Practice Questions

[Practice Questions for CSIR-NET, GATE, SET, PSC, …etc]

P. Kalika & K. Munesh
(NET(JRF), GATE, SET)
Email: maths.whisperer@gmail.com

No. of Pages: 56

Practice Questions for:

- Calculus
- Number Theory
- ODE & PDE
- Integral Equation
- Calculus of Variation
- Runge-Kutta Method
- Linear Programming
- Probability & Markov Chain

(Provide Your Feedbacks/Comments at maths.whisperer@gmail.com)

Exercise

Que (1) Find all the critical points following $f(x,y)$ and classify them as maxima, minima or saddle point.

(1). $f(x,y) = x^4 + y^4 - 2(x^2+y^2) - 5$

(2). $f(x,y) = x^{36} + y^5 + 5$

(3). $f(x,y) = -x^{30} - y^4$

(4). $f(x,y) = x^3 + y^3$

(2). Suppose that $z_1, z_2, \ldots z_n$ are vertices of a regular n-gon in order. Then the minimum value of $|z-z_1|^2 + |z-z_2|^2 + \cdots + |z-z_n|^2$, $z \in \mathbb{C}$ is — ~~Can~~ obtained at ——

(Ans: a)

(a). Centroid of n-gon

(b). $z = z_i$ for some $1 \le i \le n$

(c). Outside n-gon

(d). On the boundary of n-gon.

Hint: $z = x+iy$ & $z_k = x_k + iy_k$

$\because f(x,y) = |z-z_1|^2 + |z-z_2|^2 + \cdots + |z-z_n|^2$

$= (x-x_1)^2 + (y-y_1)^2 + (x-x_2)^2 + (y-y_2)^2 + \cdots + (x-x_n)^2 + (y-y_n)^2$

then find (x,y) by $f_x = 0$ & $f_y = 0$

further, $f_{xx} = 2n$, $f_{xy} = 0$, $f_{yy} = 2n$

then

$D = f_{xx} \cdot f_{yy} - (f_{xy})^2 = 4n^2 > 0$ & $f_{xx} = 2n > 0$

Que (3) Find the min. value of $x^2+2y^2+3z^2$, subject to the constraint $x+2y+3z = 6$.

Hint $x = 6 - 2y - 3z$, put in $f(x,y,z) \to f(y,z)$

Que(4) Find the point of the plane $x+2y+3z=12$, which is nearest to the point $(1,2,3)$.

Hint: Minimize $(x-1)^2+(y-2)^2+(z-3)^2 = f(x,y,z)$

Subject to Constraint $x+2y+3z=12$

$\Downarrow$

$x = 12-2y-3z$

thus put x in $f(x,y,z) \rightarrow f(y,z)$, further solve as usual.

Que. (5) Find the point on the plane $x+2y+3z=12$, which is nearest to the origin $(0,0,0)$

Hint: minimize $x^2+y^2+z^2 = f(x,y,z)$

Subject to $x+2y+3z=12 \Rightarrow x = 12-2y-3z$

Putting x, $f(x,y,z) \rightarrow f(y,z) = (12-2y-3z)^2+y^2+z^2$

Next $f_x = = 0$, $f_y = 0$ gives $\cancel{x} = 12/7$, $z = 18/7$

(P. Kalika Notes)

Ans (3). $(1,1)$, min. value 6 (4). $(6/7,$ (5). $(6/7, 12/7, 18/7)$

Ans (1) (a). $(0,0)$, $(1,0)$, $(-1,0)$ (b). $(0,0)$ (c) $(0,0)$ (d) $(0,0)$.

(2). $x = \frac{x_1+x_2+\cdots+x_n}{n}$, $y = \frac{y_1+y_2+\cdots+y_n}{n}$, $z = \frac{z_1+z_2+\cdots+z_n}{n}$

Exercise (max. & min.)

Que(1). Find the minimum value of $2x^2+3y^2+5z^2$ subject to the constraint $2x+3y+5z=10$.

(2). Find the point on $4x^2+3y^2+4y-6x+11$, Nearest to the origin.

(3) Find the min. & max. value of $(x-11)^2+(y-12)^2+(z-13)^2$ subject to constraint $(x-6)^2+(y-7)^2+(z-8)^2=1$.

(4). Find the min. value of $x^2+y^2+z^2$ subject to constraint $x+y+z=1$.

(5) Find the min. & max. value of $x+y+z$ subject to constraint $x^2+y^2+z^2=1$.

(6). Find the maximum value of $2x^2+3y^2+5z^2$, when given that $3x+7y+11z=1$.

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(6). pt. $\left(\frac{45}{1351}, \frac{70}{1351}, \frac{66}{1351}\right)$, Value $= \dfrac{2(45)^2+3(70)^2+5(66)^2}{(1351)^2}$

Ans. (1). pt. $(1,1,1)$, value $=10$ (2). pt. $\left(\frac{6\lambda}{2+8\lambda}, \frac{-4\lambda}{2+6\lambda}\right)$

(3). min. $(5\sqrt{3}-1)^2$, max. $(5\sqrt{3}+1)^2$ (4). $1/3$ (5). min. $-\sqrt{3}$ max. $\sqrt{3}$

(P. Kalika Notes)

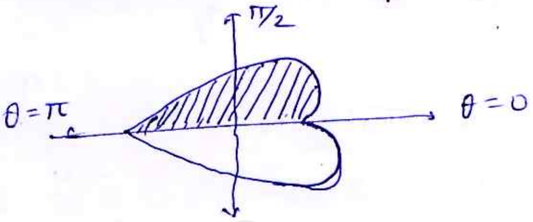
Exercise (Assignment)

Que(1). Find the area b/w the curve $y^2=4x$ & $x^2=4y$.

Ans: $A=\int_0^4\left(2\sqrt{x}-\frac{x^2}{4}\right)dx = 16/3$

Que(2). Find the volume of solid generated by revolving the cardiac $r=a(1-\cos\theta)$ about the line $y=0$.

Hint: $V=\int_{\theta_1}^{\theta_2}\frac{2\pi}{3}r^3\sin\theta\,d\theta$
$=\frac{8\pi a^3}{3}$



Que(3). Find the circumfrence of a triangle whose vertices are (0,0), (1,0) and (1,1).

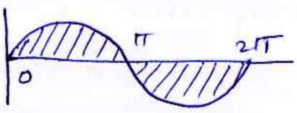
Ans: $2+\sqrt{2}$

Que(4). Find arc length of the curve which encloses region b/w parabola $y^2=x$ & $x^2=y$

Ans: $l=\int_0^1\sqrt{1+4x^2}\,dy =$
$=2\left[\frac{\sqrt{5}}{4}+\frac{1}{8}\log\left(1+\frac{\sqrt{5}}{2}\right)-\frac{1}{8}\log 2\right]$.

Que(5). Find area enclosed b/w curve $y=\sin x$ and x-axis, when x-varies 0 to 2π.

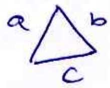
Ans:



, $A=4$.

Que(6). Find area of triangle whose sides are of length a, b and c

Ans:



, Area $=\frac{\sqrt{\left((a+b)^2-c^2\right)\left(c^2-(a-b)^2\right)}}{4}$

Que(7). Find the area of the circle with $r^2=a^2$

Ans: πa^2

Que(8). Find the sectoral Area of the curve $r=7e^{3\theta}$ b/w $\theta=0$ to π.

Ans: Area $=\int_0^{\pi}\frac{1}{2}\cdot 49\,e^{6\theta}\,d\theta=\frac{49}{12}\left[e^{6\pi}-1\right]$.

Assignment (App^n of double Integral)

(1). Using application of double integral, Find the
Ⓘ Surface area of Sphere of Radius r.

Hint: $x^2+y^2+z^2=r^2$, find, z_x, z_y, then

$$S = 2\iint_R \sqrt{1+z_x^2+z_y^2}\, dx\, dy .$$

Surface Area. $= 4\pi r^2$

Ⓘ
(2). Find the surface area of sphere $x^2+y^2+z^2=4$ lying inside cylinder $x^2+y^2=1$.

Hint: $x^2+y^2+z^2=16$, $z_x = -x/z$, $z_y = -y/z$

Surface area $S = 2\iint_R \frac{4}{z}\, dx\, dy = 16(4-\sqrt{15}) \cdot 2\pi$

Ⓘ
③ Find the Volume of Cone, whose radius is r and height is h.

Hint: $\frac{r}{\sqrt{x^2+y^2}} = \frac{h-z}{z} = \frac{h}{z} - 1 \Rightarrow z = \frac{h\sqrt{x^2+y^2}}{r+\sqrt{x^2+y^2}}$

$$\text{Vol.} = \iint_{R: x^2+y^2 \le a} z\, dx\, dy = \iint_R f(x,y)\, dy\, dx$$

$= 2$

(4). Find the Volume enclosed by plane $z=0$ & $x^2+y^2=4-z$. Also find curved surface area

Hint: Vol. $= 8\pi$ $(z = 4-x^2-y^2)$

Curved surface area $=$ 1.

* * *

Exercise (stoke's & Gauss Theorem)

Q.(1) Evaluate $\oint_C (y\,dx + z\,dy + x\,dz)$ where C is the curve of intersection of $x^2+y^2+z^2 = a^2$ & $x+z = a$

Ans: $= \int_S (\vec{\nabla} \times \vec{F}).\vec{n}\,ds = \frac{-\pi a^2}{\sqrt{2}}$

Q.(2). The value of $\int_C \vec{F}.d\vec{r}$, where $\vec{F} = -y^3\hat{i} + x^3\hat{j}$ & C is the curve ~~(~~ circular disc $x^2+y^2 \leq 1$, $z=0$ is ______

Ans: $\int_C \vec{F}.d\vec{r} = \int_S (\vec{\nabla} \times \vec{F}.)\,\vec{n}\,ds = \frac{3\pi}{2}$

Q.(3). Evaluate $\int_S \vec{\nabla} \times \vec{F}.\vec{n}\,ds$, where $\vec{F} = 4x^2z\,\hat{i} - (yz-7)\hat{j} + xy^2z\,\hat{k}$ and S is the surface b.dd by $y^2+z^2 = 25$ & $x=0$ to $x=2$.

Ans: $\int_S (\vec{\nabla} \times \vec{F})\vec{n}\,ds = \int_V \mathrm{div}\,(\vec{\nabla} \times \vec{F})\,dV = 0$

Que(4). Evaluate $\int_S \vec{F}.\vec{n}\,ds$ where $\vec{F} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ taken over the region b.dd by $x^2+y^2 = 4$ & $z=0$ to $z=3$.

Ans: $\int_S \vec{F}.\vec{n}\,ds = \int_V \vec{\nabla}.\vec{F}\,dv = 84\pi$

Q(5). The value of $\int_S \vec{F}.\vec{n}\,ds$, where $\vec{F} = 4x^2\hat{i} - 3y\hat{j} + 8xz\hat{k}$ & S is a surface $0 \leq x \leq 1$, $0 \leq y \leq 2$ & $0 \leq z \leq 3$ is ___ Ans = 30

P. Kalika Notes
(Calculus)

Q.(6). Evaluate $\int_S (x^2+2y^2+3z^2)\,ds$, where S is the surface $x^2+y^2+z^2=1$

Ans: $\int_S \vec{F}\,ds = \iiint_V \vec{\nabla}\cdot\vec{F}\,dV = 8\pi$

Q.(7). The value of $\int_S (x\,dy\,dz + y\,dx\,dz + z\,dx\,dy)$, where S is a surface

(i). of sphere $x^2+y^2+z^2=9$ Ans: $4\cdot 3^3\pi$

(ii). Cylinder $x^2+z^2=16$, $y=0$ to 3 Ans: $4^2\cdot 3^2\pi$

Q.(8). Evaluate $\int_V \nabla\cdot\vec{F}\,dv$, where $\vec{F}=(2x^2-3z)\hat{i}-2xy\hat{j}-4x\hat{k}$ and V is the region b.dd by the planes $x=0$, $y=0$, $z=0$ & $2x+2y+z=4$,

Ans: $8/3$

Q.(9). The volume of the object expressed in spherical Coordinate $V=\int_0^{2\pi}\int_0^{\pi/2}\int_0^1 r^2\sin\theta\,dr\,d\phi\,d\theta$, then V is—

(a) $\pi/3$ (b). $\pi/6$ (C). $\pi/2$ (d). $\pi/4$

Ans: (a)

Q.(10) Find the volume of cylinder, whose radius is r and height is h.

Ans: $V=\pi r^2 h$

Q.(11). The value of $\int_S \vec{F}\cdot\vec{n}\,ds$, where $\vec{F}=4xz\hat{i}-y^2\hat{j}+yz\hat{k}$ & S is surface of the cube b.dd by $x=0, x=1$ $y=0, y=1, z=0$ & $z=1$ is —

Ans: $3/2$

P.KALIKA

Assignment (Green's Theorem)

(1). Evaluate $\oint_C x^2 y\,dx + y^2 x\,dy$, where C is closed triangle formed with points (0,0), (1,0) & (1,1).

(2). Evaluate $\oint_C (x^2+y^2+xy)dx + (2x^2+3y^2)dy$, $C: x^2+y^2=1$

(3). If C is triangle with vertices (0,0), (2,0) & (1,1). then evaluate $\oint_C (x^2+xy)dx + (y^2+2xy)dy$.

(4). If C is contour of semi circular arc along with its diameter with centre at (0,0) & radius 4 unit in upper half plane. Then evaluate —

(a). $\oint_C x^2 dy + xy\,dx$

(b). $\oint_C (y^3+x^3)dy + (3x^2y+x^3)dx$

(5). Find $\oint_C (3x^2-8y^2)dx + (4y-6xy)dy$, where C is a curve bounded by $y=\sqrt{x}$ & $y=x^2$.

(6). The value of integral $\oint x\,dy - y\,dx$ along closed path $x^2+y^2=1$ is ——

(a). π (b). 2π (c). 4π (d). 8π

Ans: (1). $-1/6$ (2). 0 (3) $-1/3$ (4)(a). 0 (b). 0 (5). $3/2$
(6). b (7).

Number Theory Practice Problems

1. **NET JUNE 2019 (C):** Let $a \in \mathbb{Z}$ be such that $a = b^2 + c^2$, where $b, c \in \mathbb{Z} \setminus \{0\}$. Then a cannot be written as
   (a) pd^2, where $d \in \mathbb{Z}$ and p is a prime with $p \equiv 1 \pmod 4$
   (b) pd^2, where $d \in \mathbb{Z}$ and p is a prime with $p \equiv 3 \pmod 4$
   (c) pqd^2, where $d \in \mathbb{Z}$ and p, q are primes with $p \equiv 1 \pmod 4$, $q \equiv 3 \pmod 4$
   (d) pqd^2, where $d \in \mathbb{Z}$ and p, q are primes with $p, q \equiv 3 \pmod 4$

2. **NET DEC 2018 (B):** Given integers a and b, let $N_{a,b}$ denote the number of positive integers $k < 100$ such that $k \equiv a \pmod 9$ and $k \equiv b \pmod{11}$. Then which of the following statements is correct?
   (a) $N_{a,b} = 1$ for all integers a and b.
   (b) There exists integers a and b satisfying $N_{a,b} > 1$.
   (c) There exists integers a and b satisfying $N_{a,b} = 0$.
   (d) There exists integers a and b satisfying $N_{a,b} =$ and there exists integers c and d satisfying $N_{c,d} > 1$.

3. **NET DEC 2017 (B):** Let $f : (\mathbb{Z}/4\mathbb{Z}) \times (\mathbb{Z}/6\mathbb{Z})$ be the function $f(n) = (n \mod 4, n \mod 6)$. Then
   (a) $(0 \mod 4, 3 \mod 6)$ is in the image of f
   (b) $(a \mod 4, b \mod 6)$ is in the image of f for all even integers a and b
   (c) image of f has exactly 6 elements
   (d) kernel of $f = 24\mathbb{Z}$

4. **NET JUNE 2017 (A):** What is the remainder when 3^{256} is divided by 5?
   (a) 1. (b) 2. (c) 3. (d) 4.

5. **NET JUNE 2017 (B):** Let S be the set of all integers from 100 to 999 which are neither divisible by 3 nor by 5. The number of elements in S is
   (a) 480. (b) 420. (c) 360. (d) 240.

6. **NET JUNE 2017 (B):** The remainder obtained when 16^{2016} is divided by 9 equals
   (a) 1. (b) 2. (c) 3. (d) 7.

7. **NET DEC 2016 (B):** Given a natural number $n > 1$ such that $(n-1)! \equiv -1 \pmod n$, we can conclude that
   (a) $n = p^k$ where p is prime, $k > 1$.
   (b) $n = pq$ where p and q are distinct primes.
   (c) $n = pqr$ where p, q, r are distinct primes.
   (d) $n = p$ where p is a prime.

8. **NET JUNE 2016 (B):** Which of the following statements is FALSE? There exists an integer x such that
   (a) $x \equiv 23 \pmod{1000}$ and $x \equiv 45 \pmod{6789}$.
   (b) $x \equiv 23 \pmod{1000}$ and $x \equiv 54 \pmod{6789}$.
   (c) $x \equiv 32 \pmod{1000}$ and $x \equiv 54 \pmod{9876}$.
   (d) $x \equiv 32 \pmod{1000}$ and $x \equiv 44 \pmod{9876}$.

9. **NET DEC 2015 (C):** Which of the following intervals contains an integer satisfying the following three congruences:
   $x \equiv 2 \pmod 5$, $x \equiv 3 \pmod 7$ and $x \equiv 4 \pmod{11}$.
   (a) [401, 600]. (b) [601, 800]. (c) [801, 1000]. (d) [1001, 1200].

10. **NET JUNE 2015 (C):** Which of the following primes satisfy the congruence $a^{24} \equiv 6a + 2 \mod 13$?
    (a) 41. (b) 47. (c) 67. (d) 83.

11. **NET JUNE 2014 (B):** If n is a positive integer such that sum of all positive integers a satisfying $1 \leq a \leq n$ and $\text{GCD}(a, n) = 1$ is equal to $240n$, then the number of summands, namely, $\phi(n)$, is
    (a) 120. (b) 124. (c) 240. (d) 480.

12. **NET JUNE 2014 (C):** For positive integers m and n, let $F_n = 2^{2^n} + 1$ and $G_m = 2^{2^m} - 1$. Which of the following are true?
    (a) F_n divides G_m whenever $m > n$.
    (b) $\gcd(F_n, G_m) = 1$ whenever $m \neq n$.
    (c) $\gcd(F_n, F_m) = 1$ whenever $m \neq n$.
    (d) G_m divides F_n whenever $m < n$

13. **NET DEC 2013 (B):** For any integers a, b, let $N_{a,b}$ denote the number of positive integers $x < 1000$ such that $x \equiv a \pmod{27}$ and $x \equiv b \pmod{37}$. Then,
    (a) There exists a, b such that $N_{a,b} = 0$.
    (b) For all a, b, $N_{a,b} = 1$.
    (c) For all a, b, $N_{a,b} > 1$.
    (d) There exists a, b such that $N_{a,b} = 1$ and there exists a, b such that $N_{a,b} = 2$.

14. **NET JUNE 2013 (A):** What is the last digit of 7^{73}?
    (a) 7. (b) 9. (c) 3. (d) 1.

15. **NET JUNE 2013 (C):** Consider the congruence $x^n \equiv 2 \pmod{13}$. This congruence has a solution for x if
    (a) $n = 5$. (b) $n = 6$. (c) $n = 7$. (d) $n = 8$.

16. **NET DEC 2012 (B):** The last two digits of 7^{81} are
    (a) 07. (b) 17. (c) 37. (d) 47.

17. **NET DEC 2012 (C):** For positive integers m, let $\phi(m)$ denote the number of integer k such that $1 \leq k \leq n$ and GCD$(k, m) = 1$. Then which of the following statements are necessarily true?
   (a) $\phi(n)$ divides n for every positive integer n.
   (b) n divides $\phi\big(a^n - 1\big)$ for all positive integers a and n.
   (c) n divides $\phi\big(a^n - 1\big)$ for all positive integers a and n such that GCD$(a, n) = 1$.
   (d) a divides $\phi\big(a^n - 1\big)$ for all positive integers a and n such that GCD$(a, n) = 1$.

18. **NET JUNE 2012 (B):** The last digit of $(38)^{2011}$ is
   (a) 6. (b) 2. (c) 4. (d) 8.

19. **NET JUNE 2012 (B):** The number of positive divisors of 50000 is
   (a) 20. (b) 30. (c) 40. (d) 50.

20. **NET JUNE 2011 (B):** The number of elements in the set $\{m \mid 1 \leq m \leq 1000, m$ and 1000 are relatively prime$\}$ is
   (a) 100. (b) 250. (c) 300. (d) 400.

21. **NET JUNE 2011 (B):** The unit digit of 2^{100} is
   (a) 2. (b) 4. (c) 6. (d) 8.

22. **NBHM MSc 2018:** What is the highest power of 3 dividing 1000!?

P.Kalika

Number Theory Practice Set -1

PAGE :
DATE : / /

Instruction :- Try to solve these Questions in in less time as much as possible (max. 30 min.)

MCQ

1

(1) W.o.t.f Intervals contains an integers satisfying the following three congruences.

$x \equiv 2 \pmod 5$

$x \equiv 3 \pmod 7$

$x \equiv 4 \pmod{11}$

(a) [400, 600] (c). [801, 1000]
(b). [601, 800] (d). [1001, 1200]

(NET_2015 Dec)

(2). W.o.t.f primes satisfy the congruence

$a^{24} \equiv 6a + 2 \pmod{13}$

(a). 41 (b). 47 (c). 67 (d). 83

(NET- 2015 June)

(3)

We are given a class of 4-Boys & 4-girls. A Committee that consists of a President, a Vice-President and a Secretary is to be chosen among the 8-students of the class. Let a denote the no. of ways of choosing the committee in such a way that the committee has at least one boy and at least one girl. Let b denote the no. of ways that no. of girls is greater than or equal to that of the boys. then ___

(a) a = 288 (b). b = 168 (c). a = 144 (d). b = 192

(NET- 2014 June)

Hint: (Permutation & Combination)

PAGE :
DATE : / /

MCQ

(4). Let x & y are the values of remainder when 2^{100} is divided by 11 and 2^{105} is divided by 11. respectively. Then w.o.t.f is true ?

(a). $\phi(x+y) = 10$ (b). $\phi(xy) = 6$
(c) $\phi(xy) = 4$ (d). $\phi(x+y) = \phi(x) + \phi(y)$.

(5). Consider the congruence $x^n \equiv 2 \pmod{13}$. This congruence has a solution for x. if —

(a) $n=5$ (b). $n=6$ (c). $n=7$ (d). $n=8$

(NET-2013 June)

(6). Which of the following (w.o.t.f) is/are true?

(a). $5^6 \equiv 1 \pmod 7$
(b). $2^{\phi(7)} \equiv 1 \pmod 7$
(c). $5^7 \equiv 5 \pmod 7$
(d). $5^{\phi(7)} \equiv 5 \pmod 7$

(7). Let a, b be integers, not both zero, then w.o.t.f is/are true ?

(a) If $\gcd(a,b) = 1$, then $\gcd(a+b, a-b) = 1$ or 2.
(b). If $\gcd(a,4) = 2 = \gcd(b,4)$, then $\gcd(a+b,4) = 4$
(c). $\gcd(a,b) = \gcd(a, b+ac)$ for any integer.
(d) $\gcd(a^2, b^2) = \gcd(a,b)$.

(Number Theory Practice Test)
— P. Kalika

CSIR-NET, Number Theory
Practice Set-1

PAGE :
DATE : / /

(8). If d is a positive integers s.t $d \mid (13n+6)$, $d \mid (12n+5)$ for some integers n, then possible value of d is/are ——

(a). 1 (b) 3 (c) 5 (d) 7. (e) None

(9). How many digits are there in $2^{17} \times 3^{2} \times 5^{14} \times 7$?

(a) 14 (b) 15 (c) 16 (d). 17

(NET - 2014 June)

(10). W.o.t.f is/are perfect square ?

(a) 1022121 (b). 2042122
(c). 3063126 (d). 4083128

(NET - 2014 Dec)

(11). We define a function f(N) = sum of digit of N, expressed as decimal number. (eg. f(137) = 1+3+7 = 11. Then Evaluate $f(2^7 3^5 5^6) =$ ——

(a). 10 (b). 18 (c). 28 (d). 11

(NET 2014 Dec).

Answer key

(1). a, b, d (2). a, c (3). a, b (4). a, c
(5). a, c (6). a, b, c (7). a, b, c (8). a, d
(9). d (10). a (11). b.

CSIR-NET ODE-PDE Practice Questions

ODE and PDE Practice Questions

1.

Consider a boundary value problem (BVP) $\frac{d^2y}{dx^2} = f(x)$ with boundary conditions $y(0) = y(1) = y'(1)$, where f is a real-valued continuous function on $[0, 1]$. Then which of the following are true?

1. the given BVP has a unique solution for every f
2. the given BVP does not have a unique solution for some f
3. $y(x) = \int_0^x x\,t\,f(t)dt + \int_x^1 (t - x + xt)f(t)dt$ is a solution of the given BVP
4. $y(x) = \int_0^x (x - t + xt)f(t)dt + \int_x^1 xtf(t)dt$ is a solution of the given BVP

2.

Consider the ODE on $\mathbb{R}$ $y'(x) = f(y(x))$. If f is an even function and y is an odd function, then

1. $-y(-x)$ is also a solution.
2. $y(-x)$ is also a solution.
3. $-y(x)$ is also a solution.
4. $y(x)\,y(-x)$ is also a solution.

3.

Consider the Lagrange equation $x^2 \frac{\partial z}{\partial x} + y^2 \frac{\partial z}{\partial y} = (x + y)z$. Then the general solution of the given equation is

1. $F\left(\frac{xy}{z}, \frac{x-y}{z}\right) = 0$ for an arbitrary differentiable function F
2. $F\left(\frac{x-y}{z}, \frac{1}{x} - \frac{1}{y}\right) = 0$ for an arbitrary differentiable function F
3. $z = f\left(\frac{1}{x} - \frac{1}{y}\right)$ for an arbitrary differentiable function f
4. $z = xy\,f\left(\frac{1}{x} - \frac{1}{y}\right)$ for an arbitrary differentiable function f

4.

Consider the system of ODE in $\mathbb{R}^2$, $\frac{dY}{dt} = AY, Y(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, t > 0$ where $A = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}$ and $Y(t) = \begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix}$. Then

1. $y_1(t)$ and $y_2(t)$ are monotonically increasing for $t > 0$.
2. $y_1(t)$ and $y_2(t)$ are monotonically increasing for $t > 1$.
3. $y_1(t)$ and $y_2(t)$ are monotonically decreasing for $t > 0$.
4. $y_1(t)$ and $y_2(t)$ are monotonically decreasing for $t > 1$.

5.

The PDE $\frac{\partial^2 u}{\partial x^2} + 2\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = x$, has

1. only one particular integral.
2. a particular integral which is linear in x and y.
3. a particular integral which is a quadratic polynomial in x and y.
4. more than one particular integral.

6.

Let $y : [0, \infty) \to [0, \infty)$ be a continuously differentiable function satisfying $y(t) = y(0) + \int_0^t y(s)ds$ for $t \geq 0$. Then

1. $y^2(t) = y^2(0) + \int_0^t y^2(s)ds$.
2. $y^2(t) = y^2(0) + 2\int_0^t y^2(s)ds$.
3. $y^2(t) = y^2(0) + \int_0^t y(s)ds$.
4. $y^2(t) = y^2(0) + \left(\int_0^t y(s)ds\right)^2 + 2y(0)\int_0^t y(s)ds$.

CSIR-NET ODE-PDE Practice Questions

7. Let $a, b \in \mathbb{R}$ be such that $a^2 + b^2 \neq 0$. Then the Cauchy problem

$a\dfrac{\partial u}{\partial x} + b\dfrac{\partial u}{\partial y} = 1;\ x, y \in \mathbb{R}$

$u(x,y) = x$ on $ax + by = 1$

1. has more than one solution if either a or b is zero
2. has no solution
3. has a unique solution
4. has infinitely many solutions

8.

Let $y : [0,\infty) \to [0,\infty)$ be a continuously differentiable function satisfying

$y(t) = y(0) + \int_0^t y(s)ds$ for $t \geq 0$.

Then

1. $y^2(t) = y^2(0) + \int_0^t y^2(s)ds.$
2. $y^2(t) = y^2(0) + 2\int_0^t y^2(s)ds.$
3. $y^2(t) = y^2(0) + \int_0^t y(s)ds.$
4. $y^2(t) = y^2(0) + \left(\int_0^t y(s)ds\right)^2 + 2y(0)\int_0^t y(s)ds.$

9.

. Consider the boundary value problem

$-u''(x) = \pi^2 u(x)\ ;\ x \in (0, 1)$

$u(0) = u(1) = 0.$

If u and u' are continuous on $[0, 1]$, then

1. $u'^2(x) + \pi^2 u^2(x) = u'^2(0)$
2. $\int_0^1 u'^2(x)dx - \pi^2\int_0^1 u^2(x)dx = 0$
3. $u'^2(x) + \pi^2 u^2(x) = 0$
4. $\int_0^1 u'^2(x)dx - \pi^2\int_0^1 u^2(x)dx = u'^2(0)$

10. . The solution of the initial value problem

$$(x - y)\frac{\partial u}{\partial x} + (y - x - u)\frac{\partial u}{\partial y} = u,$$

$u(x, 0) = 1$, satisfies

1. $u^2(x - y + u) + (y - x - u) = 0.$
2. $u^2(x + y + u) + (y - x - u) = 0.$
3. $u^2(x - y + u) - (x + y + u) = 0.$
4. $u^2(y - x + u) + (x + y - u) = 0.$

11.

Let $y(x)$ be a continuous solution of the initial value problem

$y' + 2y = f(x),\quad y(0) = 0,$

where $f(x) = \begin{cases} 1, & 0 \leq x \leq 1 \\ 0, & x > 1 \end{cases}$

Then $y\left(\frac{3}{2}\right)$ is equal to

1. $\frac{\sinh(1)}{e^3}$
2. $\frac{\cosh(1)}{e^3}$
3. $\frac{\sinh(1)}{e^2}$
4. $\frac{\cosh(1)}{e^2}$

12.

Let X, Y be independent random variables and let $Z = \frac{X-Y}{2} + 3$. If X has characteristic function φ and Y has characteristic function ψ, then Z has characteristic function θ where

1. $\theta(t) = e^{-i3t}\varphi(2t)\psi(-2t).$
2. $\theta(t) = e^{i3t}\varphi\left(\frac{t}{2}\right)\psi\left(-\frac{t}{2}\right).$
3. $\theta(t) = e^{-i3t}\varphi\left(\frac{t}{2}\right)\psi\left(\frac{t}{2}\right).$
4. $\theta(t) = e^{-i3t}\varphi\left(\frac{t}{2}\right)\psi\left(\frac{-t}{2}\right).$

CSIR-NET ODE-PDE Practice Questions

13. The initial value problem $y' = 2\sqrt{y},\ y(0) = a$, has

1. a unique solution if $a < 0$
2. no solution if $a > 0$
3. infinitely many solutions if $a = 0$
4. a unique solution if $a \geq 0$

14.

Consider the initial value problem

$$\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0, \quad u(0, y) = 4e^{-2y}.$$

Then the value of $u(1,1)$ is

1. $4e^{-2}$
2. $4e^{2}$
3. $2e^{-4}$
4. $4e^{4}$

15.

Let u(x, t) satisfy the wave equation

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}; x \in (0, 2\pi), t > 0$$

$u\,(x, 0) = e^{i\omega x}$

for some $\omega \in \mathbb{R}$. Then

1. $u(x,t) = e^{i\omega x}\, e^{i\omega t}$.
2. $u(x,t) = e^{i\omega x}\, e^{-i\omega t}$.
3. $u(x,t) = e^{i\omega x}\left(\frac{e^{i\omega t} + e^{-i\omega t}}{2}\right)$.
4. $u(x,t) = t + \frac{x^2}{2}$.

16 Let u(x, y) be the solution of the equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, which tends to zero as $y \to \infty$ and has the value $\sin x$ when $y = 0$. Then

1. $u = \sum_{n=1}^{\infty} a_n \sin(nx + b_n)e^{-ny}$, where a_n are arbitrary and b_n are non-zero constants.
2. $u = \sum_{n=1}^{\infty} a_n \sin(nx + b_n)e^{-n^2 y}$, where $a_1 = 1$ and a_n $(n > 1)$, b_n are non-zero constants.
3. $u = \sum_{n=1}^{\infty} a_n \sin(nx + b_n)e^{-ny}$, where $a_1 = 1$, $a_n = 0$ for $n > 1$ and $b_n = 0$ for $n \geq 1$.
4. $u = \sum_{n=1}^{\infty} a_n \sin(nx + b_n)e^{-n^2 y}$, where $b_n = 0$ for $n \geq 0$ and a_n are all nonzero.

17.

Consider the differential equation

$$\frac{d^2y}{dx^2} - 2\tan x\,\frac{dy}{dx} - y = 0$$

defined on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Which among the following are true?

1. there is exactly one solution $y = y(x)$ with $y(0) = y'(0) = 1$ and $y\left(\frac{\pi}{3}\right) = 2\left(1 + \frac{\pi}{3}\right)$
2. there is exactly one solution $y = y(x)$ with $y(0) = 1, y'(0) = -1$ and $y\left(-\frac{\pi}{3}\right) = 2\left(1 + \frac{\pi}{3}\right)$
3. any solution $y = y(x)$ satisfies $y''(0) = y(0)$
4. if y_1 and y_2 are any two solutions then $(ax + b)y_1 = (cx + d)y_2$ for some $a, b, c, d \in \mathbb{R}$

CSIR-NET ODE-PDE Practice Questions

18. For $f \in C[0,1]$ and $n > 1$,

let $T(f) = \frac{1}{n}\left[\frac{1}{2}f(0) + \frac{1}{2}f(1) + \sum_{j=1}^{n-1} f\left(\frac{j}{n}\right)\right]$ be an approximation of the integral $I(f) = \int_0^1 f(x)dx$. For which of the following functions f is $T(f) = I(f)$?

1. $1 + \sin 2\pi nx$
2. $1 + \cos 2\pi nx$
3. $\sin^2 2\pi nx$
4. $\cos^2 2\pi(n+1)x$

19.

Let $B = \{(x_1, x_2) \in \mathbb{R}^2 \,|\, x_1^2 + x_2^2 < 1\}$, and let $C_{Id}^2(\bar{B};\mathbb{R}^2) = \{u \in C^2(\bar{B};\mathbb{R}^2) \,|\, u(x_1,x_2) = (x_1,x_2),\ \text{for } (x_1,x_2) \in \partial B\}$.

Let $u = (u_1, u_2)$ and define $J : C_{Id}^2(\bar{B};\mathbb{R}^2) \to \mathbb{R}$ by

$$J(u) = \int_B \left(\frac{\partial u_1}{\partial x_1}\frac{\partial u_2}{\partial x_2} - \frac{\partial u_1}{\partial x_2}\frac{\partial u_2}{\partial x_1}\right) dx_1 dx_2$$

Then,

1. $\inf\{J(u): u \in C_{Id}^2(\bar{B};\mathbb{R}^2)\} = 0$
2. $J(u) > 0$, for all $u \in C_{Id}^2(\bar{B}:\mathbb{R}^2)$
3. $J(u) = 1$, for infinitely many $u \in C_{Id}^2(\bar{B};\mathbb{R}^2)$
4. $J(u) = \pi$, for all $u \in C_{Id}^2(\bar{B};\mathbb{R}^2)$

20.

A solution of the PDE

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 - u = 0$$

represents

1. an ellipse in the *x*-*y* plane.
2. an ellipsoid in the *xyu* space.
3. a parabola in the *u*-*x* plane.
4. a hyperbola in the *u*-*y* plane.

20. Let $X = \{u \in C^1[0,1] \,|\, u(0) = 0\}$ and let $I: X \to \mathbb{R}$ be defined as

$$I(u) = \int_0^1 (u'(t)^2 - u(t)^2)dt$$

Which of the following are correct?

1. I is bounded below
2. I is not bounded below
3. I attains its infimum
4. I does not attain its infimum

Practice Questions of ODE & PDE for CSIR-NET Mathematics

-: P. Kalika & K. Munesh

Practice Problems

1. The critical point of the system

$$x'(t) = -4x - y$$
$$y'(t) = x - 2y \qquad \textbf{(NET-June 2015)}$$

   (a) Asymptotically stable Node
   (b) Unstable node
   (c) Asymptotically stable spiral
   (d) Unstable spiral **Answer: (a)**

2. Consider the system of differential equations

$$x'(t) = 2x - 7y$$
$$y'(t) = 3x - 8y \qquad \textbf{(NET-June 2018)}$$

   Then critical point $(0, 0)$ of the system is an

   (a) Asymptotically stable Node
   (b) Unstable node
   (c) Asymptotically stable spiral
   (d) Unstable spiral **Answer: (a)**

3. Then critical point $(0, 0)$ for the system

$$x'(t) = x - 2y + y^2 Sin(x)$$
$$y'(t) = 2x - 2y - 3yCos(y^2) \qquad \textbf{(NET-Dec 2018)}$$

   (a) is a Stable spiral point
   (b) is a Unstable spiral point
   (c) is a Saddle point
   (d) is a Stable node **Answer: (c)**

4. Consider the system of differential equations [1]

$$x'(t) = x + 4y - x^2$$
$$y'(t) = 6x - y + 2xy \qquad \textbf{(Practice Que.)}$$

   Then critical point $(0, 0)$ of the system is an

(a) Asymptotically stable Node

(b) Unstable saddle point

(c) Asymptotically stable spiral

(d) Unstable spiral **Answer: (b)**

5. Then critical point $(0, 0)$ for the system

$$x'(t) = Sin(x) - 4y$$
$$y'(t) = Sin(2x) - 5y$$

(**Practice Que.**)

(a) is a Stable spiral point

(b) is a Asymptotically stable Node

(c) is a Saddle point

(d) is a Stable node **Answer: (b)**

6. Consider the system of differential equations [1]

$$x'(t) = 8x - y^2$$
$$y'(t) = -6y + 6x^2$$

(**Practice Que.**)

Then critical point $(0, 0)$ of the system is an

(a) Asymptotically stable Node

(b) Asymptotically stable spiral

(c) Unstable saddle point

(d) Unstable spiral **Answer: (c)**

Hint: There are critical points (0,0) & (2,4)
At (0,0): Unstable saddle point
At (2,4): Unstable spiral point

7. Consider the systems of differential equations [1]

$$x'(t) = -y - x^2$$
$$y'(t) = x$$

and

$$x'(t) = -y - x^3$$
$$y'(t) = x$$

(**Practice Que.**)

Find all the critical point and nature of system on the each critical points.
Hint: Nature- Centre or Spiral Point

References

[1] S. Ross, *DIFFERENTIAL EQUATIONS, 3RD ED.* Wiley India Pvt. Limited, 2007.

[2] A. Jeffrey, *Advanced engineering mathematics.* Elsevier, 2001.

[3] E. Kreyszig, "Advanced engineering mathematics, 8-th edition," 1999.

Note: *Full Notes of dynamical system with non-linear will we available soon.*

Summary (Integrating factor)

Given: $M(x,y)\,dx + N(x,y)\,dy = 0$ (*), Then —

Type	Form (of given DE)	I.F.
Type-I	When M and N are homogeneous f^n of same deg.	$\dfrac{1}{Mx+Ny}$; if $Mx+Ny \neq 0$
Type-II	If $M = y f_1(xy)$ & $N = x f_2(xy)$	$\dfrac{1}{Mx-Ny}$; if $Mx-Ny \neq 0$
Type-III	If $\left(\dfrac{\partial M}{\partial y} - \dfrac{\partial N}{\partial x}\right)\cdot\dfrac{1}{N} = \phi(x)$	I.F. $= e^{\int \phi(x)\,dx}$
Type-IV	If $\left(\dfrac{\partial M}{\partial y} - \dfrac{\partial N}{\partial x}\right)\cdot\dfrac{1}{M} = \psi(y)$	I.F. $= e^{\int \psi(y)\,dy}$
Type-V	If $M\,dx + N\,dy = 0$ can be expressed as — $x^a y^b (my\,dx + nx\,dy) + x^r y^s (m_1 y\,dx + n_1 x\,dy) = 0$ Where $a, b, r, s, m, m_1, n, n_1$ are known constants	I.F. $= x^h y^k$, where h & k can be found by solving $\dfrac{a+h+1}{m} = \dfrac{b+k+1}{n}$ & $\dfrac{r+h+1}{m_1} = \dfrac{s+k+1}{n_1}$

once eq^n $M(x,y)\,dx + N(x,y)\,dy = 0$ (*) becomes exact diff^n eq^n (Check by $\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x}$)

It's general solution is given by —

$$\boxed{\underbrace{\int M(x,y)\,dx}_{\text{(treat } y \text{ as constant)}} + \underbrace{\int N\,dy}_{\text{(only those part of N, which doesn't contains terms of } x)} = C}$$

P. Kalika Notes

Exercise (Exact DE)

#. Find the general solⁿ of followings:—

(1). $x\,dx + y\,dy + \dfrac{x\,dy - y\,dx}{(x^2+y^2)} = 0$

(2). $e^{x/y}\left(1 + e^{-x/y}\right)dx + \left(e^{x/y} - \frac{x}{y}e^{x/y}\right)dy = 0$

(3). Value of a & b, for which diffⁿ eqⁿ is exact

$(3a^2x^2 + by\cos x)\,dx + (2\sin x - 4ay^3)\,dy = 0$

(a). $a=3,\ b=2$ (b). $a=2,\ b=3$

(c). $a=3,\ b=4$ (d). $a=2,\ b=5$

(4). Consider the D.E —

$2\cos(y^2)\,dx - xy\sin(y^2)\,dy = 0$, then I.F. can be —

(a) e^x (b). e^{-x} (c). $3x$ (d). x^3

(5). The eqⁿ $(\alpha x y^3 + y\cos x)\,dx + (x^2y^2 + \beta\sin x)\,dy = 0$ is exact if—

(a). $\alpha = 3/2,\ \beta = 1$ (b). $\alpha = 1,\ \beta = 3/2$

(c). $\alpha = 2/3,\ \beta = 1$ (d). $\alpha = 1,\ \beta = 2/3$.

---- Answer ----

(1). $x^2+y^2 - 2\tan^{-1}(x/y) = c_1$

(2). $x + y\cdot e^{x/y} = c$

(3). (a)

(4). (d)

(5).

[Integrating Factor] P. Kalika

Assignment (Exercise)

* Solve the following differential Equations.

(1). $x\,dx + y\,dy + \dfrac{x\,dy - y\,dx}{x^2+y^2} = 0$

(2). $(3x^2y^4 + 2xy)\,dx + x^2(2xy^3 - 1)\,dy = 0$

(3). $(2xy^4e^y + 2xy^3 + y)\,dx + (x^2y^4e^y - x^2y^2 - 3x)\,dy = 0$ (Hint: type-IV)

(4). $(x^2+y^2+2x)\,dx + 2y\,dy = 0$

(5). $(6y + 2y^3 + 3x^2)\,dx + \frac{3}{2}(1+y^2)x\,dy = 0$

(6). $(xy^2 + 2x^2y^3)\,dx + (x^2y - x^3y^2)\,dy = 0$

(7). $(2x^2y - 3y^4)\,dx + (3x^3 + 2xy^3)\,dy = 0$

(8). Given that I.F of $(x^7y^2 + 3y)\,dx + (3x^8y - x)\,dy = 0$ is $x^h y^k$ then —

(a) $h = -7,\ k = 1$.
(b) $h = 1,\ k = -7$
(c). $h = k = 0$
(d). $h = k = 1$
(e) NONE

(9) Given that $\frac{dy}{dx} - y/x = 0$. Then w.o.t.f. is/are true for I.F —

(a). $1/x^2$
(b). $1/y^2$
(c). $1/xy$
(d). $\frac{1}{x+y}$
(e). None

(10). $(x^2y^3 + xy^2 + y)\,dx + (x^3y^2 - x^2y + x)\,dy = 0$

(11) If x^3y^2 is an integrating factor of

$$(6y^2+axy)\,dx + (6xy+bx^2)\,dy = 0 \quad \text{where } a,b\in\mathbb{R},$$

Then —

$\boxed{\text{GATE 2017}}$

(A). $3a-5b=0$ (B). $2a-b=0$

(C). $3a+5b=0$ (D). $2a+b=0$

(12). An Integrating factor of eq^n. $\boxed{\text{IIT-JAM-2018}}$

$$\left(y+\tfrac{1}{3}y^3+\tfrac{1}{2}x^2\right)dx + \tfrac{1}{4}(x+xy^2)\,dy = 0 \quad \text{is —}$$

(a). x^2 (b). $3\log_e x$ (c). x^3 (d). $2\log_e x$ (e) None

(13). An integrating factor of $\boxed{\text{IIT–JAM 2005}}$

$$x\frac{dy}{dx} + (3x+1)y = xe^{-2x} \quad \text{is —}$$

(a) xe^{3x} (b). $3xe^{x}$ (c). xe^{x} (d). x^3e^{x}

(14). The sol^n of the DE $x\frac{dy}{dx} + (1+x)y = e^{-x}$ with the B.C $y(x=1)=0$ is —

(a). $\frac{x-1}{x}e^{-x}$ (b). $\frac{x-1}{x^2}e^{-x}$ (c). $\frac{1-x}{x^2}e^{-x}$ (d). $(x-1)^2e^{-x}$

$\boxed{\text{CSIR-NET 2019 June Physics}}$

Answer

(1) $x^2+y^2-2\tan^{-1}(x/y) = C_1$

(2). $(xy)^3 + x^2 = cy$

(3). $x^2e^y + x^2/y + x/y^3 = C$

(4). $x^2e^x + y^2e^x = C$

(5). $3x^4y + x^4y^3 + x^6 = C$

(6). $-\frac{1}{3xy} + \frac{2}{3}\log x - \frac{1}{3}\log y = C$

(7). $I.F = x^{-49/3} \cdot y^{-20/13}$

(8). (a).

(9). (a, b, c)

(10). $xy - \frac{1}{xy} + \log\frac{x}{y} = C$

(11). (A).

(12). (C).

(13). (a).

(14). (a)

P. Kalika Notes

Practice Problems

P. Kalika

Find the solutions of followings:

(1). $(x^2+y^2+2x)\,dx + 2y\,dy = 0$

(2). $\sec^2 y \frac{dy}{dx} + x\tan y = x^3$

(3). $\frac{\sin x}{y^2}\frac{dy}{dx} + \frac{3}{y} = \cos x$

W.o.t.f statements is/are true ?

(1). Solution of the problem $y'\cos x + y\sin x = 1,\ y(0)=0$ is—

(a). Periodic f^n (b). positive in $(0,\pi)$

(c). Monotonic ↑ in $(0,\pi/2)$ (d). Invertible in $(0,\pi)$

(2). The sol^n $\phi(x)$ of $x^2y'+2xy=1$ satisfies $\phi(2)=2\phi(1)$ then—

(a). $\phi(x)\to\infty$ as $x\to\infty$ (b). $\phi(x)\to -2$ as $x\to+\infty$

(c). $\phi(x)\to 0$ as $x\to\infty$ (d). None.

(3). D.E $y'+P(x)y = Q(x)y^n$ is a —

(a). linear DE for $n=0$ (c). Bernoulli eq^n for $n\neq 0,1$

(b). linear DE for $n=1$ (d). Bernoulli eq^n for $n=10$

(4). The value of 'a', for which y^a is an IF of DE $2xy\,dx - (2x^2-y^2)\,dy = 0$ is —

(a). -4 (b) 4 (c) -3 (d) 3

Answer

(1). $y^2 = -x^2 + ce^{-x}$

(2). $y = \tan^{-1}(x^2-2+ce^{-\frac{1}{2}x^2})$

(3). $IF = (\tan x/2)^{-3}$

Objective (Answer)

(1). a, b, c

(2). (c).

(3). All

(4). (c)

Assignment (Particular Integral)

Solve the following differential equations

1. $(D^2-1)y = 2^x + e^x \cos x$
2. $(D^2-4D+4)y = 8x^2 e^{2x} \sin 2x$
3. $(D^3+D^2-D-1)y = \cos 2x$
4. $(D^2-4D+4)y = e^{2x} \cos^2 x$ (H
5. $(D^2+4)y = x \cos x$ (Hint: $y_p = \frac{1}{9}[3x\cos x + 2\sin x]$)
6. $(D^2-2D+1)y = x e^x \sin 2x$ (Hint: $y_p = -\frac{e^x}{4}(x\sin 2x + \cos 2x)$)
7. $(D^2+1)y = \frac{1}{\sin x}$ (Hint: $y_p = \sin x \cdot \log \sin x - x\cos x$)
8. $(D^3-4D^2-3D+18)y = 0$ (Hint: $y_{cf} = c_1 e^{-2x} + (c_2+c_3x)e^{3x}$)
9.

Answer (Hint):

1. $y_p = e^x\left[\frac{-2\sin x + \cos x}{5}\right] + \frac{2^x}{(\log 2)^2 - 1}$
2. $y_p = -e^{2x}[2x^2\sin 2x + 4x\cos 2x - 3\sin 2x]$
3. $y_p = \frac{-1}{25}[2\sin 2x + \cos 2x]$
4. $y_p = \frac{1}{8}e^{2x}[2x^2 - \cos 2x]$

Practice Questions (Variation of Parameter)

①. Solve $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = x^2e^x$

(Ans: $y(x) = c_1x + c_2/x + \cancel{\frac{1}{2}x} + e^x - \frac{e^x}{x}$)

CSIR-NET

②. Using method of VOP, the particular solⁿ of the diffⁿ eqⁿ $y'' + 4y = \frac{3}{\sin 2x}$, $0 < x < \pi/2$ is —

ⓐ. $\frac{3}{4}\sin 2x \cdot \log(\sin 2x) - \frac{3}{2}\cos 2x$.

ⓑ. $\frac{3}{2}\sin 2x \cdot \log(\cos 2x) - \frac{3}{4}\cos 2x$

ⓒ. $\frac{3}{2}\sin 2x \log(\sin 2x) - \frac{3}{2}\cos 2x$

ⓓ. $\frac{3}{4}\sin 2x \cdot \log(\sin 2x) - \frac{3}{2}x\cos 2x$ (Ans: d)

CSIR-NET

③. The general solⁿ of the diffⁿ eqⁿ $y'' + y = f(x)$, $x \in (-\infty, \infty)$, where $f(x)$ is cts. real valued fⁿ on $(-\infty, \infty)$ is —

ⓐ. $y = A\cos x + B\sin x + \int_0^x f(x-t)\sin t\, dt$.

ⓑ. $y = A\cos x + B\sin x + \int_0^x f(x+t)\cos t\, dt$

ⓒ. $y = A\cos x + B\sin x + \int_0^x f(t)\sin(x-t)\, dt$

ⓓ. $y = \cos(x+k) + c\int_0^x f(t)\sin(x-t)\, dt$ (Ans: c).

CSIR-NET

Que(4). The gen. solⁿ of DE $y'' = -f(x)$, $x \in \mathbb{R}$, $f \in C^1[\mathbb{R}]$ then —

ⓐ. $y(x) = A + Bx + \int_0^x (x-t)f(t)\, dt$.

ⓑ $y(x) = Ax - \int_0^x x f(x-t)\, dt$

ⓒ. $y(x) = Ax + B + \int_0^x (x-t)f(x-t)\, dt$

ⓓ. $y(x) = A + Bx - \int_0^x (x-t)f(t)\, dt$. (Ans: d).

P. Kalika Notes

Practice Set (Existence, Uniqueness & Lipschitz) [ODE]

(1) Consider the IVP $y' = 2\sqrt{y}$, $y(0) = k$, then given problem has —

(a) Unique solⁿ if $a>0$
(b) Unique solⁿ if $a<0$
(c) Unique solⁿ if $a \geq 0$
(d) Unique solⁿ if $a=0$

(2). Find the largest interval predicted by Picard theorem for $\frac{dy}{dx} = 1 + y + y^2 \cos x$, $y(0)=0$ —

(a). $[-\frac{1}{3}, \frac{1}{3}]$
(b). $[0, \frac{1}{3}]$
(c). $[-1, 1]$
(d). None

(3). For $\lambda \in \mathbb{R}$, Consider DE $y'(x) = \lambda \sin(x + y(x))$, $y(0) = 1$. Then this IVP has —

(a) no solⁿ in any nbd of 0.
(b). a solⁿ in $\mathbb{R}$ if $|\lambda| < 1$ (NET-2016)
(c). a solⁿ in a nbd of 0.
(d). a solⁿ in $\mathbb{R}$ only if $|\lambda| > 1$

(4). Consider the IVP $y' = x y^{1/3}$, $y(0) = 0$, $(x, y) \in \mathbb{R} \times \mathbb{R}$. Then w.o.t.f are correct.?

(a). The fⁿ $f(x,y) = xy^{1/3}$ doesn't satisfy Lipschitz Condⁿ w.r.t. y in any nbd of $y=0$.
(b). $\exists$ a unique solⁿ for the IVP.
(c). $\exists$ no solution for the IVP
(d). $\exists$ more than one solⁿ for the IVP

(NET-2013)

(5). The diff^n eq^n $y' = 60(y^2)^{1/5}$, $x > 0$. $y(0) = 0$ has —

(a). a unique sol^n (b). two sol^n.
(c). No solution (d). infinitely many sol^n.

(NET-2012 Dec)

(6). Consider the IVP $y' = y^2$, $y(0) = 1$, $(x,y) \in \mathbb{R} \times \mathbb{R}$. Then $\exists$s a unique sol^n of the IVP on

(a). $\mathbb{R} = (-\infty, \infty)$ (b). $(-\infty, 1)$
(c). $(-2, 2)$ (d). $(-1, \infty)$

(NET-2013 June)

(7). Consider the IVP $y'(t) = f(t).y(t)$, $y(0) = 1$, where $f: \mathbb{R} \to \mathbb{R}$ is cts. Then the IVP has —

(a). infinitely many sol^n for some f.
(b). a unique sol^n in $\mathbb{R}$.
(c). no sol^n in $\mathbb{R}$ for some f.
(d). a sol^n in an interval containg 0 but not on $\mathbb{R}$ for some f.

(NET 2012 June)

(8). Find h for — $\left(h = \min\{a, \frac{b}{M}\}\right)$

(a). $y' = e^{-x^2} + y^2$, $y(0) = 1$
(b). $y' = (4y + e^{-x^2})e^y$, $y(0) = 0$

Ans:

(1). a
(2). a
(3). b, c
(4). a, d
(5). d
(6). b
(7). b
(8). (a) $h = \sqrt{2}/1+(\sqrt{2}+1)^2$
(b) $h = 1/8\sqrt{e}$

Exercise (EUT)

Find Solution

1. $\frac{dy}{dx} = 1 + y^2$, $y(1) = 0$

(Hint: Unique sol^n)

2. $\frac{dy}{dx} = 2y/x$, $y(0) = 0$

(Hint: IVP has infinite no. of sol^n.)

3. $\frac{dy}{dx} = \frac{2y}{x}$, $y(0) = 1$

(Hint: No sol^n.)

$\frac{dy}{dx} = \frac{2y}{x}$, $y(1) = 0$

(Hint: Unique sol^n.)

4. The IVP $\frac{dy}{dx} = y^2$, $y(0) = 1$ has unique sol^n in—

(a) $(-2, 2)$ (b). $(-3, 3)$

(c). $\mathbb{R}$ (d). $(-1/2, 1/2)$

(Hint: d)

5. The IVP $2x\frac{dy}{dx} = 3(2y-1)$, $y(0) = 1/2$ has —

(a) unique sol^n (b). finitely many sol^n.

(c). Infinitely many sol^n. (d). No. sol^n.

(Ans: c)

6. If $y'(t) = f(t).y(t)$, $y(0) = 1$, $f: \mathbb{R} \to \mathbb{R}$ is cts. Then this IVP has —

(a). infinitely many sol^n for some f.

(b). Unique sol^n in $\mathbb{R}$.

(c). No sol^n in $\mathbb{R}$ for some f.

(d). a sol^n. in an interval containing 0 but not in $\mathbb{R}$ for some f.

(Ans: d).

7. The IVP $\frac{dy}{dx} = xy^{1/3}$, $y(0)=1$ has —

(Hint: Unique sol^n by EUT).

8. If $y'(t) = f(t)$ s.t $y(0)=0$ where $f: \mathbb{R} \to \mathbb{R}$ is Cts., then —

(Ans: IVP has unique sol^n in an open interval containing 0 but not on $\mathbb{R}$)

9. If $\frac{dy}{dx} = y^{2/3}$ s.t $y(0)=0$, the IVP has —

(a) Unique sol^n

(b). No sol^n.

(c). More than one sol^n

(d). $y = \frac{1}{27}x^3$ is a sol^n.

(Ans: c, d)

Exercise (Self-Adjoint)

Que 1: Transform the following eq^n into an equivalent self-Adjoint equation.

(1). $x^2 \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} + 5y = 0$

(Hint: factor $= 1/x^4$)

(2). $(x^4+x^2) \frac{d^2y}{dx^2} + 2x^3 \frac{dy}{dx} + 3x = 0$

[Ans: $\frac{d}{dx}\left[(x^2+1) \frac{dy}{dx} \right] + \frac{3}{x^2} y = 0$]

(3). $a(x) \frac{d^2y}{dx^2} + b(x) y = 0$ [Ans: $\frac{d}{dx}\left[e^{\int \frac{b(x)}{a(x)}} \frac{dy}{dx} \right] = 0$]

Que 2. Consider the ODE $x''(t) + e^{t^2} x(t) = 0$ for $t \in [0, 3\pi]$ where $x : [0, 3\pi] \to \mathbb{R}$. Then what is cardinality of the set $\{ t \in [0, 3\pi] : x(t) = 0 \}$. (Ans: ≥ 3)

(3). Check (Prove) that the following Boundary Value Problem (BVP) is Strum-Liouville Problem.

(a). $\frac{d}{dt}\left[x(t) \frac{dy}{dt} \right] + [2t^2 + \lambda t^3] y = 0$

with conditions. $3y(1) + 4y'(1) = 0$

& $5y(2) - 3y'(2) = 0$

(b). $\frac{d^2y}{dx^2} + \lambda y = 0$ with conditions

$y(0) = 0$ & $y(\pi) = 0$

(4). Find the non-trivial sol^n of Strum-Liouville BVP

$y''(x) + \lambda y = 0$ with $y(0) = 0 = y(\pi)$.

(4-75M) NET-June 2013

Practice set (Green f^n)

1. Consider the BVP $u'' = -f$, $u(0) = u''(1) = 0$ on $[0,1]$ where $u' \equiv \frac{du}{dx}$ & $u'' = \frac{d^2u}{dx^2}$. Assume $f(x)$ is real-valued cts. f^n. on $[0,1]$. Then w.o.t.f is/are correct?

(a). The Green's f^n is $G(x,t) = \begin{cases} x & \text{for } 0 \le x \le t \\ t & \text{for } t \le x \le 1 \end{cases}$ (for the above BVP)

(b). Both G & $\frac{\partial G}{\partial x}$ are cts. on $[0,1] \times [0,1]$ with $\frac{\partial^2 G}{\partial x^2}$ having a discontinuity along $x = t$

(c). $G(x,t)$ satisfies the homo. eq^n. $u'' = 0$ and $0 \le x < t$ & $t \le x \le 1$

(d). The sol^n of the given BVP is

$$u(x) = \int_0^x t f(t)\,dt + \int_x^1 x f(t)\,dt$$

(NET-Dec-2012)

2. The sol^n of the diff^n eq^n $y'' = f(x)$, $x \in (0,1)$ $y(0) = y(1) = 0$ is given by $y(x) = \int_0^1 G(x,t) f(t)\,dt$

where —

(a). $G(x,t) = \begin{cases} x(t-1) & : x \le t \\ t(x-1) & : x \ge t \end{cases}$

(b). $G(x,t) = \begin{cases} x^2(t-1) & : x \le t \\ t^2(x-1) & : x > t \end{cases}$

(c). $G(x,t) = \begin{cases} x(t^2-1) & : x \le t \\ t(x^2-1) & : x > t \end{cases}$

(d). $G(x,t) = \begin{cases} \sin x(t-1) & : x \le t \\ \sin t(x-1) & : x \ge t \end{cases}$

NET-Dec-2011

(3). The green f^n $G(x,t)$ of the BVP $y'' - \frac{1}{x}y' = 1$ $y(0) = y(1) = 0$ is $G(x,t) = \begin{cases} f_1(x,t) & : x \le t \\ f_2(x,t) & : t \ge x \end{cases}$ where

(a). $f_1(x,t) = -\frac{1}{2}t(1-x^2)$, $f_2(x,t) = -\frac{1}{2t}x^2(1-t^2)$

(b). $f_1(x,t) = \frac{-1}{2x}t^2(1-x^2)$, $f_2(x,t) = -\frac{1}{2t}x^2(1-t^2)$

(c). $f_1(x,t) = -\frac{1}{2t}x^2(1-t^2)$, $f_2(x,t) = -\frac{1}{2}t(1-x^2)$

(d). $f_1(x,t) = -\frac{1}{2t}x^2(1-t^2)$, $f_2(x,t) = -\frac{1}{2x}t^2(1-x^2)$

2011 NET-June

(4). The Green's function $G(x,t)$, $0 \le x, t \le 1$ of the BVP $y'' + \lambda y = 0$, $y(0) = 0 = y(1)$ is ____

(a). Symmetric in x & t

(b). Continuous at $x = t$

(c). $\left.\frac{\partial G(x,t)}{\partial x}\right|_{x=t^-} - \left.\frac{\partial G(x,t)}{\partial x}\right|_{x=t^+} = -1$

(d). $\left.\frac{\partial G(x,t)}{\partial x}\right|_{x=t^-} - \left.\frac{\partial G(x,t)}{\partial x}\right|_{x=t^+} = 1$

GATE-2016

(5). The difference b/w the least two eigen-values of the BVP

$y'' + \lambda y = 0$, $0 < x < \pi$, $y(0) = y'(\pi) = 0$

is equal to ________

Exercise

Assignment Set

PAGE: 31

(21) Que:

(b) Convert the following DE to I.E ——

(1) $y''+y=0$ when $y(0)=0=y'(0)$

Ans: $y(x) = -\int_0^x (x-t)\, y(t)\, dt$

(2). $y''(x) - 3y'(x) + 2y(x) = 4\sin x$ with $y(0)=1, y'(0)=-2$

Ans: $y(x) = 1 - x - 4\sin x + \int_0^x (3-2(x-t))\, y(t)\, dt$

(3). The IVP Corresponding to the IE
GATE (2001)
$y(x) = 1 + \int_0^x y(t)\, dt$ is ——

✓ (a) $y'-y=0,\ y(0)=1$
(b) $y'+y=0,\ y(0)=0$
(c). $y'-y=0,\ y(0)=0$
(d). $y'+y=0,\ y(0)=1$

Hint: $\because y(0)=1 \Rightarrow$ op(b) & op(c) discarded.
$y'(x) = 0 + y(x) - y(0).0 = y(x) \Rightarrow y'(0) = y(0) = 1$
$\Rightarrow y' - y = 0 \Rightarrow$ op (a). TRUE

(4). I.E Corresponding to $y''(x) + y(x) = 0,\ x>0$
(NET)
$y(0)=1$ & $y'(0)=0$ is ——

× (a). $y(x) = \int_0^x (t-x)\, y(t)\, dt \Rightarrow y(0) \neq 1$

× (b) $y(x) = \int_0^x (t-x)\, y(t)\, dt - 1 \Rightarrow y(0) \neq 1$

✓ (c) $y(x) = \int_0^x (t-x)\, y(t)\, dt + 1$

(d) $y(x) = 1 - \frac{1}{2}\int_0^x (t-x)\, y(t)\, dt$.

Hint: $y''(x) + y = 0 \Rightarrow y'(x) - y'(0) = -\int_0^x y(t)\, dt$
$\Rightarrow y'(x) = -\int_0^x y(t)\, dt \Rightarrow y(x) - y(0) = -\int_0^x\int_0^x y(t)\, dt$
$\Rightarrow y(x) = 1 + \int_0^x (t-x)\, y(t)\, dt$

P. KALIKA

Practice Questions

PAGE :
DATE : / /

(5) Show that, sol^n $y(x)$ of $y''(x) = F(x)$ with I.C $y(0) = y_0$ and $y'(0) = y_0'$, gives

$$y(x) = y_0 + xy_0' + \int_0^x (x-t)F(t)\,dt$$

(6). Convert $y''(x) - 2xy'(x) - 3y(x) = 0$ with I.C $y(0) = 1$, $y'(0) = 0$ to Volterra IE of 2^{nd} kind.
Conversely, derive the original DE with I.C from the IE. (Ans: $y = 1 + \int_0^x (x+t)y(t)\,dt$)

(7) Reduce the following IVP into IE—

$$\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0, \quad y(0) = 1, \; y'(0) = 1$$

Ans: $y(x) = 1 + x - \int_0^x t\,y(t)\,dt$

(8) Show that, the sol^n of Volterra IE $y(x) = 1 + \int_0^x (t-x)\,y(t)\,dt$ satisfies the DE

$$y''(x) + y(x) = 0 \;\; \& \;\; \text{B.C } y(0) = 1, \; y'(0) = 1$$

(9). $y'' + xy = 1$, $y(0) = y'(0) = 0$

Ans: $u(x) = 1 - \int_0^x x(x-t)u(t)\,dt$, $u(x) = y''$.

(10). $y'' - 5y' + 6y = 0$, $y(0) = 0$, $y'(0) = -1$

Ans: $u(x) = 6x - 5 + \int_0^x (5 - 6x + 6t)u(t)\,dt$, $u(x) = y''$

(11). $y'' + y = \cos x$, $y(0) = 0$, $y'(0) = 1$

Ans: $u(x) = \cos x - x - \int_0^x (x-t)u(t)\,dt$, $u(x) = y''$

P. KALIKA

Practice Questions

PAGE :
DATE : / /

(1). Find the eigen-values & Eigen-function of
$$y(x) = \lambda \int_0^1 e^x . e^t \, y(t) \, dt .$$
(Ans: Already discussed)

(2). Show that
$$y(x) = \lambda \int_0^1 (3x-2) \, t \, y(t) \, dt$$
has no eigen value & eigen function.

GATE (3). The values of λ for which the IE
$$y(x) = \lambda \int_0^1 (6x-t) \, y(t)$$ has
non-trivial solⁿ is roots of —

(a) $(3\lambda-1)(2+\lambda) - \lambda^2 = 0$
(b) $(3\lambda-1)(2+\lambda) + 2 = 0$ [GATE-2004]
(c) $(3\lambda-1)(2+\lambda) - 4\lambda^2 = 0$
(d) $(3\lambda-1)(2+\lambda) + \lambda^3 = 0$ (Ans: C)

(4). Find the eigen-values & eigen-function of homo. IE $y(x) = \lambda \int_0^1 \sin \pi x \cos \pi t \, y(t) \, dt$

Ans: No characteristic No. & eigen function.

GATE (5). The eigen-values of IE
$$y(x) = \lambda \cdot \int_0^{2\pi} \sin(x+t) \, dt$$ has —
(a) $\pm 1/2\pi$ (b). $\pm 1/\pi$ (c). $\pm \pi$ (d). $\pm 2\pi$
Ans: (b) [GATE-2005]

(6). Solve the fredholm IE (homo.) of 2nd kind
$$y(x) = \lambda \int_0^{2\pi} \sin(x+t) \, y(t) \, dt$$
Ans: $\lambda = \pm 1/\pi$; $y(x) = \sin x \pm \cos x$.

GATE (7) The IE. $y(x) = \lambda \int_0^{2\pi} \sin(x+t) \, y(t) \, dt$ has —
[GATE-2003]

P. KALIKA

Practice Questions

PAGE :
DATE : / /

(a) two sol^{n}s for any value of λ.
(b) Unique sol^n for every value of λ.
(c) Infinitely many sol^n for only one values of λ.
(d) " " " " two values of λ.

(Ans: d)

Ans: $\left(\begin{array}{ll} \lambda_1 = 4/\pi, & y_1(x) = \cos 2x \\ \lambda_2 = 8/\pi, & y_2(x) = \cos 3x \end{array} \right)$

GATE

(8) The eigenvalue λ of the Fredholm IE

$$y(x) = \lambda \int_0^1 x^2 t\, y(t)\, dt \quad \text{is}$$

(a) 4 (b) 2 (c) -2 (d) -4 [GATE-2011]

— (Ans: a)

(9). Solve the following homo. I.E —

(a). $y(x) = \frac{1}{2}\int_0^{\pi} \sin x\, y(t)\, dt$ (y=0)

(b). $y(x) = \frac{1}{e^2-1}\int_0^1 2e^{x+t}\, y(t)\, dt$ (y=0)

(c). $y(x) = \lambda \int_0^{\pi} \cos(x+t)\, y(t)\, dt$, λ = Find

(d). $y(x) = \lambda \int_{-1}^{1} (5xt^3 + 4x^2 t)\, y(t)\, dt$, Find λ.

(e). $y(x) = \lambda \int_0^{\pi} \sin^2 x\, y(t)\, dt$.

GATE

(10) Prove that, the Linear DE of 2nd order

$$y''(x) + a_1(x) y'(x) + a_2(x) y(x) = F(x).$$

with IC $y(0) = C_0$ & $y'(0) = C_1$ can be transformed into non-homo. Volterra's IE of 2nd kind.

[GATE-2006]

(Hint: look at page-27)

H.W

Exercise Questions

PAGE :
DATE : / /

Que (40). Sol^n of I.E. $y(x) = x - \int_1^x x\, y(t)\, dt$, $y \in C^1[1,\infty)$ is ——

(a). $y = x(1 - \log x)$ (b). $y = x e^{(x-\frac{1}{2})} \cdot (x-1) + x$

(c). $y = x\, e^{\frac{1}{2}(1-x^2)}$ (d). $y = x - x(e^{x^2} - e)$

Ans: (c)

Que (41) I.E $\phi(x) = \lambda \int_0^1 [\sqrt{x}\,t - \sqrt{t}\,x]\, \phi(t)\, dt$ has—

(a). One characteristic number.

(b). Two characteristic Number

(c). Infinite no. of eigen-value

(d). No eigen-value.

Que (42) Solution of IE $y(x) = 6 - 3x^2 + \int_0^x y(t)\, dt$, $y(0) = 0$ is ——

(a). $y(x) = x^2$ (b). $y(x) = 6x$

(c). $y(x) = \sin x$ (d). $y(x) = x \sin x$.

Ans: (b)

Que (43). Eigen-values of following I.E are—

$$\phi(x) = \lambda \left[x \int_0^1 t\, \phi(t)\, dt + 2x^2 \int_0^1 \phi(t)\, dt\right]$$

(a). $\lambda_1 = -36 - 12\sqrt{10}$, $\lambda_2 = -36 + 12\sqrt{10}$

(b) $\lambda_1 = -9 - 6\sqrt{10}$, $\lambda_2 = -9 + 6\sqrt{10}$

(c). $\lambda_1 = -18 - 6\sqrt{10}$, $\lambda_2 = -18 + 6\sqrt{10}$

(d) No eigen-values.

Ans: (c)

Que (44). Solution of IE $y(x) = f(x) + \int_0^1 x t^2\, y(t)\, dt$ ~~has~~ — is given by $\phi(x) =$ —

(a) $\phi(x) = f(x)$ (b). $\phi(x) = x^3$

(c) $\phi(x) = f(x) + \frac{4}{3} x \int_0^1 x^2 f(x)\, dx$

(d) None

Ans: (c)

COV : Collection of Problem from NET

P. Kalika

(1). The variational Problem of extremizing the functional $I(y(x)) = \int_0^{2\pi} \left[\left(\frac{d}{dx}(y)\right)^2 - y^2 \right] dx$, $y(0) = 1 = y(2\pi)$ has —

(a). Unique sol^n
(b). exactly two sol^n
(c). Infinite no. of sol^n
(d). No solution

— JUNE NET-2011

(2). Suppose any two pts $P(x_0, y_0)$ & $Q(x_1, y_1)$ and $F(x, y, y')$ of 3 independent variables are given, where $y' = \frac{dy}{dx}$. In order to find among all curve $y = y(x)$ joining P & Q that one which furnished for the definite Integral $I(y) = \int_{x_0}^{x_1} F(x, y, y')\, dx$. The smallest value. W.O.t.f assumption suffices —

(a). Function F is of class c^1
(b). Functional F is of class c^2 for all system of values x, y, y' furnished by all of the admissible f^ns.
(c). - - - - - - - - c^3 - - - - - - - - -
(d). It is enough to treat y & F to be of class c^1 only with r.t. their arguments.

NET-DEC-2011

(3) The variational problem of extremizing the functional $I[y(x)] = \int_1^3 y(3x - y)\, dx$; $y(3) = 4\frac{1}{2}$, $y(1) = 1$ has —

(a). Unique sol^n
(b). exactly two sol^n
(c). Infinite no. of sol^n
(d). No solution

NET-JUNE-2012

(4). Let $J(u) = \int_0^1 \left(u_x^2 + 4\frac{u^2}{x^2} \right) x\, dx$ where $u(x)$ is smooth f^n on $[0,1]$ satisfying $u(0) = 0$ & $u(1) = 1$. W.O.t.f minimizes J ?

(a) $u(x) = x^2$
(b). $u(x) = \frac{x^2}{\sqrt{2}}$
(c). $u(x) = x^2/2$
(d). $u(x) = \frac{x^2}{4}$

NET-DEC-2012

(5) Consider the functional $I(y) = \int_a^b F(y, y')\,dx$; $y(a) = y_1$, $y(b) = y_2$, $y \in C^2[a,b]$ & F has 2nd order P.D. w.r.t y & y' and y_1 & y_2 are given real no.s. Let $y = y(x)$ be an extremizing f^n for the functional I. Then along the extremizing curve —

(a) F remains constant (c). $F - y\frac{\partial F}{\partial y'} = \text{Const}$.

(b) $\frac{\partial F}{\partial y} = 0$ (d). $F - y'\frac{\partial F}{\partial y'} = \text{Const}$.

NET- June-2013

(6). Consider the functional $J(y) = \int_a^b F(x,y,y')\,dx$, $F = y' + y$ for admissible f^ns, then J has —

(a). No extremals. (c). $y(x) = e^{-x}$ as an extremal

(b). Several extremals (d) $y = \text{const}$ " "

NET- DEC-2013

(7). The curve extremizing the f^n $I[y(x)] = \int_1^2 \frac{\sqrt{1+y'^2}}{x}\,dx$, $y(1) = 0$, $y(2) = 1$

(a) a ellipse (c). a circle

(b). a parabola (d). a st. line

NET-JUNE-2014

(8). Consider the functional $J(y) = y^2(1) + \int_0^1 y'^2\,dx$, $y(0) = 1$, $y \in C^2[0,1]$. If y extremize J then —

(a). $y(x) = 1 - \frac{1}{2}x^2$ (c). $y(x) = 1 + \frac{1}{2}x$

(b). $y(x) = 1 - \frac{1}{2}x$ (d). $y(x) = 1 + \frac{1}{2}x^2$

NET- DEC- 2014

(9). The functional $I(y(x)) = \int_a^b (y^2 + y'^2 - 2y\sin x)\,dx$ has the following extremal with C_1 and C_2 as arbitrary constants —

(a). $y = C_1e^{2x} + C_2e^{-2x} + \frac{1}{2}\sin x$ (c) $y = C_1e^{x} + C_2e^{-x} + \frac{1}{2}\sin x$

(b). $y = C_1e^{x} + C_2e^{-x} + \frac{1}{2}\sin x$ (d). $y = C_1e^{2x} + C_2e^{-2x} + \frac{1}{2}\cos x$

NET-DEC-2015

MCQ

(10). The extremal of the functional $I = \int_0^{x_1} y^2(y')^2\,dx$ that passes through $(0,0)$ & (x_1, y_1) is —

(a). a constant f^n (c). part of a parabola

(b). a linear f^n of x (d). parts of an ellipse.

NET- JUNE- 2015

P. Kalika * * * P. Kalika

P. Kalika

EXERCISE (Runge-Kutta Method)

Numerical Analysis

Que (1). (GATE 2014) The 4th order R-K Method given by $u_{j+1} = u_j + \frac{h}{6}[K_1 + 2K_2 + 2K_3 + K_4]$, $j = 0, 1, 2, \ldots$ is used to solve the IVP $u' = u$, $u(0) = \alpha$. If $u(1) = 1$ is obtained by taking step size $h = 1$, then the value of K_4 is

Que (2). (GATE 2012) Using Euler's Method, taking step size 0.1, the approximate value of y obtained corresponding to $x = 0.2$ for IVP $\frac{dy}{dx} = x^2 + y^2$ and $y(0) = 1$ is ____

(A). 1.322 (B). 1.122 (C). 1.222 (D). 1.110

Que (3). GATE 2008 Consider the IVP $\frac{dy}{dx} = f(x, y)$, $y(x_0) = y_0$. The aim is to compute the value of $y_1 = y(x_1)$, where $x_1 = x_0 + h$ $(h > 0)$. At $x = x_1$, if the value of y_1 is equal to the corresponding value of st. line passing through (x_0, y_0) and having the slope equal to the slope of the curve $y(x)$ at $x = x_0$, then the method is called —

(A). Euler's Method (B). Improved Euler's Method

(C). Backward Euler's Method (D). Taylor Series method of order

Que (4). GATE 2006 Consider the IVP $\frac{dy}{dx} = f(x, y)$, $y(x_0) = y_0$. Let $y_1 = y_0 + w_1 k_1 + w_2 k_2$ approximate the solution of the given IVP at $x_1 = x_0 + h$ with $k_1 = hf(x_0, y_0)$, $k_2 = hf(x_0 + h/6, y_0 + k_1/6)$ and h being the step-size. If the formula for y_1 yields a second order method, then the value of w_1 is ____

(A). -1 (B). -2 (C). 3 (D). $1/6$

Answer: (1). 1.017 (2). C (3). A (4). B

Assignment Problem

P. Kalika

Que (5). (NET 2013 D) Consider the IVP $\frac{dy}{dx} = x+y$, $y(0) = 1$. Then the approximate value of the sol^n $y(x)$ at $x = 0.2$, using improved Euler method with $h = 0.2$ is ——

(a) 1.11 (b). 1.20 (c). 1.24 (d) 1.48

Que (6) Using R-K Method of order 4, Find an approximate value of y at $x = 0.2$, if $\frac{dy}{dx} = x + y^2$, $y(0) = 1$.

Que (7). Solve the DE $\frac{dy}{dx} = 1-y$ with I.C $y(0) = 0$ using Euler's method & find sol^n at $x = 0.1, 0.2, 0.3$.

Ans:

x	0	0.1	0.2	0.3
sol^n	0	0.1	0.19	0.271

Que (8). Find the value of y at $x = 0.1, 0.2$ by using R-K Method for $y' = -y$, $y(0) = 1$.

(a). of 2nd order (b). of 4th order R-K method

Ans: (a)

x	0.1	0.2
y	0.905	0.899

(b)

x	0.1	0.2
y	0.965	0.8187

Que (9). Given $\frac{dy}{dx} = x^2 + y$, $y(0) = 1$, Evaluate $y(0.02)$ & $y(0.04)$ By Euler's method (Ans: 1.0202, 1.0408, 1.0619)

Que (10). Using R-K method of 4th order, Solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ with $y(0) = 1$, at $x = 0.2$ & 0.4.

(Ans: 1.196, 1.3752)

— x —

Answer: (5). (C) (6). $y(0.1) = 1.1165$, $y(0.2) = 1.2736$ (7)

LPP Practice Set - 1

(1). Consider the following Linear Programming Problem. Max $x_1 + \frac{5}{2}x_2$ subject to

$$5x_1 + 3x_2 \le 15$$
$$-x_1 + x_2 \le 1$$
$$2x_1 + 5x_2 \le 10.$$
$$x_1, x_2 \ge 0.$$

The problem
1. has no feasible solution.
2. has infinitely many optimal solutions.
3. has a unique optimal solution.
4. has an unbounded solution.

(2). **Suppose ABC is a triangle on the xy-plane with centroid D. Which of the following points can NEVER be a minimizer of the function $7x - 10y + 1$ as (x, y) runs over the triangle ABC?**

1. A **2. B**
3. C **4. D**

(3). Consider the variables $x_1 \ge 0$ and $x_2 \ge 0$ satisfying the constraints $x_1 + x_2 \ge 15$, $4x_1 - x_2 \le 15$ and $4x_2 - x_1 \le 15$. Which of the following statements is/are correct?

(1) The maximum value of $3x_1 + 2x_2$ is 25

(2) The minimum value of $3x_1 + 2x_2$ is 11

(3) $3x_1 + 2x_2$ has no finite maximum

(4) $3x_1 + 2x_2$ has no finite minimum

(4). Consider the LP problem maximize $x_1 + x_2$ subject to

$x_1 - 2x_2 \le 10$

$x_2 - 2x_1 \le 10$

$x_1, x_2 \ge 0$

Then

(1) The LP problem admits an optimal solution

(2) The LP problem is unbounded

(3) The LP problem admits no feasible solution

(4) The LP problem admits a unique feasible solution

(5). Suppose that the LP problem

maximise $c^T x$

subject to

$Ax \le b$

$x \ge 0$

admits a feasible solution and the dual minimize $b^T y$

subject to $A^T y \ge c$

$y \ge 0$

admits a feasible solution y_0. Then

(1) the dual admits an optimal solution.

(2) any feasible solution x_0 of the primal and y_0 of the dual satisfies $b^T y_0 \le c^T x_0$.

(3) the dual problem is unbounded

Ans.:. 1. 2. D

Practice set by P. Kalika

Asam PSC-19

(1) If x is a discrete r.v & $E(ax+b) = aE(x)+b$, where a & b are constants, then the variance $ax+b$ is —

(a). $a\,Var(x)$ (b). $a^2 Var(x)$ Assam PSC-19

(c). $b\,Var(x)$ (d). $ab\,Var(x)$.

NET-17D

(2) Let X & Y be independent exp. r.v. If $E(X) = 1$ & $E(Y) = 1/2$, then $P(X>2Y \mid X>Y)$ is —

(a). $1/2$ (b). $1/3$ (c). $2/3$ (d). $3/4$

Ans: (d)

NET-16D

(3) A & B play a game of tossing a fair coin. A starts the game by tossing the coin once & B then tosses the coin twice followed by A tossing the coin once & B tossing the coin twice & hits until a head turns up. whoever gets the first head wins the game. Then ——

(1). $P(B\text{ wins}) > P(A\text{ wins})$

(2). $P(B\text{ wins}) = 2P(A\text{ wins})$

(3). $P(A\text{ wins}) > P(B\text{ wins})$

(4). $P(A\text{ wins}) = 1 - P(B\text{ wins})$

Ans: 3, 4

NET-16D

(4) For any two events A & B, w.o.t.f always holds?

(1). $P^2(A\cap B^c) + P^2(A\cap B) + P^2(A^c) \geq 1/3$

(2). $P^2(A\cap B^c) + P^2(A\cap B) + P^2(A^c) = 1/3$

(3). " " " $= 1$

(4). " " " $\leq 1/3$

Ans: 1

Probability Practice Set- 1

NET-2018D

(5) Suppose $X \sim$ Cauchy(0,1). Then the distⁿ of $\frac{1-X}{1+X}$ is ——

(1). Uniform (0,1) (2). Normal (0,1)

(3). Double exp. (0,1) (4). Cauchy (0,1)

NET-2018J

(6). In a data set with mean 2.5 & std. deviation 0.5. The median must be

(1) bigger than 2.5 (2). smaller than 2.5

(3). smaller than 3 (4). bigger than 2.

Ans: 3,4

NET-2018J

(7). Let X & Y be two random variables satisfying for $x,y \geq 0$, $E(X)=3$, $V(X)=9$, $E(Y)=2$ and $V(Y)=4$, w.o.t.f statements are correct?

(1). $0 \leq Cov(X,Y) \leq 4$ (2). $E(XY) \leq 6$

(3). $V(X+Y) \leq 25$ (4). $E(X+Y)^2 \geq 25$

Ans: 3, 4

NET-2018J

(8) Let X & Y be two r.v with joint p.d.f —

$$f(x,y)=\begin{cases} 1/\pi & ; 0 \leq x^2+y^2 \leq 1 \\ 0 & ; o.w \end{cases}$$

Then w.o.t.f statements are true?

(1). X & Y are independent (2). $P(X>0)=1/2$

(3). $E(Y)=0$ (4). $Cov(X,Y)=0$

Ans: 2,3,4

By: P. Kalika (maths.whisperer@gmail.com)

NET-18J

(9). A std. fair die is rolled until some face other than 5 or 6 turns up. Let X denote the face value of the last roll and $A = \{X \text{ is even}\}$ & $B = \{X \text{ is at most } 2\}$. Then—

P. Kalika

(1). $P(A \cap B) = 0$ (2). $P(A \cap B) = 1/6$

(3). $P(A \cap B) = 1/4$ (4). $P(A \cap B) = 1/3$

Ans: 3

NET-18J

(10). A simple random sample of size n will be drawn from a class of 125 students, and the mean maths score of the sample will be computed. If the std. error of the sample mean for "with replacement sampling" is twice as much as the std. error of the sample mean for "without replacement" sampling the value of n is —

(1). 32 (2). 63 (3). 79 (4). 94

Ans:

NET 2011-June

(11). w.o.t.f is/are c.d.f of random variables —

(1). $F_1(x) = \begin{cases} 0 & : x \le 0 \\ e^{-x} & : x > 0 \end{cases}$ (2). $F_2(x) = \begin{cases} 0 & : x \le 0 \\ 1-e^{-x} & : x > 0 \end{cases}$

(3). $F_3(x) = \begin{cases} 0 & : x \le 0 \\ 1 & : x > 0 \end{cases}$ (4). $F_4(x) = \begin{cases} 0 & : x < 0 \\ 1/2 & ; 0 \le x < 1 \\ 1 & , x \ge 0 \end{cases}$

Ans: 1

NET-2011J

(12). Let X be a r.v taking values in set E. Let $P(X > a+b \mid X > a) = P(X > b)$ $\forall\, a, b \in E$. Then w.o.t.f is a possible distn of X ?

(1). Poisson (2). Geometric

(3). Log-normal (4). Exponential. Ans:

Probability Practice Set- 2

(1). Five numbers 10, 7, 5, 4 and 2 are to be arranged in a sequence from left to right following the directions given below:

1. No two odd or even numbers are next to each other.
2. The second number from the left is exactly half of the left-most number.
3. The middle number is exactly twice the right-most number.

GATE 2019

Which is the second number from the right?

(A) 2 (B) 4 (C) 7 (D) 10

(2). Forty students watched films A, B and C over a week. Each student watched either only one film or all three. Thirteen students watched film A, sixteen students watched film B and nineteen students watched film C. How many students watched all three films?

GATE 2018

(A) 0 (B) 2 (C) 4 (D) 8

(3). Let X and Y have joint probability density function given by

$$f_{X,Y}(x,y) = \begin{cases} 2, & 0 \le x \le 1-y, \ 0 \le y \le 1 \\ 0, & \text{otherwise.} \end{cases}$$

If f_Y denotes the marginal probability density function of Y, then $f_Y(1/2) =$ 1 .

(4). Let the cumulative distribution function of the random variable X be given by

$$F_X(x) = \begin{cases} 0, & x < 0, \\ x, & 0 \le x < 1/2, \\ (1+x)/2, & 1/2 \le x < 1, \\ 1, & x \ge 1. \end{cases}$$

Then $\mathbb{P}(X = 1/2) =$ 1/4 .

(5). Let $\{X_j\}$ be a sequence of independent Bernoulli random variables with $\mathbb{P}(X_j = 1) = 1/4$ and let $Y_n = \frac{1}{n}\sum_{j=1}^{n} X_j^2$. Then Y_n converges, in probability, to 1/4 .

(6).

Let X be the number of heads in 4 tosses of a fair coin by Person 1 and let Y be the number of heads in 4 tosses of a fair coin by Person 2. Assume that all the tosses are independent. Then the value of $\mathbb{P}(X = Y)$ correct up to three decimal places is .273.

(7).

Let X_1 and X_2 be independent geometric random variables with the same probability mass function given by $\mathbb{P}(X = k) = p(1-p)^{k-1}$, $k = 1, 2, \ldots$ Then the value of $\mathbb{P}(X_1 = 2 | X_1 + X_2 = 4)$ correct up to three decimal places is 1/3.

(8).

An urn contains four balls, each ball having equal probability of being white or black. Three black balls are added to the urn. The probability that five balls in the urn are black is

(A) 2/7 (B) 3/8 (C) 1/2 (D) 5/7

GATE 2018

(9).

An unbiased coin is tossed six times in a row and four different such trials are conducted. One trial implies six tosses of the coin. If H stands for head and T stands for tail, the following are the observations from the four trials:
(1) HTHTHT (2) TTHHHT (3) HTTHHT (4) HHHT__ __.

Which statement describing the last two coin tosses of the fourth trial has the highest probability of being correct?

(A) Two T will occur.
(B) One H and one T will occur.
(C) Two H will occur.
(D) One H will be followed by one T.

GATE 2018

Probability & Distribution Practice Set -3

(1). An urn has 3 red and 6 black balls. Balls are drawn at random one by one without replacement. The probability that second red ball appears at the fifth draw is

1. $\frac{1}{9!}$
2. $\frac{4!}{9!}$
3. $4\left(\frac{6!4!}{9!}\right)$
4. $\frac{6!4!}{9!}$

(2). From the six letters A, B, C, D, E and F, three letters are chosen at random with replacement. What is the probability that either the word BAD or the word CAD can be formed from the chosen letters?

1. $\frac{1}{216}$
2. $\frac{3}{216}$
3. $\frac{6}{216}$
4. $\frac{12}{216}$

(3). Let X be a random variable which is symmetric about 0. Let F be the cumulative distribution function of X. Which of the following statements is always true?

1. $F(x) + F(-x) = 1$ for all $x \in \mathbb{R}$.
2. $F(x) - F(-x) = 0$ for all $x \in \mathbb{R}$.
3. $F(x) + F(-x) = 1 + P(X = x)$ for all $x \in \mathbb{R}$.
4. $F(x) + F(-x) = 1 - P(X = -x)$ for all $x \in \mathbb{R}$.

(4). Suppose X has density $f(x|\theta) = \frac{1}{\theta}e^{-x/\theta}, x > 0$ where $\theta > 0$ is unknown. Define Y as follows:

$Y = k$ if $k \le X < k+1$, $\quad k = 0, 1, 2, \cdots$

Then the distribution of Y is

1. normal.
2. binomial.
3. Poisson.
4. geometric.

(5). Let X and Y be independent exponential random variables. If $E[X] = 1$ and $E[Y] = \frac{1}{2}$ then $P(X > 2Y | X > Y)$ is

1. $\frac{1}{2}$
2. $\frac{1}{3}$
3. $\frac{2}{3}$
4. $\frac{3}{4}$

(6). Let S be the set of all 3×3 matrices having 3 entries equal to 1 and 6 entries equal to 0. A matrix M is picked uniformly at random from the set S. Then

1. $P\{M \text{ is nonsingular}\} = \frac{1}{14}$
2. $P\{M \text{ has rank } 1\} = \frac{1}{14}$
3. $P\{M \text{ is identity}\} = \frac{1}{14}$
4. $P\{\text{trace}(M) = 0\} = \frac{1}{14}$

P. Kalika

Practice / Asked Que-Set

Assam PSC-19

(1) Consider a Markov Chain with state space $\{0,1,2\}$ and the transition prob. matrix
The the period of 0 is —

$$P = \begin{bmatrix} 0 & 1/2 & 1/2 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

(A). 0 (B). 1
(C). 2 (D). None

Ans : 2

NET-17 June

(2) Let $\{X_n\}$ be a Markov chain on $\{0,1,2,\ldots\}$ with
[4.75] $P_{00} = 2/3$, $P_{01} = 1/3$, $P_{i,i+1} = 2/3$, $P_{i,i-1} = 1/3$, $i \geq 1$,
$P_{ij} = 0$ otherwise. W.o.t.f statements are correct

(a) $\{X_n\}$ is recurrent
(b) $\{X_n\}$ is transient.
(c). $P(\lim_{n\to\infty} X_n = 0) > 0$
(d). $P(\lim_{n\to\infty} X_n = +\infty) > 0$

Ans: 2, 4

NET-17 June

(3). W.o.t.f statements are correct ?
(4.75) (a). For a finite state Markov chain there is at least one transient state.
(b). For a finite state markov chain there is at least one stationary distⁿ.
(c). For a countable state Markov chain, every state can be transient.
(d). For an aperiodic countable state Markov chain there is at least one stationary distⁿ.

Ans: 2, 3

NET-11D

(4). An aperiodic Markov chain with stationary distⁿ prob. on the state space $\{1,2,3,4,5\}$ must have at least one
(a). null recurrent state (b). positive recurrent state
(c). +ve recc. & null recc. (d). transient state.

Ans: (b)

Markov Chain Practice Set - 1

NET 2011-Dec

(5). Let $\Pi = (\pi_1, \pi_2, \pi_3, \pi_4)$ be the stationary dist^n for a markov chain on the state space $\{1,2,3,4\}$ with transition prob. matrix P. Suppose that states 1 & 2 are transient & states 3 & 4 form a communicatory class. w.o.t.f is/are true?

(1) $\pi P^3 = \pi P^5$ (2). $\pi_1 = 0$ & $\pi_2 = 0$

(3) $\pi_3 + \pi_4 = 1$ (4). One of π_3 & π_4 is 0.

Ans: 2,3

NET 2011-June

(6) ~~w.o.t.f are c.d.f of random~~

Let $\{X_n\}$ be stationary ~~dist~~^{n} Markov Chain s.t —

$P(X_{i+1} = 1 \mid X_i = 1) = p_1 = 1 - P(X_{i+1} = 0 \mid X_i = 1)$,

$P(X_{i+1} = 1 \mid X_i = 0) = p_0 = 1 - P(X_{i+1} = 0 \mid X_i = 0)$ & $P(X_i = 1) = \pi_1 = 1 - P(X = 0)$. Then —

(a) $\pi_1 = p_1$ (b). $\pi_1 = p_0$

(c). $\pi_1 = p_0 / (1 - p_0 + p_1)$ (d). $\pi_1 = 1/2$.

Ans: All

Markov Chain Practice Set - 2

(1). Let $(X_n)_{n\geq 0}$ be a Markov chain on the state space $S := \{1, 2, \cdots, 23\}$ with transition probability given by

$$p_{i,i+1} = p_{i,i-1} = \frac{1}{2} \quad \forall\, 2 \leq i \leq 22$$
$$p_{1,2} = p_{1,23} = \frac{1}{2}$$
$$p_{23,1} = p_{23,22} = \frac{1}{2}.$$

Then, which of the following statements are true?

1. $(X_n)_{n\geq 0}$ has a unique stationary distribution.
2. $(X_n)_{n\geq 0}$ is irreducible.
3. $\mathbb{P}(X_n = 1) \longrightarrow \frac{1}{23}$.
4. $(X_n)_{n\geq 0}$ is recurrent.

(2). Consider a Markov chain $\{X_n \mid n \geq 0\}$ with state space $\{1, 2, 3\}$ and transition matrix

$$P = \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}.$$ Then $P(X_3 = 1 \mid X_0 = 1)$ equals

1. 0
2. $\frac{1}{4}$
3. $\frac{1}{2}$
4. $\frac{1}{8}$

(3). Consider a Markov Chain with state space $S = \{0, 1, 2, 3\}$ and with transition probability matrix P given by

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{pmatrix} 2/3 & 0 & 1/3 & 0 \\ 1 & 0 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 \\ 1/4 & 1/4 & 1/4 & 1/4 \end{pmatrix} \end{array}$$

Then

1. 1 is a recurrent state.
2. 0 is a recurrent state.
3. 3 is a recurrent state.
4. 2 is a recurrent state.

(4). Consider a Markov chain with five states $\{1,2,3,4,5\}$ and transition matrix

$$P = \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{7} & 0 & 0 & \frac{6}{7} \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \\ \frac{1}{3} & 0 & 0 & \frac{2}{3} & 0 \\ 0 & \frac{5}{8} & 0 & 0 & \frac{3}{8} \end{pmatrix}$$

Which of the following are true?

1. 3 and 1 are in the same communicating class
2. 1 and 4 are in the same communicating class
3. 4 and 2 are in the same communicating class
4. 2 and 5 are in the same communicating class

Some Useful Links:

1. Free Maths Study Materials (https://pkalika.in/2020/04/06/free-maths-study-materials/**)**

2. BSc/MSc Free Study Materials (https://pkalika.in/2019/10/14/study-material/**)**

3. MSc Entrance Exam Que. Paper: (https://pkalika.in/2020/04/03/msc-entrance-exam-paper/**)**
[JAM(MA), JAM(MS), BHU, CUCET, ...etc]

4. PhD Entrance Exam Que. Paper: (https://pkalika.in/que-papers-collection/**)**
[CSIR-NET, GATE(MA), BHU, CUCET,IIT, NBHM, ...etc]

5. CSIR-NET Maths Que. Paper: (https://pkalika.in/2020/03/30/csir-net-previous-yr-papers/**)**
[Upto 2019 Dec]

6. Practice Que. Paper: (https://pkalika.in/2019/02/10/practice-set-for-net-gate-set-jam/**)**
[Topic-wise/Subject-wise]