiple Choice Questions (MCQ)**. Each MCQ type question has four choices out of which only one choice is the correct answer. This section has 30 Questions and carry a total of 50 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.30 carry 2 marks each.
5. **Section – B** contains **Multiple Select Questions (MSQ)**. Each MSQ type question is similar to MCQ but with a difference that there may be one or more than one choice(s) that are correct out of the four given choices. The candidate gets full credit if he/she selects all the correct choices only and no wrong choices. This section has 10 Questions and carry 2 marks each with a total of 20 marks.
6. **Section – C** contains **Numerical Answer Type (NAT)** questions. For these NAT type questions, the answer is a real number which needs to be entered using the virtual numerical keypad on the monitor. No choices will be shown for these type of questions. This section has 20 Questions and carry a total of 30 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.20 carry 2 marks each.
7. Depending upon the JAM test paper, there may be useful common data that may be required for answering the questions. If the paper has such useful data, the same can be viewed by clicking on the **Useful Data** button that appears at the top, right hand side of the screen.
8. The computer allotted to you at the examination centre runs specialized software that permits only one choice to be selected as answer for multiple choice questions using a mouse, one or more than one choices to be selected as answer for multiple select questions using a mouse and to enter a suitable number for the numerical answer type questions using the virtual numeric keypad and mouse.
9. Your answers shall be updated and saved on a server periodically and also at the end of the examination. The examination will **stop automatically** at the end of **180 minutes**.
10. Multiple choice questions (Section-A) will have four choices against A, B, C, D, out of which only **ONE** choice is the correct answer. The candidate has to choose the correct answer by clicking on the bubble (○) placed before the choice.
11. Multiple select questions (Section-B) will also have four choices against A, B, C, D, out of which **ONE OR MORE THAN ONE** choice(s) is /are the correct answer. The candidate has to choose the correct answer by clicking on the checkbox (□) placed before the choices for each of the selected choice(s).
12. For numerical answer type questions (Section-C), each question will have a numerical answer and there will not be any choices**. For these questions, the answer should be entered** by using the mouse and the virtual numerical keypad that appears on the monitor.
13. In all questions, questions not attempted will result in zero mark. In **Section – A** (MCQ), wrong answer will result in **NEGATIVE** marks. For all 1 mark questions, 1/3 marks will be deducted for each wrong answer. For all 2 marks questions, 2/3 marks will be deducted for each wrong answer. In **Section – B** (MSQ), there is **NO NEGATIVE** and **NO PARTIAL** marking provisions. There is **NO NEGATIVE** marking in **Section – C** (NAT) as well.

14. **Non-programmable calculators** are allowed but sharing of calculators is not allowed.

15. Mobile phones, electronic gadgets other than calculators, charts, graph sheets, and mathematical tables are **NOT** allowed in the examination hall.

16. You can use the scribble pad provided to you at the examination centre for all your rough work. The scribble pad has to be returned at the end of the examination.

Declaration by the candidate:

"I have read and understood all the above instructions. I have also read and understood clearly the instructions given on the admit card and shall follow the same. I also understand that in case I am found to violate any of these instructions, my candidature is liable to be cancelled. I also confirm that at the start of the examination all the computer hardware allotted to me are in proper working condition".

Notation

$\mathbb{N}$	- The set of natural numbers = $\{1,2,3,\ldots\}$
$\mathbb{Z}$	- The set of integers
$\mathbb{Q}$	- The set of rational numbers
$\mathbb{R}$	- The set of real numbers
$\mathbb{C}$	- The set of complex numbers
S_n	- The group of permutations of n distinct symbols
$\mathbb{Z}_n$	- The group of integers modulo n
$M_n(\mathbb{R})$	- The vector space of $n \times n$ real matrices
$\hat{\imath}, \hat{\jmath}, \hat{k}$	- Standard mutually orthogonal unit vectors
i	- Imaginary number $\sqrt{-1}$
$\bar{a}$	- Complex conjugate of a
$\bar{A}$	- Complex conjugate of matrix A
A^T	- Transpose of matrix A
$\emptyset$	- Empty set
sup	- supremum
inf	- infimum
y'	- Derivative of y

SECTION – A

MULTIPLE CHOICE QUESTIONS (MCQ)

Q. 1 – Q.10 carry one mark each.

Q.1 Suppose N is a normal subgroup of a group G. Which one of the following is true?

(A) If G is an infinite group then G/N is an infinite group
(B) If G is a nonabelian group then G/N is a nonabelian group
(C) If G is a cyclic group then G/N is an abelian group
(D) If G is an abelian group then G/N is a cyclic group

Q.2 Let $y(x) = u(x)\sin x + v(x)\cos x$ be a solution of the differential equation $y'' + y = \sec x$. Then $u(x)$ is

(A) $\ln|\cos x| + C$ (B) $-x + C$
(C) $x + C$ (D) $\ln|\sec x| + C$

Q.3 Let a, b, c, d be distinct non-zero real numbers with $a + b = c + d$. Then an eigenvalue of the matrix $\begin{bmatrix} a & b & 1 \\ c & d & 1 \\ 1 & -1 & 0 \end{bmatrix}$ is

(A) $a + c$ (B) $a + b$ (C) $a - b$ (D) $b - d$

Q.4 Let S be a nonempty subset of $\mathbb{R}$. If S is a finite union of disjoint bounded intervals, then which one of the following is true?

(A) If S is not compact, then $\sup S \notin S$ and $\inf S \notin S$
(B) Even if $\sup S \in S$ and $\inf S \in S$, S need not be compact
(C) If $\sup S \in S$ and $\inf S \in S$, then S is compact
(D) Even if S is compact, it is not necessary that $\sup S \in S$ and $\inf S \in S$

Q.5 Let $\{x_n\}$ be a convergent sequence of real numbers. If $x_1 > \pi + \sqrt{2}$ and $x_{n+1} = \pi + \sqrt{x_n - \pi}$ for $n \geq 1$, then which one of the following is the limit of this sequence?

(A) $\pi + 1$ (B) $\pi + \sqrt{2}$ (C) π (D) $\pi + \sqrt{\pi}$

Q.6 The volume of the portion of the solid cylinder $x^2 + y^2 \leq 2$ bounded above by the surface $z = x^2 + y^2$ and bounded below by the xy-plane is

(A) π (B) 2π (C) 3π (D) 4π

Q.7 Let $f: \mathbb{R} \to \mathbb{R}$ be a differentiable function with $f(0) = 0$. If for all $x \in \mathbb{R}$, $1 < f'(x) < 2$, then which one of the following statements is true on $(0, \infty)$?

(A) f is unbounded
(B) f is increasing and bounded
(C) f has at least one zero
(D) f is periodic

Q.8 If an integral curve of the differential equation $(y - x)\frac{dy}{dx} = 1$ passes through $(0, 0)$ and $(\alpha, 1)$, then α is equal to

(A) $2 - e^{-1}$
(B) $1 - e^{-1}$
(C) e^{-1}
(D) $1 + e$

Q.9 An integrating factor of the differential equation

$$\frac{dy}{dx} = \frac{2xy^2 + y}{x - 2y^3}$$

is

(A) $\frac{1}{y}$
(B) $\frac{1}{y^2}$
(C) y
(D) y^2

Q.10 Let A be a nonempty subset of $\mathbb{R}$. Let $I(A)$ denote the set of interior points of A. Then $I(A)$ can be

(A) empty
(B) singleton
(C) a finite set containing more than one element
(D) countable but not finite

Q. 11 – Q. 30 carry two marks each.

Q.11 Let S_3 be the group of permutations of three distinct symbols. The direct sum $S_3 \oplus S_3$ has an element of order

(A) 4
(B) 6
(C) 9
(D) 18

Q.12 The orthogonal trajectories of the family of curves $y = C_1 x^3$ are

(A) $2\,x^2 + 3\,y^2 = C_2$
(B) $3\,x^2 + y^2 = C_2$
(C) $3\,x^2 + 2\,y^2 = C_2$
(D) $x^2 + 3\,y^2 = C_2$

Q.13 Let G be a nonabelian group. Let $\alpha \in G$ have order 4 and let $\beta \in G$ have order 3. Then the order of the element $\alpha\beta$ in G

(A) is 6
(B) is 12
(C) is of the form $12k$ for $k \geq 2$
(D) need not be finite

Q.14 Let S be the bounded surface of the cylinder $x^2 + y^2 = 1$ cut by the planes $z = 0$ and $z = 1 + x$. Then the value of the surface integral $\iint_S 3z^2 d\sigma$ is equal to

(A) $\int_0^{2\pi} (1 + \cos\theta)^3 d\theta$

(B) $\int_0^{2\pi} \sin\theta \cos\theta \, (1 + \cos\theta)^2 d\theta$

(C) $\int_0^{2\pi} (1 + 2\cos\theta)^3 d\theta$

(D) $\int_0^{2\pi} \sin\theta \cos\theta \, (1 + 2\cos\theta)^2 d\theta$

Q.15 Suppose that the dependent variables z and w are functions of the independent variables x and y, defined by the equations $f(x, y, z, w) = 0$ and $g(x, y, z, w) = 0$, where $f_z g_w - f_w g_z = 1$. Which one of the following is correct?

(A) $z_x = f_w g_x - f_x g_w$

(B) $z_x = f_x g_w - f_w g_x$

(C) $z_x = f_z g_x - f_x g_z$

(D) $z_x = f_z g_w - f_z g_x$

Q.16

Let $A = \begin{bmatrix} 0 & 1 - i \\ -1 - i & i \end{bmatrix}$ and $B = A^T \overline{A}$. Then

(A) an eigenvalue of B is purely imaginary

(B) an eigenvalue of A is zero

(C) all eigenvalues of B are real

(D) A has a non-zero real eigenvalue

Q.17 The limit

$$\lim_{x \to 0+} \frac{1}{\sin^2 x} \int_{\frac{x}{2}}^{x} \sin^{-1} t \, dt$$

is equal to

(A) 0

(B) $\frac{1}{8}$

(C) $\frac{1}{4}$

(D) $\frac{3}{8}$

Q.18 Let $P_2(\mathbb{R})$ be the vector space of polynomials in x of degree at most 2 with real coefficients. Let $M_2(\mathbb{R})$ be the vector space of 2×2 real matrices. If a linear transformation $T: P_2(\mathbb{R}) \to M_2(\mathbb{R})$ is defined as

$$T(f) = \begin{bmatrix} f(0) - f(2) & 0 \\ 0 & f(1) \end{bmatrix}$$

then

(A) T is one-one but not onto

(B) T is onto but not one-one

(C) Range (T) = span $\left\{ \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} -2 & 0 \\ 0 & 1 \end{bmatrix} \right\}$

(D) Null (T) = span $\{x^2 - 2x, \ 1 - x\}$

Q.19

Let $B_1 = \{(1,2),\ (2,-1)\}$ and $B_2 = \{(1,0),\ (0,1)\}$ be ordered bases of $\mathbb{R}^2$. If $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation such that $[T]_{B_1,B_2}$, the matrix of T with respect to B_1 and B_2, is $\begin{bmatrix} 4 & 3 \\ 2 & -4 \end{bmatrix}$, then $T(5,5)$ is equal to

(A) $(-9,\ 8)$ (B) $(9,\ 8)$ (C) $(-15,\ -2)$ (D) $(15,\ 2)$

Q.20

Let $S = \bigcap_{n=1}^{\infty}\left(\left[0,\ \frac{1}{2n+1}\right] \cup \left[\frac{1}{2n},\ 1\right]\right)$. Which one of the following statements is **FALSE**?

(A) There exist sequences $\{a_n\}$ and $\{b_n\}$ in $[0,1]$ such that $S = [0,1] \setminus \bigcup_{n=1}^{\infty}(a_n,\ b_n)$
(B) $[0,1]\setminus S$ is an open set
(C) If A is an infinite subset of S, then A has a limit point
(D) There exists an infinite subset of S having no limit points

Q.21 Let $f : \mathbb{R} \to \mathbb{R}$ be a strictly increasing continuous function. If $\{a_n\}$ is a sequence in $[0,1]$, then the sequence $\{f(a_n)\}$ is

(A) increasing
(B) bounded
(C) convergent
(D) not necessarily bounded

Q.22

Which one of the following statements is true for the series $\sum_{n=1}^{\infty}(-1)^n \frac{(2n)!}{n^{2n}}$?

(A) The series converges conditionally but not absolutely
(B) The series converges absolutely
(C) The sequence of partial sums of the series is bounded but not convergent
(D) The sequence of partial sums of the series is unbounded

Q.23

The sequence $\left\{\cos\left(\frac{1}{2}\tan^{-1}\left(-\frac{n}{2}\right)^n\right)\right\}$ is

(A) monotone and convergent
(B) monotone but not convergent
(C) convergent but not monotone
(D) neither monotone nor convergent

Q.24

If $y(t)$ is a solution of the differential equation $y'' + 4y = 2e^t$, then

$$\lim_{t\to\infty} e^{-t}\, y(t)$$

is equal to

(A) $\frac{2}{3}$ (B) $\frac{2}{5}$ (C) $\frac{2}{7}$ (D) $\frac{2}{9}$

Q.25

For what real values of x and y, does the integral $\int_x^y (6-t-t^2)dt$ attain its maximum?

(A) $x=-3,\ y=2$
(B) $x=2,\ y=3$
(C) $x=-2,\ y=2$
(D) $x=-3,\ y=4$

Q.26 The area of the planar region bounded by the curves $x=6y^2-2$ and $x=2y^2$ is

(A) $\frac{\sqrt{2}}{3}$
(B) $\frac{2\sqrt{2}}{3}$
(C) $\frac{4\sqrt{2}}{3}$
(D) $\sqrt{2}$

Q.27

For $n \geq 2$, let $f_n: \mathbb{R} \to \mathbb{R}$ be given by $f_n(x) = x^n \sin x$. Then at $x=0$, f_n has a

(A) local maximum if n is even

(B) local maximum if n is odd

(C) local minimum if n is even

(D) local minimum if n is odd

Q.28

For $m, n \in \mathbb{N}$, define $f_{m,n}(x) = \begin{cases} x^m \sin\left(\frac{1}{x^n}\right), & x \neq 0 \\ 0 & x = 0 \end{cases}$

Then at $x=0$, $f_{m,n}$ is

(A) differentiable for each pair m, n with $m > n$

(B) differentiable for each pair m, n with $m < n$

(C) not differentiable for each pair m, n with $m > n$

(D) not differentiable for each pair m, n with $m < n$

Q.29

Let G and H be nonempty subsets of $\mathbb{R}$, where G is connected and $G \cup H$ is not connected. Which one of the following statements is true for all such G and H ?

(A) If $G \cap H = \emptyset$, then H is connected

(B) If $G \cap H = \emptyset$, then H is not connected

(C) If $G \cap H \neq \emptyset$, then H is connected

(D) If $G \cap H \neq \emptyset$, then H is not connected

Q.30

Let $f : \{ (x,y) \in \mathbb{R}^2 : x > 0,\ y > 0 \} \to \mathbb{R}$ be given by

$$f(x,y) = x^{\frac{1}{3}} y^{\frac{-4}{3}} \tan^{-1}\left(\frac{y}{x}\right) + \frac{1}{\sqrt{x^2+y^2}}$$

Then the value of

$$g(x,y) = \frac{x f_x(x,y) + y f_y(x,y)}{f(x,y)}$$

(A) changes with x but not with y

(B) changes with y but not with x

(C) changes with x and also with y

(D) neither changes with x nor with y

SECTION - B

MULTIPLE SELECT QUESTIONS (MSQ)

Q. 1 – Q. 10 carry two marks each.

Q.1

Let $f: \mathbb{R} \to \mathbb{R}$ be a function defined by $f(x) = \int_{-5}^{x} (t-1)^3 dt$.
In which of the following interval(s), f takes the value 1?

(A) $[-6,\ 0]$ (B) $[-2,\ 4]$ (C) $[2,\ 8]$ (D) $[6,\ 12]$

Q.2

Which of the following statements is (are) true?

(A) $\mathbb{Z}_2 \oplus \mathbb{Z}_3$ is isomorphic to $\mathbb{Z}_6$

(B) $\mathbb{Z}_3 \oplus \mathbb{Z}_3$ is isomorphic to $\mathbb{Z}_9$

(C) $\mathbb{Z}_4 \oplus \mathbb{Z}_6$ is isomorphic to $\mathbb{Z}_{24}$

(D) $\mathbb{Z}_2 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_5$ is isomorphic to $\mathbb{Z}_{30}$

Q.3

Which of the following conditions implies (imply) the convergence of a sequence $\{x_n\}$ of real numbers?

(A) Given $\varepsilon > 0$ there exists an $n_0 \in \mathbb{N}$ such that for all $n \geq n_0$, $|x_{n+1} - x_n| < \varepsilon$

(B) Given $\varepsilon > 0$ there exists an $n_0 \in \mathbb{N}$ such that for all $n \geq n_0$, $\frac{1}{(n+1)^2}|x_{n+1} - x_n| < \varepsilon$

(C) Given $\varepsilon > 0$ there exists an $n_0 \in \mathbb{N}$ such that for all $n \geq n_0$, $(n+1)^2|x_{n+1} - x_n| < \varepsilon$

(D) Given $\varepsilon > 0$ there exists an $n_0 \in \mathbb{N}$ such that for all m, n with $m > n \geq n_0$, $|x_m - x_n| < \varepsilon$

Q.4

Let $\vec{F}$ be a vector field given by $\vec{F}(x,y,z) = -y\,\hat{\imath} + 2\,x\,y\,\hat{\jmath} + z^3\hat{k}$, for $(x,y,z) \in \mathbb{R}^3$. If C is the curve of intersection of the surfaces $x^2 + y^2 = 1$ and $y + z = 2$, then which of the following is (are) equal to $\left|\int_C \vec{F}\,.\,d\vec{r}\right|$?

(A) $\int_0^{2\pi}\int_0^1 (1 + 2r\sin\theta) r\, dr d\theta$

(B) $\int_0^{2\pi}\left(\frac{1}{2} + \frac{2}{3}\sin\theta\right) d\theta$

(C) $\int_0^{2\pi}\int_0^1 (1 + 2r\sin\theta)\ dr d\theta$

(D) $\int_0^{2\pi}(1 + \sin\theta)\, d\theta$

Q.5

Let V be the set of 2×2 matrices $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ with complex entries such that $a_{11} + a_{22} = 0$. Let W be the set of matrices in V with $a_{12} + \overline{a_{21}} = 0$. Then, under usual matrix addition and scalar multiplication, which of the following is (are) true?

(A) V is a vector space over $\mathbb{C}$
(B) W is a vector space over $\mathbb{C}$
(C) V is a vector space over $\mathbb{R}$
(D) W is a vector space over $\mathbb{R}$

Q.6

The initial value problem

$$y' = \sqrt{y},\ \ y(0) = \alpha,\ \ \alpha \geq 0$$

has

(A) at least two solutions if $\alpha = 0$

(B) no solution if $\alpha > 0$

(C) at least one solution if $\alpha > 0$

(D) a unique solution if $\alpha = 0$

Q.7 Which of the following statements is (are) true on the interval $\left(0, \frac{\pi}{2}\right)$?

(A) $\cos x < \cos(\sin x)$

(B) $\tan x < x$

(C) $\sqrt{1+x} < 1 + \frac{x}{2} - \frac{x^2}{8}$

(D) $\frac{1-x^2}{2} < \ln(2+x)$

Q.8 Let $f: \mathbb{R}^2 \longrightarrow \mathbb{R}$ be defined by

$$f(x,y) = \begin{cases} xy\dfrac{x^2-y^2}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

At $(0,0)$,

(A) f is not continuous

(B) f is continuous, and both f_x and f_y exist

(C) f is differentiable

(D) f_x and f_y exist but f is not differentiable

Q.9

Let $f, g : [0,1] \to [0,1]$ be functions. Let $R(f)$ and $R(g)$ be the ranges of f and g, respectively. Which of the following statements is (are) true?

(A) If $f(x) \le g(x)$ for all $x \in [0,1]$, then $\sup R(f) \le \inf R(g)$

(B) If $f(x) \le g(x)$ for some $x \in [0,1]$, then $\inf R(f) \le \sup R(g)$

(C) If $f(x) \le g(y)$ for some $x, y \in [0,1]$, then $\inf R(f) \le \sup R(g)$

(D) If $f(x) \le g(y)$ for all $x, y \in [0,1]$, then $\sup R(f) \le \inf R(g)$

Q.10

Let $f: (-1,1) \longrightarrow \mathbb{R}$ be the function defined by

$$f(x) = x^2 e^{1/(1-x^2)}$$

Then

(A) f is decreasing in $(-1,0)$

(B) f is increasing in $(0,1)$

(C) $f(x) = 1$ has two solutions in $(-1,1)$

(D) $f(x) = 1$ has no solutions in $(-1,1)$

SECTION – C

NUMERICAL ANSWER TYPE (NAT)

Q. 1 – Q. 10 carry one mark each.

Q.1

Let C be the straight line segment from $P(0,\ \pi)$ to $Q\left(4,\ \frac{\pi}{2}\right)$, in the xy-plane. Then the value of $\int_C\ e^x(\cos y\ dx - \sin y\ dy)$ is ______________

Q.2

Let S be the portion of the surface $z = \sqrt{16 - x^2}$ bounded by the planes $x = 0,\ x = 2,\ y = 0,$ and $y = 3$. The surface area of S, correct upto three decimal places, is ______________

Q.3

The number of distinct normal subgroups of S_3 is ______________

Q.4

Let $f: \mathbb{R}^2 \longrightarrow \mathbb{R}$ be defined by

$$f(x,y) = \begin{cases} \left(1 + \frac{x}{y}\right)^2, & y \neq 0 \\ 0, & y = 0 \end{cases}$$

If the directional derivative of f at $(0,0)$ exists along the direction $\cos\alpha\ \hat{\imath} + \sin\alpha\ \hat{\jmath}$, where $\sin\alpha \neq 0$, then the value of $\cot\alpha$ is ______________

Q.5

Let $f: \mathbb{R}^3 \longrightarrow \mathbb{R}$ be defined by

$$f(x,y,z) = \sin x + 2\,e^{\frac{y}{2}} + z^2$$

The maximum rate of change of f at $\left(\frac{\pi}{4},\ 0,\ 1\right)$, correct upto three decimal places, is ______________

Q.6

If the power series

$$\sum_{n=0}^{\infty} \frac{n!}{n^n} x^{2n}$$

converges for $|x| < c$ and diverges for $|x| > c$, then the value of c, correct upto three decimal places, is ______________

Q.7

If $5^{2015} \equiv n\ (\bmod\ 11)$ and $n \in \{0,1,2,3,4,5,6,7,8,9,10\}$, then n is equal to _________

Q.8

If the set $\left\{ \begin{bmatrix} x & -x \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ x & x \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix} \right\}$ is linearly dependent in the vector space of all 2×2 matrices with real entries, then x is equal to ______________

Q.9

Let $f: \mathbb{R} \longrightarrow \mathbb{R}$ be defined by

$$f(x) = \begin{cases} x^6 - 1, & x \in \mathbb{Q} \\ 1 - x^6, & x \notin \mathbb{Q} \end{cases}$$

The number of points at which f is continuous, is ______________

Q.10

Let $f: (0, 1) \to \mathbb{R}$ be a continuously differentiable function such that f' has finitely many zeros in $(0, 1)$ and f' changes sign at exactly two of these points. Then for any $y \in \mathbb{R}$, the maximum number of solutions to $f(x) = y$ in $(0, 1)$ is ______________

Q. 11 – Q. 20 carry two marks each.

Q.11 Let R be the planar region bounded by the lines $x = 0$, $y = 0$ and the curve $x^2 + y^2 = 4$, in the first quadrant. Let C be the boundary of R, oriented counter-clockwise. Then the value of

$$\oint_C x(1 - y)\, dx + (x^2 - y^2)\, dy$$

is ______________

Q.12 Suppose G is a cyclic group and $\sigma, \tau \in G$ are such that order(σ) = 12 and order(τ) = 21. Then the order of the smallest group containing σ and τ is ______________

Q.13 The limit

$$\lim_{n\to\infty} \sum_{k=2}^{n} \frac{1}{k^3 - k}$$

is equal to ______________

Q.14 Let $M_2(\mathbb{R})$ be the vector space of 2×2 real matrices. Let V be a subspace of $M_2(\mathbb{R})$ defined by

$$V = \left\{ A \in M_2(\mathbb{R}) : A \begin{bmatrix} 0 & 2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 3 & 1 \end{bmatrix} A \right\}$$

Then the dimension of V is ______________

Q.15

Let $f: \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x,y) = \begin{cases} \frac{y}{\sin y}, & y \neq 0 \\ 1, & y = 0 \end{cases}$$

Then the integral

$$\frac{1}{\pi^2} \int_{x=0}^{1} \int_{y=\sin^{-1} x}^{\frac{\pi}{2}} f(x,y)\, dy\, dx$$

correct upto three decimal places, is ______________

Q.16

The coefficient of $\left(x - \frac{\pi}{4}\right)^3$ in the Taylor series expansion of the function

$$f(x) = 3 \sin x \cos\left(x + \frac{\pi}{4}\right), \qquad x \in \mathbb{R}$$

about the point $\frac{\pi}{4}$, correct upto three decimal places, is ______________

Q.17

If $\int_0^x \left(e^{-t^2} + \cos t\right) dt$ has the power series expansion $\sum_{n=1}^{\infty} a_n x^n$, then a_5, correct upto three decimal places, is equal to ______________

Q.18

Let ℓ be the length of the portion of the curve $x = x(y)$ between the lines $y = 1$ and $y = 3$, where $x(y)$ satisfies

$$\frac{dx}{dy} = \frac{\sqrt{1 + y^2 + y^4}}{y}, \qquad x(1) = 0$$

The value of ℓ, correct upto three decimal places, is ______________

Q.19 The limit

$$\lim_{x \to 0+} \frac{9}{x}\left(\frac{1}{\tan^{-1} x} - \frac{1}{x}\right)$$

is equal to ______________

Q.20 Let P and Q be two real matrices of size 4×6 and 5×4, respectively. If $\text{rank}(Q) = 4$ and $\text{rank}(QP) = 2$, then $\text{rank}(P)$ is equal to ______________

END OF THE QUESTION PAPER

JAM 2015

Answer Keys for the Test Paper: Mathematics (MA)

Section A - MCQ Multiple Choice Questions			Section B - MSQ Multiple Select Questions			Section C - NAT Numerical Answer Type Questions		
Q. No.	Key	Marks	Q. No.	Key	Marks	Q. No.	Range	Marks
1	C	1	1	A;C;D	2	1	1 to 1	1
2	C	1	2	A;D	2	2	6.28 to 6.29	1
3	B	1	3	C;D	2	3	3 to 3	1
4	B	1	4	A;B	2	4	-1 to -1	1
5	A	1	5	A;C;D	2	5	2.34 to 2.35	1
6	B	1	6	A;C	2	6	1.64 to 1.65	1
7	A	1	7	A;D	2	7	1 to 1	1
8	C	1	8	B;C	2	8	-1 to -1	1
9	B	1	9	B;C;D	2	9	2 to 2	1
10	A	1	10	A;B;C	2	10	3 to 3	1
11	B	2				11	8 to 8	2
12	D	2				12	84 to 84	2
13	D	2				13	0.25 to 0.25	2
14	A	2				14	2 to 2	2
15	A	2				15	0.125 to 0.125	2
16	C	2				16	1.41 to 1.42	2
17	D	2				17	0.10 to 0.11	2
18	C	2				18	5.09 to 5.10	2
19	D	2				19	3 to 3	2
20	D	2				20	2 to 2	2
21	B	2						
22	B	2						
23	A	2						
24	B	2						
25	A	2						
26	C	2						
27	D	2						
28	A	2						
29	D	2						
30	D	2						

2014 – MA

Booklet No. 501870

A

Test Paper Code : MA

QUESTION BOOKLET CODE **A**

Reg. No.

Time : 3 Hours

Name :

Maximum Marks : 100

SEAL

GENERAL INSTRUCTIONS

1. This Question-cum-Answer Booklet has **28** pages consisting of Part-I and Part-II.
2. An **ORS** (Optical Response Sheet) is inserted inside the Question-cum-Answer Booklet for filling in the answers of Part-I. Verify that the CODE and NUMBER Printed on the ORS matches with the CODE and NUMBER Printed on the **Question-cum-Answer Booklet.**
3. Based on the performance of Part-I, a certain number of candidates will be shortlisted. Part-II will be evaluated only for those shortlisted candidates.
4. The merit list of the qualified candidates will depend on the performance in both the parts.
5. Write your **Registration Number and Name** on the top right corner of this page as well as on the right hand side of the **ORS**. Also fill the appropriate bubbles for your registration number in the **ORS**.
6. The Question Booklet contains blank spaces for your rough work. No additional sheets will be provided for rough work.
7. **Non-Programmable Calculator is ALLOWED. But clip board, log tables, slide rule, cellular phone and other electronic gadgets are NOT ALLOWED.**
8. The Question-cum-Answer Booklet and the ORS must be returned in its entirety to the Invigilator before leaving the examination hall. **Do not remove any page from this Booklet.**
9. Refer to special instructions/useful data on the reverse of this page.

Instructions for Part-I

10. Part-I consists of **35** objective type questions. The first 10 questions carry **ONE** mark each and the rest 25 questions carry **TWO** marks each.
11. Each question has 4 choices for its answer: (A), (B), (C) and (D). Only **ONE** of the four choices is correct.
12. Fill the correct answer on the left hand side of the included **ORS** by darkening the appropriate bubble with a black ink ball point pen as per the instructions given therein.
13. There will be **negative marks for wrong answers**. For each 1 mark question the negative mark will be 1/3 and for each 2 mark question it will be 2/3.

Instructions for Part-II

14. Part-II has **8** subjective type questions. Answers to this part must be written in blue/black/blue-black ink only. The use of sketch pen, pencil or ink of any other color is not permitted.
15. Do not write more than one answer for the same question. In case you attempt a descriptive question more than once, please cancel the answer(s) you consider wrong. Otherwise, the answer appearing last only will be evaluated.

SEAL

A

Special Instructions / Useful Data

$\mathbb{N}$	: The set of all positive integers
$\mathbb{R}$	: The set of all real numbers
f', f''	: First and second derivatives respectively of a real function f
$\frac{\partial g}{\partial x}, \frac{\partial g}{\partial y}, \frac{\partial g}{\partial z}$	: Partial derivatives of g with respect to x, y and z respectively
log	: The logarithm to the base e
i, j, k	: Standard unit orthogonal vectors

A

IMPORTANT NOTE FOR CANDIDATES

- Part-I consists of 35 objective type questions. The first ten questions carry ***one*** mark each and the rest of the objective questions carry ***two*** marks each. There will be negative marks for wrong answers. For each 1 mark question the negative mark will be 1/3 and for each 2 mark question it will be 2/3.
- Write the answers to the objective questions by filling in the appropriate bubble on the left hand side of the included ORS.
- Part-II consists of 8 descriptive type questions each carrying ***five*** marks.

PART- I

Objective Questions

Q. 1 – Q. 10 carry one mark each.

Q.1 Let $f(x)=|x^2-25|$ for all $x\in\mathbb{R}$. The total number of points of $\mathbb{R}$ at which f attains a local extremum (minimum or maximum) is

(A) 1 (B) 2 (C) 3 (D) 4

Q.2 The coefficient of $(x-1)^2$ in the Taylor series expansion of $f(x)=xe^x$ $(x\in\mathbb{R})$ about the point $x=1$ is

(A) $\frac{e}{2}$ (B) $2e$ (C) $\frac{3e}{2}$ (D) $3e$

Q.3 Let $f(x,y)=\sum_{k=1}^{10}\left(x^2-y^2\right)^k$ for all $(x,y)\in\mathbb{R}^2$. Then for all $(x,y)\in\mathbb{R}^2$,

(A) $x\frac{\partial f}{\partial x}(x,y)-y\frac{\partial f}{\partial y}(x,y)=0$

(B) $x\frac{\partial f}{\partial x}(x,y)+y\frac{\partial f}{\partial y}(x,y)=0$

(C) $y\frac{\partial f}{\partial x}(x,y)-x\frac{\partial f}{\partial y}(x,y)=0$

(D) $y\frac{\partial f}{\partial x}(x,y)+x\frac{\partial f}{\partial y}(x,y)=0$

Q.4 For $a, b, c\in\mathbb{R}$, if the differential equation

$$\left(ax^2+bxy+y^2\right)dx+\left(2x^2+cxy+y^2\right)dy=0$$

is exact, then

(A) $b=2, c=2a$ (B) $b=4, c=2$ (C) $b=2, c=4$ (D) $b=2, a=2c$

A

Q.5 If $f(x,y,z)=x^2y+y^2z+z^2x$ for all $(x,\ y,\ z)\in\mathbb{R}^3$ and $\nabla=\frac{\partial}{\partial x}\mathbf{i}+\frac{\partial}{\partial y}\mathbf{j}+\frac{\partial}{\partial z}\mathbf{k}$, then the value of $\nabla\bullet(\nabla\times\nabla f)+\nabla\bullet(\nabla f)$ at $(1,1,1)$ is

(A) 0 (B) 3 (C) 6 (D) 9

Q.6 The radius of convergence of the power series $\sum_{n=0}^{\infty}2^{2n}x^{n^2}$ is

(A) $\frac{1}{4}$ (B) 1 (C) 2 (D) 4

Q.7 Let G be a group of order 17. The total number of non-isomorphic subgroups of G is

(A) 1 (B) 2 (C) 3 (D) 17

Q.8 Which one of the following is a subspace of the vector space $\mathbb{R}^3$?

(A) $\{(x,y,z)\in\mathbb{R}^3 : x+2y=0,\ 2x+3z=0\}$

(B) $\{(x,y,z)\in\mathbb{R}^3 : 2x+3y+4z-3=0,\ z=0\}$

(C) $\{(x,y,z)\in\mathbb{R}^3 : x\geq 0,\ y\geq 0\}$

(D) $\{(x,y,z)\in\mathbb{R}^3 : x-1=0,\ y=0\}$

Q.9 Let $T:\mathbb{R}^3\to\mathbb{R}^3$ be the linear transformation defined by $T(x,y,z)=(x+y,\ y+z,\ z+x)$ for all $(x,\ y,\ z)\in\mathbb{R}^3$. Then

(A) $\text{rank}(T)=0$, $\text{nullity}(T)=3$

(B) $\text{rank}(T)=2$, $\text{nullity}(T)=1$

(C) $\text{rank}(T)=1$, $\text{nullity}(T)=2$

(D) $\text{rank}(T)=3$, $\text{nullity}(T)=0$

Q.10 Let $f:\mathbb{R}\to\mathbb{R}$ be a continuous function satisfying $x+\int_0^x f(t)\,dt=e^x-1$ for all $x\in\mathbb{R}$. Then the set $\{x\in\mathbb{R} : 1\leq f(x)\leq 2\}$ is the interval

(A) $[\log 2,\ \log 3]$

(B) $[2\log 2,\ 3\log 3]$

(C) $[e-1,\ e^2-1]$

(D) $[0,\ e^2]$

A

Q. 11 – Q. 35 carry two marks each.

Q.11 The system of linear equations

$$\begin{aligned} x-y+2z &= b_1 \\ x+2y-z &= b_2 \\ 2y-2z &= b_3 \end{aligned}$$

is inconsistent when (b_1, b_2, b_3) equals

(A) (2, 2, 0) (B) (0, 3, 2) (C) (2, 2, 1) (D) (2, −1, −2)

Q.12 Let $A=\begin{bmatrix} a & -1 & 4 \\ 0 & b & 7 \\ 0 & 0 & 3 \end{bmatrix}$ be a matrix with real entries. If the sum and the product of all the eigenvalues of A are 10 and 30 respectively, then a^2+b^2 equals

(A) 29 (B) 40 (C) 58 (D) 65

Q.13 Consider the subspace $W=\left\{(x_1, x_2, \ldots, x_{10}) \in \mathbb{R}^{10} : x_n = x_{n-1}+x_{n-2} \text{ for } 3 \le n \le 10\right\}$ of the vector space $\mathbb{R}^{10}$. The dimension of W is

(A) 2 (B) 3 (C) 9 (D) 10

Q.14 Let $y_1(x)$ and $y_2(x)$ be two linearly independent solutions of the differential equation

$$x^2 y''(x) - 2xy'(x) - 4y(x) = 0 \text{ for } x \in [1, 10].$$

Consider the Wronskian $W(x) = y_1(x)y_2'(x) - y_2(x)y_1'(x)$. If $W(1)=1$, then $W(3)-W(2)$ equals

(A) 1 (B) 2 (C) 3 (D) 5

Q.15 The equation of the curve passing through the point $\left(\frac{\pi}{2}, 1\right)$ and having slope $\frac{\sin(x)}{x^2} - \frac{2y}{x}$ at each point (x, y) with $x \neq 0$ is

(A) $-x^2y+\cos(x) = \frac{-\pi^2}{4}$

(B) $x^2y+\cos(x) = \frac{\pi^2}{4}$

(C) $x^2y-\sin(x) = \frac{\pi^2}{4} - 1$

(D) $x^2y+\sin(x) = \frac{\pi^2}{4} + 1$

A

Q.16 The value of $\alpha \in \mathbb{R}$ for which the curves $x^2 + \alpha y^2 = 1$ and $y = x^2$ intersect orthogonally is

(A) -2 (B) $\frac{-1}{2}$ (C) $\frac{1}{2}$ (D) 2

Q.17 Let $x_n = 2^{2n}\left(1 - \cos\left(\frac{1}{2^n}\right)\right)$ for all $n \in \mathbb{N}$. Then the sequence $\{x_n\}$

(A) does NOT converge

(B) converges to 0

(C) converges to $\frac{1}{2}$

(D) converges to $\frac{1}{4}$

Q.18 Let $\{x_n\}$ be a sequence of real numbers such that $\lim_{n\to\infty}(x_{n+1} - x_n) = c$, where c is a positive real number. Then the sequence $\left\{\frac{x_n}{n}\right\}$

(A) is NOT bounded

(B) is bounded but NOT convergent

(C) converges to c

(D) converges to 0

Q.19 Let $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ be two series, where $a_n = \frac{(-1)^n n}{2^n}$, $b_n = \frac{(-1)^n}{\log(n+1)}$ for all $n \in \mathbb{N}$. Then

(A) both $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are absolutely convergent

(B) $\sum_{n=1}^{\infty} a_n$ is absolutely convergent but $\sum_{n=1}^{\infty} b_n$ is conditionally convergent

(C) $\sum_{n=1}^{\infty} a_n$ is conditionally convergent but $\sum_{n=1}^{\infty} b_n$ is absolutely convergent

(D) both $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are conditionally convergent

Q.20 The set $\left\{\frac{x^2}{1+x^2} : x \in \mathbb{R}\right\}$ is

(A) connected but NOT compact in $\mathbb{R}$

(B) compact but NOT connected in $\mathbb{R}$

(C) compact and connected in $\mathbb{R}$

(D) neither compact nor connected in $\mathbb{R}$

A

Q.21 The set of all limit points of the set $\left\{ \dfrac{2}{x+1} : x \in (-1, 1) \right\}$ in $\mathbb{R}$ is

(A) $[1, \infty)$ (B) $(1, \infty)$ (C) $[-1, 1]$ (D) $[-1, \infty)$

Q.22 Let $S = [0, 1] \cup [2, 3)$ and let $f : S \to \mathbb{R}$ be defined by $f(x) = \begin{cases} 2x & \text{if } x \in [0, 1], \\ 8 - 2x & \text{if } x \in [2, 3). \end{cases}$

If $T = \{ f(x) : x \in S \}$, then the inverse function $f^{-1} : T \to S$

(A) does NOT exist (B) exists and is continuous

(C) exists and is NOT continuous (D) exists and is monotonic

Q.23 Let $f(x) = x^3 + x$ and $g(x) = x^3 - x$ for all $x \in \mathbb{R}$. If f^{-1} denotes the inverse function of f, then the derivative of the composite function $g \circ f^{-1}$ at the point 2 is

(A) $\dfrac{2}{13}$ (B) $\dfrac{1}{2}$ (C) $\dfrac{11}{13}$ (D) $\dfrac{11}{4}$

Q.24 For all $(x, y) \in \mathbb{R}^2$, let $f(x, y) = \begin{cases} x & \text{if } y = 0, \\ x - y^3 \sin(1/y) & \text{if } y \neq 0. \end{cases}$

Then at the point (0, 0),

(A) f is NOT continuous

(B) f is continuous but NOT differentiable

(C) $\dfrac{\partial f}{\partial x}$ exists but $\dfrac{\partial f}{\partial y}$ does NOT exist

(D) f is differentiable

Q.25 For all $(x, y) \in \mathbb{R}^2$, let $f(x, y) = \begin{cases} \dfrac{x}{|x|}\sqrt{x^2 + y^2} & \text{if } x \neq 0, \\ 0 & \text{if } x = 0. \end{cases}$

Then $\dfrac{\partial f}{\partial x}(0, 0) + \dfrac{\partial f}{\partial y}(0, 0)$ equals

(A) -1 (B) 0 (C) 1 (D) 2

A

Q.26 Let $f:\mathbb{R}\to\mathbb{R}$ be a function with continuous derivative such that $f(\sqrt{2})=2$ and $f(x)=\lim_{t\to 0}\frac{1}{2t}\int_{x-t}^{x+t} s\,f'(s)\,ds$ for all $x\in\mathbb{R}$. Then $f(3)$ equals

(A) $\sqrt{3}$ (B) $3\sqrt{2}$ (C) $3\sqrt{3}$ (D) 9

Q.27 The value of $\int_{x=0}^{1}\int_{y=0}^{x^2}\int_{z=0}^{y}(y+2z)\,dz\,dy\,dx$ is

(A) $\frac{1}{53}$ (B) $\frac{2}{21}$ (C) $\frac{1}{6}$ (D) $\frac{5}{3}$

Q.28 If C is a smooth curve in $\mathbb{R}^3$ from $(-1, 0, 1)$ to $(1, 1, -1)$, then the value of $\int_C (2xy+z^2)\,dx+(x^2+z)\,dy+(y+2xz)\,dz$ is

(A) 0 (B) 1 (C) 2 (D) 3

Q.29 Let C be the boundary of the region $R=\left\{(x, y)\in\mathbb{R}^2 : -1\le y\le 1,\ 0\le x\le 1-y^2\right\}$ oriented in the counterclockwise direction. Then the value of $\oint_C y\,dx+2x\,dy$ is

(A) $\frac{-4}{3}$ (B) $\frac{-2}{3}$ (C) $\frac{2}{3}$ (D) $\frac{4}{3}$

Q.30 Let G be a cyclic group of order 24. The total number of group isomorphisms of G onto itself is

(A) 7 (B) 8 (C) 17 (D) 24

Q.31 Let S_n be the group of all permutations on the set $\{1,2,\ldots,n\}$ under the composition of mappings. For $n>2$, if H is the smallest subgroup of S_n containing the transposition (1, 2) and the cycle $(1, 2, \ldots, n)$, then

(A) $H=S_n$ (B) H is abelian

(C) the index of H in S_n is 2 (D) H is cyclic

A

Q.32 Let S be the oriented surface $x^2+y^2+z^2=1$ with the unit normal $\mathbf{n}$ pointing outward. For the vector field $\mathbf{F}(x,y,z)=x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$, the value of $\iint_S \mathbf{F}\cdot\mathbf{n}\,dS$ is

(A) $\frac{\pi}{3}$ (B) 2π (C) $\frac{4\pi}{3}$ (D) 4π

Q.33 Let $f:(0,\infty)\to\mathbb{R}$ be a differentiable function such that $f'(x^2)=1-x^3$ for all $x>0$ and $f(1)=0$. Then $f(4)$ equals

(A) $\frac{-47}{5}$ (B) $\frac{-47}{10}$ (C) $\frac{-16}{5}$ (D) $\frac{-8}{5}$

Q.34 Which one of the following conditions on a group G implies that G is abelian?

(A) The order of G is p^3 for some prime p

(B) Every proper subgroup of G is cyclic

(C) Every subgroup of G is normal in G

(D) The function $f:G\to G$, defined by $f(x)=x^{-1}$ for all $x\in G$, is a homomorphism

Q.35 Let $S=\left\{x\in\mathbb{R} : x^6-x^5\le 100\right\}$ and $T=\left\{x^2-2x : x\in(0,\ \infty)\right\}$. The set $S\cap T$ is

(A) closed and bounded in $\mathbb{R}$

(B) closed but NOT bounded in $\mathbb{R}$

(C) bounded but NOT closed in $\mathbb{R}$

(D) neither closed nor bounded in $\mathbb{R}$

A

PART - II

Descriptive Questions

Q. 36 – Q. 43 carry five marks each.

Q.36 Find all the critical points of the function $f:\mathbb{R}^2 \to \mathbb{R}$ defined by $f(x, y) = x^3 + xy + y^3$ for all $(x, y) \in \mathbb{R}^2$. Also, examine whether the function f attains a local maximum or a local minimum at each of these critical points.

A

Q.37 Given that there is a common solution to the following equations:

$$\mathbf{P}: y' + 2y = e^x y^2, \quad y(0) = 1,$$
$$\mathbf{Q}: y'' - 2y' + \alpha y = 0,$$

find the value of α and hence find the general solution of $\mathbf{Q}$.

A

Q.38 Let $f:\mathbb{R}\to\mathbb{R}$ be a twice differentiable function such that $f\left(\frac{1}{2^n}\right)=0$ for all $n\in\mathbb{N}$. Show that $f'(0)=0=f''(0)$.

A

Q.39 Let A be an $n \times n$ matrix with real entries such that $A^2 = A$. If I denotes the $n \times n$ identity matrix, then show that $\text{rank}(A - I) = \text{nullity}(A)$.

A

Q.40 Evaluate $\iint\limits_S \frac{xy}{\sqrt{1+2x^2}}\,dS$, where the surface $S = \left\{ (x,\ y,\ x^2 + y) \in \mathbb{R}^3 \ :\ 0 \le x \le y,\ x + y \le 1 \right\}$.

A

Q.41 Let $f:(0,1)\to\mathbb{R}$ be a differentiable function such that $|f'(x)|\le 5$ for all $x\in(0,1)$. Show that the sequence $\left\{f\left(\frac{1}{n+1}\right)\right\}$ converges in $\mathbb{R}$.

A

Q.42 Let H be a subgroup of the group $(\mathbb{R}, +)$ such that $H \cap [-1, 1]$ is a finite set containing a nonzero element. Show that H is cyclic.

A

Q.43 If K is a nonempty closed subset of $\mathbb{R}$, then show that the set $\{x+y : x \in K,\ y \in [1, 2]\}$ is closed in $\mathbb{R}$.

Solution Keys for MA Test Paper - JAM 2014

Code - A		Code - B		Code - C		Code - D	
C	1	C	1	C	1	A	1
C	2	B	2	B	2	D	2
D	3	D	3	C	3	A	3
B	4	B	4	D	4	B	4
C	5	A	5	B	5	B	5
B	6	A	6	A	6	C	6
B	7	C	7	A	7	B	7
A	8	D	8	C	8	D	8
D	9	C	9	B	9	C	9
A	10	B	10	D	10	C	10
C	11	C	11	B	11	A	11
A	12	B	12	C	12	D	12
A	13	D	13	B	13	A	13
D	14	C	14	B	14	D	14
B	15	A	15	D	15	C	15
D	16	B	16	A	16	D	16
C	17	D	17	D	17	B	17
C	18	A	18	D	18	C	18
B	19	B	19	A	19	A	19
A	20	C	20	A	20	A	20
A	21	B	21	D	21	B	21
C	22	C	22	C	22	D	22
B	23	A	23	A	23	C	23
D	24	B	24	C	24	B	24
C	25	A	25	B	25	B	25
B	26	D	26	C	26	B	26
B	27	A	27	D	27	D	27
C	28	A	28	A	28	A	28
D	29	D	29	B	29	A	29
B	30	D	30	C	30	C	30
A	31	A	31	A	31	A	31
D	32	B	32	D	32	B	32
A	33	C	33	C	33	C	33
D	34	C	34	A	34	C	34
A	35	D	35	B	35	D	35

A

2013 – MA

Test Paper Code : MA

Time : 3 Hours Maximum Marks : 100

INSTRUCTIONS

1. This question-cum-answer booklet has **32** pages and has **30** questions. Please ensure that the copy of the question-cum-answer booklet you have received contains all the questions.
2. Write your **Registration Number, Name and the name of the Test Centre** in the appropriate space provided on the right side.
3. Write the answers to the objective questions against each Question No. in the **Answer Table for Objective Questions**, provided on Page No. **4**. Do not write anything else on this page.
4. Each objective question has **4 choices** for its answer: (A), (B), (C) and (D). Only **ONE** of them is the correct answer. There will be **negative marking** for wrong answers to objective questions. The following marking scheme for objective questions shall be used :
   (a) For each correct answer, you will be awarded **2 (Two)** marks.
   (b) For each wrong answer, you will be awarded **–0.5 (Negative 0.5)** mark.
   (c) Multiple answers to a question will be treated as a wrong answer .
   (d) For each un-attempted question, you will be awarded **0 (Zero)** mark.
   (e) Negative marks for objective part will be carried over to total mark.
5. Answer the fill in the blank type and descriptive type questions only in the **space provided after each question**. No negative marks for fill in the blank type questions.
6. Do not write more than one answer for the same question. In case you attempt a fill in the blank or a descriptive question more than once, please cancel the answer(s) you consider wrong. Otherwise, the answer appearing last only will be evaluated.
7. All answers must be written in blue/black/ blue-black ink only. Sketch pen, pencil or ink of any other colour should not be used.
8. All rough work should be done in the space provided and scored out finally.
9. No supplementary sheets will be provided to the candidates.
10. **Clip board, log tables, slide rule, cellular phone and electronic gadgets in any form are NOT allowed. Non Programmable Calculator is allowed.**
11. The question-cum-answer booklet must be returned in its entirety to the Invigilator before leaving the examination hall. Do not remove any page from this booklet.
12. Refer to special instructions/useful data on the reverse.

A

2013 – MA

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

REGISTRATION NUMBER						

Name :

Test Centre :

Do not write your Registration Number or Name anywhere else in this question-cum-answer booklet.

I have read all the instructions and shall abide by them.

....

Signature of the Candidate

I have verified the information filled by the Candidate above.

....

Signature of the Invigilator

A

Special Instructions/ Useful Data

$\mathbb{R}$	: The set of all real numbers
$\mathbb{N}$	: The set of all positive integers
f'	: First derivative of a real function f of single variable
$\mathbb{Z}_p$	: $\{0,1,...,p-1\}$ with addition and multiplication modulo p
S^o	: Interior of a set $S \subseteq \mathbb{R}$
$\overline{S}$	: Closure of a set $S \subseteq \mathbb{R}$

A

IMPORTANT NOTE FOR CANDIDATES

- **Questions 1-10 (objective questions) carry *two* marks each, questions 11-20 (fill in the blank questions) carry *three* marks each and questions 21-30 (descriptive questions) carry *five* marks each.**
- **The marking scheme for the objective type question, is as follows:**
  (a) For each correct answer, you will be awarded **2 (Two)** marks.
  (b) For each wrong answer, you will be awarded **-0.5 (Negative 0.5)** mark.
  (c) Multiple answers to a question will be treated as a wrong answer.
  (d) For each un-attempted question, you will be awarded **0 (Zero)** mark.
  (e) Negative marks for objective part will be carried over to total marks.
- **There is no negative marking for fill in the blank questions.**
- **Write the answers to the objective questions in the *Answer Table for Objective Questions* provided on page 4 only.**

Objective Questions

Q.1 Let $A=\begin{pmatrix}1 & 1 & 1\\ 3 & -1 & 1\\ 1 & 5 & 3\end{pmatrix}$ and V be the vector space of all $X\in\mathbb{R}^3$ such that $AX=0$. Then $\dim(V)$ is

(A) 0 (B) 1 (C) 2 (D) 3

Q.2 The value of n for which the divergence of the function $\vec{F}=\frac{\vec{r}}{|\vec{r}|^n}$, $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$, $|\vec{r}|\neq 0$, vanishes is

(A) 1 (B) -1 (C) 3 (D) -3

Q.3 Let A and B be subsets of $\mathbb{R}$. Which of the following is **NOT** necessarily true?

(A) $(A\cap B)^o\subseteq A^o\cap B^o$ (B) $A^o\cup B^o\subseteq (A\cup B)^o$

(C) $\overline{A}\cup\overline{B}\subseteq\overline{A\cup B}$ (D) $\overline{A}\cap\overline{B}\subseteq\overline{A\cap B}$

Q.4 Let $[x]$ denote the greatest integer function of x. The value of α for which the function

$$f(x)=\begin{cases}\dfrac{\sin[-x^2]}{[-x^2]}, & x\neq 0\\ \alpha, & x=0\end{cases}$$

is continuous at $x=0$ is

(A) 0 (B) $\sin(-1)$ (C) $\sin 1$ (D) 1

A

Q.5 Let the function $f(x)$ be defined by

$$f(x)=\begin{cases} e^{x}, & x \text{ is rational} \\ e^{1-x}, & x \text{ is irrational} \end{cases}$$

for x in $(0, 1)$. Then

(A) f is continuous at every point in $(0, 1)$
(B) f is discontinuous at every point in $(0, 1)$
(C) f is discontinuous only at one point in $(0, 1)$
(D) f is continuous only at one point in $(0, 1)$

Q.6 The value of the integral

$$\iint_D \sqrt{x^2+y^2}\,dx\,dy, \quad D=\left\{(x,y)\in\mathbb{R}^2:\ x\le x^2+y^2\le 2x\right\}$$

is

(A) 0 (B) $\frac{7}{9}$ (C) $\frac{14}{9}$ (D) $\frac{28}{9}$

Q.7 Let

$$x_n=\left(1-\frac{1}{3}\right)^2\left(1-\frac{1}{6}\right)^2\left(1-\frac{1}{10}\right)^2\cdots\left(1-\frac{1}{\frac{n(n+1)}{2}}\right)^2, \quad n\ge 2.$$

Then $\lim_{n\to\infty} x_n$ is

(A) $\frac{1}{3}$ (B) $\frac{1}{9}$ (C) $\frac{1}{81}$ (D) 0

Q.8 Let p be a prime number. Let G be the group of all 2×2 matrices over $\mathbb{Z}_p$ with determinant 1 under matrix multiplication. Then the order of G is

(A) $(p-1)p(p+1)$
(B) $p^2(p-1)$
(C) p^3
(D) $p^2(p-1)+p$

A

Q.9 Let V be the vector space of all 2×2 matrices over $\mathbb{R}$. Consider the subspaces

$$W_1 = \left\{ \begin{pmatrix} a & -a \\ c & d \end{pmatrix} : a, c, d \in \mathbb{R} \right\} \text{ and } W_2 = \left\{ \begin{pmatrix} a & b \\ -a & d \end{pmatrix} : a, b, d \in \mathbb{R} \right\}.$$

If $m = \dim(W_1 \cap W_2)$ and $n = \dim(W_1 + W_2)$, then the pair (m, n) is

(A) $(2, 3)$ (B) $(2, 4)$ (C) $(3, 4)$ (D) $(1, 3)$

Q.10 Let $\wp_n$ be the real vector space of all polynomials of degree at most n. Let $D : \wp_n \to \wp_{n-1}$ and $T : \wp_n \to \wp_{n+1}$ be the linear transformations defined by

$$D\left(a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n\right) = a_1 + 2a_2 x + \ldots + n a_n x^{n-1},$$

$$T\left(a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n\right) = a_0 x + a_1 x^2 + a_2 x^3 + \ldots + a_n x^{n+1},$$

respectively. If A is the matrix representation of the transformation $DT - TD : \wp_n \to \wp_n$ with respect to the standard basis of $\wp_n$, then the trace of A is

(A) $-n$ (B) n (C) $n+1$ (D) $-(n+1)$

A

Answer Table for Objective Questions

Write the Code of your chosen answer only in the 'Answer' column against each Question Number. Do not write anything else on this page.

Question Number	**Answer**	**Do not write in this column**
01		
02		
03		
04		
05		
06		
07		
08		
09		
10		

FOR EVALUATION ONLY

Number of Correct Answers		Marks	(+)
Number of Incorrect Answers		Marks	(−)
Total Marks in Question Nos. 1-10			( )

A

Fill in the blank questions

Q.11 The equation of the curve satisfying $\sin y \dfrac{dy}{dx} = \cos y(1 - x\cos y)$ and passing through the origin is

Ans:

Q.12 Let f be a continuously differentiable function such that $\int_0^{2x^2} f(t)\,dt = e^{\cos x^2}$ for all $x \in (0, \infty)$. The value of $f'(\pi)$ is

Ans:

Q.13 Let $u = \dfrac{y^2 - x^2}{x^2 y^2}$, $v = \dfrac{z^2 - y^2}{y^2 z^2}$ for $x \neq 0$, $y \neq 0$, $z \neq 0$. Let $w = f(u, v)$, where f is a real valued function defined on $\mathbb{R}^2$ having continuous first order partial derivatives. The value of $x^3 \dfrac{\partial w}{\partial x} + y^3 \dfrac{\partial w}{\partial y} + z^3 \dfrac{\partial w}{\partial z}$ at the point $(1, 2, 3)$ is

Ans:

A

Q.14 The set of points at which the function $f(x,y)=x^4+y^4-x^2-y^2+1$, $(x,y)\in\mathbb{R}^2$ attains local maximum is

Ans:

Q.15 Let C be the boundary of the region in the first quadrant bounded by $y=1-x^2$, $x=0$ and $y=0$, oriented counter-clockwise. The value of $\oint_C (xy^2dx-x^2y\,dy)$ is

Ans:

Q.16 Let $f(x)=\begin{cases}0, & -1\le x\le 0\\ x^4, & 0<x\le 1\end{cases}$. If $f(x)=\sum_{k=0}^{n}\frac{f^{(k)}(0)}{k!}x^k+\frac{f^{(n+1)}(\xi)}{(n+1)!}x^{n+1}$ is the Taylor's formula for f about $x=0$ with maximum possible value of n, then the value of ξ for $0<x\le 1$ is

Ans:

Q.17 Let $\vec{F}=2z\hat{i}+4x\hat{j}+5y\hat{k}$, and let C be the curve of intersection of the plane $z=x+4$ and the cylinder $x^2+y^2=4$, oriented counter-clockwise. The value of $\oint_C \vec{F}\cdot d\vec{r}$ is

Ans:

A

Q.18 Let f and g be the functions from $\mathbb{R}\setminus\{0, 1\}$ to $\mathbb{R}$ defined by $f(x)=\frac{1}{x}$ and $g(x)=\frac{x-1}{x}$ for $x\in \mathbb{R}\setminus\{0, 1\}$. The smallest group of functions from $\mathbb{R}\setminus\{0, 1\}$ to $\mathbb{R}$ containing f and g under composition of functions is isomorphic to

Ans:

Q.19 The orthogonal trajectory of the family of curves $\frac{x^2}{2}+y^2=c$, which passes through (1, 1) is

Ans:

Q.20 The function to which the power series $\sum_{n=1}^{\infty}(-1)^{n+1} n\, x^{2n-2}$ converges is

Ans:

A

Descriptive questions

Q.21 Let $0 < a \leq 1$, $s_1 = \frac{a}{2}$ and for $n \in \mathbb{N}$, let $s_{n+1} = \frac{1}{2}\left(s_n^2 + a\right)$. Show that the sequence $\{s_n\}$ is convergent, and find its limit.

Space for the answer

A

Q.22 Evaluate

$$\int_{1/4}^{1} \int_{\sqrt{x-x^2}}^{\sqrt{x}} \frac{x^2 - y^2}{x^2}\, dy\, dx$$

by changing the order of integration.

Space for the answer

A

Q.23 Find the general solution of the differential equation

$$x^2\,\frac{d^3y}{dx^3}+x\,\frac{d^2y}{dx^2}-6\,\frac{dy}{dx}+6\,\frac{y}{x}=\frac{x\ln x+1}{x^2},\quad x>0.$$

Space for the answer

A

Q.24 Let S_1 be the hemisphere $x^2+y^2+z^2=1,\ z>0$ and S_2 be the closed disc $x^2+y^2\le 1$ in the xy plane. Using Gauss' divergence theorem, evaluate $\iint_S \vec{F}\cdot d\bar{S}$, where $\vec{F}=z^2x\,\hat{i}+\left(\frac{y^3}{3}+\tan z\right)\hat{j}+(x^2z+y^2)\,\hat{k}$ and $S=S_1\cup S_2$.

Also evaluate $\iint_{S_1} \vec{F}\cdot d\bar{S}$.

Space for the answer

A

Q.25 Let

$$f(x,y)=\begin{cases}\dfrac{2\left(x^3+y^3\right)}{x^2+2y}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0).\end{cases}$$

Show that the first order partial derivatives of f with respect to x and y exist at $(0,0)$. Also show that f is not continuous at $(0,0)$.

Space for the answer

A

Q.26 Let A be an $n \times n$ diagonal matrix with characteristic polynomial $(x-a)^p (x-b)^q$, where a and b are distinct real numbers. Let V be the real vector space of all $n \times n$ matrices B such that $AB = BA$. Determine the dimension of V.

Space for the answer

A

Q.27 Let A be an $n \times n$ real symmetric matrix with n distinct eigenvalues. Prove that there exists an orthogonal matrix P such that $AP = PD$, where D is a real diagonal matrix.

Space for the answer

A

Q.28 Let K be a compact subset of $\mathbb{R}$ with nonempty interior. Prove that K is of the form $[a, b]$ or of the form $[a, b] \setminus \bigcup I_n$, where $\{I_n\}$ is a countable disjoint family of open intervals with end points in K.

Space for the answer

A

Q.29 Let $f:[a, b]\to\mathbb{R}$ be a continuous function such that f is differentiable in (a,c) and (c, b), $a<c<b$. If $\lim_{x\to c} f'(x)$ exists, then prove that f is differentiable at c and $f'(c)=\lim_{x\to c} f'(x)$.

Space for the answer

A

Q.30 Let G be a finite group, and let φ be an automorphism of G such that $\varphi(x) = x$ if and only if $x = e$, where e is the identity element in G. Prove that every $g \in G$ can be represented as $g = x^{-1}\varphi(x)$ for some $x \in G$. Moreover, if $\varphi(\varphi(x)) = x$ for every $x \in G$, then show that G is abelian.

Space for the answer

A

2013 - MA Objective Part (Question Number 1 – 10)	
Total Marks	Signature

Fill in the blanks Part and Descriptive Part					
Question Number	Marks		Question Number	Marks	
11			21		
12			22		
13			23		
14			24		
15			25		
16			26		
17			27		
18			28		
19			29		
20			30		
Total Marks in Fill in the blanks Part and Descriptive Part					

Total (Objective Part)	:	
Total (Fill in the blanks Part and Descriptive Part)	:	
Grand Total	:	
Total Marks (in words)	:	
Signature of Examiner(s)	:	
Signature of Head Examiner(s)	:	
Signature of Scrutinizer	:	
Signature of Chief Scrutinizer	:	
Signature of Coordinating Head Examiner	:	

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2012- MA

Test Paper Code: MA

2012- MA

Time: 3 Hours **Maximum Marks: 300**

INSTRUCTIONS

1. This question-cum-answer booklet has **36** pages and has **29** questions. Please ensure that the copy of the question-cum-answer booklet you have received contains all the questions.
2. Write your **Registration Number, Name and the name of the Test Centre** in the appropriate space provided on the right side.
3. Write the answers to the objective questions against each Question Number in the **Answer Table for Objective Questions**, provided on Page **7**. Do not write anything else on this page.
4. Each objective question has **4 choices** for its answer: (A), (B), (C) and (D). Only **ONE** of them is the correct answer. There will be **negative marking** for wrong answers to objective questions. The following marking scheme for objective questions shall be used:
   (a) For each correct answer, you will be awarded **6 (Six)** marks.
   (b) For each wrong answer, you will be awarded **-2 (Negative two)** mark.
   (c) Multiple answers to a question will be treated as a wrong answer.
   (d) For each un-attempted question, you will be awarded **0 (Zero)** mark.
   (e) Negative marks for objective part will be carried over to total marks.
5. Answer the subjective question only in the space provided after each question.
6. Do not write more than one answer for the same question. In case you attempt a subjective question more than once, please cancel the answer(s) you consider wrong. Otherwise, the answer appearing last only will be evaluated.
7. All answers must be written in blue/black/blue-black ink only. Sketch pen, pencil or ink of any other colour should not be used.
8. All rough work should be done in the space provided and scored out finally.
9. No supplementary sheets will be provided to the candidates.
10. **Clip board, log tables, slide rule, calculator, cellular phone and electronic gadgets in any form are NOT allowed.**
11. The question-cum-answer booklet must be returned in its entirety to the Invigilator before leaving the examination hall. Do not remove any page from this booklet.
12. Refer to special instructions/useful data on the reverse.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

REGISTRATION NUMBER						

Name:

Test Centre:

Do not write your Registration Number or Name anywhere else in this question-cum-answer booklet.

I have read all the instructions and shall abide by them.

………………………………………………..
Signature of the Candidate

I have verified the information filled by the Candidate above.

………………………………………………..
Signature of the Invigilator

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Special Instructions/ Useful Data

$\mathbb{N}$	:	The set of all natural numbers, that is, the set of all positive integers 1, 2, 3, …
$\mathbb{Z}$	:	The set of all integers
$\mathbb{Q}$	:	The set of all rational numbers
$\mathbb{R}$	:	The set of all real numbers
$\{e_1, e_2, \ldots, e_n\}$	:	The standard basis of the real vector space $\mathbb{R}^n$
f', f''	:	First and second derivatives respectively of a real function f
$f_x(a,b),\ f_y(a,b)$	:	Partial derivatives with respect to x and y respectively of $f:\mathbb{R}^2 \to \mathbb{R}$ at (a,b)
$R \times S$	:	Product ring of rings R, S with component-wise operations of addition and multiplication

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IMPORTANT NOTE FOR CANDIDATES

- **Questions 1-15 (objective questions) carry *six* marks each and questions 16-29 (subjective questions) carry *fifteen* marks each.**
- **Write the answers to the objective questions in the *Answer Table for Objective Questions* provided on page 7 only.**

Q.1 Let $\{x_n\}$ be the sequence $+\sqrt{1}, -\sqrt{1}, +\sqrt{2}, -\sqrt{2}, +\sqrt{3}, -\sqrt{3}, +\sqrt{4}, -\sqrt{4}, \ldots$.
If

$$y_n = \frac{x_1 + x_2 + \cdots + x_n}{n} \text{ for all } n \in \mathbb{N},$$

then the sequence $\{y_n\}$ is

(A) monotonic
(B) NOT bounded
(C) bounded but NOT convergent
(D) convergent

Q.2 The number of distinct real roots of the equation $x^9 + x^7 + x^5 + x^3 + x + 1 = 0$ is

(A) 1 (B) 3 (C) 5 (D) 9

Q.3 If $f : \mathbb{R}^2 \to \mathbb{R}$ is defined by

$$f(x,y) = \begin{cases} \dfrac{x^3}{x^2 + y^4} & \text{if } (x,y) \neq (0,0), \\ 0 & \text{if } (x,y) = (0,0), \end{cases}$$

then

(A) $f_x(0,0) = 0$ and $f_y(0,0) = 0$
(B) $f_x(0,0) = 1$ and $f_y(0,0) = 0$
(C) $f_x(0,0) = 0$ and $f_y(0,0) = 1$
(D) $f_x(0,0) = 1$ and $f_y(0,0) = 1$

Q.4 The value of $\int_{z=0}^{1} \int_{y=0}^{z} \int_{x=0}^{y} x\, y^2\, z^3 \, dx\, dy\, dz$ is

(A) $\frac{1}{90}$ (B) $\frac{1}{50}$ (C) $\frac{1}{45}$ (D) $\frac{1}{10}$

A

Q.5 The differential equation $\left(1+x^2y^3+\alpha x^2y^2\right)dx+\left(2+x^3y^2+x^3y\right)dy=0$ is exact if α equals

(A) $\frac{1}{2}$ (B) $\frac{3}{2}$ (C) 2 (D) 3

Q.6 An integrating factor for the differential equation $\left(2xy+3x^2y+6y^3\right)dx+\left(x^2+6y^2\right)dy=0$ is

(A) x^3 (B) y^3 (C) e^{3x} (D) e^{3y}

Q.7 For $c>0$, if $a\hat{i}+b\hat{j}+c\hat{k}$ is the unit normal vector at $\left(1,1,\sqrt{2}\right)$ to the cone $z=\sqrt{x^2+y^2}$, then

(A) $a^2+b^2-c^2=0$ (B) $a^2-b^2+c^2=0$
(C) $-a^2+b^2+c^2=0$ (D) $a^2+b^2+c^2=0$

Q.8 Consider the quotient group $\mathbb{Q}/\mathbb{Z}$ of the additive group of rational numbers. The order of the element $\frac{2}{3}+\mathbb{Z}$ in $\mathbb{Q}/\mathbb{Z}$ is

(A) 2 (B) 3 (C) 5 (D) 6

Q.9 Which one of the following is TRUE ?

(A) The characteristic of the ring $6\mathbb{Z}$ is 6
(B) The ring $6\mathbb{Z}$ has a zero divisor
(C) The characteristic of the ring $\left(\mathbb{Z}/6\mathbb{Z}\right)\times 6\mathbb{Z}$ is zero
(D) The ring $6\mathbb{Z}\times 6\mathbb{Z}$ is an integral domain

Q.10 Let W be a vector space over $\mathbb{R}$ and let $T:\mathbb{R}^6\to W$ be a linear transformation such that $S=\{Te_2, Te_4, Te_6\}$ spans W. Which one of the following must be TRUE ?

(A) S is a basis of W
(B) $T\left(\mathbb{R}^6\right)\neq W$
(C) $\{Te_1, Te_3, Te_5\}$ spans W
(D) $\ker(T)$ contains more than one element

A

Q.11 Consider the following subspace of $\mathbb{R}^3$:

$$W = \left\{ (x, y, z) \in \mathbb{R}^3 \,\middle|\, 2x + 2y + z = 0,\ 3x + 3y - 2z = 0,\ x + y - 3z = 0 \right\}.$$

The dimension of W is

(A) 0 (B) 1 (C) 2 (D) 3

Q.12 Let P be a 4×4 matrix whose determinant is 10. The determinant of the matrix $-3P$ is

(A) –810 (B) –30 (C) 30 (D) 810

Q.13 If the power series $\sum_{n=0}^{\infty} a_n x^n$ converges for $x = 3$, then the series $\sum_{n=0}^{\infty} a_n x^n$

(A) converges absolutely for $x = -2$
(B) converges but not absolutely for $x = -1$
(C) converges but not absolutely for $x = 1$
(D) diverges for $x = -2$

Q.14 If $Y = \left\{ \frac{x}{1+|x|} \,\middle|\, x \in \mathbb{R} \right\}$, then the set of all limit points of Y is

(A) $(-1,1)$ (B) $(-1,1]$ (C) $[0,1]$ (D) $[-1,1]$

Q.15 If C is a smooth curve in $\mathbb{R}^3$ from $(0,0,0)$ to $(2,1,-1)$, then the value of

$$\int_C (2xy + z)\,dx + (z + x^2)\,dy + (x + y)\,dz$$

is

(A) –1 (B) 0 (C) 1 (D) 2

A

Answer Table for Objective Questions

Write the Code of your chosen answer only in the 'Answer' column against each Question Number. Do not write anything else on this page.

Question Number	**Answer**	**Do not write in this column**
01		
02		
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FOR EVALUATION ONLY

Number of Correct Answers		Marks	(+)
Number of Incorrect Answers		Marks	(–)
Total Marks in Questions 1-15			( )

A

Q.16 (a) Examine whether the following series is convergent:

$$\sum_{n=1}^{\infty} \frac{n!}{1\cdot 3\cdot 5\cdots(2n-1)}.$$

(6)

(b) For each $x \in \mathbb{R}$, let $[x]$ denote the greatest integer less than or equal to x. Further, for a fixed $\beta \in (0,1)$, define $a_n = \frac{1}{n}[n\beta] + n^2\beta^n$ for all $n \in \mathbb{N}$. Show that the sequence $\{a_n\}$ converges to β.

(9)

A

Q.17 (a) Evaluate $\lim_{x \to 0} \frac{\int_0^{x^2} \sqrt{4+t^3}\, dt}{x^2}$. (6)

(b) For $a, b \in \mathbb{R}$ with $a < b$, let $f : [a,b] \to \mathbb{R}$ be continuous on $[a,b]$ and twice differentiable on (a,b). Further, assume that the graph of f intersects the straight line segment joining the points $(a, f(a))$ and $(b, f(b))$ at a point $(c, f(c))$ for $a < c < b$. Show that there exists a real number $\xi \in (a,b)$ such that $f''(\xi) = 0$. (9)

A

Q.18 (a) Show that the point (0,0) is neither a point of local minimum nor a point of local maximum for the function $f:\mathbb{R}^2 \to \mathbb{R}$ given by $f(x,y) = 3x^4 - 4x^2y + y^2$ for $(x,y) \in \mathbb{R}^2$. (6)

(b) Find all the critical points of the function $f:\mathbb{R}^2 \to \mathbb{R}$ given by $f(x,y) = x^3 + y^3 - 3x - 12y + 40$ for $(x,y) \in \mathbb{R}^2$. Also, examine whether the function f attains a local maximum or a local minimum at each of these critical points. (9)

A

Q.19 (a) Evaluate $\int_{x=0}^{4} \int_{y=\sqrt{4-x}}^{2} e^{y^3}\, dy\, dx.$ (6)

(b) Using multiple integral, find the volume of the solid region in $\mathbb{R}^3$ bounded above by the hemisphere $z = 1 + \sqrt{1 - x^2 - y^2}$ and bounded below by the cone $z = \sqrt{x^2 + y^2}$. (9)

A

Q.20 Find the area of the portion of the surface $z = x^2 - y^2$ in $\mathbb{R}^3$ which lies inside the solid cylinder $x^2 + y^2 \leq 1$. (15)

A

Q.21 Let $y(x)$ be the solution of the differential equation $\frac{d^2y}{dx^2} - y = 0$ such that $y(0) = 2$ and $y'(0) = 2\alpha$. Find all values of $\alpha \in [0,1)$ such that the infimum of the set $\{y(x) \mid x \in \mathbb{R}\}$ is greater than or equal to 1. (15)

A

Q.22 (a) Assume that $y_1(x) = x$ and $y_2(x) = x^3$ are two linearly independent solutions of the homogeneous differential equation $x^2 \frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 3y = 0$. Using the method of variation of parameters, find a particular solution of the differential equation $x^2 \frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 3y = x^5$. (6)

(b) Solve the differential equation $\frac{dy}{dx} + \frac{5y}{6x} = \frac{5x^4}{y^5}$ subject to the condition $y(1) = 1$. (9)

A

Q.23 (a) Let $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ be the position vector field in $\mathbb{R}^3$ and let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function. Show that $\vec{\nabla} \times \left\{ f\left(|\vec{r}|\right)\vec{r} \right\} = \vec{0}$ for $\vec{r} \neq \vec{0}$. (6)

(b) Let W be the region inside the solid cylinder $x^2 + y^2 \leq 4$ between the plane $z = 0$ and the paraboloid $z = x^2 + y^2$. Let S be the boundary of W. Using Gauss's divergence theorem, evaluate $\iint_S \vec{F} \cdot \hat{n}\, dS$, where

$$\vec{F} = \left(x^2 + y^2 - 4\right)\hat{i} + \left(3xy\right)\hat{j} + \left(2xz + z^2\right)\hat{k}$$

and $\hat{n}$ is the outward unit normal vector to S. (9)

A

Q.24 (a) Let G be a finite group whose order is not divisible by 3. Show that for every $g \in G,$ there exists an $h \in G$ such that $g = h^3$. (6)

(b) Let A be the group of all rational numbers under addition, B be the group of all non-zero rational numbers under multiplication and C the group of all positive rational numbers under multiplication. Show that no two of the groups A, B and C are isomorphic. (9)

A

Q.25 (a) Let I be an ideal of a commutative ring R. Define

$$A=\{r\in R \mid r^n\in I \text{ for some } n\in\mathbb{N}\}.$$

Show that A is an ideal of R. (6)

(b) Let F be a field. For each $p(x)\in F[x]$ (the polynomial ring in x over F) define $\varphi: F[x]\to F\times F$ by $\varphi(p(x))=(p(0),p(1))$.

(i) Prove that φ is a ring homomorphism.

(ii) Prove that the quotient ring $F[x]/(x^2-x)$ is isomorphic to the ring $F\times F$. (9)

A

Q.26 (a) Let P, D and A be real square matrices of the same order such that P is invertible, D is diagonal and $D = PAP^{-1}$. If $A^n = 0$ for some $n \in \mathbb{N}$, then show that $A = 0$. (6)

(b) Let $T : V \to W$ be a linear transformation of vector spaces. Prove the following:

(i) If $\{v_1, v_2, \ldots, v_k\}$ spans V, and T is onto, then $\{Tv_1, Tv_2, \ldots, Tv_k\}$ spans W.

(ii) If $\{v_1, v_2, \ldots, v_k\}$ is linearly independent in V, and T is one-one, then $\{Tv_1, Tv_2, \ldots, Tv_k\}$ is linearly independent in W.

(iii) If $\{v_1, v_2, \ldots, v_k\}$ is a basis of V, and T is bijective, then $\{Tv_1, Tv_2, \ldots, Tv_k\}$ is a basis of W. (9)

A

Q.27 (a) Let $\{v_1, v_2, v_3\}$ be a basis of a vector space V over $\mathbb{R}$. Let $T : V \to V$ be the linear transformation determined by

$$Tv_1 = v_1,\ Tv_2 = v_2 - v_3 \text{ and } Tv_3 = v_2 + 2v_3.$$

Find the matrix of the transformation T with $\{v_1 + v_2,\ v_1 - v_2,\ v_3\}$ as a basis of both the domain and the co-domain of T. (6)

(b) Let W be a three dimensional vector space over $\mathbb{R}$ and let $S : W \to W$ be a linear transformation. Further, assume that every non-zero vector of W is an eigenvector of S. Prove that there exists an $\alpha \in \mathbb{R}$ such that $S = \alpha I$, where $I : W \to W$ is the identity transformation. (9)

A

Q.28 (a) Show that the function $f:\mathbb{R}\to\mathbb{R}$, defined by $f(x)=x^2$ for $x\in\mathbb{R}$, is not uniformly continuous. (6)

(b) For each $n\in\mathbb{N}$, let $f_n:\mathbb{R}\to\mathbb{R}$ be a uniformly continuous function. If the sequence $\{f_n\}$ converges uniformly on $\mathbb{R}$ to a function $f:\mathbb{R}\to\mathbb{R}$, then show that f is uniformly continuous. (9)

A

Q.29 (a) Let A be a nonempty bounded subset of $\mathbb{R}$. Show that $\{x \in \mathbb{R} \mid x \geq a \text{ for all } a \in A\}$ is a closed subset of $\mathbb{R}$. (6)

(b) Let $\{x_n\}$ be a sequence in $\mathbb{R}$ such that $|x_{n+1} - x_n| < \frac{1}{n^2}$ for all $n \in \mathbb{N}$. Show that the sequence $\{x_n\}$ is convergent. (9)

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2012 - MA Objective Part (Question Number 1 – 15)	
Total Marks	**Signature**

Subjective Part					
Question Number	**Marks**		**Question Number**	**Marks**	
16			**23**		
17			**24**		
18			**25**		
19			**26**		
20			**27**		
21			**28**		
22			**29**		
Total Marks in Subjective Part					

Total (Objective Part) :	
Total (Subjective Part) :	
Grand Total :	
Total Marks (in words) :	
Signature of Examiner(s) :	
Signature of Head Examiner(s) :	
Signature of Scrutinizer :	
Signature of Chief Scrutinizer :	
Signature of Coordinating Head Examiner :	

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2011 MA

Test Paper Code: MA

Time: 3 Hours **Maximum Marks: 300**

INSTRUCTIONS

1. This question-cum-answer booklet has **40** pages and has **29** questions. Please ensure that the copy of the question-cum-answer booklet you have received contains all the questions.
2. Write your **Registration Number, Name and the name of the Test Centre** in the appropriate space provided on the right side.
3. Write the answers to the objective questions against each Question No. in the **Answer Table for Objective Questions**, provided on Page No. **7**. Do not write anything else on this page.
4. Each objective question has **4 choices** for its answer: (A), (B), (C) and (D). Only **ONE** of them is the correct answer. There will be **negative marking** for wrong answers to objective questions. The following marking scheme for objective questions shall be used:
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2011 MA

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

REGISTRATION NUMBER						

Name:

Test Centre:

Do not write your Registration Number or Name anywhere else in this question-cum-answer booklet.

I have read all the instructions and shall abide by them.

……………………………………………...
Signature of the Candidate

I have verified the information filled by the Candidate above.

……………………………………………...
Signature of the Invigilator

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Special Instructions/ Useful Data

$\mathbb{R}$: The set of all real numbers

$\mathbb{N}$: The set of all natural numbers, that is, the set of all positive integers 1, 2, …. .

$\mathbb{Z}$: The set of all integers

A

IMPORTANT NOTE FOR CANDIDATES

- **Questions 1-15 (objective questions) carry *six* marks each and questions 16-29 (subjective questions) carry *fifteen* marks each.**
- **Write the answers to the objective questions in the *Answer Table for Objective Questions* provided on page 7 only.**

Q.1 Let $a_n = \sum_{k=1}^{n} \frac{n}{n^2+k}$, for $n \in \mathbb{N}$. Then the sequence $\{a_n\}$ is

(A) Convergent
(B) Bounded but not convergent
(C) Diverges to ∞
(D) Neither bounded nor diverges to ∞

Q.2 The number of real roots of the equation $x^3 + x - 1 = 0$ is

(A) 0
(B) 1
(C) 2
(D) 3

Q.3 The value of $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{\sqrt{n^2+kn}}$ is

(A) $2(\sqrt{2}-1)$
(B) $2\sqrt{2}-1$
(C) $2-\sqrt{2}$
(D) $\frac{1}{2}(\sqrt{2}-1)$

Q.4 Let V be the region bounded by the planes $x=0$, $x=2$, $y=0$, $z=0$ and $y+z=1$. Then the value of the integral $\iiint_V y \, dx\, dy\, dz$ is

(A) $\frac{1}{2}$
(B) $\frac{4}{3}$
(C) 1
(D) $\frac{1}{3}$

Q.5 The solution $y(x)$ of the differential equation $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0$ satisfying the conditions $y(0)=4$, $\frac{dy}{dx}(0) = 8$ is

(A) $4e^{2x}$
(B) $(16x+4)e^{-2x}$
(C) $4e^{-2x} + 16x$
(D) $4e^{-2x} + 16xe^{2x}$

A

Q.6 If y^a is an integrating factor of the differential equation $2xy\,dx - (3x^2 - y^2)\,dy = 0$, then the value of a is

(A) -4 (B) 4 (C) -1 (D) 1

Q.7 Let $\vec{F} = ay\,\hat{i} + z\,\hat{j} + x\,\hat{k}$ and C be the positively oriented closed curve given by $x^2 + y^2 = 1,\ z = 0$. If $\oint_C \vec{F} \cdot d\vec{r} = \pi$, then the value of a is

(A) -1 (B) 0 (C) $\frac{1}{2}$ (D) 1

Q.8 Consider the vector field $\vec{F} = (ax + y + a)\,\hat{i} + \hat{j} - (x + y)\,\hat{k}$, where a is a constant. If $\vec{F} \cdot \text{curl}\,\vec{F} = 0$, then the value of a is

(A) -1 (B) 0 (C) 1 (D) $\frac{3}{2}$

Q.9 Let G denote the group of all 2×2 invertible matrices with entries from $\mathbb{R}$. Let

$$H_1 = \{A \in G : \det(A) = 1\} \text{ and } H_2 = \{A \in G : A \text{ is upper triangular}\}.$$

Consider the following statements:

P : H_1 is a normal subgroup of G

Q : H_2 is a normal subgroup of G.

Then

(A) Both P and Q are true (B) P is true and Q is false

(C) P is false and Q is true (D) Both P and Q are false

Q.10 For $n \in \mathbb{N}$, let $n\mathbb{Z} = \{nk : k \in \mathbb{Z}\}$. Then the number of units of $\mathbb{Z}/11\mathbb{Z}$ and $\mathbb{Z}/12\mathbb{Z}$, respectively, are

(A) 11, 12 (B) 10, 11 (C) 10, 4 (D) 10, 8

A

Q.11 Let A be a 3×3 matrix with trace $(A) = 3$ and $\det(A) = 2$. If 1 is an eigenvalue of A, then the eigenvalues of the matrix $A^2 - 2I$ are

(A) $1,\ 2(i-1),\ -2(i+1)$
(B) $-1,\ 2(i-1),\ 2(i+1)$
(C) $1,\ 2(i+1),\ -2(i+1)$
(D) $-1,\ 2(i-1),\ -2(i+1)$

Q.12 Let $T:\mathbb{R}^n \to \mathbb{R}^n$ be a linear transformation, where $n \geq 2$. For $k \leq n$, let

$$E = \{v_1, v_2, \ldots, v_k\} \subseteq \mathbb{R}^n \text{ and } F = \{Tv_1, Tv_2, \ldots, Tv_k\}.$$

Then

(A) If E is linearly independent, then F is linearly independent
(B) If F is linearly independent, then E is linearly independent
(C) If E is linearly independent, then F is linearly dependent
(D) If F is linearly independent, then E is linearly dependent

Q.13 For $n \neq m$, let $T_1:\mathbb{R}^n \to \mathbb{R}^m$ and $T_2:\mathbb{R}^m \to \mathbb{R}^n$ be linear transformations such that T_1T_2 is bijective. Then

(A) rank $(T_1) = n$ and rank $(T_2) = m$
(B) rank $(T_1) = m$ and rank $(T_2) = n$
(C) rank $(T_1) = n$ and rank $(T_2) = n$
(D) rank $(T_1) = m$ and rank $(T_2) = m$

Q.14 The set of all x at which the power series $\sum_{n=1}^{\infty} \frac{n}{(2n+1)^2} (x-2)^{3n}$ converges is

(A) $[-1, 1)$ (B) $[-1, 1]$ (C) $[1, 3)$ (D) $[1, 3]$

Q.15 Consider the following subsets of $\mathbb{R}$:

$$E = \left\{\frac{n}{n+1} : n \in \mathbb{N}\right\}, \qquad F = \left\{\frac{1}{1-x} : 0 \leq x < 1\right\}.$$

Then

(A) Both E and F are closed
(B) E is closed and F is NOT closed
(C) E is NOT closed and F is closed
(D) Neither E nor F is closed

A

Answer Table for Objective Questions

Write the Code of your chosen answer only in the 'Answer' column against each Question No. Do not write anything else on this page.

Question Number	Answer	Do not write in this column
01		
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FOR EVALUATION ONLY

Number of Correct Answers		Marks	(+)
Number of Incorrect Answers		Marks	(–)
Total Marks in Questions 1-15			( )

A

Q.16 (a) Let $\{a_n\}$ be a sequence of non-negative real numbers such that $\sum_{n=1}^{\infty} a_n$ converges, and let $\{k_n\}$ be a strictly increasing sequence of positive integers. Show that $\sum_{n=1}^{\infty} a_{k_n}$ also converges. (9)

(b) Suppose $f : [0,1] \to \mathbb{R}$ is differentiable and $f'(x) \leq 1$ at every $x \in (0,1)$. If $f(0) = 0$ and $f(1) = 1$, show that $f(x) = x$ for all $x \in [0,1]$. (6)

A

Q.17 (a) Suppose f is a real valued function defined on an open interval I and differentiable at every $x \in I$. If $[a, b] \subset I$ and $f'(a) < 0 < f'(b)$, then show that there exists $c \in (a,b)$ such that $f(c) = \min_{a \leq x \leq b} f(x)$. (9)

(b) Let $f:(a, b) \to \mathbb{R}$ be a twice differentiable function such that f'' is continuous at every point in (a, b). Prove that

$$\lim_{h \to 0} \frac{f(x+h) - 2f(x) + f(x-h)}{h^2} = f''(x)$$

for every $x \in (a, b)$. (6)

A

Q.18 Find all critical points of the following function and check whether the function attains maximum or minimum at each of these points:

$$u(x,y) = x^4 + y^4 - 2x^2 - 2y^2 + 4xy, \qquad (x,y) \in \mathbb{R}^2.$$

. (15)

A

Q.19 (a) Let $\varphi:[a,b]\to\mathbb{R}$ be differentiable and $[c,d] = \{\varphi(x):\ a\le x\le b\}$, and let $f:[c,d]\to\mathbb{R}$ be continuous. Let $g:[a,b]\to\mathbb{R}$ be defined by $g(x) = \int\limits_{c}^{\varphi(x)} f(t)\,dt$ for $x\in[a,b]$. Then show that g is differentiable and $g'(x) = f(\varphi(x))\,\varphi'(x)$ for all $x\in[a,b]$. (9)

(b) If $f:[0,1]\to\mathbb{R}$ is such that $\int\limits_{0}^{\sin x} f(t)dt = \frac{\sqrt{3}}{2}\,x$ for all $x\in\mathbb{R}$, then find $f\left(\frac{1}{2}\right)$. (6)

A

Q.20 Find the area of the surface of the solid bounded by the cone $z = 3 - \sqrt{x^2 + y^2}$ and the paraboloid $z = 1 + x^2 + y^2$. (15)

A

Q.21 Obtain the general solution of each of the following differential equations:

(a) $y - x\dfrac{dy}{dx} = \dfrac{dy}{dx}\, y^2 e^y$. (6)

(b) $\dfrac{dy}{dx} = \dfrac{x+2y+8}{2x+y+7}$. (9)

A

Q.22 (a) Determine the values of $b > 1$ such that the differential equation

$$x^2 \frac{d^2 y}{dx^2} + x\frac{dy}{dx} + y = 0, \quad 1 < x < b$$

satisfying the conditions $y(1) = 0 = y(b)$ has a nontrivial solution. (9)

(b) Find $v(x)$ such that $y(x) = e^{4x} v(x)$ is a particular solution of the differential equation

$$\frac{d^2 y}{dx^2} - 8\frac{dy}{dx} + 16y = \left(2x + 11x^{10} + 21x^{20}\right)e^{4x}.$$

(6)

A

Q.23 (a) Change the order of integration in the double integral

$$\int_{-1}^{2}\left(\int_{-x}^{2-x^2} f(x,y)\,dy\right)dx.$$

(6)

(b) Let $\vec{F}=(x^2-xy^2)\hat{i}+y^2\,\hat{j}$. Using Green's theorem, evaluate the line integral $\int_C \vec{F}\cdot d\vec{r}$, where C is the positively oriented closed curve which is the boundary of the region enclosed by the $x-$axis and the semi-circle $y=\sqrt{1-x^2}$ in the upper half plane.

(9)

A

Q.24 (a) If $\vec{F} = (x^2 + y - 4)\hat{i} + 3xy\,\hat{j} + (2xz + z^2)\hat{k}$, then evaluate the surface integral $\iint_S (\nabla \times \vec{F}) \cdot \hat{n}\, dS$, where S is the surface of the cone $z = 1 - \sqrt{x^2 + y^2}$ lying above the $xy-$plane and $\hat{n}$ is the unit normal to S making an acute angle with $\hat{k}$. (9)

(b) Show that the series $\sum_{n=1}^{\infty} \frac{x}{\sqrt{n}(1 + n^p x^2)}$ converges uniformly on $\mathbb{R}$ for $p > 1$. (6)

A

Q.25 (a) Find a value of c such that the following system of linear equations has no solution:

$$x+2y+3z=1,$$
$$3x+7y+cz=2,$$
$$2x+cy+12z=3.$$

(6)

(b) Let V be the vector space of all polynomials with real coefficients of degree at most n, where $n \geq 2$. Considering elements of V as functions from $\mathbb{R}$ to $\mathbb{R}$, define

$$W=\left\{ p \in V: \int_0^1 p(x)dx = 0 \right\}.$$

Show that W is a subspace of V and $\dim(W)=n$. (9)

A

Q.26 (a) Let A be a 3×3 real matrix with $\det(A)=6$. Then find $\det(\text{adj}\,A)$. (6)

(b) Let v_1 and v_2 be non-zero vectors in $\mathbb{R}^n$, $n\geq3$, such that v_2 is not a scalar multiple of v_1. Prove that there exists a linear transformation $T:\mathbb{R}^n\rightarrow\mathbb{R}^n$ such that $T^3=T$, $Tv_1=v_2$, and T has at least three distinct eigenvalues. (9)

A

Q.27 (a) If E is a subset of $\mathbb{R}$ that does not contain any of its limit points, then prove that E is a countable set. (9)

(b) Let $f:(a,b)\to\mathbb{R}$ be a continuous function. If f is uniformly continuous, then prove that there exists a continuous function $g:[a,b]\to\mathbb{R}$ such that $g(x)=f(x)$ for all $x\in(a,b)$. (6)

A

Q.28 (a) On $\mathbb{R}^3$, define a binary operation $*$ as follows: For $(x,y,t),\ (x',y',t')$ in $\mathbb{R}^3$,

$$(x,y,t)*(x',y',t')=\left(x+x',\ \ y+y',\ \ t+t'+\frac{1}{2}(x'y-xy')\right).$$

Then show that $(\mathbb{R}^3,\ *)$ is a group, and find its center. (9)

(b) For $k\in\mathbb{N}$, let $k\mathbb{Z}=\{kn:\ n\in\mathbb{Z}\}$. For any $m,n\in\mathbb{N}$, show that $I=m\mathbb{Z}\cap n\mathbb{Z}$ is an ideal of $\mathbb{Z}$. Further, find the generators of I. (6)

A

Q.29 Let G be a group of order p^2, where p is a prime number. Let $x \in G$. Prove that $\{y \in G : xy = yx\} = G$. (15)

A

2011 MA Objective Part (Question Number 1 – 15)	
Total Marks	**Signature**

Subjective Part					
Question Number	**Marks**		**Question Number**	**Marks**	
16			**23**		
17			**24**		
18			**25**		
19			**26**		
20			**27**		
21			**28**		
22			**29**		
Total Marks in Subjective Part					

Total (Objective Part) :	
Total (Subjective Part) :	
Grand Total :	
Total Marks (in words) :	
Signature of Examiner(s) :	
Signature of Head Examiner(s) :	
Signature of Scrutinizer :	
Signature of Chief Scrutinizer :	
Signature of Coordinating Head Examiner :	

A

2010 – MA

Test Paper Code : MA

Time : 3 Hours **Max. Marks : 300**

INSTRUCTIONS

1. The question-cum-answer booklet has **40** pages and has **29** questions. Please ensure that the copy of the question-cum-answer booklet you have received contains all the questions.
2. Write your **Registration Number, Name and the Name of the Test Centre** in the appropriate space provided on the right side.
3. Write the answers to the objective questions against each Question No. in the **Answer Table for Objective Questions**, provided on Page No. **7**. Do not write anything else on this page.
4. Each objective question has **4 choices** for its answer : (A), (B), (C) and (D). Only **ONE** of them is the correct answer. There will be **negative marking** for wrong answers to objective questions. The following marking scheme for objective questions shall be used :
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   (b) For each wrong answer, you will be awarded **–2 (Negative two)** marks.
   (c) Multiple answers to a question will be treated as a wrong answer.
   (d) For each un-attempted question, you will be awarded **0 (Zero)** mark.
   (e) Negative marks for objective part will be carried over to total marks.
5. Answer the subjective question only in the space provided after each question.
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7. All answers must be written in blue/black/blue-black ink only. Sketch pen, pencil or ink of any other colour should not be used.
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11. The question-cum-answer booklet must be returned in its entirety to the Invigilator before leaving the examination hall. Do not remove any page from this booklet.
12. Refer to special instructions/useful data on the reverse.

A

2010 – MA

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

REGISTRATION NUMBER						

Name :

Test Centre :

Do not write your Registration Number or Name anywhere else in this question-cum-answer booklet.

I have read all the instructions and shall abide by them.

....

Signature of the Candidate

I have verified the information filled by the Candidate above.

....

Signature of the Invigilator

A

Special Instructions / Useful Data

R	:	The set of all real numbers
Q	:	The set of all rational numbers
N	:	The set $\{1,2,3,...\}$ of all natural numbers
ϕ	:	The empty set
$E \setminus F$	:	The set $\{x \in E : x \notin F\}$, where E and F are sets
$\ln(x)$	:	The logarithm of x with respect to the base e

DO NOT WRITE ON THIS PAGE

A

IMPORTANT NOTE FOR CANDIDATES

- **Questions 1-15 (objective questions) carry *six* marks each and questions 16-29 (subjective questions) carry *fifteen* marks each.**
- **Write the answers to the objective questions in the *Answer Table for Objective Questions* provided on page 7 only.**

Q.1 Which of the following conditions does NOT ensure the convergence of a real sequence $\{a_n\}$?

(A) $|a_n - a_{n+1}| \to 0$ as $n \to \infty$

(B) $\sum_{n=1}^{\infty} |a_n - a_{n+1}|$ is convergent

(C) $\sum_{n=1}^{\infty} n\, a_n$ is convergent

(D) The sequences $\{a_{2n}\}$, $\{a_{2n+1}\}$ and $\{a_{3n}\}$ are convergent

Q.2 The value of $\iint_G \frac{\ln(x^2+y^2)}{x^2+y^2}\, dx\, dy$, where $G = \{(x,y) \in \mathbf{R}^2 : 1 \le x^2+y^2 \le e^2\}$, is

(A) π (B) 2π (C) 3π (D) 4π

Q.3 The number of elements of S_5 (the symmetric group on 5 letters) which are their own inverses equals

(A) 10 (B) 11 (C) 25 (D) 26

Q.4 Let S be an infinite subset of $\mathbf{R}$ such that $S \cap \mathbf{Q} = \phi$. Which of the following statements is true?

(A) S must have a limit point which belongs to $\mathbf{Q}$

(B) S must have a limit point which belongs to $\mathbf{R} \setminus \mathbf{Q}$

(C) S cannot be a closed set in $\mathbf{R}$

(D) $\mathbf{R} \setminus S$ must have a limit point which belongs to S

Space for rough work

A

Q.5 Let $f:(1,4)\rightarrow \mathbf{R}$ be a uniformly continuous function and let $\{a_n\}$ be a Cauchy sequence in $(1,2)$. Let $x_n = a_n^2 f(a_n^2)$ and $y_n = \frac{1}{1+a_n^2} f(a_n^2)$, for all $n \in \mathbf{N}$. Which of the following statements is true?

(A) Both $\{x_n\}$ and $\{y_n\}$ must be Cauchy sequences in $\mathbf{R}$

(B) $\{x_n\}$ must be a Cauchy sequence in $\mathbf{R}$ but $\{y_n\}$ need not be a Cauchy sequence in $\mathbf{R}$

(C) $\{y_n\}$ must be a Cauchy sequence in $\mathbf{R}$ but $\{x_n\}$ need not be a Cauchy sequence in $\mathbf{R}$

(D) Neither $\{x_n\}$ nor $\{y_n\}$ needs to be a Cauchy sequence in $\mathbf{R}$

Q.6 Let $\vec{F} = 2xyz\,e^{x^2}\hat{i} + z\,e^{x^2}\hat{j} + y\,e^{x^2}\hat{k}$ be the gradient of a scalar function. The value of $\int_L \vec{F}\cdot d\vec{r}$ along the oriented path L from $(0,0,0)$ to $(1,0,2)$ and then to $(1,1,2)$ is

(A) 0 (B) $2e$ (C) e (D) e^2

Q.7 Let $\vec{F} = xy\,\hat{i} + y\,\hat{j} - yz\,\hat{k}$ denote the force field on a particle traversing the path L from $(0,0,0)$ to $(1,1,1)$ along the curve of intersection of the cylinder $y = x^2$ and the plane $z = x$. The work done by $\vec{F}$ is

(A) 0 (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) 1

Q.8 Let $\mathbf{R}[X]$ be the ring of real polynomials in the variable X. The number of ideals in the quotient ring $\mathbf{R}[X]/(X^2 - 3X + 2)$ is

(A) 2 (B) 3 (C) 4 (D) 6

Space for rough work

A

Q.9 Consider the differential equation $\frac{dy}{dx} = ay - by^2$, where $a, b > 0$ and $y(0) = y_0$. As $x \to +\infty$, the solution $y(x)$ tends to

(A) 0 (B) $\frac{a}{b}$ (C) $\frac{b}{a}$ (D) y_0

Q.10 Consider the differential equation $(x+y+1)\,dx + (2x+2y+1)\,dy = 0$. Which of the following statements is true?

(A) The differential equation is linear
(B) The differential equation is exact
(C) e^{x+y} is an integrating factor of the differential equation
(D) A suitable substitution transforms the differentiable equation to the variables separable form

Q.11 Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be a linear transformation such that $T((1,2)) = (2,3)$ and $T((0,1)) = (1,4)$. Then $T((5,6))$ is

(A) $(6,-1)$ (B) $(-6,1)$ (C) $(-1,6)$ (D) $(1,-6)$

Q.12 The number of 2×2 matrices over $\mathbf{Z}_3$ (the field with three elements) with determinant 1 is

(A) 24 (B) 60 (C) 20 (D) 30

Space for rough work

A

Q.13 The radius of convergence of the power series $\sum_{n=0}^{\infty} a_n z^{n^2}$, where $a_0 = 1$, $a_n = 3^{-n} a_{n-1}$ for $n \in \mathbf{N}$, is

(A) 0 (B) $\sqrt{3}$ (C) 3 (D) ∞

Q.14 Let $T : \mathbf{R}^3 \to \mathbf{R}^3$ be the linear transformation whose matrix with respect to the standard basis $\{e_1, e_2, e_3\}$ of $\mathbf{R}^3$ is $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$. Then T

(A) maps the subspace spanned by e_1 and e_2 into itself
(B) has distinct eigenvalues
(C) has eigenvectors that span $\mathbf{R}^3$
(D) has a non-zero null space

Q.15 Let $T : \mathbf{R}^3 \to \mathbf{R}^3$ be the linear transformation whose matrix with respect to the standard basis of $\mathbf{R}^3$ is $\begin{pmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{pmatrix}$, where a, b, c are real numbers not all zero. Then T

(A) is one-to-one
(B) is onto
(C) does not map any line through the origin onto itself
(D) has rank 1

Space for rough work

A

Answer Table for Objective Questions

Write the Code of your chosen answer only in the 'Answer' column against each Question No. Do not write anything else on this page.

Question No.	Answer	Do not write in this column
01		
02		
03		
04		
05		
06		
07		
08		
09		
10		
11		
12		
13		
14		
15		

FOR EVALUATION ONLY

No. of correct answers		Marks	(+)
No. of incorrect answers		Marks	(−)
Total marks in question nos. 1-15			( )

A

Q.16 (a) Obtain the general solution of the following system of differential equations:

$$\frac{dx}{dt} = x + 2y$$

$$\frac{dy}{dt} = 4x - y + e^{3t}$$

(9)

(b) Find the curve passing through $\left(\frac{1}{2}, 0\right)$ and having slope at (x, y) given by the differential equation $2(1+y^2)\,dx + (2x - \tan^{-1} y)\,dy = 0$. (6)

A

Q.17 (a) Find the volume of the region in the first octant bounded by the surfaces $x = 0, y = x, y = 2 - x^2, z = 0$ and $z = x^2$. (6)

(b) Suppose $f : \mathbf{R} \to \mathbf{R}$ is a non-constant continuous function satisfying $f(x + y) = f(x)f(y)$ for all $x, y \in \mathbf{R}$.

(i) Show that $f(x) \neq 0$ for all $x \in \mathbf{R}$.

(ii) Show that $f(x) > 0$ for all $x \in \mathbf{R}$.

(iii) Show that there exists $\beta \in \mathbf{R}$ such that $f(x) = \beta^x$ for all $x \in \mathbf{R}$. (9)

A

Q.18 (a) Let $f(x)$ and $g(x)$ be real valued functions continuous in $[a, b]$, differentiable in (a, b) and let $g'(x) \neq 0$ for all $x \in (a, b)$. Show that there exists $c \in (a, b)$ such that $\frac{f(c)-f(a)}{g(b)-g(c)} = \frac{f'(c)}{g'(c)}$. (9)

(b) Let $0 < \lambda < 4$ and let $\{a_n\}$ be a sequence of positive real numbers satisfying $a_{n+1} = \lambda a_n^2(1-a_n)$ for $n \in \mathbf{N}$. Prove that $\lim\limits_{n \to \infty} a_n$ exists and determine this limit. (6)

A

Q.19 Let G be an open subset of $\mathbf{R}$.

(a) If $0 \notin G$, then show that $H = \{xy : x, y \in G\}$ is an open subset of $\mathbf{R}$. (9)

(b) If $0 \in G$ and if $x + y \in G$ for all $x, y \in G$, then show that $G = \mathbf{R}$. (6)

A

Q.20 Let $p(x)$ be a non-constant polynomial with real coefficients such that $p(x) \neq 0$ for all $x \in \mathbf{R}$. Define $f(x) = \dfrac{1}{p(x)}$ for all $x \in \mathbf{R}$. Prove that

(i) for each $\varepsilon > 0$, there exists $\alpha > 0$ such that $|f(x)| < \varepsilon$ for all $x \in \mathbf{R}$ satisfying $|x| > \alpha$, and

(ii) $f : \mathbf{R} \to \mathbf{R}$ is a uniformly continuous function. (15)

A

Q.21 (a) Let $M(k)$ and $m(k)$ denote respectively the absolute maximum and the absolute minimum values of $x^3+9x^2-21x+k$ in the closed interval $[-10,2]$. Find all the real values of k for which $|M(k)|=|m(k)|$. (6)

(b) Let $\alpha_1=0, \beta_1=1$; $\alpha_2=1, \beta_2=1$, and for $n\geq 3$,

$\alpha_n=\alpha_{n-1}+2\alpha_{n-2}$,

$\beta_n=\beta_{n-1}+2\beta_{n-2}$.

Prove that, for $n\in\mathbf{N}$

(i) $\beta_n=2\alpha_n+(-1)^{n-1}$

(ii) $\alpha_n+\beta_n=2^{n-1}$

Deduce that $\lim_{n\to\infty}\frac{\alpha_n a+\beta_n b}{2^{n-1}}=\frac{a+2b}{3}$ for any $a,b\in\mathbf{R}$. (9)

A

Q.22 (a) Let $f(x,y) = \alpha x^2 + xy + \beta y^2$, $\alpha \neq 0, \beta \neq 0, 4\alpha\beta \neq 1$. Find sufficient conditions on α, β such that $(0,0)$ is

(i) a point of local maxima of $f(x, y)$

(ii) a point of local minima of $f(x, y)$

(iii) a saddle point of $f(x, y)$. (9)

(b) Find the derivative of $f(x, y, z) = 7x^3 - x^2 z - z^2 + 28y$ at the point $A = (1, -1, 0)$ along the unit vector $\frac{1}{7}(6\hat{i} - 2\hat{j} + 3\hat{k})$. What is the unit vector along which f decreases most rapidly at A? Also, find the rate of this decrease. (6)

A

Q.23 Using $x = e^z$, transform the differential equation $x^2 \frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = \cos x$ to a second order differential equation with constant coefficients. Obtain the general solution of the transformed differential equation. **(15)**

A

Q.24 Let G be a group and let $A(G)$ denote the set of all automorphisms of G, i.e., all one-to-one, onto, group homomorphisms from G to G. An automorphism $f: G \to G$ of the form $f(x) = axa^{-1}$, $x \in G$ (for some $a \in G$) is called an inner automorphism. Let $I(G)$ denote the set of all inner automorphisms of G.

(a) Show that $A(G)$ is a group under composition of functions and that $I(G)$ is a normal subgroup of $A(G)$. (9)

(b) Show that $I(G)$ is isomorphic to $G/Z(G)$, where $Z(G) = \{g \in G : xg = gx \text{ for all } x \in G\}$ is the center of G. (6)

A

Q.25 (a) Give an example of a linear transformation $T:\mathbf{R}^2 \to \mathbf{R}^2$ such that $T^2(v) = -v$ for all $v \in \mathbf{R}^2$. (6)

(b) Let V be a real n-dimensional vector space and let $T:V \to V$ be a linear transformation satisfying $T^2(v) = -v$ for all $v \in V$.

(i) Show that n is even.

(ii) Use T to make V into a complex vector space such that the multiplication by complex numbers extends the multiplication by real numbers.

(iii) Show that, with respect to the complex vector space structure on V obtained in (ii), $T:V \to V$ is a complex linear transformation. (9)

A

Q.26 Let W be the region bounded by the planes $x = 0, y = 0, y = 3, z = 0$ and $x + 2z = 6$. Let S be the boundary of this region. Using Gauss' divergence theorem, evaluate $\iint_S \vec{F} \cdot \hat{n}\, dS$, where $\vec{F} = 2xy\hat{i} + yz^2\hat{j} + xz\hat{k}$ and $\hat{n}$ is the outward unit normal vector to S.

(15)

A

Q.27 (a) Using Stokes' theorem evaluate the line integral $\int_L (y\hat{i} + z\hat{j} + x\hat{k}) \cdot \vec{dr}$, where L is the intersection of $x^2 + y^2 + z^2 = 1$ and $x + y = 0$ traversed in the clockwise direction when viewed from the point $(1, 1, 0)$. (9)

(b) Change the order of integration in the integral $\int_0^1 \int_{x-1}^{\sqrt{1-x^2}} f(x, y)\, dy\, dx$. (6)

A

Q.28 In a group G, $x \in G$ is said to be conjugate to $y \in G$, written $x \sim y$, if there exists $z \in G$ such that $x = zyz^{-1}$.

(a) Show that $\sim$ is an equivalence relation on G. Show that a subgroup N of G is a normal subgroup of G if and only if N is a union of equivalence classes of $\sim$. (6)

(b) Consider the group of all non-singular 3×3 real matrices under matrix multiplication. Show that $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 3 & 0 \\ 1 & 2 & 1 \end{bmatrix} \sim \begin{bmatrix} 3 & 0 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (i.e., the two matrices are conjugate). (9)

A

Q.29 Let S denote the commutative ring of all continuous real valued functions on $[0,1]$, under pointwise addition and multiplication. For $a \in [0,1]$, let $M_a = \{f \in S \mid f(a) = 0\}$.

(a) Show that M_a is an ideal in S. (6)

(b) Show that M_a is a maximal ideal in S. (9)

C

2009 – MA

Test Paper Code : MA

Time : 3 Hours Maximum Marks : 300

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Somesh

Somesh

C

2009 – MA

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

ROLL NUMBER							

Name :

Test Centre :

Do not write your Roll Number or Name anywhere else in this question-cum-answer booklet.

I have read all the instructions and shall abide by them.

....

Signature of the Candidate

I have verified the information filled by the Candidate above.

....

Signature of the Invigilator

C

Special Instructions/Useful Data

A^T	-	Transpose of the matrix A
$\mathbb{C}$	-	Set of complex numbers
$\mathbb{N}$	-	Set of natural numbers
$\mathbb{Q}$	-	Set of rational numbers
$\mathbb{R}$	-	Set of real numbers
$\mathbb{Z}$	-	Set of integers

DO NOT WRITE ON THIS PAGE

C

IMPORTANT NOTE FOR CANDIDATES

- **Questions 1-15 (objective questions) carry *six* marks each and questions 16-29 (subjective questions) carry *fifteen* marks each.**
- **Write the answers to the objective questions in the *Answer Table for Objective Questions* provided on page 7 only.**

Q.1 Let V be the vector space of all 6×6 real matrices over the field $\mathbb{R}$. Then the dimension of the subspace of V consisting of all symmetric matrices is

(A) 15 (B) 18 (C) 21 (D) 35

Q.2 Let R be the ring of all functions from $\mathbb{R}$ to $\mathbb{R}$ under point-wise addition and multiplication. Let $I = \{f : \mathbb{R} \to \mathbb{R} \mid f \text{ is a bounded function}\}$, $J = \{f : \mathbb{R} \to \mathbb{R} \mid f(3) = 0\}$. Then

(A) J is an ideal of R but I is not an ideal of R
(B) I is an ideal of R but J is not an ideal of R
(C) both I and J are ideals of R
(D) neither I nor J is an ideal of R

Q.3 Which of the following sequences of functions is uniformly convergent on $(0, 1)$?

(A) x^n (B) $\frac{n}{nx+1}$ (C) $\frac{x}{nx+1}$ (D) $\frac{1}{nx+1}$

Q.4 Let $T : \mathbb{R}^4 \to \mathbb{R}^4$ be a linear transformation satisfying $T^3 + 3T^2 = 4I$, where I is the identity transformation. Then the linear transformation $S = T^4 + 3T^3 - 4I$ is

(A) one-one but not onto (B) onto but not one-one
(C) invertible (D) non-invertible

Q.5 The number of all subgroups of the group $(Z_{60}, +)$ of integers modulo 60 is

(A) 2 (B) 10 (C) 12 (D) 60

C

Q.6 Let $a_n = \begin{cases} \frac{1}{3^n} & \text{if } n \text{ is a prime,} \\ \frac{1}{4^n} & \text{if } n \text{ is not a prime.} \end{cases}$

Then the radius of convergence of the power series $\sum_{n=1}^{\infty} a_n x^n$ is

(A) 4 (B) 3 (C) $\frac{1}{3}$ (D) $\frac{1}{4}$

Q.7 The set of all limit points of the sequence $1, \frac{1}{2}, \frac{1}{4}, \frac{3}{4}, \frac{1}{8}, \frac{3}{8}, \frac{5}{8}, \frac{7}{8}, \frac{1}{16}, \frac{3}{16}, \frac{5}{16}, \frac{7}{16}, \frac{9}{16}, \ldots$ is

(A) $[0, 1]$
(B) $(0, 1]$
(C) the set of all rational numbers in $[0, 1]$
(D) the set of all rational numbers in $[0, 1]$ of the form $\frac{m}{2^n}$ where m and n are integers

Q.8 Let $F : \mathbb{R} \to \mathbb{R}$ be a continuous function and $a > 0$. Then the integral $\int_0^a \left[\int_0^x F(y)\, dy \right] dx$ equals

(A) $\int_0^a y F(y)\, dy$ (B) $\int_0^a (a - y) F(y)\, dy$

(C) $\int_0^a (y - a) F(y)\, dy$ (D) $\int_a^0 y F(y)\, dy$

Q.9 The set of all positive values of a for which the series $\sum_{n=1}^{\infty} \left(\frac{1}{n} - \tan^{-1}\left(\frac{1}{n}\right) \right)^a$ converges, is

(A) $\left(0, \frac{1}{3}\right]$ (B) $\left(0, \frac{1}{3}\right)$ (C) $\left[\frac{1}{3}, \infty\right)$ (D) $\left(\frac{1}{3}, \infty\right)$

Q.10 Let a be a non-zero real number. Then $\lim_{x \to a} \frac{1}{x^2 - a^2} \int_a^x \sin(t^2)\, dt$ equals

(A) $\frac{1}{2a} \sin(a^2)$ (B) $\frac{1}{2a} \cos(a^2)$ (C) $-\frac{1}{2a} \sin(a^2)$ (D) $-\frac{1}{2a} \cos(a^2)$

C

Q.11 Let $T(x, y, z) = xy^2 + 2z - x^2z^2$ be the temperature at the point (x, y, z). The unit vector in the direction in which the temperature decreases most rapidly at $(1, 0, -1)$ is

(A) $-\frac{1}{\sqrt{5}}\hat{i} + \frac{2}{\sqrt{5}}\hat{k}$ (B) $\frac{1}{\sqrt{5}}\hat{i} - \frac{2}{\sqrt{5}}\hat{k}$

(C) $\frac{2}{\sqrt{14}}\hat{i} + \frac{3}{\sqrt{14}}\hat{j} + \frac{1}{\sqrt{14}}\hat{k}$ (D) $-\left(\frac{2}{\sqrt{14}}\hat{i} + \frac{3}{\sqrt{14}}\hat{j} + \frac{1}{\sqrt{14}}\hat{k}\right)$

Q.12 Consider the differential equation $2\cos(y^2)\,dx - xy\sin(y^2)\,dy = 0$. Then

(A) e^x is an integrating factor (B) e^{-x} is an integrating factor

(C) $3x$ is an integrating factor (D) x^3 is an integrating factor

Q.13 Suppose $\vec{V} = p(x, y)\hat{i} + q(x, y)\hat{j}$ is a continuously differentiable vector field defined in a domain D in $\mathbb{R}^2$. Which one of the following statements is NOT equivalent to the remaining ones?

(A) There exists a function $\phi(x, y)$ such that $\frac{\partial \phi}{\partial x} = p(x, y)$ and $\frac{\partial \phi}{\partial y} = q(x, y)$ for all $(x, y) \in D$

(B) $\frac{\partial q}{\partial x} = \frac{\partial p}{\partial y}$ holds at all points of D

(C) $\oint_C \vec{V} \cdot d\vec{r} = 0$ for every piecewise smooth closed curve C in D

(D) The differential $p\,dx + q\,dy$ is exact in D

Q.14 Let $f, g : [-1, 1] \to \mathbb{R}$, $f(x) = x^3$, $g(x) = x^2|x|$. Then

(A) f and g are linearly independent on $[-1, 1]$

(B) f and g are linearly dependent on $[-1, 1]$

(C) $f(x)g'(x) - f'(x)g(x)$ is NOT identically zero on $[-1, 1]$

(D) there exist continuous functions $p(x)$ and $q(x)$ such that f and g satisfy $y'' + py' + qy = 0$ on $[-1, 1]$

Q.15 The value of c for which there exists a twice differentiable vector field $\vec{F}$ with $\text{curl}\,\vec{F} = 2x\hat{i} - 7y\hat{j} + cz\hat{k}$ is

(A) 0 (B) 2 (C) 5 (D) 7

C

Q.16 Container A contains 100 cc of milk and container B contains 100 cc of water. 5 cc of the liquid in A is transferred to B, the mixture is thoroughly stirred and 5 cc of the mixture in B is transferred back into A. Each such two-way transfer is called a dilution. Let a_n be the percentage of water in container A after n such dilutions, with the understanding that $a_0 = 0$.

(a) Prove that $a_1 = \dfrac{100}{21}$ and that, in general, $a_n = \dfrac{100}{21} + \dfrac{19}{21} a_{n-1}$ for $n = 1, 2, 3, \ldots$. (6)

(b) Using (a) prove that $a_n = 50\left[1 - \left(\dfrac{19}{21}\right)^n\right]$ for $n = 1, 2, 3, \ldots$.

Find $\lim_{n\to\infty} a_n$ and explain why the answer is intuitively obvious. (9)

C

Q.17 (a) Let $f: \mathrm{N} \times \mathrm{N} \to \mathrm{R}$ be a non-negative function. Assume that for every $m \in \mathrm{N}$, the series $\sum_{n=1}^{\infty} f(m, n)$ is convergent and has sum a_m and further that the series $\sum_{m=1}^{\infty} a_m$ is also convergent and has sum L. Prove that for every n, the series $\sum_{m=1}^{\infty} f(m, n)$ is convergent and if we denote its sum by b_n then the series $\sum_{n=1}^{\infty} b_n$ is also convergent and has sum L. (9)

(b) Define $f: \mathrm{N} \times \mathrm{N} \to \mathrm{R}$ by

$$f(m, n) = \begin{cases} 0 & \text{if } n > m, \\ \dfrac{-1}{2^{m-n}} & \text{if } n < m, \\ 1 & \text{if } n = m. \end{cases}$$

Show that $\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} f(m, n) = 2$ and $\sum_{n=1}^{\infty} \sum_{m=1}^{\infty} f(m, n) = 0$. (6)

C

Q.18 (a) Evaluate $\iint_R \cos\left(\max\left\{x^3, y^{3/2}\right\}\right) dx\, dy$, where $R = [0,1] \times [0,1]$. (9)

(b) Let $S = \sqrt{1} + \sqrt{2} + \sqrt{3} + ... + \sqrt{10000}$ and $I = \int_0^{10000} \sqrt{x}\, dx$. Show that $I \le S \le I + 100$. (6)

C

Q.19 Let $D = \{(x, y) : x \geq 0, y \geq 0\}$. Let $f(x, y) = (x^2 + y^2)\, e^{-x-y}$ for $(x, y) \in D$.
Prove that f attains its maximum on D at two boundary points.
Deduce that $\frac{x^2 + y^2}{4} \leq e^{x+y-2}$ for all $x \geq 0,\ y \geq 0$. (15)

C

Q.20 (a) Let $a_1, b_1, a_2, b_2 \in \mathbb{R}$. Show that the condition $a_2 b_1 > 0$ is sufficient but not necessary for the system

$$\frac{dx}{dt} = a_1 x + b_1 y,$$
$$\frac{dy}{dt} = a_2 x + b_2 y,$$

to have two linearly independent solutions of the form $x = c_1 e^{\lambda_1 t}$, $y = d_1 e^{\lambda_1 t}$ and $x = c_2 e^{\lambda_2 t}$, $y = d_2 e^{\lambda_2 t}$ with $c_1, d_1, c_2, d_2, \lambda_1, \lambda_2 \in \mathbb{R}$. (9)

(b) Show that the differential equation representing the family of all straight lines which have an intercept of constant length L between the coordinate axes is

$$x\frac{dy}{dx} - y = \frac{L\frac{dy}{dx}}{\sqrt{1+\left(\frac{dy}{dx}\right)^2}}.$$ (6)

C

Q.21 Let $A, B, k > 0$. Solve the initial value problem

$$\frac{dy}{dx} - Ay + By^3 = 0,\ x > 0,\ y(0) = k.$$

Also show that

(a) if $k < \sqrt{\frac{A}{B}}$, then the solution $y(x)$ is monotonically increasing on $(0, \infty)$ and tends to $\sqrt{\frac{A}{B}}$ as $x \to \infty$;

(b) if $k > \sqrt{\frac{A}{B}}$, then the solution $y(x)$ is monotonically decreasing on $(0, \infty)$ and tends to $\sqrt{\frac{A}{B}}$ as $x \to \infty$. (15)

C

Q.22 (a) Evaluate

$$\int_C \left(3y^2 + 2z^2\right) dx + (6x - 10z)y\, dy + \left(4xz - 5y^2\right) dz$$

along the portion from $(1, 0, 1)$ to $(3, 4, 5)$ of the curve C, which is the intersection of the two surfaces $z^2 = x^2 + y^2$ and $z = y + 1$. (6)

(b) A particle moves counterclockwise along the curve $3x^2 + y^2 = 3$ from $(1, 0)$ to a point P, under the action of the force $\vec{F}(x, y) = \frac{x}{y}\hat{i} + \frac{y}{x}\hat{j}$. Prove that there are two possible locations of P such that the work done by $\vec{F}$ is 1. (9)

C

Q.23 Verify Stokes's theorem for the hemisphere $x^2+y^2+z^2=9,\ z\geq 0$ and the vector field $\vec{F}=\left(z^2-y\right)\hat{i}+(x-2yz)\,\hat{j}+\left(2xz-y^2\right)\hat{k}$. (15)

C

Q.24 (a) Let $T: \mathbb{R}^3 \to \mathbb{R}^2$ be the linear transformation defined by $T(x, y, z) = (x+2y, x-z)$. Let $N(T)$ be the null space of T and $W = \left\{ \vec{v} \in \mathbb{R}^3 \mid \vec{v} \cdot \vec{u} = 0, \text{ for all } \vec{u} \in N(T) \right\}$. Find a linear transformation $S: \mathbb{R}^2 \to W$ such that $TS = I$, where I is the identity transformation on $\mathbb{R}^2$. (9)

(b) Suppose A is a real square matrix of odd order such that $A + A^T = 0$. Prove that A is singular. (6)

C

Q.25 (a) Find all pairs (a, b) of real numbers for which the system of equations

$$x+3y=1$$
$$4x+ay+z=0$$
$$2x+3z=b$$

has (i) a unique solution, (ii) infinitely many solutions, (iii) no solution. (9)

(b) Let A be an $n \times n$ matrix such that $A^n = 0$ and $A^{n-1} \neq 0$. Show that there exists a vector $v \in \mathbb{R}^n$ such that $\{v, Av, \dots, A^{n-1}v\}$ forms a basis for $\mathbb{R}^n$. (6)

C

Q.26 (a) In which of the following pairs are the two groups isomorphic to each other? Justify your answers.

(i) $\mathbb{R}/\mathbb{Z}$ and S^1, where $\mathbb{R}$ is the additive group of real numbers and $S^1 = \{z \in \mathbb{C} : |z| = 1\}$ under complex multiplication.

(ii) $(\mathbb{Z}, +)$ and $(\mathbb{Q}, +)$. (9)

(b) Prove or disprove that if G is a finite abelian group of order n, and k is a positive integer which divides n, then G has at most one subgroup of order k. (6)

C

Q.27 Let I and J be ideals of a ring R. Let IJ be the set of all possible sums $\sum_{i=1}^{n} a_i b_i$, where $a_i \in I, b_i \in J$ for $i = 1, 2, \ldots, n$ and $n \in \mathbb{N}$.

(a) Prove that IJ is an ideal of R and $IJ \subseteq I \cap J$. (9)

(b) Is it true that $IJ = I \cap J$? Justify your answer. (6)

C

Q.28 A sequence $\{f_n\}$ of functions defined on an interval I is said to be uniformly bounded on I if there exists some M such that $|f_n(x)| \le M$ for all $x \in I$ and for all $n \in \mathbb{N}$.

(a) Prove that if a sequence of functions $\{f_n\}$ converges to a function f on I and $\{f_n\}$ is uniformly bounded on I, then f is bounded on I. (6)

(b) Suppose the sequences $\{f_n\}$ and $\{g_n\}$ of functions converge uniformly to f and g respectively on I and both are uniformly bounded on I. Prove that the product sequence $\{f_n g_n\}$ converges to fg uniformly on I. Show by an example that this may fail if only one of $\{f_n\}$ and $\{g_n\}$ is uniformly bounded on I. (9)

C

Q.29 (a) Prove that if f is a real-valued function which is uniformly continuous on an interval (a, b), then f is bounded on (a, b). (9)

(b) Let f be a differentiable function on an interval (a, b). Assume that f' is bounded on (a, b). Prove that f is uniformly continuous on (a, b). (6)

A

2008-MA

Test Paper Code: MA

Time: 3 Hours **Maximum Marks: 300**

INSTRUCTIONS

1. The question-cum-answer booklet has **36** pages and has **29** questions. Please ensure that the copy of the question-cum-answer booklet you have received contains all the questions.
2. Write your **Roll Number, Name and the name of the Test Centre** in the appropriate space provided on the right side.
3. Write the answers to the objective questions against each Question No. in the **Answer Table for Objective Questions**, provided on Page No. **5**. Do not write anything else on this page.
4. Each objective question has **4 choices** for its answer: (A), (B), (C) and (D). Only **ONE** of them is the correct answer. There will be **negative marking** for wrong answers to objective questions. The following marking scheme for objective questions shall be used:
   (a) For each correct answer, you will be awarded **6 (Six)** marks.
   (b) For each wrong answer, you will be awarded **-2 (Negative two)** marks.
   (c) Multiple answers to a question will be treated as a wrong answer.
   (d) For each un-attempted question, you will be awarded **0 (Zero)** mark.
   (e) Negative marks for objective part will be carried over to total marks.
5. Answer the subjective question only in the space provided after each question.
6. Do not write more than one answer for the same question. In case you attempt a subjective question more than once, please cancel the answer(s) you consider wrong. Otherwise, the answer appearing last only will be evaluated.
7. All answers must be written in blue/black/blue-black ink only. Sketch pen, pencil or ink of any other colour should not be used.
8. All rough work should be done in the space provided and scored out finally.
9. No supplementary sheets will be provided to the candidates.
10. **Clip board, log tables, slide rule, calculator, cellular phone, pager and electronic gadgets in any form are NOT allowed.**
11. The question-cum-answer booklet must be returned in its entirety to the Invigilator before leaving the examination hall. Do not remove any page from this booklet.
12. Refer to special instructions/useful data on the reverse.

A

2008-MA

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

ROLL NUMBER							

Name:

Test Centre:

Do not write your Roll Number or Name anywhere else in this question-cum-answer booklet.

I have read all the instructions and shall abide by them.

....

Signature of the Candidate

I have verified the information filled by the Candidate above.

....

Signature of the Invigilator

A

Special Instructions/ Useful Data

$\mathbb{Z}$ - Set of all integers
$\mathbb{Q}$ - Set of all rational numbers
$\mathbb{R}$ - Set of all real numbers
$\mathbb{C}$ - Set of all complex numbers
A^T - Transpose of the matrix A
$\overline{A}^T$ - Conjugate transpose of the matrix A
det A - Determinant of the matrix A

DO NOT WRITE ON THIS PAGE

A

IMPORTANT NOTE FOR CANDIDATES

- **Questions 1-15 (objective questions) carry *six* marks each and questions 16-29 (subjective questions) carry *fifteen* marks each.**
- **Write the answers to the objective questions in the *Answer Table for Objective Questions* provided on page 5 only.**

Q.1 The least positive integer n, such that $\begin{pmatrix} \cos\frac{\pi}{4} & \sin\frac{\pi}{4} \\ -\sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{pmatrix}^n$ is the identity matrix of order 2, is

(A) 4 (B) 8 (C) 12 (D) 16

Q.2 Let $S = \{T : \mathbb{R}^3 \to \mathbb{R}^3 \mid T$ is a linear transformation with $T(1,0,1) = (1,2,3)$, $T(1,2,3) = (1,0,1)\}$. Then S is

(A) a singleton set
(B) a finite set containing more than one element
(C) a countably infinite set
(D) an uncountable set

Q.3 Let $s_n = \int_0^1 \frac{n\,x^{n-1}}{(1+x)}\,dx$ for $n \geq 1$. Then as $n \to \infty$, the sequence $\{s_n\}$ tends to

(A) 0 (B) $1/2$ (C) 1 (D) $+\infty$

Q.4 The work done by the force $\vec{F} = 4y\,\hat{i} - 3xy\,\hat{j} + z^2\,\hat{k}$ in moving a particle over the circular path $x^2 + y^2 = 1$, $z = 0$ from $(1,0,0)$ to $(0,1,0)$ is

(A) $\pi + 1$ (B) $\pi - 1$ (C) $-\pi + 1$ (D) $-\pi - 1$

Q.5 The set of all boundary points of $\mathbb{Q}$ in $\mathbb{R}$ is

(A) $\mathbb{R}$ (B) $\mathbb{R} \setminus \mathbb{Q}$ (C) $\mathbb{Q}$ (D) $\varnothing$

Q.6 Let $V = \left\{(x,y,z) \in \mathbb{R}^3 : \frac{1}{4} \leq x^2 + y^2 + z^2 \leq 1\right\}$ and $\vec{F} = \frac{x\,\hat{i} + y\,\hat{j} + z\,\hat{k}}{(x^2 + y^2 + z^2)^2}$ for $(x,y,z) \in V$. Let $\hat{n}$ denote the outward unit normal vector to the boundary of V and S denote the part $\left\{(x,y,z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = \frac{1}{4}\right\}$ of the boundary of V. Then $\iint_S \vec{F} \cdot \hat{n}\, dS$ is equal to

(A) -8π (B) -4π (C) 4π (D) 8π

A

Q.7 The set $U=\left\{x\in\mathbb{R} \mid \sin x=\frac{1}{2}\right\}$ is

(A) open
(B) closed
(C) both open and closed
(D) neither open nor closed

Q.8 Let $f(x)=\int_0^x (x^2-t^2)\,g(t)\,dt$, where g is a real valued continuous function on $\mathbb{R}$. Then $f'(x)$ is equal to

(A) 0
(B) $x^3\,g(x)$
(C) $\int_0^x g(t)\,dt$
(D) $2x\int_0^x g(t)\,dt$

Q.9 Let $y_1(x)$ and $y_2(x)$ be linearly independent solutions of the differential equation $y''+P(x)y'+Q(x)y=0$, where $P(x)$ and $Q(x)$ are continuous functions on an interval I. Then $y_3(x)=a\,y_1(x)+b\,y_2(x)$ and $y_4(x)=c\,y_1(x)+d\,y_2(x)$ are linearly independent solutions of the given differential equation if

(A) $ad=bc$
(B) $ac=bd$
(C) $ad\neq bc$
(D) $ac\neq bd$

Q.10 The set $R=\left\{f \mid f \text{ is a function from } \mathbb{Z} \text{ to } \mathbb{R}\right\}$ under the binary operations $+$ and $\cdot$ defined as $(f+g)(n)=f(n)+g(n)$ and $(f\cdot g)(n)=f(n)\,g(n)$ for all $n\in\mathbb{Z}$ forms a ring. Let $S_1=\left\{f\in R \mid f(-n)=f(n) \text{ for all } n\in\mathbb{Z}\right\}$ and $S_2=\left\{f\in R \mid f(0)=0\right\}$. Then

(A) S_1 and S_2 are both ideals in R
(B) S_1 is an ideal in R while S_2 is not
(C) S_2 is an ideal in R while S_1 is not
(D) neither S_1 nor S_2 is an ideal in R

Q.11 Let $T:\mathbb{R}^3\to\mathbb{R}^3$ be a linear transformation such that $T(1,2,3)=(1,2,3)$, $T(1,5,0)=(2,10,0)$ and $T(-1,2,-1)=(-3,6,-3)$. The dimension of the vector space spanned by all the eigenvectors of T is

(A) 0
(B) 1
(C) 2
(D) 3

A

Q.12 Let $\{a_n\}$ and $\{b_n\}$ be sequences of real numbers defined as $a_1 = 1$ and for $n \geq 1$, $a_{n+1} = a_n + (-1)^n 2^{-n}$, $b_n = \dfrac{2a_{n+1} - a_n}{a_n}$. Then

(A) $\{a_n\}$ converges to zero and $\{b_n\}$ is a Cauchy sequence
(B) $\{a_n\}$ converges to a non-zero number and $\{b_n\}$ is a Cauchy sequence
(C) $\{a_n\}$ converges to zero and $\{b_n\}$ is not a convergent sequence
(D) $\{a_n\}$ converges to a non-zero number and $\{b_n\}$ is not a convergent sequence

Q.13 Let $f : (-1,1) \to \mathbb{R}$ be defined as $f(x) = \dfrac{x^2}{1-\cos x}$ for $x \neq 0$ and $f(0) = 2$. If $f(x) = \sum_{n=0}^{\infty} a_n x^n$ is the Taylor expansion of f for all x in $(-1,1)$, then $\sum_{n=0}^{\infty} a_{2n+1}$ is

(A) 0 (B) $1/2$ (C) 1 (D) 2

Q.14 Let $y_1(x)$ and $y_2(x)$ be twice differentiable functions on an interval I satisfying the differential equations $\dfrac{dy_1}{dx} - y_1 - y_2 = e^x$ and $2\dfrac{dy_1}{dx} + \dfrac{dy_2}{dx} - 6y_1 = 0$. Then $y_1(x)$ is

(A) $C_1 e^{-2x} + C_2 e^{3x} - \frac{1}{4}e^x$
(B) $C_1 e^{2x} + C_2 e^{-3x} + \frac{1}{4}e^x$
(C) $C_1 e^{2x} + C_2 e^{-3x} - \frac{1}{4}e^x$
(D) $C_1 e^{-2x} + C_2 e^{3x} + \frac{1}{4}e^x$

Q.15 Let G be a finite group and H be a normal subgroup of G of order 2. Then the order of the center of G is

(A) 0 (B) 1 (C) an even integer ≥ 2 (D) an odd integer ≥ 3

Space for rough work

A

Answer Table for Objective Questions

Write the Code of your chosen answer only in the 'Answer' column against each Question No. Do not write anything else on this page.

Question No.	Answer	Do not write in this column
01		
02		
03		
04		
05		
06		
07		
08		
09		
10		
11		
12		
13		
14		
15		

FOR EVALUATION ONLY

No. of Correct Answers		Marks	(+)
No. of Incorrect Answers		Marks	(–)
Total Marks in Question Nos. 1-15			( )

A

Q.16 (a) Let f and g be continuous functions on $\mathbb{R}$ such that $f(x)=\int_0^x g(t)\,dt$ and $g(x)=\int_x^0 f(t)\,dt+1$. Prove that $(f(x))^2+(g(x))^2=1$ for all $x\in\mathbb{R}$. (6)

(b) Let $f:\mathbb{R}\to\mathbb{R}$ be a function such that f' is continuous on $\mathbb{R}$. Show that the series $\sum_{n=1}^{\infty}\left(f\left(\frac{x}{2n}\right)-f\left(\frac{x}{2n+1}\right)\right)$ converges uniformly on $[0,1]$. (9)

A

Q.17 (a) Find the maxima, minima and saddle points, if any, for the function $f(x,y)=(y-x^2)(y-2x^2)$ on $\mathbb{R}^2$. (6)

(b) Let $P(x)=a_0+a_1x^2+a_2x^4+\cdots+a_nx^{2n}$, where $n\geq 1$ and $a_k>0$ for $k=0$, $1,\cdots,n$. Show that $P(x)-x\,P'(x)=0$ has exactly two real roots. (9)

A

Q.18 (a) Given that $y_1(x)=x$ is a solution of $(1+x^2)y''-2xy'+2y=0,\ x>0$, find a second linearly independent solution. (6)

(b) Solve $x^2y''+xy'-y=4x\log x,\ x>0$. (9)

A

Q.19 (a) Let ϕ be a differentiable function on $[0,1]$ satisfying $\phi'(x) \le 1 + 3\phi(x)$ for all $x \in [0,1]$ with $\phi(0) = 0$. Show that $3\phi(x) \le e^{3x} - 1$. **(6)**

(b) If $y_1(x) = x(1-2x)$, $y_2(x) = 2x(1-x)$ and $y_3(x) = x(e^x - 2x)$ are three solutions of a non-homogeneous linear differential equation $y'' + P(x)y' + Q(x)y = R(x)$, where $P(x)$, $Q(x)$ and $R(x)$ are continuous functions on $[a,b]$ with $a > 0$, then find its general solution. **(9)**

A

Q.20 (a) Evaluate $\int_1^4 \int_0^1 \int_{2y}^2 \frac{\cos x^2}{\sqrt{z}}\, dx\, dy\, dz$. **(6)**

(b) Find the surface area of the portion of the cone $z^2 = x^2 + y^2$ that is inside the cylinder $z^2 = 2y$. **(9)**

A

Q.21 (a) Use Green's theorem to evaluate the integral $\oint_C x^2\,dx + (x + y^2)\,dy$, where C is the closed curve given by $y = 0$, $y = x$ and $y^2 = 2 - x$ in the first quadrant, oriented counter clockwise. (6)

(b) Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function. Use change of variables to prove that $\iint_D f(x-y)\,dx\,dy = \int_{-1}^{1} f(u)\,du$, where $D = \left\{(x,y) \in \mathbb{R}^2 \,:\, |x| + |y| \le 1\right\}$. (9)

A

Q.22 Using Gauss's divergence theorem, evaluate the integral $\iint_S \vec{F} \cdot \hat{n}\, dS$, where $\vec{F} = 4xz\,\hat{i} - y^2\,\hat{j} + 4yz\,\hat{k}$, S is the surface of the solid bounded by the sphere $x^2 + y^2 + z^2 = 10$ and the paraboloid $x^2 + y^2 = z - 2$, and $\hat{n}$ is the outward unit normal vector to S. (15)

A

Q.23 (a) A square matrix M of order n with complex entries is called skew Hermitian if $M+\overline{M}^T=0$, where 0 is the zero matrix of order n.

Determine whether $V=\{M \mid M$ is a 2×2 skew Hermitian matrix $\}$ is a vector space over (i) the field $\mathbb{R}$ and (ii) the field $\mathbb{C}$ with the usual operations of addition and scalar multiplication for matrices? (6)

(b) Let $V=\{P(x) \mid P(x)$ is a polynomial of degree $\leq n$ with real coefficients $\}$ and $T: V \rightarrow \mathbb{R}^m$ be defined as $T(P(x))=(P(1),P(2),\cdots,P(m))$. Then show that T is linear and determine the Nullity of T. (9)

A

Q.24 Let G be the set of all 3×3 real matrices M such that $MM^T = M^T M = I_3$ and let $H = \{M \in G \mid \det M = 1\}$, where I_3 is the identity matrix of order 3. Then show that

(i) G is a group under matrix multiplication,

(ii) H is a normal subgroup of G,

(iii) $\phi : G \to \{-1, 1\}$ given by $\phi(M) = \det M$ is onto,

(iv) G / H is abelian. (15)

A

Q.25 (a) Suppose that $(R,+,\cdot)$ is a ring having the property $a\cdot b=c\cdot a \Rightarrow b=c$, when $a \neq 0$. Then prove that $(R,+,\cdot)$ is a commutative ring. (6)

(b) Let R be a commutative ring with identity. For $a_1, a_2, \cdots, a_n \in R$, the ideal generated by $\{a_1, a_2, \cdots, a_n\}$ is given by

$$\langle a_1, a_2, \cdots, a_n \rangle = \{r_1 a_1 + r_2 a_2 + \cdots + r_n a_n \mid r_i \in R, 1 \leq i \leq n\}.$$

Let $\mathbb{Z}[x]$ be the set of all polynomials with integer coefficients. Consider the ideal $I = \{f \in \mathbb{Z}[x] \mid f(0) \text{ is an even integer}\}$. Prove that $I = \langle 2, x \rangle$ and that it is a maximal ideal. (9)

A

Q.26 For a given positive integer $n > 1$, show that there exist subspaces $X_1, X_2, \cdots, X_n$ of $\mathbb{R}^m$ for some integer $m > n$ and a linear transformation $T : \mathbb{R}^m \to \mathbb{R}^m$ such that

- dim $X_k = k$, $k = 1, 2, \cdots, n$,
- for $i \neq j$, $X_i \cap X_j = \{\vec{0}\}$, where $\vec{0}$ is the zero vector of $\mathbb{R}^m$,
- $T(X_k) = X_{k-1}$, $k = 1, 2, \cdots, n$, where $X_0 = \{\vec{0}\}$.

Also, find the rank of T. (15)

A

Q.27 Let $f:(0,\infty)\rightarrow(0,\infty)$ be a continuously differentiable function and let $z=\dfrac{xy}{f(x^2+y^2)}$ be defined for $xy\neq 0$.

(a) Prove that $\dfrac{\partial z}{\partial x}+\dfrac{\partial z}{\partial y}=\dfrac{(x+y)}{[f(x^2+y^2)]^2}\left\{f(x^2+y^2)-2xy\,f'(x^2+y^2)\right\}$. (6)

(b) Further, if f is homogeneous of degree $\dfrac{1}{2}$, then verify that $x\dfrac{\partial z}{\partial x}+y\dfrac{\partial z}{\partial y}=z$. (9)

A

Q.28 Determine the interval of convergence of the power series $\sum_{n=1}^{\infty} n(2n-1)x^{2n}$ and show that its sum is $\frac{x^2(1+3x^2)}{\left(1-x^2\right)^3}$ at any point x in its interval of convergence. (15)

A

Q.29 (a) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined as $f(x, y) = x^2 \cos(y/x)$ for $x \neq 0$ and $f(x, y) = 0$ for $x = 0$. Compute $\frac{\partial f}{\partial x}$ at all points in $\mathbb{R}^2$ and show that it is continuous at the origin. (6)

(b) Let $f : (0,1) \to (0, \infty)$ be a uniformly continuous function. If $\{x_n\}$ is a Cauchy sequence in $(0,1)$, then prove that $\{f(x_n)\}$ is a Cauchy sequence in $(0, \infty)$. Hence deduce that for any two Cauchy sequences $\{x_n\}$ and $\{y_n\}$ in $(0,1)$, $\{|f(x_n) - f(y_n)|\}$ is a Cauchy sequence in $(0, \infty)$. (9)

B

2007 – MA

Test Paper Code : MA

Time : 3 Hours Maximum Marks : 300

INSTRUCTIONS

1. The question-cum-answer book has **40** pages and has **29** questions. Please ensure that the copy of the question-cum-answer book you have received contains all the questions.
2. Write your **Roll Number, Name** and **the Name of the Test Centre** in the appropriate space provided on the right side.
3. Write the answers to the objective questions against each Question No. in the **Answer Table for Objective Questions**, provided on Page No. **7**. Do not write anything else on this page.
4. Each objective question has **4 choices** for its answer: (A), (B), (C) and (D). Only **ONE** of them is the correct answer. There will be **negative marking** for wrong answers to objective questions. The following marking scheme for objective questions shall be used :
   (a) For each objective question, you will be awarded **6 (six)** marks if you have written only the correct answer.
   (b) In case you have not written any answer for a question, you will be awarded **0 (zero)** mark for that question.
   (c) In all other cases, you will be awarded **–2 (minus two)** marks for the question.
   (d) Negative marks for objective part will be carried over to total marks.
5. Answer the subjective question only in the space provided after each question.
6. Do not write more than one answer for the same question. In case you attempt a subjective question more than once, please cancel the answer(s) you consider wrong. Otherwise, the answer appearing later only will be evaluated.
7. All answers must be written in blue/ black/blue-black ink only. Sketch pen, pencil or ink of any other colour should not be used.
8. All rough work should be done in the space provided and scored out finally.
9. No supplementary sheets will be provided to the candidates.
10. **Logarithmic Tables / Calculator of any kind / cellular phone / pager / electronic gadgets are not allowed.**
11. The question-cum-answer book must be returned in its entirety to the Invigilator before leaving the examination hall. Do not remove any page from this book.
12. Refer to notations used on the reverse.

B

2007 – MA

READ THE INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

ROLL NUMBER						

Name :

Test Centre :

Do not write your Roll Number or Name anywhere else in this question-cum-answer book.

I have read all the instructions and shall abide by them.

....

Signature of the Candidate

I have verified the information filled by the Candidate above.

....

Signature of the Invigilator

B

NOTATIONS USED

R : The set of all real numbers

Z : The set of all integers

N : The set of all natural numbers 1, 2, 3, ...

$i = \sqrt{-1}$

DO NOT WRITE ON THIS PAGE

B

IMPORTANT NOTE FOR CANDIDATES

- **Attempt ALL the 29 questions.**
- **Questions 1-15 (objective questions) carry *six* marks each and questions 16-29 (subjective questions) carry *fifteen* marks each.**
- **Write the answers to the objective questions in the *Answer Table for Objective Questions* provided on page 7 only.**

1. Let $A(t)$ denote the area bounded by the curve $y=e^{-|x|}$, the x-axis and the straight lines $x=-t$ and $x=t$. Then $\lim_{t\to\infty} A(t)$ is equal to

(A) 2
(B) 1
(C) 1/2
(D) 0

2. If k is a constant such that $xy+k=e^{(x-1)^2/2}$ satisfies the differential equation

$x\dfrac{dy}{dx}=(x^2-x-1)y+(x-1)$, then k is equal to

(A) 1
(B) 0
(C) –1
(D) –2

3. Which of the following functions is uniformly continuous on the domain as stated?

(A) $f(x)=x^2, \quad x\in \mathbf{R}$

(B) $f(x)=\dfrac{1}{x}, \quad x\in[1,\infty)$

(C) $f(x)=\tan x, \quad x\in(-\pi/2,\pi/2)$

(D) $f(x)=[x], \quad x\in[0,1]$

([x] is the greatest integer less than or equal to x)

Space for rough work

B

4. Let R be the ring of polynomials over $\mathbf{Z}_2$ and let I be the ideal of R generated by the polynomial x^3+x+1. Then the number of elements in the quotient ring R/I is

(A) 2
(B) 4
(C) 8
(D) 16

5. Which of the following sets is a basis for the subspace

$W=\left\{\begin{bmatrix} x & y \\ 0 & t \end{bmatrix} : x+2y+t=0,\ y+t=0\right\}$

of the vector space of all real 2×2 matrices?

(A) $\left\{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right\}$

(B) $\left\{\begin{bmatrix} 2 & 1 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}\right\}$

(C) $\left\{\begin{bmatrix} -1 & 1 \\ 2 & -1 \end{bmatrix}\right\}$

(D) $\left\{\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}\right\}$

6. Let G be an Abelian group of order 10. Let $S=\{g\in G : g^{-1}=g\}$. Then the number of non-identity elements in S is

(A) 5
(B) 2
(C) 1
(D) 0

Space for rough work

B

7. Let (a_n) be an increasing sequence of positive real numbers such that the series $\sum_{k=1}^{\infty} a_k$ is divergent. Let $s_n = \sum_{k=1}^{n} a_k$ for $n = 1,2,\dots$ and $t_n = \sum_{k=2}^{n} \frac{a_k}{s_{k-1}s_k}$ for $n = 2,3,\dots$. Then $\lim_{n\to\infty} t_n$ is equal to

(A) $1/a_1$

(B) 0

(C) $1/(a_1 + a_2)$

(D) $a_1 + a_2$

8. For every function $f:[0,1] \to \mathbf{R}$ which is twice differentiable and satisfies $f'(x) \geq 1$ for all $x \in [0,1]$, we must have

(A) $f''(x) \geq 0$ for all $x \in [0,1]$

(B) $f(x) \geq x$ for all $x \in [0,1]$

(C) $f(x_2) - x_2 \leq f(x_1) - x_1$ for all $x_1, x_2 \in [0,1]$ with $x_2 \geq x_1$

(D) $f(x_2) - x_2 \geq f(x_1) - x_1$ for all $x_1, x_2 \in [0,1]$ with $x_2 \geq x_1$

9. Let $f:\mathbf{R}^2 \to \mathbf{R}$ be defined by

$$f(x,y) = \begin{cases} \dfrac{x^2 y}{x^4 + y^2} & \text{if } (x,y) \neq (0,0), \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$$

Which of the following statements holds regarding the continuity and the existence of partial derivatives of f at $(0,0)$?

(A) Both partial derivatives of f exist at $(0,0)$ and f is continuous at $(0,0)$

(B) Both partial derivatives of f exist at $(0,0)$ and f is NOT continuous at $(0,0)$

(C) One partial derivative of f does NOT exist at $(0,0)$ and f is continuous at $(0,0)$

(D) One partial derivative of f does NOT exist at $(0,0)$ and f is NOT continuous at $(0,0)$

Space for rough work

B

10. Suppose (c_n) is a sequence of real numbers such that $\lim_{n \to \infty} |c_n|^{1/n}$ exists and is non-zero. If the radius of convergence of the power series $\sum_{n=0}^{\infty} c_n x^n$ is equal to r, then the radius of convergence of the power series $\sum_{n=1}^{\infty} n^2 c_n x^n$ is

(A) less than r
(B) greater than r
(C) equal to r
(D) equal to 0

11. The rank of the matrix $\begin{bmatrix} 1 & 4 & 8 \\ 2 & 10 & 22 \\ 0 & 4 & 12 \end{bmatrix}$ is

(A) 3
(B) 2
(C) 1
(D) 0

12. Let $f : \mathbf{R} \to \mathbf{R}$ be a continuous function. If $\int_0^x f(2t)\,dt = \frac{x}{\pi}\sin(\pi x)$ for all $x \in \mathbf{R}$, then $f(2)$ is equal to

(A) -1
(B) 0
(C) 1
(D) 2

Space for rough work

B

13. Let $\vec{u} = (a e^x \sin y - 4x)\hat{\mathrm{i}} + (2y + e^x \cos y)\hat{\mathrm{j}} + az\,\hat{\mathrm{k}}$, where a is a constant. If the line integral $\oint_C \vec{u}\cdot d\vec{r}$ over every closed curve C is zero, then a is equal to

(A) -2

(B) -1

(C) 0

(D) 1

14. One of the integrating factors of the differential equation $(y^2 - 3xy)dx + (x^2 - xy)dy = 0$ is

(A) $1/(x^2 y^2)$

(B) $1/(x^2 y)$

(C) $1/(x y^2)$

(D) $1/(x y)$

15. Let C denote the boundary of the semi-circular disk $D = \{(x,y) \in \mathbf{R}^2 : x^2 + y^2 \le 1,\ y \ge 0\}$ and let $\varphi(x,y) = x^2 + y$ for $(x,y) \in D$. If $\hat{n}$ is the outward unit normal to C, then the integral $\oint_C (\vec{\nabla}\varphi)\cdot\hat{n}\,ds$, evaluated counter-clockwise over C, is equal to

(A) 0

(B) $\pi - 2$

(C) π

(D) $\pi + 2$

Space for rough work

QUESTION-CUM-ANSWER BOOK CODE **B**

Answer Table for Objective Questions

Write the Code of your chosen answer only in the 'Answer' column against each Question No. Do not write anything else on this page:

Question No.	Answer	Do not write in this column
01		
02		
03		
04		
05		
06		
07		
08		
09		
10		
11		
12		
13		
14		
15		

FOR EVALUATION ONLY

No. of Correct Answers		Marks	(+)
No. of Incorrect Answers		Marks	(–)
Total Marks in Question Nos. 1-15			( )

B

16. (a) Let $M = \begin{bmatrix} 1+i & 2i & i+3 \\ 0 & 1-i & 3i \\ 0 & 0 & i \end{bmatrix}$. Determine the eigenvalues of the matrix $B = M^2 - 2M + I$. (9)

(b) Let N be a square matrix of order 2. If the determinant of N is equal to 9 and the sum of the diagonal entries of N is equal to 10, then determine the eigenvalues of N. (6)

17. (a) Using the method of variation of parameters, solve the differential equation

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = x^2,$$

given that x and $\frac{1}{x}$ are two solutions of the corresponding homogeneous equation. (9)

(b) Find the real number α such that the differential equation

$$\frac{d^2y}{dx^2} + 2(\alpha-1)(\alpha-3)\frac{dy}{dx} + (\alpha-2)y = 0$$

has a solution $y(x) = a\cos(\beta x) + b\sin(\beta x)$ for some non-zero real numbers a, b, β. (6)

18. (a) Let a, b, c be non-zero real numbers such that $(a-b)^2 = 4ac$. Solve the differential equation $a\left(x+\sqrt{2}\right)^2 \frac{d^2y}{dx^2} + b\left(x+\sqrt{2}\right)\frac{dy}{dx} + cy = 0$. (9)

(b) Solve the differential equation

$dx + (e^{y\sin y} - x)(y\cos y + \sin y)dy = 0$. (6)

19. Let $f(x, y) = x(x-2y^2)$ for $(x, y) \in \mathbf{R}^2$. Show that f has a local minimum at $(0, 0)$ on every straight line through $(0, 0)$. Is $(0, 0)$ a critical point of f? Find the discriminant of f at $(0, 0)$. Does f have a local minimum at $(0, 0)$? Justify your answers. (15)

20. (a) Find the finite volume enclosed by the paraboloids $z=2-x^2-y^2$ and $z=x^2+y^2$. (9)

(b) Let $f:[0,3]\to\mathbf{R}$ be a continuous function with $\int_0^3 f(x)dx=3$. Evaluate

$$\int_0^3 [x f(x)+\int_0^x f(t)dt]\,dx.$$ (6)

21. (a) Let S be the surface $\{(x,y,z)\in\mathbf{R}^3:\ x^2+y^2+2z=2,\ z\geq 0\}$, and let $\hat{n}$ be the outward unit normal to S. If $\vec{F}=y\,\hat{\mathrm{i}}+xz\,\hat{\mathrm{j}}+(x^2+y^2)\,\hat{\mathrm{k}}$, then evaluate the integral $\iint_S \vec{F}\cdot\hat{n}\,dS$. (9)

(b) Let $\vec{r}=x\,\hat{\mathrm{i}}+y\,\hat{\mathrm{j}}+z\,\hat{\mathrm{k}}$ and $r=|\vec{r}|$. If a scalar field φ and a vector field $\vec{u}$ satisfy $\vec{\nabla}\varphi=\vec{\nabla}\times\vec{u}+f(r)\,\vec{r}$, where f is an arbitrary differentiable function, then show that $\nabla^2\varphi=r f'(r)+3f(r)$. (6)

22. (a) Let D be the region bounded by the concentric spheres $S_1 : x^2+y^2+z^2=a^2$ and $S_2 : x^2+y^2+z^2=b^2$, where $a<b$. Let $\hat{n}$ be the unit normal to S_1 directed away from the origin. If $\nabla^2\varphi=0$ in D and $\varphi=0$ on S_2, then show that

$$\iiint_D \left|\vec{\nabla}\varphi\right|^2 dV + \iint_{S_1} \varphi\,(\vec{\nabla}\varphi)\cdot\hat{n}\,dS = 0. \qquad (9)$$

(b) Let C be the curve in $\mathbf{R}^3$ given by $x^2+y^2=a^2$, $z=0$ traced counter-clockwise, and let $\vec{F}=x^2y^3\,\hat{\mathrm{i}}+\hat{\mathrm{j}}+z\,\hat{\mathrm{k}}$. Using Stokes' theorem, evaluate $\oint_C \vec{F}\cdot d\vec{r}$. (6)

23. Let V be the subspace of $\mathbf{R}^4$ spanned by the vectors (1,0,1,2), (2,1,3,4) and (3,1,4,6). Let $T:V\to\mathbf{R}^2$ be a linear transformation given by $T(x,y,z,t)=(x-y,\,z-t)$ for all $(x,y,z,t)\in V$. Find a basis for the null space of T and also a basis for the range space of T. (15)

24. (a) Compute the double integral $\iint_D (x+2y)\,dx\,dy$, where D is the region in the xy-plane bounded by the straight lines $y=x+3$, $y=x-3$, $y=-2x+4$ and $y=-2x-2$. (9)

(b) Evaluate $\int_0^{\pi/2}\left[\int_{\pi/2}^{\pi}\frac{\sin x}{x}dx\right]dy + \int_{\pi/2}^{\pi}\left[\int_{y}^{\pi}\frac{\sin x}{x}dx\right]dy$. (6)

25. (a) Does the series $\sum_{k=1}^{\infty}\frac{(-1)^k k + x^k}{k^2}$ converge uniformly for $x \in [-1, 1]$? Justify. (9)

(b) Suppose (f_n) is a sequence of real-valued functions defined on $\mathbf{R}$ and f is a real-valued function defined on $\mathbf{R}$ such that $|f_n(x) - f(x)| \le |a_n|$ for all $n \in \mathbf{N}$ and $a_n \to 0$ as $n \to \infty$. Must the sequence (f_n) be uniformly convergent on $\mathbf{R}$? Justify. (6)

26. (a) Suppose f is a real-valued thrice differentiable function defined on $\mathbf{R}$ such that $f'''(x)>0$ for all $x \in \mathbf{R}$. Using Taylor's formula, show that

$f(x_2)-f(x_1)>(x_2-x_1)f'\left(\dfrac{x_1+x_2}{2}\right)$ for all x_1 and x_2 in $\mathbf{R}$ with $x_2>x_1$. (9)

(b) Let (a_n) and (b_n) be sequences of real numbers such that $a_n \le a_{n+1} \le b_{n+1} \le b_n$ for all $n \in \mathbf{N}$. Must there exist a real number x such that $a_n \le x \le b_n$ for all $n \in \mathbf{N}$? Justify your answer. (6)

27. Let G be the group of all 2×2 matrices with real entries with respect to matrix multiplication. Let G_1 be the smallest subgroup of G containing $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$, and G_2 be the smallest subgroup of G containing $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ and $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$. Determine all elements of G_1 and find their orders. Determine all elements of G_2 and find their orders. Does there exist a one-to-one homomorphism from G_1 onto G_2? Justify. (15)

28. (a) Let p be a prime number and let $\mathbf{Z}$ be the ring of integers. If an ideal J of $\mathbf{Z}$ contains the set $p\mathbf{Z}$ properly, then show that $J = \mathbf{Z}$. (Here $p\mathbf{Z} = \{ px : x \in \mathbf{Z} \}$.) (9)

(b) Consider the ring $R = \{ a + ib : a, b \in \mathbf{Z} \}$ with usual addition and multiplication. Find all invertible elements of R. (6)

29. (a) Suppose E is a non-empty subset of $\mathbf{R}$ which is bounded above, and let $\alpha = \sup E$. If E is closed, then show that $\alpha \in E$. If E is open, then show that $\alpha \notin E$. (9)

(b) Find all limit points of the set $E = \left\{ n + \dfrac{1}{2\,m} : n, m \in \mathbf{N} \right\}$. (6)

B

2007 – MA Objective Part (Q. Nos. 1 – 15)	
Total Marks	Signature

Subjective Part					
Q. No.	Marks	Signature	Q. No.	Marks	Signature
16			23		
17			24		
18			25		
19			26		
20			27		
21			28		
22			29		
Total Marks in Subjective Part					

Total (Objective Part) :	
Total (Subjective Part) :	
Grand Total :	
Total Marks (in words) :	
Signature of Examiner(s) :	
Signature of Head Examiner(s) :	
Signature of Scrutinizer :	
Signature of Chief Scrutinizer :	
Signature of Coordinating Head Examiner :	

JAM 2006

MATHEMATICS TEST PAPER

NOTATIONS USED

ℝ : The set of all real numbers
Z : The set of all integers

IMPORTANT NOTE FOR CANDIDATES

Objective Part:
Attempt ALL the objective questions (Questions 1-15). Each of these questions carries *six* marks. Each incorrect answer carries *minus two*. Write the answers to the objective questions in the *Answer Table for Objective Questions* provided on page 7 only.

Subjective Part:
Attempt ALL subjective questions (Questions 16-29). Each of these questions carries *fifteen* marks.

1. $\lim_{n\to\infty} \frac{2^{n+1}+3^{n+1}}{2^n+3^n}$ equals

(A) 3
(B) 2
(C) 1
(D) 0

2. Let $f(x) = (x-2)^{17}(x+5)^{24}$. Then

(A) f does not have a critical point at 2
(B) f has a minimum at 2
(C) f has a maximum at 2
(D) f has neither a minimum nor a maximum at 2

3. Let $f(x,y) = x^5 y^2 \tan^{-1}\left(\frac{y}{x}\right)$. Then $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}$ equals

(A) $2f$
(B) $3f$
(C) $5f$
(D) $7f$

4. Let G be the set of all irrational numbers. The interior and the closure of G are denoted by G^0 and $\overline{G}$, respectively. Then

(A) $G^0 = \phi, \quad \overline{G} = G$
(B) $G^0 = \mathbb{R}, \quad \overline{G} = \mathbb{R}$
(C) $G^0 = \phi, \quad \overline{G} = \mathbb{R}$
(D) $G^0 = G, \quad \overline{G} = \mathbb{R}$

5. Let $f(x)=\int_{\sin x}^{\cos x} e^{-t^2}\,dt$. Then $f'(\pi/4)$ equals

(A) $\sqrt{1/e}$

(B) $-\sqrt{2/e}$

(C) $\sqrt{2/e}$

(D) $-\sqrt{1/e}$

6. Let C be the circle $x^2+y^2=1$ taken in the anti-clockwise sense. Then the value of the integral

$$\int_C \left[\left(2xy^3+y\right)dx+\left(3x^2y^2+2x\right)dy\right]$$

equals

(A) 1

(B) $\pi/2$

(C) π

(D) 0

7. Let r be the distance of a point $P(x,y,z)$ from the origin O. Then ∇r is a vector

(A) orthogonal to $\overrightarrow{OP}$

(B) normal to the level surface of r at P

(C) normal to the surface of revolution generated by OP about x-axis

(D) normal to the surface of revolution generated by OP about y-axis

8. Let $T:\mathbb{R}^3\to\mathbb{R}^3$ be defined by

$$T(x_1,\,x_2,\,x_3)=(x_1-x_2,\,x_1-x_2,\,0).$$

If $N(T)$ and $R(T)$ denote the null space and the range space of T respectively, then

(A) $\dim N(T)=2$

(B) $\dim R(T)=2$

(C) $R(T)=N(T)$

(D) $N(T)\subset R(T)$

9. Let S be a closed surface for which $\iint_S \vec{r}.\hat{n}\,d\sigma=1$. Then the volume enclosed by the surface is

(A) 1

(B) 1/3

(C) 2/3

(D) 3

10. If $(c_1 + c_2 \ln x)/x$ is the general solution of the differential equation

$$x^2 \frac{d^2y}{dx^2} + kx\frac{dy}{dx} + y = 0, \qquad x > 0,$$

then k equals

(A) 3
(B) −3
(C) 2
(D) −1

11. If A and B are 3×3 real matrices such that $rank(AB)=1$, then $rank(BA)$ **cannot** be

(A) 0
(B) 1
(C) 2
(D) 3

12. The differential equation representing the family of circles touching y-axis at the origin is

(A) linear and of first order
(B) linear and of second order
(C) nonlinear and of first order
(D) nonlinear and of second order

13. Let G be a group of order 7 and $\phi(x) = x^4$, $x \in G$. Then ϕ is

(A) not one – one
(B) not onto
(C) not a homomorphism
(D) one – one, onto and a homomorphism

14. Let R be the ring of all 2×2 matrices with integer entries. Which of the following subsets of R is an integral domain?

(A) $\left\{\begin{pmatrix} 0 & x \\ y & 0 \end{pmatrix} : x, y \in \mathbf{Z}\right\}$

(B) $\left\{\begin{pmatrix} x & 0 \\ 0 & y \end{pmatrix} : x, y \in \mathbf{Z}\right\}$

(C) $\left\{\begin{pmatrix} x & 0 \\ 0 & x \end{pmatrix} : x \in \mathbf{Z}\right\}$

(D) $\left\{\begin{pmatrix} x & y \\ y & z \end{pmatrix} : x, y, z \in \mathbf{Z}\right\}$

15. Let $f_n(x) = n \sin^{2n+1} x \cos x$. Then the value of

$$\lim_{n\to\infty} \int_0^{\pi/2} f_n(x)dx - \int_0^{\pi/2} \left(\lim_{n\to\infty} f_n(x)\right) dx$$

is

(A) $1/2$
(B) 0
(C) $-1/2$
(D) $-\infty$

16. (a) Test the convergence of the series

$$\sum_{n=1}^{\infty} \frac{n^n}{n!\, 3^n}$$ (6)

(b) Show that

$$\ln(1+\cos x) \le \ln 2 - \frac{x^2}{4}$$

for $0 \le x \le \pi/2$. (9)

17. Find the critical points of the function

$$f(x, y) = x^3 + y^2 - 12x - 6y + 40.$$

Test each of these for maximum and minimum. (15)

18. (a) Evaluate $\iint_R xe^{y^2}\, dx\, dy$, where R is the region bounded by the lines $x = 0$, $y = 1$ and the parabola $y = x^2$. (6)

(b) Find the volume of the solid bounded above by the surface $z = 1 - x^2 - y^2$ and below by the plane $z = 0$. (9)

19. Evaluate the surface integral

$$\iint_S x\left(12y - y^4 + z^2\right) d\sigma,$$

where the surface S is represented in the form $z = y^2$, $0 \le x \le 1$, $0 \le y \le 1$. (15)

20. Using the change of variables, evaluate $\iint_R xy\, dx\, dy$, where the region R is bounded by the curves $xy = 1$, $xy = 3$, $y = 3x$ and $y = 5x$ in the first quadrant. (15)

21. (a) Let u and v be the eigenvectors of A corresponding to the eigenvalues 1 and 3 respectively. Prove that $u + v$ is not an eigenvector of A. (6)

(b) Let A and B be real matrices such that the sum of each row of A is 1 and the sum of each row of B is 2. Then show that 2 is an eigenvalue of AB. (9)

22. Suppose W_1 and W_2 are subspaces of $\mathbb{R}^4$ spanned by $\{(1,2,3,4),(2,1,1,2)\}$ and $\{(1,0,1,0),(3,0,1,0)\}$ respectively. Find a basis of $W_1 \cap W_2$. Also find a basis of $W_1 + W_2$ containing $\{(1,0,1,0),(3,0,1,0)\}$. (15)

23. Determine y_0 such that the solution of the differential equation

$$y' - y = 1 - e^{-x},\ y(0) = y_0$$

has a finite limit as $x \to \infty$. (15)

24. Let $\phi(x,y,z) = e^x \sin y$. Evaluate the surface integral $\iint_S \frac{\partial \phi}{\partial n}\, d\sigma$, where S is the surface of the cube $0 \le x \le 1,\ 0 \le y \le 1,\ 0 \le z \le 1$ and $\frac{\partial \phi}{\partial n}$ is the directional derivative of ϕ in the direction of the unit outward normal to S. Verify the divergence theorem. (15)

25. Let $y = f(x)$ be a twice continuously differentiable function on $(0,\infty)$ satisfying

$$f(1) = 1 \text{ and } f'(x) = \frac{1}{2} f\left(\frac{1}{x}\right),\ x > 0.$$

Form the second order differential equation satisfied by $y = f(x)$, and obtain its solution satisfying the given conditions. (15)

26. Let $G = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in \mathbf{Z} \right\}$ be the group under matrix addition and H be the subgroup of G consisting of matrices with even entries. Find the order of the quotient group G/H. (15)

27. Let

$$f(x) = \begin{cases} x^2 & 0 \le x \le 1 \\ \sqrt{x} & x > 1. \end{cases}$$

Show that f is uniformly continuous on $[0, \infty)$. (15)

28. Find $M_n = \max_{x \ge 0} \left\{ \frac{x}{n(1+nx^3)} \right\}$, and hence prove that the series

$$\sum_{n=1}^{\infty} \frac{x}{n\left(1+nx^3\right)}$$

is uniformly convergent on $[0, \infty)$. (15)

29. Let R be the ring of polynomials with real coefficients under polynomial addition and polynomial multiplication. Suppose

$$I = \{ p \in R : \text{ sum of the coefficients of } p \text{ is zero} \}.$$

Prove that I is a maximal ideal of R. (15)

A

2005 – MA

Test Paper Code : MA

Time : 3 Hours **Max. Marks : 300**

INSTRUCTIONS

1. The question-cum-answer book has **44** pages and has **32** questions. Please ensure that the copy of the question-cum-answer book you have received contains all the questions.
2. Write your **Roll Number, Name and the name of the Test Centre** in the appropriate space provided on the right side.
3. Write the answers to the objective questions against each Question No. in the **Answer Table for Objective Questions,** provided on page No. **7**. Do not write anything else on this page.
4. Each objective question has **4 choices** for its answer: (A), (B), (C) and (D). Only **ONE** of them is the correct answer. There will be **negative marking** for wrong answers to objective questions. The following marking scheme for objective questions shall be used :
   (a) For each objective question, you will be awarded **6 (six)** marks if you have written only the correct answer.
   (b) In case you have not written any answer for a question you will be awarded **0 (zero)** mark for that question.
   (c) In all other cases, you will be awarded **–2 (minus two)** marks for the question.
5. Answer the subjective question only in the space provided after each question.
6. Do not write more than one answer for the same question. In case you attempt a subjective question more than once, please cancel the answer(s) you consider wrong. Otherwise, the answer appearing later only will be evaluated.
7. All answers must be written in blue/black/blue-black ink only. Sketch pen, pencil or ink of any other colour should not be used.
8. All rough work should be done in the space provided and scored out finally.
9. No supplementary sheets will be provided to the candidates.
10. **Logarithmic Tables / Calculator of any kind / cellular phone / pager / electronic gadgets are not allowed.**
11. The question-cum-answer book must be returned in its entirety to the Invigilator before leaving the examination hall. Do not remove any page from this book.
12. Refer to special instruction/useful data on the reverse.

A

2005 – MA

READ THE INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

ROLL NUMBER					

Name :

Test Centre :

Do not write your Roll Number or Name anywhere else in this question-cum-answer book.

I have read all the instructions and shall abide by them.

....

Signature of the Candidate

I have verified the information filled by the Candidate above.

....

Signature of the Invigilator

A

<u>**IMPORTANT NOTE FOR CANDIDATES**</u>

Objective Part :

Attempt ALL the objective questions (Questions 1 – 15). Each of these questions carries <u>six</u> marks. Write the answers to the objective questions ONLY in the *Answer Table for Objective Questions* provided on page 7.

Subjective Part :

Attempt ALL questions in the *Core Section* (Questions 16 – 22). Questions 16 – 21 carry *<u>twenty one</u>* marks each and Question 22 carries *<u>twelve</u>* marks. There are Five Optional Sections (A, B, C, D and E). Each Optional Section has two questions, each of the questions carries *<u>eighteen</u>* marks. Attempt both questions from *<u>any two</u>* Optional Sections. Thus in the Subjective Part, attempt a total of 11 questions.

1. Let $\{a_n\}$, $\{b_n\}$ and $\{c_n\}$ be sequences of real numbers such that $b_n = a_{2n}$ and $c_n = a_{2n+1}$. Then $\{a_n\}$ is convergent

   (A) implies $\{b_n\}$ is convergent but $\{c_n\}$ need not be convergent

   (B) implies $\{c_n\}$ is convergent but $\{b_n\}$ neet not be convergent

   (C) implies both $\{b_n\}$ and $\{c_n\}$ are convergent

   (D) if both $\{b_n\}$ and $\{c_n\}$ are convergent

2. An integrating factor of $x\frac{dy}{dx} + (3x+1)y = xe^{-2x}$ is

   (A) $x\,e^{3x}$

   (B) $3x\,e^{x}$

   (C) $x\,e^{x}$

   (D) x^3e^{x}

A

3. The general solution of $x^2 \dfrac{d^2 y}{dx^2} - 5x \dfrac{dy}{dx} + 9y = 0$ is

(A) $(c_1 + c_2 x)e^{3x}$

(B) $(c_1 + c_2 \ln x)x^3$

(C) $(c_1 + c_2 x)x^3$

(D) $(c_1 + c_2 \ln x)e^{x^3}$

(Here c_1 and c_2 are arbitrary constants.)

4. Let $\vec{r} = x\,\hat{i} + y\,\hat{j} + z\,\hat{k}$. If $\phi(x, y, z)$ is a solution of the Laplace equation then the vector field $\left(\vec{\nabla}\phi + \vec{r}\right)$ is

(A) neither solenoidal nor irrotational

(B) solenoidal but not irrotational

(C) both solenoidal and irrotational

(D) irrotational but not solenoidal

5. Let $\vec{F} = x\,\hat{i} + 2y\,\hat{j} + 3z\,\hat{k}$, S be the surface of the sphere $x^2 + y^2 + z^2 = 1$ and $\hat{n}$ be the inward unit normal vector to S. Then $\iint_S \vec{F} \cdot \hat{n}\, dS$ is equal to

(A) 4π

(B) -4π

(C) 8π

(D) -8π

A

6. Let A be a 3×3 matrix with eigenvalues $1, -1$ and 3. Then

(A) $A^2 + A$ is non-singular

(B) $A^2 - A$ is non-singular

(C) $A^2 + 3A$ is non-singular

(D) $A^2 - 3A$ is non-singular

7. Let $T : \boldsymbol{R}^3 \longrightarrow \boldsymbol{R}^3$ be a linear transformation and I be the identity transformation of $\boldsymbol{R}^3$. If there is a scalar c and a non-zero vector $x \in \boldsymbol{R}^3$ such that $T(x) = cx$, then rank$(T - cI)$

(A) cannot be 0

(B) cannot be 1

(C) cannot be 2

(D) cannot be 3

8. In the group $\{1, 2, \ldots, 16\}$ under the operation of multiplication modulo 17, the order of the element 3 is

(A) 4

(B) 8

(C) 12

(D) 16

9. A ring R has maximal ideals

(A) if R is infinite

(B) if R is finite

(C) if R is finite with at least 2 elements

(D) only if R is finite

A

10. The integral $\int_0^1 \left[\int_0^{1-z}\left(\int_0^2 dx\right)dy\right]dz$ is equal to

(A) $\int_0^1 \left[\int_0^{1-y}\left(\int_0^2 dx\right)dz\right]dy$

(B) $\int_0^1 \left[\int_0^{1-y}\left(\int_0^1 dx\right)dz\right]dy$

(C) $\int_0^2 \left[\int_0^2\left(\int_0^{1-z} dx\right)dz\right]dy$

(D) $\int_0^2 \left[\int_0^2\left(\int_0^{1-y} dx\right)dz\right]dy$

11. Let $f:\boldsymbol{R}\longrightarrow\boldsymbol{R}$ be continuous and $g,h:\boldsymbol{R}^2\longrightarrow\boldsymbol{R}$ be differentiable. Let $F(u,v)=\int\limits_v^u f(t)dt$, where $u=g(x,y)$ and $v=h(x,y)$. Then $\frac{\partial F}{\partial x}+\frac{\partial F}{\partial y}=$

(A) $f(g(x,y))\left[\frac{\partial g}{\partial x}+\frac{\partial g}{\partial y}\right]-f(h(x,y))\left[\frac{\partial h}{\partial x}+\frac{\partial h}{\partial y}\right]$

(B) $f(h(x,y))\left[\frac{\partial g}{\partial x}+\frac{\partial g}{\partial y}\right]-f(g(x,y))\left[\frac{\partial h}{\partial x}-\frac{\partial h}{\partial y}\right]$

(C) $f(h(x,y))\left[\frac{\partial g}{\partial x}+\frac{\partial g}{\partial y}\right]+f(g(x,y))\left[\frac{\partial h}{\partial x}-\frac{\partial h}{\partial y}\right]$

(D) $f(g(x,y))\left[\frac{\partial g}{\partial x}-\frac{\partial g}{\partial y}\right]+f(g(x,y))\left[\frac{\partial h}{\partial x}-\frac{\partial h}{\partial y}\right]$

A

12. Let $y = f(x)$ be a smooth curve such that $0 < f(x) < K$ for all $x \in [a,b]$. Let

L = length of the curve between $x = a$ and $x = b$

A = area bounded by the curve, x-axis, and the lines $x = a$ and $x = b$

S = area of the surface generated by revolving the curve about x-axis between $x = a$ and $x = b$

Then

(A) $2\pi KL < S < 2\pi A$

(B) $S \leq 2\pi A < 2\pi KL$

(C) $2\pi A \leq S < 2\pi KL$

(D) $2\pi A < 2\pi KL < S$

13. Let $f : \boldsymbol{R} \longrightarrow \boldsymbol{R}$ be defined by $f(t) = t^2$ and let U be any non-empty open subset of $\boldsymbol{R}$. Then

(A) $f(U)$ is open

(B) $f^{-1}(U)$ is open

(C) $f(U)$ is closed

(D) $f^{-1}(U)$ is closed

A

14. Let $f : (-1, 1) \longrightarrow \boldsymbol{R}$ be such that $f^{(n)}(x)$ exists and $\left|f^{(n)}(x)\right| \leq 1$ for every $n \geq 1$ and for every $x \in (-1, 1)$. Then f has a convergent power series expansion in a neighbourhood of

(A) every $x \in (-1, 1)$

(B) every $x \in \left(\frac{-1}{2}, 0\right)$ only

(C) no $x \in (-1, 1)$

(D) every $x \in \left(0, \frac{1}{2}\right)$ only

15. Let $a > 1$ and $f, g, h : [-a, a] \longrightarrow \boldsymbol{R}$ be twice differentiable functions such that for some c with $0 < c < 1 < a$,

$f(x) = 0$ only for $x = -a, 0, a$;
$f'(x) = 0 = g(x)$ only $x = -1, 0, 1$;
$g'(x) = 0 = h(x)$ only for $x = -c, c$.

The possible relations between f, g, h are

(A) $f = g'$ and $h = f'$

(B) $f' = g$ and $g' = h$

(C) $f = -g'$ and $h' = g$

(D) $f = -g'$ and $h' = f$

A

CORE SECTION

16. (a) Solve the initial value problem

$$\frac{d^2y}{dx^2} - y = x\left(\sin x + e^x\right), \qquad y(0) = y'(0) = 1 \qquad (12)$$

(b) Solve the differential equation

$$(2y\sin x + 3y^4\sin x\ \cos x)dx - (4y^3\cos^2 x + \cos x)dy = 0 \qquad (9)$$

A

17. Let G be a finite abelian group of order n with identity e. If for all $a \in G$, $a^3 = e$, then, by induction on n, show that $n = 3^k$ for some nonnegative integer k. (21)

A

18. (a) Let $f:[a,b]\longrightarrow \boldsymbol{R}$ be a differentiable function. Show that there exist points $c_1, c_2 \in (a,b)$ such that

$$2f(c_1)f'(c_1) = f'(c_2)\,[f(a)+f(b)] \qquad (9)$$

(b) Let

$$f(x,y)=\begin{cases}(x^2+y^2)\left[\ln(x^2+y^2)+1\right] & \text{for } (x,y)\neq(0,0)\\ \alpha & \text{for } (x,y)=(0,0)\end{cases}$$

Find a suitable value for α such that f is continuous. For this value of α, is f differentiable at $(0,0)$? Justify your claim. (12)

A

19. (a) Let S be the surface $x^2+y^2+z^2=1,\ z\geq 0$. Use Stoke's theorem to evaluate

$$\int_C [(2x-y)\,dx - y\,dy - z\,dz]$$

where C is the circle $x^2+y^2=1,\ z=0$, oriented anticlockwise. (12)

(b) Show that the vector field $\vec{F}=(2xy-y^4+3)\hat{\mathrm{i}}+(x^2-4xy^3)\hat{\mathrm{j}}$ is conservative. Find its potential and also the work done in moving a particle from $(1,0)$ to $(2,1)$ along some curve. (9)

A

20. Let $T : \boldsymbol{R}^3 \longrightarrow \boldsymbol{R}^3$ be defined by $T(x,y,z) = (y+z, z, 0)$. Show that T is a linear transformation. If $v \in \boldsymbol{R}^3$ is such that $T^2(v) \neq 0$, then show that $B = \{v, T(v), T^2(v)\}$ forms a basis of $\boldsymbol{R}^3$. Compute the matrix of T with respect to B. Also find a $v \in \boldsymbol{R}^3$ such that $T^2(v) \neq 0$. (21)

A

21. (a) For each $n \in \boldsymbol{N}$, define $f_n : [-1, 1] \longrightarrow \boldsymbol{R}$ by

$$f_n(x) = \begin{cases} 4n^2 x & \text{for } x \in \left[0, \dfrac{1}{2n}\right) \\ -4n^2\left(x - \dfrac{1}{n}\right) & \text{for } x \in \left[\dfrac{1}{2n}, \dfrac{1}{n}\right) \\ 0 & \text{for } x \in \left[\dfrac{1}{n}, 1\right] \end{cases}$$

Compute $\int_0^1 f_n(x)\,dx$ for each n. Analyse pointwise and uniform convergence of the sequence of functions $\{f_n\}$. (12)

(b) Let $f : \boldsymbol{R} \longrightarrow \boldsymbol{R}$ be a continuous function with $|f(x) - f(y)| \geq |x - y|$ for every $x, y \in \boldsymbol{R}$. Is f one-one? Show that there cannot exist three points $a, b, c \in \boldsymbol{R}$ with $a < b < c$ such that $f(a) < f(c) < f(b)$. (9)

A

22. Find the volume of the cylinder with base as the disk of unit radius in the xy-plane centred at (1, 1, 0) and the top being the surface $z = \left[(x-1)^2 + (y-1)^2\right]^{3/2}$. (12)

A

OPTIONAL SECTION : A

23. (a) Bag A contains 3 white and 4 red balls, and bag B contains 6 white and 3 red balls. A biased coin, twice as likely to come up heads as tails, is tossed once. If it shows head, a ball is drawn from bag A, otherwise, from bag B. Given that a white ball was drawn, what is the probability that the coin came up tail? (9)

(b) Let the random variables X and Y have the joint probability density function $f(x,y)$ given by

$$f(x,y) = \begin{cases} y^2 e^{-y(x+1)} & x \geq 0,\ y \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Are the random variables X and Y independent? Justify your answer. (9)

A

24. (a) Let $X_1, X_2, \ldots, X_n$ be independently identically distributed random variables (rv's) with common probability density function (pdf) $f_X(x,\theta) = \frac{1}{\theta} e^{-x/\theta}$; $x > 0$, $\theta > 0$. Obtain the moment generating function (mgf) of $\overline{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$. Also find the mgf of the rv $Y = 2n\overline{X}/\theta$. (9)

(b) Let $X_1, X_2, \ldots, X_9$ be an independent random sample from $N(2,4)$ and Y_1, Y_2, Y_3, Y_4 be an independent random sample from $N(1,1)$. Find $P(\overline{X} > \overline{Y})$, where $\overline{X}$ and $\overline{Y}$ are sample means.

[Given $P(Z > 1.2) = 0.1151$, where $Z \sim N(0,1)$] (9)

A

OPTIONAL SECTION : B

25. (a) Let $X_1, X_2, \ldots, X_n$ be a random sample from a distribution having pdf

$$f(x; x_0, \alpha) = \begin{cases} \dfrac{\alpha x_0^{\alpha}}{x^{\alpha+1}} & \text{for } x > x_0 \\ 0 & \text{otherwise} \end{cases}$$

where $x_0 > 0$, $\alpha > 0$. Find the maximum likelihood estimator of α if x_0 is known. (9)

(b) Let $X_1, X_2, \ldots, X_5$ be a random sample from the standard normal population. Determine the constant c such that the random variable

$$Y = \frac{c(X_1 + X_2)}{\sqrt{X_3^2 + X_4^2 + X_5^2}}$$

will have a t-distribution. (9)

A

26. (a) A random sample of size $n = 1$ is drawn from pdf $f_X(x,\theta) = \frac{1}{\theta} e^{-x/\theta}$; $x > 0$, $\theta > 0$. It is decided to test $H_0 : \theta = 5$ against $H_1 : \theta = 7$ based on the criterion: reject H_0 if the observed value is greater than 10. Obtain the probabilities of type I and type II errors. (9)

(b) Let $X_1, X_2, \ldots, X_n$ be a random sample from a normal population $N(\mu, \sigma^2)$. Find the best test for testing $H_0 : \mu = 0, \sigma^2 = 1$ against $H_1 : \mu = 1, \sigma^2 = 4$. (9)

A

OPTIONAL SECTION : C

27. (a) Let $f, g : \boldsymbol{R} \longrightarrow \boldsymbol{R}$ be such that for $x, y \in \boldsymbol{R}$,

$$\phi(x+iy) = e^x [f(y) + ig(y)]$$

is an analytic function. Find a differential equation of order 2 satisfied by f. (9)

(b) Compute $\int_{|z+1|=2} (2z+1) e^{\left(\sqrt{2} + 1/z\right)} \, dz$. (9)

A

28. (a) Let $f(z)$ be analytic in the whole complex plane such that for all $r > 0$,

$$\int_0^{2\pi} \left| f(re^{i\theta}) \right| d\theta \leq \sqrt{r}$$

Find $\dfrac{f^{(n)}(0)}{n!}$ for all $n \geq 0$. (9)

(b) Find all values of $\alpha \in \boldsymbol{C}$ such that $f(z) = (z + \bar{z})^2 + 2\alpha |z|^2 + \alpha(\bar{z})^2$ is analytic at some point z having non-zero real part. (9)

A

OPTIONAL SECTION : D

29. A hemispherical bowl of radius 12 cm is fixed such that its rim is horizontal. A light rod of length 20 cm with weights w and W attached to its two ends is placed inside the bowl. In equilibrium, the weight w is just touching the rim of the bowl. Find the ratio $w : W$. (18)

A

30. A uniform ladder of length $2a$ and mass m lies in a vertical plane with one end against a smooth wall, the other end being supported on a horizontal floor. The ladder is released from rest when inclined at an angle α to the horizontal. Find the inclination of the ladder to the horizontal when it ceases to touch the wall. (18)

A

OPTIONAL SECTION : E

31. (a) Estimate the error in evaluating the integral $\int_0^8 (1+x^2)e^{-x}dx$ by Simpson's $\frac{1}{3}$rd rule with spacing $h = 0.25$. (9)

(b) Using Newton-Raphson method, compute the point of intersection of the curves $y = x^3$ and $y = 8x + 4$ near the point $x = 3$, correct up to 2 decimal places.

[Round-off the first iteration up to 2 decimal places for further computation] (9)

A

32. The polynomial $p_3(x) = x^3 + x^2 - 2$ interpolates the function $f(x)$ at the points $x = -1, 0, 1$ and 2. If the data $f(3) = -14$ is added, find the new interpolating polynomial by using Newton's forward difference formula. Also find $f(2.5)$ by using Newton's backward difference formula with pivot value 3. Justify whether the value obtained will be the same if pivot value 2 is taken. (18)

Some Useful Links:

1. Free Study Materials(By P Kalika) (https://pkalika.in/2019/10/14/study-material/**)**

2. Free Maths Study Materials(Donated) (https://pkalika.in/2020/04/06/free-maths-study-materials/**)**

3. MSc Entrance Exam Que. Paper: (https://pkalika.in/2020/04/03/msc-entrance-exam-paper/**)**
[JAM(MA), JAM(MS), BHU, CUCET, ...etc]

4. PhD Entrance Exam Que. Paper: (https://pkalika.in/que-papers-collection/**)**
[CSIR-NET, GATE(MA), BHU, CUCET,IIT, NBHM, ...etc]

5. CSIR-NET Maths Que. Paper: (https://pkalika.in/2020/03/30/csir-net-previous-yr-papers/**)**
[Upto 2019 Dec]

6. Practice Que. Paper: (https://pkalika.in/2019/02/10/practice-set-for-net-gate-set-jam/**)**
[Topic-wise/Subject-wise]

(Provide your Feedbacks/Comments at maths.whisperer@gmail.com)