BHU Mathematics

Research Entrance Test Papers

Contents:

No. of Pages: 99

A124

Set No. : 1

Question Booklet No.

RET/17/TEST-B

904 | Mathematics (Arts)

(To be filled up by the candidate by blue/black ball point pen)

Roll No. | | | | | | | | | | |

Roll No. (Write the digits in words) .....

Serial No. of OMR Answer Sheet .....

Day and Date .....

(Signature of Invigilator)

INSTRUCTIONS TO CANDIDATES

(Use only blue/black ball-point pen in the space above and on both sides of the Answer Sheet)

1. Within 30 minutes of the issue of the Question Booklet, Please ensure that you have got the correct booklet and it contains all the pages in correct sequence and no page/question is missing. In case of faulty Question Booklet, Bring it to the notice of the Superintendent/Invigilators immediately to obtain a fesh Question Booklet.
2. Do not bring any loose paper, written or blank, inside the Examination Hall ***except the Admit Card without its envelope.***
3. ***A separate Answer Sheet is given. It should not be folded or mutilated. A second Answer Sheet shall not be provided.***
4. Write your Roll Number and Serial Number of the Answer Sheet by pen in the space provided above.
5. ***On the front page of the Answer Sheet, write by pen your Roll Number in the space provided at the top, and by darkening the circles at the bottom. Also, wherever applicable, write the Question Booklet Number and the Set Number in appropriate places.***
6. ***No overwriting is allowed in the entries of Roll No., Question Booklet No. and Set No. (if any) on OMR sheet and Roll No. and OMR sheet no. on the Question Booklet.***
7. ***Any change in the aforesaid entries is to be verified by the invigilator, otherwise it will be taken as unfair means.***
8. ***This Booklet contains 40 multiple choice questions followed by 10 short answer questions. For each MCQ, you are to record the correct option on the Answer Sheet by darkening the appropriate circle in the corresponding row of the Answer Sheet, by pen as mentioned in the guidelines given on the first page of the Answer Sheet. For answering any five short Answer Questions use five Blank pages attached at the end of this Question Booklet.***
9. For each question, darken only **one** circle on the Answer Sheet. If you darken more than one circle or darken a circle partially, the answer will be treated as incorrect.
10. ***Note that the answer once filled in ink cannot be changed. If you do not wish to attempt a question, leave all the circles in the corresponding row blank (such question will be awarded zero marks).***
11. For rough work, use the inner back pages of the title cover and the blank page at the end of this Booklet.
12. ***Deposit both OMR Answer Sheet and Question Booklet at the end of the Test.***
13. You are not permitted to leave the Examination Hall until the end of the Test.
14. If a candidate attempts to use any form of unfair means, he/she shall be liable to such punishment as the University may determine and impose on him/her.

Total No. of Printed Pages : 28

Research Entrance Test-2017

No. of Questions : 50

प्रश्नों की संख्या : 50

Time : 2 Hours ***Full Marks : 200***

समय : 2 घण्टे ***पूर्णाङ्क : 200***

Note: (1) This Question Booklet contains **40** Multiple Choice Questions followed by **10** Short Answer Questions.

इस प्रश्न पुस्तिका में **40** वस्तुनिष्ठ व **10** लघु उत्तरीय प्रश्न हैं।

(2) Attempt as many MCQs as you can. Each MCQ carries **3 (Three)** marks. **1 (One)** mark will be deducted for each incorrect answer. **Zero** mark will be awarded for each unattempted question. If more than one alternative answers of MCQs seem to be approximate to the correct answer, choose the closest one.

अधिकाधिक वस्तुनिष्ठ प्रश्नों को हल करने का प्रयत्न करें। प्रत्येक वस्तुनिष्ठ प्रश्न **3** (तीन) अंकों का है। प्रत्येक गलत उत्तर के लिए **1** (एक) अंक काटा जायेगा। प्रत्येक अनुत्तरित प्रश्न का प्राप्तांक शून्य होगा। यदि वस्तुनिष्ठ प्रश्नों के एकाधिक वैकल्पिक उत्तर सही उत्तर के निकट प्रतीत हों, तो निकटतम सही उत्तर दें।

(3) Answer only **5** Short Answer Questions. Each question carries **16 (Sixteen)** marks and should be answered in **150-200** words. Blank **5 (Five)** pages attached with this booklet shall only be used for the purpose. Answer each question on separate page, after writing Question No.

केवल **5** (पाँच) लघुउत्तरीय प्रश्नों के उत्तर दें। प्रत्येक प्रश्न **16** (सोलह) अंकों का है तथा उनका उत्तर **150-200** शब्दों के बीच होना चाहिए। इसके लिए इस पुस्तिका में लगे हुए सादे **5** (पाँच) पृष्ठों का ही उपयोग आवश्यक है। प्रत्येक प्रश्न का उत्तर एक नए पृष्ठ से, प्रश्न संख्या लिखकर शुरू करें।

01. Who wrote Svapnavasavadattam ?

(1) Kalidasa (2) Bhasa

(3) Banabhatta (4) Jayadev

स्वप्नवासवदत्तम् किसकी रचना है ?

(1) कालिदास (2) भास

(3) बाणभट्ट (4) जयदेव

02. Who was the founder of Satyashodhak Samaj ?

(1) Atmaram Pandurang (2) Jyotiba Phule

(3) Mahadev Govind Ranade (4) Ayyankali

सत्यशोधक समाज की स्थापना किसने की थी ?

(1) आत्माराम पांडुरंग (2) ज्योतिबा फुले

(3) महादेव गोविन्द रानाडे (4) अय्यंकली

03. Who discovered the caves of Bhimbetka ?

(1) Paramananda Acharya (2) V.D. Krishnaswami

(3) Rajendra Prasad Das (4) V.S. Wakankar

भीमबेटका शैलाश्रयों की खोज किसने की थी ?

(1) परमानन्द आचार्य (2) वी.डी. कृष्णास्वामी

(3) राजेन्द्र प्रसाद दास (4) वी.एस. वाकणकर

04. Umakant and Ramakant Gundecha Brothers are associated with :

(1) Dhrupad (2) Kathak

(3) Sarod (4) Tabla

उमाकांत एवं रमाकांत गुंदेचा बंधु का सम्बन्ध किससे है ?

(1) ध्रुपद (2) कथक

(3) सरोद (4) तबला

05. Who among the following is **NOT** a winner of Nobel Prize in Literature ?

(1) Kazuo Ishiguro (2) Bob Dylan
(3) Rabindranath Tagore (4) Ruskin Bond

अधोलिखित में से कौन साहित्य में नोबेल पुरस्कार विजेता **नहीं** है ?

(1) काजू इंशिगुरो (2) बॉब डिलन
(3) रबीन्द्रनाथ टैगोर (4) रस्किन बॉन्ड

06. In which State of India is Lomus Rishi cave situated ?

(1) Uttarakhand (2) Maharashtra
(3) Madhya Pradesh (4) Bihar

लोमस ऋषि की गुफा भारत के किस राज्य में स्थित है ?

(1) उत्तराखंड (2) महाराष्ट्र
(3) मध्य प्रदेश (4) बिहार

07. Which of the following languages is NOT declared by Government of India as Classical Language of India ?

(1) Tamil (2) Oria
(3) Kannada (4) Bengali

निम्नलिखित में से भारत की किस भाषा को भारत सरकार द्वारा शास्त्रीय भाषा का दर्जा प्राप्त **नहीं** है ?

(1) तमिल (2) उड़िया
(3) कन्नड़ (4) बंगाली

08. To whom does the Speaker of the Loksabha submit his/her resignation ?

(1) The Prime Minister (2) The Deputy Speaker
(3) The President (4) The Vice President

लोकसभा अध्यक्ष अपना त्यागपत्र किसे देता है ?

(1) प्रधान मंत्री (2) लोकसभा उपाध्यक्ष
(3) राष्ट्रपति (4) उपराष्ट्रपति

09. Who has been appointed as Chairman of the Committee assigned to prepare the final draft of the National Education Policy in 2017 ?

(1) Ram Shanker Kureel

(2) Krishnawamy Kasturirangan

(3) T.V. Kattimani

(4) Manjul Bhargava

राष्ट्रीय शिक्षा नीति, 2017 का मसौदा तैयार करने के लिए गठित समिति के अध्यक्ष कौन हैं ?

(1) राम शंकर कुरील (2) कृष्णास्वामी कस्तूरीरंगन

(3) टी. वी. कट्टीमनी (4) मंजुल भार्गव

10. Who is the author of 'Dast e Saba' ?

(1) Sahir Ludhiyanvi (2) Faiz Ahmed Faiz

(3) Muhammad Iqbal (4) Ziauddin Barani

'दस्त ए सबा' के लेखक कौन हैं ?

(1) साहिर लुधियानवी (2) फैज़ अहमद फैज़

(3) मुहम्मद इकबाल (4) जिआउद्दीन बरानी

11. Which of the following statement is not **true** ?

(1) If domain of a function is countable, then its range is also countable.

(2) The set of all real numbers IR is a counteble set.

(3) The set of all rational numbers Q is a countable set.

(4) The family of all finite subsets of a countable set is conntable

निम्नलिखित में से कौन सत्य **नहीं** है ?

(1) यदि किसी फलन का प्रांत गणनीय हो, तो उसका परिसर भी गणनीय होगा।

(2) समस्त वास्तविक संख्याओं IR का समुच्चय गणनीय है।

(3) समस्त रेशनल संख्याओं Q का समुच्चय गणनीय है।

(4) किसी गणनीय समुच्चय के सभी परिमित उपसमुच्चयों का समूह भी गणनीय है।

12. Let $f : \mathbb{R} \to \mathbb{R}$ be a function such that $f(f(x)) = -x$ for all $x \in \mathbb{R}$. Then

(1) f is 1-1 but not onto.

(2) f is onto but not 1-1.

(3) f is both 1-1 and onto.

(4) f need not be either 1-1 or onto.

माना $f : \mathbb{R} \to \mathbb{R}$ एक ऐसा फलन है कि $f(f(x)) = -x$ सभी $x \in \mathbb{R}$ के लिए तब :

(1) f एकाकी है, परन्तु आच्छादक नहीं।

(2) f आच्छादक है, परन्तु एकाकी नहीं।

(3) f एकाकी तथा आच्छादक है।

(4) ऐसा जरूरी नहीं कि f आच्छादक या एकांकी हो।

13. Let G be a finite group and p be a prime number. The number of p-sylow subgroups of G is of the form :

माना G एक परिमित समूह है तथा P एक अविभाज्य संख्या है। p- सैलो उपपरिमित समूहों की संख्या का रूप होगा :

(1) $1 + np$ (2) $n-p$

(3) $np - 1$ (4) $n + p^2$

14. A Group G of order 35 :

(1) is abelian but not cyclic (2) is cyclic

(3) is non- abelian (4) does not exist

एक 35 आर्डर का परिमित समूह :

(1) अबीलियन है पर साइक्लिक नहीं

(2) साइक्लिक है

(3) नान अबीलियन है

(4) सम्भव नहीं है

15. If p is a prime number, then the splitting field over the field of rational numbers, of the polynomial x^p-1 is of degree :

यदि P एक अभाज्य संख्या है, तो परिमेय संख्याओं के क्षेत्र पर आधारित बहुपद x^p-1 के विभाजक क्षेत्र की डिग्री होगी :

(1) $p + 1$ (2) $2p + 1$

(3) p (4) $p - 1$

16. Which of the following is **correct** ?

(1) There is a field of 35 elements.

(2) There is a field of 48 elements.

(3) There is a field of 64 elements.

(4) There is a field of 80 elements

निम्न में से कौन-सा कथन सही है ?

(1) एक ऐसा क्षेत्र है जिसमें कुल 35 सदस्य हैं।

(2) एक ऐसा क्षेत्र है जिसमें कुल 48 सदस्य हैं।

(3) एक ऐसा क्षेत्र है जिसमें कुल 64 सदस्य हैं।

(4) एक ऐसा क्षेत्र है जिसमें कुल 80 सदस्य हैं।

17. Let A and B be $n \times n$ matrices such that $BA + B^2 = I - BA^2$, where I is the $n \times n$ identity matrix. Which of the following is always **true** ?

(1) A is non-singular. (2) B is non-singular.

(3) A + B is non-singular. (4) AB is non-singular.

माना A तथा B $n \times n$ आव्यूह है तथा $BA + B^2 = I - BA^2$, जहाँ I $n \times n$ तत्समक आव्यूह है। निम्न में से कौन हमेशा **सही** है ?

(1) A व्युत्क्रमणीय है। (2) B व्युत्क्रमणीय है।

(3) A + B व्युत्क्रमणीय है। (4) AB व्युत्क्रमणीय है।

18. Let $M_n(K)$ denote the vector space of all $n \times n$ matrices with entries in the field K. Fix a matrix $A = (\alpha_{ij}) \in M_n(K)$ and consider the linear map $T : M_n(K) \to M_n(K)$ given by

$T(X) = AX$

Then :

माना कि $M_n(K)$ क्षेत्र K से प्रविष्टियों से युक्त सभी $n \times n$ आव्यूहों की समष्टि को निर्दिष्ट करता है। एक आव्यूह $A = (\alpha_{ij})$ $M_n(K)$ को स्थिर करें, तथा रैखिक मानचित्र $T : M_n(K) \to M_n(K)$ जो $T(x) = AX$ से दिया जाता है, पर विचार करें। तब :

(1) Trace $(T) = n \sum_{i=1}^{n} \alpha_{ij}$ (2) Trace $(T) = \sum_{i=1}^{n} \sum_{j=1}^{n} \alpha_{ij}$

(3) Rank $(T) = n^2$ (4) T is non-singular

19. Let V be a finite dimensional vector space over IR. Let $T : V \to V$ be a linear transformation such that rank (T) = rank (T^2). Then which of the following is **not** correct ?

माना कि IR पर V एक परिमित विमीय सदिश समष्टि है तथा $T : V \to V$ एक रैखिक रूपान्तर है तथा जाति (T^2) = जाति (T) है। तब निम्न में से कौन सही **नहीं** है ?

(1) Kernel (T^2) = Kernel (T) (2) Range (T^2) = Range (T)

(3) Kernel (T) $\cap$ Kernel $(T^2) = \{0\}$ (4) Range (T) $\cap$ Kernel $(T) = \{0\}$

20. Let X be a metric space and $f : X \to IR$ is a continuous function. Let

$G = \{(x, f(x)) : x \in X\}$ be the graph of f,

Then :

(1) G is homeomorphic to $X \times IR$. (2) G is homeomorphic to $IR \times X$.

(3) G is homeomorphic to IR. (4) G is homeomorphic to X.

माना कि X एक मानक समष्टि है तथा $f : X \to \mathbb{R}$ एक सतत फलन है। माना $G = \{(x, f(x)) : x \in X\}$ f का लेखाचित्र है। तो :

(1) G तथा $X \times \mathbb{R}$ समरूपी हैं। (2) G तथा $\mathbb{R} \times X$ समरूपी हैं।

(3) G तथा $\mathbb{R}$ समरूपी हैं। (4) G तथा X समरूपी हैं।

21. Let $X = \{a, b, c\}$. Then which of the following classes of subsets of X is a topology on X ?

माना $X = \{a, b, c\}$, तब निम्नलिखित में से कौन X उपसमुच्चय के लिए X पर एक टोपोलॉजी है ?

(1) $\{\phi, \{a\}, \{b\}, X\}$ (2) $\{\phi, \{a\}, \{b\}, \{c\}, \{a,b\}, X\}$

(3) $\{\phi, \{a\}, \{a,b\}, X\}$ (4) $\{\phi, \{a,b\}, \{b,c\}, X\}$

22. Every finite Hausdorff is :

(1) a discrete space (2) an indiscrete space

(3) not a T_1 – space (4) not a T_4 – space

प्रत्येक परिमित हाउसडार्फ समष्टि है :

(1) एक तुच्छ समष्टि (2) एक विविक्त समष्टि

(3) T_1 समष्टि नहीं (4) T_4 समष्टि नहीं

23. Let M denote the family of all Lebesgue measurable subsets of $\mathbb{R}$. Which of the followings is **not** true ?

(1) $A_1, A_2 \in M \Rightarrow A_1 \cup A_2 \in M$

(2) $A_1, A_2 \in M \Rightarrow A_1 \cap A_2 \in M$

(3) If the Lebesgue out measure of a subset A of $\mathbb{R}$ is zero, then $A \in M$.

(4) If $A_n \in M$ for each $n \in \mathbb{N}$, then $m\left(\bigcup_{n=1}^{\infty} A_n\right) = \sum_{n=1}^{\infty} m(A_n)$, where m denotes the Lebesgue measure.

मान लीजिए M, $\mathbb{R}$ के सभी लेवेग मेजरेबिल के सबसमुच्चयों के समूह को प्रर्दशित करता है। तब निम्नलिखित में से कौन सत्य **नहीं** है ?

(1) $A_1, A_2 \in M \Rightarrow A_1 \cup A_2 \in M$

(2) $A_1, A_2 \in M \Rightarrow A_1 \cap A_2 \in M$

(3) यदि किसी $\mathbb{R}$ के उपसमुच्चय A और लेबेग आऊट्स मेजर जीरो है, तब $A \in M$

(4) यदि $A_n \in M$ के लिए प्रत्येक $n \in \mathbb{N}$, तब $m\left(\bigcup_{n=1}^{\infty} A_n\right) = \sum_{n=1}^{\infty} m(A_n)$ यहाँ m लेबेग मेजर को दिखाता है।

24. Let X be a normed linear space consider the following statements.

I : If X is a Banach space then $\{x \in X : \|x\| = 1\}$ is complete.

II : If $\{x \in X : \|x\| = 1\}$ is complete, then X is a Banach space.

Then :

(1) Both I and II are true. (2) Neither I nor II is true.

(3) I is true but II is false. (4) II is true but I is false.

मान लीजिए X एक नार्म्ड लीनियर स्पेस है। निम्नलिखित कथनों पर विचार कीजिए।

I. यदि X बानाक स्पेस है तो $\{x \in X : \|x\| = 1$ पूर्ण है।

II. यदि $\{x \in X : \|x\| = 1\}$ पूर्ण है, तो X बानाक स्पेस है।

(1) I तथा II दोनों सही हैं।

(2) I तथा II में से कोई भी सही नही है।

(3) I सही है, II गलत है।

(4) II सही है, I गलत है।

25. Let $l^2 = \{a = (a_k)_{k\geq 1} : a_k \in \mathbb{C}$ and $\left\{\sum_{k=1}^{\infty} |a_k|^2\right\}^{1/2} = \|a\|_2 < \infty\}$ and

$l^\infty \{a = (a_k)_{k\geq 1} : a_k \in \mathbb{C} \sup_{k\geq 1} |a_k| < \infty\}$

Define a map $T : l^\infty \to l^2$ as

$T(a) = \{a_1, \frac{a_2}{2}, \frac{a_3}{2^2}, \frac{a_4}{2^3}, \ldots\ldots\ldots\}$

Which of the following statements in **true** ?

(1) T is discontinuous linear map.

(2) T maps l^∞ anto l^2.

(3) T^{-1} exists and is continuons.

(4) T is uniformal continuous.

माना कि $l^2 = \{a = (a_k)_{k\geq 1} : a_k \in \mathbb{C}$ and $\left\{\sum_{k=1}^{\infty} |a_k|^2\right\}^{1/2} = \|a\|_2 < \infty\}$, और

$l^\infty \{a = (a_k)_{k\geq 1} : a_k \in \mathbb{C} \sup_{k\geq 1} |a_k| < \infty\}$

Define a map $T : l^\infty \to l^2$ as

$T(a) = \{a_1, \frac{a_2}{2}, \frac{a_3}{2^2}, \frac{a_4}{2^3}, \ldots\ldots\ldots\}$

तथा T उपरोक्त हैं। तब कौन-सा कथन सही है ?

(1) T एक असंतत रैखित फलन है।

(2) T l^∞ तथा l^2 के बीच में आच्छादक फलन है।

(3) T^{-1} का अस्तित्व है तथा वह संतत है।

(4) T एक समानतः संतत है।

26. Let $f : IR \to IR$ be twice differentiable function, with $f(0) = f(1) = f'(0) = 0$. Then :

(1) f'' is a zero function (2) $f''(0) = 0$

(3) $f''(x) = 0$ for some $x \in (0, 1)$ (4) f' never vanishes

माना कि $f : IR \to IR$ एक दो बार संततः अवकलनीय फलन है तथा $f(0) = f(1) = f'(0) = 0$ तब :

(1) f'' शून्यक फलन है

(2) $f'(0) = 0$

(3) किसी $x \in (0,1)$ के लिए $f''(x) = 0$

(4) f' कभी लुप्त नहीं होता

27. Let A be subset of IR, $A \neq \phi$, $A \neq IR$ and A is closed, then A is :

(1) the closure of interior of A (2) a countable set

(3) a compact set (4) not opoen

मना कि A, IR का एक उपसमुच्चय है तथा $A \neq \phi, A \neq IR$ व A संवृत्त है, तव A है :

(1) A के आंतरिक का संवरक है। (2) एक गणनीय समुच्चय है।

(3) एक संहत समुच्चय है। (4) विवृत नहीं है।

28. Let $P > 1$ then $\lim_{n\to\infty} \left(\frac{1}{1^p} + \frac{1}{2^p} + \ldots\ldots + \frac{1}{n^p}\right)$ is :

माना कि $p > 1$, तब $\lim_{n\to\infty} \left(\frac{1}{1^p} + \frac{1}{2^p} + \ldots\ldots + \frac{1}{n^p}\right)$ है :

(1) $\frac{1}{1-p}$ (2) $\frac{1}{1+p}$

(3) $\frac{1}{p-1}$ (4) $\frac{1}{p}$

29. The radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{1}{n^{10}} x^n$ is :

घात श्रेणी $\sum_{n=1}^{\infty} \frac{1}{n^{10}} x^n$ की अभिसरण त्रिज्या है :

(1) 1 (2) 2

(3) 0 (4) ∞

30. The residue of the function $\frac{e^{2z}}{(z+1)^3}$ at $z = -1$ is :

$\frac{e^{2z}}{(z+1)^3}$ पर $z = -1$ का अवशेष है :

(1) e^2 (2) $2e^2$

(3) $2e^{-2}$ (4) 0

31. The real projective space IRP^n is :

(1) orientable if n is even.

(2) always orientable.

(3) not orientable if n is even.

(4) always non – orientable.

वास्तविक त्रोजेक्टिव स्पेस IRP^n है।

(1) ओरिएन्टेबल यदि n सम हो

(2) हमेशा ओरिएन्टेबल

(3) ओरिएन्टेबल नहीं यदि n सम हो

(4) हमेशा नान-ओरिएन्टेबल

32. Which one of the following statements is **not** correct ?

(1) Every smooth manifold satisfies the first axiom of countablity.

(2) Every smooth manifold is locally connected.

(3) If a smooth manifold is Hausdorff, then it is locally compact.

(4) Every smooth manifold is para compact.

निम्न में से कौन सही **नहीं** है ?

(1) प्रत्येक स्मूथ मैनिफोल्ड सैटिसफायर फस्ट काउन्टेबल का एम्ज़िम है।

(2) प्रत्येक स्मूथ मैनिफोल्ड लोकली जुड़ा है।

(3) यदि एक स्मूथ मैनिफोल्ड हाउसर्डाफ हो तो वह लोकली कम्पैक्ट है।

(4) प्रत्येक स्मूथ मैनिफोल्ड पैरा कम्पैक्टट है।

33. Which of the following statements is **not** true ?

(1) The Lie bracket operation is IR – linear.

(2) The Lie bracket operation is not skew – symmetric.

(3) The Lie bracket operation satisfies the Jacobi identity.

(4) The Lie bracket operation satisfies

$[f X, g Y] = f g [X, Y] + f (X g) Y - g (Y f) X$

निम्न में से कौन सही नहीं है ?

(1) ली ब्रेकेट IR–रेखिक है।

(2) ली ब्रेकेट स्क्यू-सिमैट्रिक नहीं है।

(3) ली ब्रेकेट जैकोबी आइडेन्टिटी को सन्तुष्ट करता है।

(4) ली ब्रेकेट संतुष्ट करता है :

$[f X, g Y] = f g [X, Y] + f (X g) Y - g (Y f) X$

34. Suppose $f : D - \mathbb{C}$ is of class C^1 on some open subset D of $\mathbb{C}^n$. Then f is holomorphic on D if and only if :

माना $f : D \to \mathbb{C}$ एक क्लास C^1 का फलन है, जहाँ D, $\mathbb{C}^n$ का एक विवृत उपसमुच्चय है। तब f, D में होलोमार्फिक तभी और केवल तभी होगा यदि :

(1) $\frac{\partial f}{\partial \bar{z}^k} = 0, \quad K = 1, 2, \ldots\ldots, n,$

(2) $\frac{\partial f}{\partial z^k} = 0, \quad K = 1, 2, \ldots\ldots, n,$

(3) $\frac{\partial f}{\partial z^k} + \frac{\partial f}{\partial \bar{z}^k} = 0, \quad K = 1, 2, \ldots\ldots, n.$

(4) $\frac{\partial f}{\partial z^k} = \frac{\partial f}{\partial \bar{z}^k}, \quad K = 1, 2, \ldots\ldots, n.$

35. For the Problem : Maximise $\left(x_1^{1/3} . x_2^{2/3}\right)$ subject to $x_1 + 2x_2 = 100$, the optimal value is :

समस्या के लिए अधिकतम $\left(x_1^{1/3} . x_2^{2/3}\right)$ विषय अनुरूप $x_1 + 2x_2 = 100$, का सर्वोकृष्ट वैल्यू हैं :

(1) $\frac{100}{3}$ (2) $\frac{3}{100}$

(3) $\frac{50}{3}$ (4) $\frac{200}{3}$

36. The function $f(x) = \begin{cases} x, & 0 \le x \le 1 \\ 1; & 1 \le x \le 2 \end{cases}$ is :

(1) Convex
(2) Quasi – convex
(3) Semi – strictly quasi convex
(4) All above are true

फलन $f(x) = \begin{cases} x, & 0 \le x \le 1 \\ 1; & 1 \le x \le 2 \end{cases}$ है :

(1) कॉनवेक्स (2) क्वासी-कॉनवेक्स
(3) सेमी स्ट्रिक्टली क्वासी-कन्वेक्स (4) उपरोक्त में सभी सही

37. The partial differential equation
$f(x, y, u)\, u_x + g(x, y, u)\, u_y = h(x, y, u)$ is a :

(1) Linear equation (2) Semi-linear equation
(3) Quasilinear equation (4) None of the above

निम्न आंशिक अवकलन समीकरण दिया है :
$f(x, y, u)\, u_x + g(x, y, u)\, u_y = h(x, y, u)$
उपरोक्त समीकरण है।

(1) रेखिक समीकरण (2) अर्द्धरेखिक समीकरण
(3) क्वासी रेखिक समीकरण (4) उपरोक्त में कोई नही

38. A solution of PDE

$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 - 4 = 0$ represents :

(1) an ellipse in xy – plane. (2) an ellipsoid in xyu space.
(3) a parabola in u – x plane. (4) a hyperbola in u – y plane.

पी०डी०ई० का समाधान है :

$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2-4=0$ निरूपित करता है :

(1) एक xy समतल (प्लेन) पर दीर्घवृत्त

(2) एक xyu की जगह पर दीर्घवृत्ताकार

(3) एक u – x समतल (प्लेन) में पैरावोला (उद्विकास)

(4) एक u – y समतल (प्लेन) में अतिशयोक्ति (हायपरबोला)

39. For an irrotational motion of a fluid, which of the following statements is always **true** ?

(1) The vorticity vector is zero.

(2) The vorticity vector is non-zero.

(3) Divergence of velocity is non-zero.

(4) Divergence of velocity is zero.

द्रव के अचक्रीय गति के लिए निम्न में से कौन हमेशा सत्य है ?

(1) वोर्टिसिटी वेक्टर शून्य है।

(2) वोर्टिसिटी वेक्टर शून्य नहीं है।

(3) गति का डाइवरजैन्स शून्य नहीं है।

(4) गति का डाइवरजैन्स शून्य है।

40. The complex potential of a uniform flow U making an angle α with x-axis is :

सतत प्रवाह U जो कि x-axis से α कोण बनाता है, उसका कॉम्प्लेक्स पोटेन्सियल है :

(1) $Ue^{i(\pi+\alpha)}z$ (2) $Ue^{i(\pi-\alpha)}z$

(3) $-Ue^{i(\pi+\alpha)}z$ (4) $-Ue^{i(\pi-\alpha)}z$

Short Answer Questions

लघु उत्तरीय प्रश्न

Note: Attempt any **five** questions. Write answer in **150-200** words. Each question carries **16** marks. Answer each question on separate page, after writing Question Number.

किन्हीं **पाँच** प्रश्नों के उत्तर दीजिए। प्रत्येक का उत्तर **150-200** शब्दों में दीजिए। प्रत्येक प्रश्न **16** अंकों का है। प्रत्येक प्रश्न का उत्तर अलग पृष्ठ पर प्रश्न संख्या लिखकर शुरु करें।

01. If G is a finite group with O (G) = P^n, for some prime number p then prove that, the centre Z (G) ≠ {e}. Hence show that O (G) = P^2 implies that G is abelion.

यदि G has निश्चित समुच्चय है तथा O(G) = p^n, जहाँ p एक अविभाज्य संख्या है, तब सिद्ध कीजिए कि Z (G) ≠ {e} । उपरोक्त तथ्य का उपयोग करते हुए दिखाइये कि यदि O (G) = p^2 तो G एबीलिएन है।

02. If R is a commutative ring with identity and M is an ideal of R, then prove that M is a maximal ideal if and only if R/M is a field.

यदि R कम्यूटेटिव रिंग है तथा उसके पास यूनिट एलीमैन्ट है तथा M, R का आइडियल है तो सिद्ध कीजिए M मैक्सिमल तभी व केवल तभी है जबकि R/M क्षेत्र हो जाये।

03. Using the Contour integration, evaluate the integral $\int_0^{2\pi} \frac{\cos 3\theta}{5-4\cos\theta} d\theta$.

कन्टूर समाकलन का प्रयोग करते हुए, समाकलन $\int_0^{2\pi} \frac{\cos 3\theta}{5-4\cos\theta} d\theta$ का मान ज्ञात कीजिए।

04. Let X be a non-empty convex subset of IR^n and f, g : X → IR be functions. If f is non-positive and convex and g is positive and convex, then show that $\frac{f}{g}$ is semi-strictly quesi convex.

माना X एक नॉन-एम्पष्टी कॉनवेक्स उपसमुच्चय है IR^n का, तथा $f, g : X \to IR$ में फलन हैं। यदि f नान पोजिटिव तथा कॉनवेक्स, तथा g पॉजिटिव व कॉनवेक्स हों तो दिखाइये कि $\frac{f}{g}$ अर्द्ध-स्ट्रिक्टली क्वासी कॉनवेक्स है।

05. (a) Describe a smooth structure on real projective plane.

रीयल प्रोजेक्टिव प्लेन में स्मूथ स्ट्रक्चर की व्याख्या कीजिए।

(b) Let (M,g) be an n-dimensianal para compact Riemannian manifold. Prove that it always contains a unique metris symmetric connection.

माना (M,g) एक n- विमीय रीमानियन मैनीफोल्ड है। सिद्ध कीजिए कि वह हमेशा एक विशिष्ट मैट्रिक सिमैट्रिक कनैक्सन रखता है।

06. Define a topological property. Prove that compactness is a topological property.

टोपोलाजिकल प्रापर्टी को परिभाषित कीजिए। सिद्ध कीजिए कि कम्पेक्टनेस एक टोपोलाजिकल प्रापर्टी है।

07. Prove that a subset of IR is connected if and only if it is an intervel.

सिद्ध कीजिए कि IR का कोई उपसमुच्चय कनक्टेड तभी और सिर्फ तभी है जबकि वह एक अन्तराल हो।

08. Define a normed linear space and a Banach space. State and prove a necessary and sufficient for a normed linear space to be a Banach space.

नार्मड लीनियर स्पेस की तथा बानाक स्पेस की परिभाषा दीजिए। एक आवश्यक व पर्याप्त शर्त जो उसे बानाक स्पेस बनाती है को लिखिये तथा उसे सिद्ध कीजिए।

09. If both the ends of a bar of length as are at temperature zero and the initial temperature is to be prescribed by function f(x) in the bar. Then find the temperature at subsequent time t.

कोई छड़ जिसके दोनों अन्तिम छोर के बिन्दुओं पर तापमान शून्य है तथा जिसकी लम्बाई a है, तथा उसका आरम्भिक तापमान f(x) से प्रस्क्राइब्ड है। उसके बाद टाइम t पर उसका तापमान ज्ञात कीजिए।

10. Describe Euler's equation of motion for incompressible inviscus fluid.

आयलर के सततता समीकरण न दबाया जा सकने वाले इनविस्कस द्रव के लिए ज्ञात कीजिए।

2016.

Set No. : 1

Question Booklet No.

RET/16/TEST-B

895 | Mathematics (Science)

(To be filled up by the candidate by blue/black ball point pen)

Roll No. | | | | | | | | | | |

Roll No. (Write the digits in words) .....

Serial No. of OMR Answer Sheet .....

Day and Date .....

(Signature of Invigilator)

INSTRUCTIONS TO CANDIDATES

(Use only blue/black ball-point pen in the space above and on both sides of the Answer Sheet)

1. Within 30 minutes of the issue of the Question Booklet, Please ensure that you have got the correct booklet and it contains all the pages in correct sequence and no page/question is missing. In case of faulty Question Booklet, Bring it to the notice of the Superintendent/Invigilators immediately to obtain a fesh Question Booklet.
2. Do not bring any loose paper, written or blank, inside the Examination Hall ***except the Admit Card without its envelope.***
3. ***A separate Answer Sheet is given. It should not be folded or mutilated. A second Answer Sheet shall not be provided.***
4. Write your Roll Number and Serial Number of the Answer Sheet by pen in the space provided above.
5. ***On the front page of the Answer Sheet, write by pen your Roll Number in the space provided at the top, and by darkening the circles at the bottom. Also, wherever applicable, write the Question Booklet Number and the Set Number in appropriate places.***
6. ***No overwriting is allowed in the entries of Roll No., Question Booklet No. and Set No. (if any) on OMR sheet and Roll No. and OMR sheet no. on the Question Booklet.***
7. ***Any change in the aforesaid entries is to be verified by the invigilator, otherwise it will be taken as unfair means.***
8. ***This Booklet contains 40 multiple choice questions followed by 10 short answer questions. For each MCQ, you are to record the correct option on the Answer Sheet by darkening the appropriate circle in the corresponding row of the Answer Sheet, by pen as mentioned in the guidelines given on the first page of the Answer Sheet. For answering any five short Answer Questions use five Blank pages attached at the end of this Question Booklet.***
9. For each question, darken only **one** circle on the Answer Sheet. If you darken more than one circle or darken a circle partially, the answer will be treated as incorrect.
10. ***Note that the answer once filled in ink cannot be changed. If you do not wish to attempt a question, leave all the circles in the corresponding row blank (such question will be awarded zero marks)***
11. For rough work, use the inner back pages of the title cover and the blank page at the end of this Booklet.
12. ***Deposit both OMR Answer Sheet and Question Booklet at the end of the Test.***
13. You are not permitted to leave the Examination Hall until the end of the Test.
14. If a candidate attempts to use any form of unfair means, he/she shall be liable to such punishment as the University may determine and impose on him/her.

Total No. of Printed Pages : 24

39.

Research Entrance Test-2016

No. of Questions : 50

प्रश्नों की संख्या : 50

Time : 2 Hours *Full Marks : 200*

समय : 2 घण्टे *पूर्णाङ्क : 200*

Note: (1) This Question Booklet contains **40** Multiple Choice Questions followed by **10** Short Answer Questions.

इस प्रश्न पुस्तिका में **40** वस्तुनिष्ठ व **10** लघु उत्तरीय प्रश्न हैं।

(2) Attempt as many MCQs as you can. Each MCQ carries **3 (Three)** marks. **1 (One)** mark will be deducted for each incorrect answer. **Zero** mark will be awarded for each unattempted question. If more than one alternative answers of MCQs seem to be approximate to the correct answer, choose the closest one.

अधिकाधिक वस्तुनिष्ठ प्रश्नों को हल करने का प्रयत्न करें। प्रत्येक वस्तुनिष्ठ प्रश्न **3 (तीन)** अंकों का है। प्रत्येक गलत उत्तर के लिए **1 (एक)** अंक काटा जायेगा। प्रत्येक अनुत्तरित प्रश्न का प्राप्तांक **शून्य** होगा। यदि वस्तुनिष्ठ प्रश्नों के एकाधिक वैकल्पिक उत्तर सही उत्तर के निकट प्रतीत हों, तो निकटतम सही उत्तर दें।

(3) Answer only **5** Short Answer Questions. Each question carries **16 (Sixteen)** marks and should be answered in **150-200** words. Blank **5 (Five)** pages attached with this booklet shall only be used for the purpose. Answer each question on separate page, after writing Question No.

केवल **5 (पाँच)** लघुउत्तरीय प्रश्नों के उत्तर दें। प्रत्येक प्रश्न **16 (सोलह)** अंकों का है तथा उनका उत्तर **150-200** शब्दों के बीच होना चाहिए। इसके लिए इस पुस्तिका में लगे हुए सादे **5 (पाँच)** पृष्ठों का ही उपयोग आवश्यक है। प्रत्येक प्रश्न का उत्तर एक नए पृष्ठ से, प्रश्न संख्या लिखकर शुरू करें।

01. Which is not true for reactions by the S_N2 mechanism ?

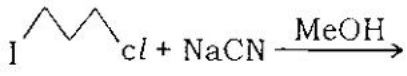
(1) proceeds through a backside attack and results in inversion

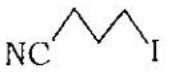
(2) tends to proceed with weak nucleophiles solvents like CH_3OH, H_2O, CH_3CH_2OH.

(3) rate of reaction proceeds from primary (fastest) > secondary >> tertiary (slowest)

(4) occurs in one step

02. Which is the main product of the following reaction ?



(1) 

(2) 

(3) 

(4) 

03. Which of the following conditions is necessary for a reaction to be spontaneous ?

(1) $\Delta S_{sur} > 0$ (2) $\Delta S_{sys} > 0$

(3) $\Delta S_{sur} + \Delta S_{sys} > 0$ (4) $\Delta S_{sur} + \Delta S_{sys} < 0$

04. Dead organs are generally stored in formalin. Formalin is :

(1) aqueous formaldehyde (2) aqueous ferrous sulphate

(3) aqueous formic acid (4) aqueous ferric alum

05. Regarding "carbon credits", which one of the following statement is **not** correct :

(1) The carbon credit system was ratified in conjunction with the Kyoto Protocol.

(2) Carbon credits are awarded to countires or groups that have reduced greenhouse gases below their emission quota.

(3) The goal of the carbon credit system is to limit the increase of carbon dioxide emission.

(4) Carbon credits are traded at a price fixed from time to time by the United Nations Environment Progrmme.

06. Ball bearings are used in bicycles, cars, etc., because :

(1) the actual area of contact between the wheel and axle is increased.

(2) the effective area of contact between the wheel and axle is increased

(3) the effective area of contact between the wheel and axle is reduced

(4) the actual area of contact between the wheel and axle is reduced.

07. During respiration, energy is released. It is stored in the form of :

(1) ADP (2) ATP (3) NADP (4) APP

08. Which of the following is known as Royal disease :

(1) Sickle cell anemia (2) Haemophilia

(3) Alzheimers disease (4) Colour blindness

09. The xylem in plants is responsible for :

(1) transport of water (2) transport of food

(3) transport of oxygen (4) transport of amino acids

10. Two wires, of the same material, have their lenghts in the ratio 1:2 and their diameters in the ratio 2:1. If both are stretched separately by equal weights, the ratio of increase in their lengths, $L_1 : L_2$ would be :

(1) 1:2 (2) 2:1 (3) 1:8 (4) 8:1

11. A sphere starts rotating under no forces about its centre with angular velocity components 2n, n and 3n about its principal axes respectively. During rotaion :

(1) each angular velocity component remains same

(2) only second angular velocity component remains same and other two change.

(3) first and third angular velocity components become periodic

(4) all the three components change but resultant of angular velocity remains same.

12. If q and p are generalized co-ordinate and momentum variable respectivly, then.

(1) $p = \frac{\partial T}{\partial q}$ (2) $p = \frac{\partial V}{\partial q}$

(3) $p = \frac{\partial L}{\partial q}$ (4) $p = \frac{\partial S}{\partial q}$

Where T,V,L and S are the Kinetic energy, Potential energy, Lagrange function and Hamilton's principal function respectively.

13. If transformation equations from one phase space (q_r, p_r) to another phase space (Q_r, P_r) are canonical, then :

(1) $\frac{\partial P_r}{\partial q_r} = \frac{\partial p_r}{\partial Q_r}$ (2) $\frac{\partial P_r}{\partial Q_r} = \frac{\partial p_r}{\partial q_r}$

(3) $\frac{\partial P_r}{\partial q_r} = -\frac{\partial p_r}{\partial Q_r}$ (4) $\frac{\partial Q_r}{\partial P_r} = \frac{\partial p_r}{\partial q_r}$

14. The family of curves given by ϕ = constant and ψ = constant in fluid dynamics intersects at :

(1) 45^0 (2) 60^0 (3) 90^0 (4) 75^0

15. Streamlines and pathlines become same when the motion is :

(1) Steady (2) Unsteady

(3) Turbulent (4) Larminar

16. If U_{max} is the maximum velocity in the plane Poiseulle flow, then the average velocity in the flow is given by

(1) $\frac{1}{3} U_{max}$ (2) $\frac{2}{3} U_{max}$

(3) $\frac{3}{2} U_{max}$ (4) $\frac{1}{2} U_{max}$

17. The difference equation at the boundary point x = 0 of the boundary value problem $y'' - y = x$, $y'(0) = 0$, $y(1) = 1$ by using the central difference schemes for the derivatives when width of each sub-interval is 0.25 is :

(1) $32 y_0 + 33 y_1 = 0$ (2) $33 y_0 - 32 y_1 = 0$

(3) $33 y_0 + 32 y_1 = 0$ (4) $32 y_0 - 33 y_1 = 0$

18. Initial value problem $y'' + y = 0$, $y(0) = y'(0) = 0$ is equivalent to the integral equation :

(1) $y(x) = -\int_0^x (x - t) y(t)\, dt$ (2) $y(x) = \int_0^x (x - t) y(t)\, dt$

(3) $y(x) = -\int_0^x (x + t) y(t)\, dt$ (4) $y(x) = \int_0^x (x + t) y(t)\, dt$

19. The solution of partial differential equation $\frac{\partial u}{\partial u} + 2\frac{\partial u}{\partial x} + 3u = 0$, with $u(x, 0) = \sin x$ is :

(1) $\sin(x - 2t)$ (2) $e^{-t} \sin(x - 2t)$

(3) $e^{-3t} \sin(x - 2t)$ (4) $e^{2t} \sin x$

20. The nature of partial differential equation

$$\cos^2 x \frac{\partial^2 u}{\partial x^2} + \sin 2x \frac{\partial^2 u}{\partial xy} + \cos^2 x \frac{\partial^2 u}{\partial y^2} = x^2 \text{ is :}$$

(1) parabolic (2) hyperbolic

(3) elliptic (4) circular

21. The number of zeros of the polynomial $p(z) = z^4 - 4z^3 + z - 1 = 0$ lying inside the unit disc is :

(1) 4 (2) 3 (3) 2 (4) 1

22. The value of integral $\int_{z=n\,2} \frac{dz}{z^2 \sin z}$ is :

(1) $\frac{\pi i}{3}$ (2) $\frac{2\pi i}{3}$ (3) πi (4) 0

23. A finite abelion group of prime power order is internal direct product of :

(1) its subgroups (2) its normal subgroups

(3) cyclic groups (4) subgroups of prime order

24. Total number of elements of order 5 in the group $\mathbb{Z}_{25} \oplus \mathbb{Z}_5$ is :

(1) 25 (2) 24 (3) 5 (4) 6

25. U (15), the set of all integers less than 15 and relative prime to 15 is a group under multiplication module 15. Number of cyclic subgroups of U (15) is :

(1) 2 (2) 3 (3) 4 (4) 6

26. The number of cyclic subgroups of order 15 in the group $\mathbb{Z}_{30} \oplus \mathbb{Z}_{20}$ is :

(1) 6 (2) 5 (3) 10 (4) 11

27. If A^0 and B^0 denote interior of A and B respectively, then :

(1) $A^0 \cup B^0 = (A \cup B)^0$ (2) $A^0 \cup B^0 = (A \cap B)^0$

(3) $A^0 \cup B^0 \supseteq (A \cup B)^0$ (4) $A^0 \cup B^0 \subseteq (A \cup B)^0$

28. If Ω is a convex subset of $\mathbb{R}^n$, then which one of the following is false :

(1) int Ω is a convex set

(2) $\bar{\Omega}$ is a convex set

(3) $\overline{\text{int}\,\Omega} = \bar{\Omega}$

(4) for any $a \in \text{int}\ \Omega$ and $b \in \bar{\Omega}$, $[a, b] \subset \text{int}\ \Omega$.

29. Let $c_{[a,b]}$ be a vector space of all real valued continuous functions defined on closed interval [a,b], $0 < a < b < \infty$. if

for $f \in c_{[a,b]}$, $||f||_3 = \left(\int_a^b |f(x)|^{1/3}\,dx\right)^3$, then

(1) $(c_{[a,b]}, ||\ ||_3)$ is a normed linear space but not a Banach space over a set of all real numbers IR

(2) $(c_{[a,b]}, ||\ ||_3)$ is a Banach space over IR

(3) $(c_{[a,b]}, ||\ ||_3)$ is not a normed linear space over IR

(4) In $(c_{[a,b]}, ||\ ||_3)$ every absolutely convergent series is convergent IR.

30. Which one of the following is false :

(1) $f(x) = \begin{cases} 1 \text{ for } -1 \le x \le x \ne 0 \\ 2 \text{ for } x = 0 \end{cases}$

is quasi convex

(2) $f(x) = \begin{cases} \dfrac{|x|}{x} \text{ for } x \ne 0 \\ 0 \quad \text{for } x = 0 \end{cases}$

is quasi convex

(3) $f(x) = \begin{cases} x \text{ for } 0 \le x \le 1 \\ 1 \text{ for } 1 < x \le 2 \end{cases}$

is quasi convex

(4) $f(x) = \begin{cases} x \text{ for } 0 \le x \le 1 \\ 1 \text{ for } 1 < x \le 2 \end{cases}$

is not semi-strictly quasi convex

31. If C is a closed convex cone in $\mathbb{R}^n$, then which one of the following is false :

(1) C^* is a closed convex cone.

(2) $x \in C \Leftrightarrow \alpha^t x \geq 0 \ \forall \ \alpha \in C^*$

(3) $x \in \text{int } C \Leftrightarrow \alpha^t x > 0 \ \forall \ \alpha \in C^* \setminus \{0\}$

(4) $\alpha \in \text{int } C^* \Leftrightarrow \alpha^t x \geq 0 \ \forall x \in C \setminus \{0\}$

32. The radius of convergence of power series $\sum_{n=0}^{\infty} n!\, z^{n!}$ is :

(1) 1 (2) e (3) $\frac{1}{e}$ (4) 0

33. If $a > 0$, $b > 0$, $1 < p < \infty$, $\frac{1}{p^2} + \frac{1}{q^2} = 1$ and $ab = \frac{a^{p^2}}{p^2} + \frac{b^{q^2}}{q^2}$ then :

(1) $a^{p^2} = b^{q^2}$ (2) $a^{p^2} = b^{q^2}$

(3) $a^{\frac{1}{p}} = b^{\frac{1}{q}}$ (4) $a^{\frac{1}{p3}} = b^{\frac{1}{q2}}$

34. The series :

$1 - 3 + 6 - 10 + 15 - \ldots\ldots\ldots\ldots$

(1) is convergent but not absolutely convergent.

(2) is absolutely convergent

(3) oscillates finitely

(4) oscillates infinitely

35. The value of $\int_0^{\infty} \frac{dx}{(1+x)x^{1/2}}$ is :

(1) π (2) $\frac{\pi}{2}$ (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{3}$

36. The general linear group GL (n, IR) is differentiable manifold of dimension :

(1) $\frac{n(n+1)}{2}$ (2) $\frac{n(n-1)}{2}$

(3) n^2 (4) $n^2/2$

37. For any x, y, z $\in \chi$ (m) and f, g, $\in c^{\infty}$(m), [f x, g y] =

(1) fg [x, y] + f (xg) y - g(yf) x (2) fg [x, y] + f (xg) y + g(yf) x

(3) fg [x, y] + (xg) (yf) - (yf) (xg) (4) fg [x, y] + (xg) (yf) + (yf) (xg)

38. The integral curves for the vector field in IR^2 given by $X = x^2 \frac{\partial}{\partial x^1} - (x^2)^3 \frac{\partial}{\partial x^2}$ is :

(1) parabola (2) hyperbola

(3) ellipse (4) circle

39. If $w = xdx + ydy + (x + z)\, dz$ be 1 – form in IR^3, then the exterior differential dw is:

(1) $dx \wedge dy$ (2) $dx \wedge dz$

(3) 0 (4) $dy \wedge dx$

40. If an open set $U \subset \mathbb{R}^n$ is diffeomorphic to an open set $V \subset \mathbb{R}^m$, then :

(1) $n = m$ (2) $n > m$

(3) $n < m$ (4) $n = m + 1$

Short Answer Questions

Note: Attempt any **five** questions. Write answer in **150-200** words. Each question carries **16** marks. Answer each question on separate page, after writing Question Number.

1. Derive Hamilton's equations of motion from Hamilton-Jacobi theory in generalised system of rigid body motion.

2. Derive the equation of continuity in cylindrical coordinate system.

3. Use the Galerkin method to solve the boundary value problem $y'' + y = \frac{1}{4}(7x = 12x^2)$, $y(0) = 0$, $y(2) = 0$ by taking the approximate function as $c_1 x(x - 2) + c_2 x^2 (x - 2)$.

4. Using the contour integration, evaluate the integral $\int_0^{2\pi} \frac{\cos 3\theta}{5 - 4\cos\theta} d\theta$.

5. (a) Prove that continuous image of a compact space is compact.

(b) Prove that continuous image of a connected space is connected.

6. Let G be a finite group and it $p^m \mid 0(G)$, then show that G has a sub group of order p^m, where p is prime and $m \geq 1$.

7. Let X be an open convex subset of $\mathbb{R}^n$ and $f, g : x \to \mathbb{R}$ be differentiable on X. If f is convex and g is positive and affine on X, then show that $\frac{f}{g}$ is pseudo convex on X.

8. Show that the n-sphere S^n is an n-dimensional differentiable manifold.

9. Prove that the zeros of

$f(z) = f(x + iy) = u(x, y) + i\,v(x, y)$, $x, y \in \mathbb{R}$, $i = \sqrt{-1}$ are the insections of the curves $u(x, y) = 0$ and $v(x, y) = 0$. Also show that at a double zero of $f(z)$ each of the curves $u(x, y) = 0$, $v(x, y) = 0$ has a double point and the two curves intersect at an angle $\pi/4$

10. Let Ω be the set of all invertible linear operators on $\mathbb{R}^n$ onto itself. If $T \in \Omega$, $S \in L(\mathbb{R}^n, \mathbb{R}^n)$ and $||S-T||\,||T^{-1}|| < 1$, then prove that $S \in \Omega$ and Ω is an open subset of $L(\mathbb{R}^n, \mathbb{R}^n)$. Also prove that the mapping $T \to T^{-1}$ is continuous on Ω.

2016.

Set No. : 1

Question Booklet No.

RET/16/TEST-B

988 | Mathematical Science

(To be filled up by the candidate by blue/black ball point pen)

Roll No. | | | | | | | | | | |

Roll No. (Write the digits in words) .....

Serial No. of OMR Answer Sheet .....

Day and Date .....

(Signature of Invigilator)

INSTRUCTIONS TO CANDIDATES

(Use only blue/black ball-point pen in the space above and on both sides of the Answer Sheet)

1. Within 30 minutes of the issue of the Question Booklet, Please ensure that you have got the correct booklet and it contains all the pages in correct sequence and no page/question is missing. In case of faulty Question Booklet, Bring it to the notice of the Superintendent/Invigilators immediately to obtain a fesh Question Booklet.
2. Do not bring any loose paper, written or blank, inside the Examination Hall ***except the Admit Card without its envelope.***
3. ***A separate Answer Sheet is given. It should not be folded or mutilated. A second Answer Sheet shall not be provided.***
4. Write your Roll Number and Serial Number of the Answer Sheet by pen in the space provided above.
5. ***On the front page of the Answer Sheet, write by pen your Roll Number in the space provided at the top, and by darkening the circles at the bottom. Also, wherever applicable, write the Question Booklet Number and the Set Number in appropriate places.***
6. ***No overwriting is allowed in the entries of Roll No., Question Booklet No. and Set No. (if any) on OMR sheet and Roll No. and OMR sheet no. on the Question Booklet.***
7. ***Any change in the aforesaid entries is to be verified by the invigilator, otherwise it will be taken as unfair means.***
8. ***This Booklet contains 40 multiple choice questions followed by 10 short answer questions. For each MCQ, you are to record the correct option on the Answer Sheet by darkening the appropriate circle in the corresponding row of the Answer Sheet, by pen as mentioned in the guidelines given on the first page of the Answer Sheet. For answering any five short Answer Questions use five Blank pages attached at the end of this Question Booklet.***
9. For each question, darken only **one** circle on the Answer Sheet. If you darken more than one circle or darken a circle partially, the answer will be treated as incorrect.
10. ***Note that the answer once filled in ink cannot be changed. If you do not wish to attempt a question, leave all the circles in the corresponding row blank (such question will be awarded zero marks).***
11. For rough work, use the inner back pages of the title cover and the blank page at the end of this Booklet.
12. ***Deposit both OMR Answer Sheet and Question Booklet at the end of the Test.***
13. You are not permitted to leave the Examination Hall until the end of the Test.
14. If a candidate attempts to use any form of unfair means, he/she shall be liable to such punishment as the University may determine and impose on him/her.

Total No. of Printed Pages : 40

38.

SEAL

Research Entrance Test-2016

No. of Questions : 50

Time : 2 Hours ***Full Marks : 200***

Note: **(1)** This Question Booklet contains **40** Multiple Choice Questions followed by **10** Short Answer Questions.

(2) Attempt as many MCQs as you can. Each MCQ carries **3 (Three)** marks. **1 (One)** mark will be deducted for each incorrect answer. **Zero** mark will be awarded for each unattempted question. If more than one alternative answers of MCQs seem to be approximate to the correct answer, choose the closest one.

(3) Answer only **5** Short Answer Questions. Each question carries **16 (Sixteen)** marks and should be answered in **150-200** words. Blank **5 (Five)** pages attached with this booklet shall only be used for the purpose. Answer each question on separate page, after writing Question No.

(4) For mathematical Science Students only :

(i) This paper contains **three** sections :

(A) **Mathematical Section** (Q. No. **11-40** & Short Answer Questions)

(B) **Statistics Section** (Q. N0. **41-70** & Short Answer Questions)

(C) **Computer Science Section** (Q. No. **71-100** & Short Answer Questions) **A** candidate has to attempt only **one** section.

(ii) Q. No. **1 to 10** are compulsory to **all**.

(A) Mathematics Section

01. Which is not true for reactions by the S_N2 mechanism ?

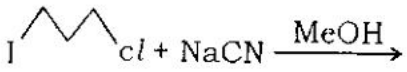
(1) proceeds through a backside attack and results in inversion

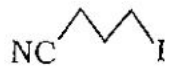
(2) tends to proceed with weak nucleophiles solvents like CH_3OH, H_2O, CH_3CH_2OH.

(3) rate of reaction proceeds from primary (fastest) > secondary >> tertiary (slowest)

(4) occurs in one step

02. Which is the main product of the following reaction ?



(1) 

(2) 

(3) 

(4) 

03. Which of the following conditions is necessary for a reaction to be spontaneous ?

(1) $\Delta S_{sur} > 0$ (2) $\Delta S_{sys} > 0$

(3) $\Delta S_{sur} + \Delta S_{sys} > 0$ (4) $\Delta S_{sur} + \Delta S_{sys} < 0$

04. Dead organs are generally stored in formalin. Formalin is :

(1) aqueous formaldehyde (2) aqueous ferrous sulphate

(3) aqueous formic acid (4) aqueous ferric alum

05. Regarding "carbon credits", which one of the following statement is **not** correct :

(1) The carbon credit system was ratified in conjunction with the Kyoto Protocol.

(2) Carbon credits are awarded to countires or groups that have reduced greenhouse gases below their emission quota.

(3) The goal of the carbon credit system is to limit the increase of carbon dioxide emission.

(4) Carbon credits are traded at a price fixed from time to time by the United Nations Environment Progrmme.

06. Ball bearings are used in bicycles, cars, etc., because :

(1) the actual area of contact between the wheel and axle is increased.

(2) the effective area of contact between the wheel and axle is increased

(3) the effective area of contact between the wheel and axle is reduced

(4) the actual area of contact between the wheel and axle is reduced.

07. During respiration, energy is released. It is stored in the form of :

(1) ADP (2) ATP (3) NADP (4) APP

08. Which of the following is known as Royal disease :

(1) Sickle cell anemia (2) Haemophilia

(3) Alzheimers disease (4) Colour blindness

09. The xylem in plants is responsible for :

(1) transport of water (2) transport of food

(3) transport of oxygen (4) transport of amino acids

10. Two wires, of the same material, have their lenghts in the ratio 1:2 and their diameters in the ratio 2:1. If both are stretched separately by equal weights, the ratio of increase in their lengths, $L_1 : L_2$ would be :

(1) 1:2 (2) 2:1 (3) 1:8 (4) 8:1

11. The dimension of the vector space $V = \{A = (a_{ij})_{n\times n};\ a_{ij} \in C,\ a_{ij} = -\overline{a_{ji}}\}$ over field IR is :

(1) n^2 (2) $n^2 - 1$ (3) $n^2 - n$ (4) $n^2/2$

12. Let f is differentiable and $f(x) \neq 1$ for any real x then :

(1) f is monotonic (2) f has unique fixed point

(3) f has atmost one fixed point (4) f has no fixed point

13. Choose the incorrect, for the Initial Value Problem (IVP) :

$\frac{dy}{dx} = \frac{(y-x)}{x}$ with $y(x_0) = y_0$ has :

(1) No solutionif $y_0 = 0, x_0 = 0$.

(2) Unique solution if $y_0 = 1, x_0 = 1$

(3) Unique solution if $y_0 = 1, x_0 = 0$

(4) Infinite solution if $y_0 = 1, x_0 = 0$

14. If $(x-1)^2 u_{xx} - (y-2)^2 u_{yy} + 2xu_y + 2xyu = 0$ is parabolic in $S \subseteq R^2$ but not in $R^2 - \{S\}$ then S is :

(1) $\{(x, y) \in R^2; x = 1 \text{ or } y = 2\}$ (2) $\{(x, y) \in R^2; x = 1 \text{ and } y = 2\}$

(3) $\{(x, y) \in R^2; x = 1\}$ (4) $\{(x, y) \in R^2; x = 2\}$

15. The possible value of α for which the variational problem $I(y(x)) = \int_0^1 (3y^2 + 2x^3 y^1)\,dx,\ y(\alpha) = 1$ has extremals are :

(1) $-1, 0$ (2) $0, 1$ (3) $-1, 1$ (4) $0, -1$

16. If a holomonic system defined by n-generalized coordinates $q_1, q_2, \ldots\ldots, q_n$ moves under the influence of conservative system, then Hamilton's equation of motion is :

(1) $\dot{p}_j = -\frac{\partial H}{\partial q_j}, \dot{q}_j = \frac{\partial H}{\partial p_j}$ (2) $\dot{p}_j = -\frac{\partial H}{\partial q_j}, \dot{q}_j = -\frac{\partial H}{\partial p_j}$

(3) $\dot{p}_j = \frac{\partial H}{\partial p_j}, \dot{q}_j = \frac{\partial H}{\partial q_j}$ (4) $\dot{p}_j = \frac{\partial H}{\partial p_j}, q_j = -\frac{\partial H}{\partial q_j}$

17. Let $P_n(x)$ be the Legendre polynomial of degree n such that $P_n(1) = 1$, $n = 1, 2, \ldots\ldots$ If $\int_{-1}^{1} \sum_{j=1}^{n} j(2_j+1)P_j(x)^2 dx = 20$ n is e qual to :

(1) 2 (2) 3 (3) 4 (4) 5

18. Consider the topology $\tau = \{G \subseteq R : R\backslash G\}$ is compact in $(R, \tau_U) \cup \{\phi, R\}$ on where is the usual topology on R and ϕ is the empty set. Then (R, τ) is :

(1) A connected Hausdorff space

(2) Connected but not Hausdorff

(3) Hausdorff but not connected

(4) Neither connected nor Hausdorff

19. The geometrical representation of a complex number divided by 'i' is :

(1) Reflection abot the x-axis

(2) Reflection abot the y-axis

(3) Rotation throgh 90^0 about the origin in the counter clockwise direction

(4) Rotation through 90^0 about the origin in the clockwise direction

20. The truncation error of the following method $u_{j+1} = u_j + \frac{1}{6}(k_1 + 2k_2 + 2K_3 + k_4)$ is of order :

(1) 3 (2) 4 (3) 5 (4) 6

21. The sum of residues of the function $f(t) = \frac{z^2}{(z-1)(z-2)(z-3)}$ at $z = 1, 2, 3$ and ∞ is :

(1) 1/2 (2) 1 (3) -8 (4) 0

22. If $f : R \to R$ is continous then which one of the following cannot be the image of [0, 1] under f :

(1) {0} (2) (0, 1) (3) [0, 1) (4) [0,1]

23. The value of the integral of $\frac{1}{z}$ along the semi circular arc $|z| = 1$ form $z = -1$ to $z = +1$, lies below the real axis, is :

(1) pi (2) - pi (3) 0 (4) 2 pi

24. If $A = \begin{pmatrix} 2 & 3 \\ 0 & 2 \end{pmatrix}$ then the value of e^A is :

(1) $\begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix} e^2$ (2) $\begin{pmatrix} e^2 & e^3 \\ 1 & e^2 \end{pmatrix}$

(3) $\begin{pmatrix} 2 & 3 \\ 0 & 2 \end{pmatrix} e^2$ (4) $\begin{pmatrix} e & e^3 \\ 1 & e \end{pmatrix}$

25. The maximum number of independent components of tensor of rank three, symmetry in last two indices in V_n, are :

(1) $\frac{n(n+1)}{2}$ (2) $\frac{n^2(n+1)}{2}$

(3) $\frac{n(n-1)}{2}$ (4) $\frac{n^2(n-1)}{2}$

26. The solution of the differential equation $\frac{d^3y}{dx^3} - 2y\frac{d^2y}{dx^2} - \frac{dy}{dx} + 2y = e^{5x}$ is :

(1) $y = Ae^x + Be^{-x} + Ce^{2x} + \frac{e^{5x}}{72}$ (2) $y = Ae^{-x} + Be^x + Ce^{2x} + \frac{e^{5x}}{72}$

(3) $y = Ae^x + Be^{-x} + Ce^{2x} + \frac{xe^{5x}}{72}$ (4) $y = Ae^x + Be^{-x} + Ce^{2x} - \frac{xe^{5x}}{72}$

27. The locus of complex number, z, satisfying $|z - 2| + |z + 2| = 6$ is :

(1) Line segment (2) Circle

(3) Ellipse (4) Straight Line

28. For a continuous function $f : R \to R$, let $Z(f) = \{x \in R : f(x) = 0\}$. Then Z (f) is always :

(1) compact (2) open

(3) connected (4) closed

29. Let $f(x) = \max(\sin x, \cos x)$, for all $x \in R$. Then :

(1) f is differentiable on R

(2) f is differentiable on R except at 0

(3) f is nowhere differentiable

(4) f is differentiable except at a countable set of points

30. For the metric $ds^2 = -a(dx^1)^2 + b(dx^2)^2 + c(dx^3)^2 + d(dx^4)^2$, where a,b,c,d are functions of x^1 (i = 1, 2, 3, 4), the value of the Christoffel symbol [2,11] is :

(1) $\frac{1}{2}\frac{\partial a}{\partial x^2}$ (2) $-\frac{1}{2}\frac{\partial a}{\partial x^2}$

(3) $\frac{1}{2}\frac{\partial b}{\partial x^1}$ (4) $-\frac{1}{2}\frac{\partial b}{\partial x^2}$

31. The function $f(x, y) = \begin{cases} \frac{x^2+y^2}{|x|+|y|}, (x,y) \neq (0,0) \\ 0, \quad (x,y) = (0,0) \end{cases}$ is :

(1) continuous at (0,0) but its partial derivatives f_x and f_y do not exist at (0,0)

(2) discontinuous at (0,0) but f_x and f_y exist at (0,0)

(3) continuous at (0,0) and f_x exists but f_y does not exist at (0,0)

(4) continuous at (0,0) and f_y exists but f_x does not exist at (0,0)

32. The boundary of $\{r + is : r, s \in Q\}$ in the topological space C (under usual topology) is :

(1) the empty set (2) $\{x + iy : x, y \in R - Q\}$

(3) $\{is : s \in R - Q\}$ (4) C

33. For a homogeneous medium containing charges and currents, the value of $\nabla^2 E - \mu \in \frac{\partial^2 E}{\partial t^2}$ is :

(1) $\in \nabla p + \mu \frac{\partial}{\partial t} J$ (2) $\frac{1}{\in}\nabla p + \mu \frac{\partial}{\partial t} J$

(3) $\in \nabla p + \frac{1}{\mu}\frac{\partial}{\partial t} J$ (4) $\frac{1}{\in}\nabla p + \frac{1\partial}{\mu \partial t} J$

34. The continuous function f:R → R defined by $f(x) = (1 + x^2)^{2016}$ is :

(1) onto but not one-one (2) one-one but not onto

(3) both one-one and onto (4) neither one-one nor onto

35. The number of elements of order 5 in $Z_{25} \oplus Z_5$ are :

(1) 24 (2) 25 (3) 30 (4) 100

36. Consider the following LPP min $z = 10x_1 + x_2 + 5x_3$ such that $5x_1 - 7x_2 + 3x_3 \geq 50$, $x_1, x_2, x_3 \geq 0$, then its optimal value is :

(1) 50/3 (2) 250/3 (3) 10/3 (4) 100/3

37. The value of cos θ for the angle θ between $f(t) = 2t - 1$ and $g(t) = t^2$ in the vector space V of polynomial with inner product $\{f,g\} = \int_0^1 f(t)\, g(t)\, dt$ is :

(1) $\frac{\sqrt{15}}{4}$ (2) $\frac{\sqrt{15}}{6}$ (3) $\frac{1}{\sqrt{15}}$ (4) $\frac{1}{\sqrt{6}}$

38. The Fourier transform of $f(x) = \begin{cases} 1 \text{ for } |x| < a \\ 0 \text{ for } |x| > a \end{cases}$ is :

(1) $\sqrt{\frac{\pi}{2}} \frac{\text{Sin sa}}{\text{s}}$ (2) $\sqrt{\frac{\pi}{2}} \frac{\text{Cos sa}}{\text{s}}$

(3) $\sqrt{\frac{2}{\pi}} \frac{\text{Sin sa}}{\text{s}}$ (4) $\sqrt{\frac{2}{\pi}} \frac{\text{Cos sa}}{\text{s}}$

39. The value of integral $\int_1^4 [\log_e x]\, dx$ where [ ] denotes greatest integer function is :

(1) log 4 (2) 1/2 (3) 4 + e (4) 4 - e

40. Solution of the Poission's equation $u_{xx} + u_{yy} = -1$ in the square $|x| \leq 1$, $|y| \leq 1$ with $u = 0$ at $x = \pm 1$ and $y = \pm 1$ is :

(1) $u(x, y) = \frac{1}{16}(1 - x^2)(1 - y^2)$

(2) $u(x, y) = \frac{5}{16}(1 - x^2)(1 - y^2)$

(3) $u(x, y) = \frac{5}{16}(1 + x^2)(1 + y^2)$

(4) $u(x, y) = \frac{1}{16}(1 + x^2)(1 + y^2)$

Short Answer Questions

Note: Attempt any **five** questions. Write answer in **150-200** words. Each question carries **16** marks. Answer each question on separate page, after writing Question Number.

01. Define gamma function on complex plane and show that it has only simple poles as singularities.

02. Find the nature of extremals of the functional $\int_1^2 \frac{\left[1+y'^2\right]^{\frac{1}{2}}}{x}\,dx$ under the boundary condition y(1) = 0, y(2) = 2.

03. Define a regular surface. Show that graph of a smooth function is a regular surface.

04. Show that a necessary and sufficienct condition for a curve to be a spherical is that its radius of curvature p and radius torsion satisfy $\frac{p}{\sigma}+\frac{d}{ds}(p'\sigma) = 0$ where s is arc lenght parameter and $p' = \frac{dp}{ds}$

05. State and prove Euler formula for normal curvature at point on a regular surface.

06. Show that the equation $z^5 + 15z + 1 = 0$ has four roots in the annulus $\frac{2}{3} < |z| < 2$

07. Show that the age of the universe (t_0) in terms of observable parameters for open universe is

$$t_0 = \frac{q_0}{H_0(1-2q_0)^{2/3}} + \left[\frac{\sqrt{1-2q_0}}{q_0} - \cosh^{-1}\left(\frac{1-q_0}{q_0}\right)\right]$$; where H_0 and q_0 are the Hubble constant and deceleration parameter at present.

08. Evaluate the intergral $\int_0^1 \frac{dx}{1+x^2}$ by Simpson's one-third rule taking k= 8. The arguments and corresponding values of the function f(x) are tabulate below :

x	0.000	0.125	0.250	0.375	0.500	0.625	0.875	1.000
f (x)	1.00000	0.98461	0.94117	0.87671	0.80000	0.64000	0.56637	0.50000

09. Show that the transformation $w = z^2$ of complex plane transform circle centred on real line in z-plane to limacon's in w-plane.

10. If $f_n : R \to R$ be the function $fn(x) = \frac{1}{n^3\left(x-\frac{1}{n}\right)^2+1}$ and $f : R \to R$ be the zero function then show that $f_n(x) \to f(x)$ for each $x \in R$ but f_n does not converge uniformly to f.

(B) Statistics Section

41. Which of following is not a correct statement about Bivariate Normal Distribution ?

(1) Marginaldistributions are normal

(2) Both variables are linearly related

(3) Conditional variance is constant

(4) Conditional variance is not constant

42. A sample of size $n(\geq 2)$ is drawn without replacement from a finite population os size N, using an arbitrary sampling scheme. Let πi denote the inclusion probability of the i-th unit and πij, the joint probability of units i and , $1 \leq i < j \leq N$. Which of the following statement is always true ?

(1) $\sum_{i=1}^{N} \pi i = n$

(2) $\sum_{i=1}^{N} \pi ij = n\pi i,\ 1 \leq i \leq N$

(3) $\pi ij > 0$ for all I, j, $1 \leq i < j \leq N$

(4) $\pi i \pi j - \pi ij > 0$ for all I, j, $1 \leq i < j \leq N$

43. Hundred (100) tickets are marked 1,2, ... 100 and are arranged at random. Four tickets are picked from these tickets and are gtiven to four persons A,B, C and D. What is the probability that A gets the ticket with the largest value (among A, B, C, D) and D gets the ticket with the smallest value (among A, B, C, D) ?

(1) $\frac{1}{4}$ (2) $\frac{1}{6}$ (3) $\frac{1}{2}$ (4) $\frac{1}{12}$

44. Suppose $X_1, X_2, \ldots\ldots X_n$ is a random sample from $U(0, \theta)$, > 0. Let $X_{(1)} \leq X_{(2)} \leq \ldots\ldots \leq X_{(n)}$ be the order statistics and S be sample mean. Consider the two unbiasedestimators for θ : T1 = 2S and T2 = $\left(\frac{n+1}{n}\right) X_{(n)}$ Then $\lim_{n \to \infty} \frac{VAR(T2)}{VAR(T1)}$ =

(1) 0 (2) 1 (3) ∞ (4) 12

45. Let $X_1, X_2, \ldots\ldots X_n$ be a random sample of size n from a p-variate Normal distribution with mean μ and positive definite covariance matrix $\sum$. Choose the correct statement.

(1) $(X_1 - \mu)' \sum^{-1} (X_1 - \mu)$ has chi-square distribution with 1 d.f.

(2) $\sum_{i=1}^{n} (Xi - \mu)(Xi - \mu)$ has Wishart distribution with n d. f.

(3) SS' (S is sample mean vector) has Wishart distribution with pd.f.

(4) $X_1 + X_2$ and $X_1 - X_2$ are independently distributed

46. A factorial experiment invlving four factors F1, F2, F3, and F4 each at 2 levels, 0 and 1, is planned in 4 blocks each of size 4. One of these block has the following contents :

F1	F2	F3	F4
0	0	0	0
0	0	0	0
1	0	1	1
1	1	1	0

The confounded factorial effect are :

(1) F1F2, F1F3, F2F3

(2) F1F3, F1F2F4, F2F3F4

(3) F3F4, F1F2F3, F1F2F4

(4) F1F4, F2F3, F1F2F3F4

47. Suppose the comulative distribution function of failure time T of a component is :

$1 - \exp(-ct^{\alpha})$, $t > 0$, $\alpha > 1$, $c > 0$

Then the hazard rate of $\lambda(t)$ is :

(1) Constant

(2) Monotone increasing in t

(3) Monotone decreasing in t

(4) Non-monotone function in t

48. Consider the following probability mass function P(x) where the parameters (θ_1, θ_2) take values in parameter space $\left\{\left(\frac{1}{3},3\right),\left(\frac{1}{2},2,\right),\left(2,\frac{1}{2}\right),\left(3,\frac{1}{3}\right)\right\}$

$(?_1,?_2)$ x	$\left(\frac{1}{3},3\right)$	$\left(\frac{1}{2},2,\right)$	$\left(2,\frac{1}{2}\right)$	$\left(3,\frac{1}{3}\right)$
1	1/11	1/7	1/8	1/9
2	1/11	1/14	1/16	1/9
3	8/11	5/7	3/4	2/3
4	1/11	1/14	1/16	1/9

Let X be a random observation from this distribution. If the observed value of X is 3, then

(1) MLE of $\theta_1 = 1/3$, MLE of $\theta_2 = 3$

(2) MLE of $\theta_1 = 1/2$, MLE of $\theta_2 = 2$

(3) MLE of $\theta_1 = 2$, MLE of $\theta_2 = 1/2$

(4) MLE of $\theta_1 = 3$, MLE of $\theta_2 = 1/3$

49. A symmetric die is thrown 720 times. The lower bound for the probability of getting 100 to 140 sixes is :

(1) 0.25 (2) 0.50 (3) 0.75 (4) 0.05

50. 10 If X_1 follows B (n_1, p_1) and X_2 follows B(n_2, p_2) independently of X_1 then $Z = x_1 + x_2$ will follow :

(1) Binomial distribution if $n_1 = n_2$.

(2) Binomial distribution if $p_1 = p_2$.

(3) Poisson distribution if $n_1 = n_2$.

(4) Poisson distribution if $p_1 = p_2$.

51. Suppose X has a geometrice distribution with parameter p :

$P(X = t) = q^t p$; t = 0, 1, 2 .....

Which of the following is correct :

(1) $P(y = t/X > k) = pq^t$ (2) $P(y = t/X < k) = pq^t$

(3) $P(y = t/X \geq k) = pq^t$ (4) P(y =t/X k) = pq^t

52. let X has the M.G.F. as : $M(t) = \frac{\left(1+2e^t\right)^3}{27}$

The mean and variance of X are

(1) 1/2 and 9/4 (2) 2 and 4/9

(3) 2 and 2/3 (4) 1 and 2/3

53. If X~ N (0, 1) then $Z = \frac{1}{2} X^2$ follows :

(1) Chi-square with 1 d.f. (2) Gamma $\left(\frac{1}{2}\right)$

(3) $\beta\left(\frac{1}{2}, \frac{1}{2}\right)$ of first kind (4) $\beta\left(\frac{1}{2}, \frac{1}{2}\right)$ of secondt kind

54. Which of the following statements is correct ?

(1) Convergence in Probability is stronger than the almost sure convergent.

(2) Convergence in Probability is stronger than the Convergence in law.

(3) Convergence in law is stronger than the Convergence in Probability.

(4) Convergence in law is stronger than the almost sure convergent.

55. Let $X_1, X_2, \ldots\ldots, X_n$ are iid variables from N $(?, ?^2)$, then the statistics $\sum Xi \sum Xi^2$ is :

(1) Jointly sufficient and complete

(2) Sufficient but not complete

(3) Complete but not sufficient

(4) Neither sufficient nor complete

56. A sequence $\{A_n\}$ defined as :

$$A_n\left\{\omega \text{ s. t. } 0 < \omega < b + \frac{(-1)^n}{n}\right\};\ (b > 1)$$

(1) Monotone non-decreasing.

(2) Monotone non-increasing.

(3) Non-monotone but convergent.

(4) Non-monotone and non-convergent.

57. Which of the following is/are correct ?

(i) Every monotone field is a sigma field

(ii) Every field containing infinite elements is always a sigma field

(1) (i) and (ii) both are correct
(2) (i) and (ii) both are incorrect
(3) (i) is correct but (ii) is incorrect
(4) (i) is incorrect but (ii) is correct

58. Consider the following assertions :

(i) A minimal sufficient statistic is always complete.

(ii) A minimal sufficient statistic may not be complete.

(iii) A complete sufficient statistic is always minimal sufficient.

(iv) A complete sufficient statistic may not be minimal sufficient.

(1) Only (i) is true (2) Only (ii) and (iv) are true
(3) Only (ii) and (iii) are true (4) Only (i) and (iii) are true

59. Suppose $X \sim n(0, \sigma^2)$; then $Y = X^2$ will follow :

(1) Pareto distribution (2) Cauchy distribution
(3) Rayleigh distribution (4) Student's t- distribution

60. A Bayes decision rule :

(1) is never admissible
(2) is always admissible
(3) admissible only if it is unique
(4) admissible if it is unique except under risk equivalence

61. What is the effect of an outlier on the value of a correlation coefficeint ?

(1) An outlier willalways decrease a correlation coefficient.

(2) An outlier willalways increase a correlation coefficient.

(3) An outlier might either decrease or increase a correlation coefficient, depending on where it is in relation to the other points

(4) An outlier will have no effect on a correlation coefficient

62. The lifetime of a bulb is exponentially distributed with mean 100 hours. The bulb remains switched on for exactly 4 hours everyday and remains switched off the remaining time. What is the probability that the bulb stops working on or before the 25th day ?

(1) $\frac{1-e^{-1}}{1-e^{\frac{1}{25}}}$ (2) $1-e^{\frac{1}{25}}$

(3) $1 - e^{-1}$ (4) e^{-1}

63. A Markow chain is said to be irreducible if :

(1) Possible to get from any where to any where

(2) Chain has probability one of returning to the initial state

(3) Chain has probability zero of returning to the initial state

(4) None of the above statement is correct.

64. In acceptance-rejection method a function t(x) that majorizes the density function f(x) for all x is :

(1) A density function

(2) A density function only for some x

(3) A density function only for values less than x

(4) Not necessarily a density function

65. The samples from a two-parameter Weibull distribution for some known values of shape and scale parameters can be generated with the help of inverse transfrom method by using the transformation :

(1) $\{-\text{scale} \log \{1 - u\}\}^{\text{shape}}$ (2) $\{1 - \exp(-x^{\text{shape}}/\text{scale})\}u$

(3) $\{-\text{scale} \log \{1 - u\}\}^{1/\text{shape}}$ (4) $\{\exp(-x^{\text{shape}}/\text{scale})\}u$

66. Suppose you have a posterior distribution $p(\theta/x)$ based a sample of size n i. e., $x : x_1, x_2 \ldots\ldots, x_n \sim f(x/\theta)$ and a prior distribution $g(\theta)$. If an additional observation x_{n+1} is added at a later stage, then the updated posterior will be :

(1) Same as $p(\theta/x)$

(2) Proportional to $f(x_{n+1})/\theta)\, p(\theta/x)$

(3) Proportional to $\{f(x/\theta) + f(x_{n+1})/\theta)\, p(\theta/x)$

(4) Cannot be updated

67. Consider an independent Bernoulli trails with success probability θ Using the prior proportional to $\theta^{a-1}(1-\theta)^{b-1}$. What is the Bayes estimator of θ when the loss is squared error loss ?

(1) $\frac{a+t}{a+b+n}$ (2) $\frac{a-t}{a+b+n}$

(3) $\frac{a+t}{n}$ (4) $\left(\frac{a+t}{n}\right)^2$

Where is the number of success in n independent trials.

68. For a Likelihood ratio test, let us define $\lambda(x) = \frac{\sup f_{\theta \in \Theta_0}(x/0)}{\sup f_{\theta \in \Theta}(x/0)}, \Theta = \Theta_0 \cup \Theta_1$. Then assuming some regularity conditions, which statement about the quantity $k = -2\log \lambda(x)$ is true ?

(1) k follows chi-square distribution with d.f. equals to the number of independent parameters in Θ_0

(2) k follows chi-square distribution with d.f. equals to the difference between the number of independent parameters in Θ and the number in Θ_0

(3) k follows f distribution with n_1 and n_2 degrees of freedom where n_1 and n_2 are the number of parameters in Θ_0 and Θ respectively.

(4) k follows chi-square distribution with d.f. equals to the number of independent parameters in Θ

69. WHO recommended method of sampling to estimate immunization coverage in developing countrie is :

(1) Simple random sampling

(2) Stratified random sampling

(3) 30 cluster random sampling

(4) Muli-stage random sampling

70. Calculation of relative risk requires a :

(1) Prospective study (2) Retrospective study

(3) Cross sectional study (4) Exploratory study

Short Answer Questions

Note: Attempt any **five** questions. Write answer in **150-200** words. Each question carries **16** marks. Answer each question on separate page, after writing Question Number.

1. What is complete and partial confounding ?

2. Explain the basic principle on which quadrature formulae are developed.

3. Explain by means of an example the conditional transfer in R or any high level language.

4. What is data warehousing ?

5. State and prove Rao-Blackwell theorem.

6. Define odds ratio in case-control study design by means of an example.

7. Define Jeffreys prior and work out Jeffreys prior for success probability p in Bernoulli distribution.

8. What do you mean by stochastic simulation ?

9. What is robust regression ?

10. Define Cox's proportional hazards model for several covariates.

(C) Computer Science Section

71. Consider the following segment of C program :

```
int x, y, n;
x = 1;
y = 1;
If (n > 0)
x = x + 1 ;
else
y = y - 1 ;
```

After execution of above program segment the value of x and y if n = 1 is :

(1) x = 2, y = 0; (2) x = 1, y = 0;

(3) x = 1, y = 1; (4) x = 2, y = 1;

72. Consider the following segment of C program :

```
int i, j:
j = 0;
for (i =0; i < = 5; i = i + 2/3)
{
j = j + 1 ;
}
```

The number of times the body of for loop is executd :

(1) 9 (2) 8

(3) infinite (4) 11

73. How many of the followig declarations are correct ?

```
Int x ;
float letter, DIGIT ;
double = p, q
m, n, z : INTEGER
long int m; count;
long float temp;
```

(1) Three are correct (2) One is correct
(3) Two are correct (4) All six ar incorrect

74. Consider the following C program :

```
main ( )
{
int num 1. num 2;
scanf ("%2d%5d", & num 1, & num 2);
print f(:%d%d", num 1, num 2);
}
```

If the data input to the program 31426 and 50, then the output will be :

(1) 31426,50 (2) 50,31426
(3) 31,426 (4) 3142,650

75. Consider the following ANSI union :

```
union item
{
int m;
float x;
char c;
};
```

Total memory location required to store any union variable of type item is :

(1) 2 byte (2) 4 byte
(3) 6 byte (4) 7 byte

76. When simplified with Boolean Algebra, the expression $(x + y)(x + z)$ simplifies to :

(1) x (2) $x + x(y + z)$

(3) $x(1 + yz)$ (4) $x + yz$

77. How many 1's are present in the binary representation of $3 \times 512 + 7 \times 64 + 5 \times 8 + 3$:

(1) 8 (2) 9 (3) 10 (4) 11

78. The decimal number equivalent of $(4057.06)_8$ is :

(1) 2095, 75 (2) 2095, 075

(3) 2095, 937 (4) 2095, 0937

79. $(734)_8 = (\ )_{16}$:

(1) CID (2) DCI (3) ICD (4) IDC

80. Two fuzzy sets A and B are given with membership :

$\mu_A(x) = \{0.2, 0.4, 0.8, 0.5, 0.1\}$

$\mu_B(x) = \{0.1, 0.3, 0.6, 0.3, 0.2\}$

Then the value of $\mu_{A \cup B}$ will be :

(1) {0.9, 0.7, 0.4, 0.8, 0.9} (2) {0.2, 0.4, 0.8, 0.5, 0.2}

(3) {0.3, 0.7, 1.4, 0.8, 0.3} (4) {0.1, 0.3, 0.6, 0.3, 0.1}

81. If a pixel of an image is shuffled, then he parameter that not change is :

(1) Histogram, Mean, Covariance

(2) Histogram, Mean, Entropy

(3) Histogram, Covariance

(4) Covariance, Entropy

82. The information is accessed in stack data structure is :

(1) LIFO (2) FIFO

(3) Random (4) LIFO and Random Both

83. The preorder traversal of abinary tree is DEBFCA. The root node of binary tree is :

(1) B (2) C (3) A (4) D

84. Chromatic number of bipartite graph is :

(1) 4 (2) 2 (3) 3 (4) 1

85. Which of the following pair (A,B) of events is mutually exculusive, in the random experiment of tossing of a coin four times ?

(1) A : Obtaining at least three heads, B : Obtaining at least three tails.

(2) A : Obtaining at least two heads, B : Obtaining at most two tails.

(3) A : Obtaining at least two heads, B : Obtaining at least two tails.

(4) A : Obtaining at least three heads, B : Obtaining at most three tails.

86. Assuming the normal distribution, suppose that a 95% confidence interval for mean μ is (50, 60). Which of the following could possibly be a 99% confidence interval for the same ?

(1) (52, 58) (2) (52, 62) (3) (48, 58) (4) (48, 62)

87. Let T(n) be the function defined by T(0) = 1 and T(n) = T(n - 1) + n, n > = 1. Which of the following is true :

(1) $T(n) = O(n^2)$ (2) $T(n) = O(\sqrt{n})$

(3) $T(n) = O(\log_2 n)$ (4) $T(n) = O(n)$

88. In AVL tree, difference between height of left and right subtrees is :

(1) Less than 1 (2) Less than equal to 1

(3) equal to 1 (4) greater than equal to 1

89. The output of D-flip flop :
(1) same as input
(2) complement of input
(3) not depend on input
(4) depend on past input and clock

90. Fundamental period of $x(n) = 10 \cos(\pi n/3)$ is :
(1) 3 (2) 1/3 (3) 6 (4) $\pi n/3$

91. A and B are two logical statements. Statement B is logical equivalent to statement A iff :
(1) $(A \rightarrow B) \wedge (B \rightarrow A)$ is tautology
(2) $(A \rightarrow B) \mid\mid (B \rightarrow A)$ is tautology
(3) $A \rightarrow B$ is contradiction
(4) $A \rightarrow B$ is tautology

92. Nayquist rate of the signal $x(t) = 3 \cos(50\pi t) + 10 \sin(30\pi t) - \cos(100\pi t)$ is :
(1) 50 Hertz (2) 300 Hertz
(3) 100 Hertz (4) 150 Hertz

93. Postfix expression equivalent to infix expression (A - B)* (D/E) is :
(1) ABDE*/- (2) ABDE-/*
(3) AB - DE/* (4) None of these

94. The Laplace transform of $\sin \omega t$ is :
(1) $\frac{s^2}{s^2+\omega^2}$ (2) $\frac{\omega^2}{s^2+\omega^2}$
(3) $\frac{\omega}{s^2+\omega^2}$ (4) $\frac{\omega}{s^2+\omega}$

95. A casual signal has Z-transform with complex conjugate pole located at unit circle ($|z|=1$). The signal is :

(1) Oscillatory and decaying (2) Oscillatory and increasing

(3) Oscillating (4) Constant

96. The function $f(x) = \sin(x)$ is decreasing in :

(1) $(0, \pi)$ (2) $(0, \pi/2)$

(3) $(\pi/2, 3\pi/2)$ (4) $(\pi, 2\pi)$

97. If $f(x) = \sin 2x$, $0 \leq x \leq \pi/2$, the value x at which tangent is parallel to x-axis :

(1) $\frac{p}{4}$ (2) $\frac{p}{3}$ (3) $\frac{p}{2}$ (4) p

98. If A and B are two square matrices of same order, then which of the following is not correct ?

(1) $\det(AB) = \det(A) + \det(B)$ (2) $\det(A^T) = \det(A)$

(3) $\det(A^{-1}) = 1/\det(A)$ (4) $\det(AB) = \det(A).\det(B)$

99. Worst case time complexity of heap sort is :

(1) $O(n)$ (2) $O(n^2)$

(3) $O(n^3)$ (4) $O(n\log_2 n)$

100. The probability mass function of a random variable X is given below :

$f(x) = x/15$; $x = 1, 2, 3, 4, 5$

$= 0$; otherwise

Then the conditional probability that X lies between 1/2 and 5/2 given that X is greater than 1 is :

(1) 1/7 (2) 3/7 (3) 2/15 (4) 1/5

Short Answer Questions

Note: Attempt any **five** questions. Write answer in **150-200** words. Each question carries **16** marks. Answer each question on separate page, after writing Question Number.

1. (a) Give data structure in which linear list is implemented using random access. Using this representation/data structure write an algorithm for inserting and deleting an element form the list.

(b) Describe stack and queue data structure. Explain PUSH and POP operation on a stack.

2. (a) Suppose the following list of letters is inserted in order into an empty Binary search tree :

J, R, D, G, T, E, M, H, P, A, F, Q

(i) Find the final Binary tree

(ii) Give sequential representation of the tree in (i)

(iii) Give the link representation of the tree in (i) using three array.

(iv) Find the inorder, preorder, and postorder traversal of Binary tree.

(v) Consider the Binary tree (i), give tree after the node M and D is deleted.

(b) A Binary tree T has 9 nobes. The inorder and preorder traversal of T yield the following sequence of nodes :

INORDER : E, A, C, K, F, H, D, B, G

PREORDER : F, A, E, K, C, D, H, G, B

Draw the Binary tree and also give its postorder traversal

3. Discuss maximum likelihood method of estimation. Obtain estimators of μ and σ^2 on the basis of random samples of size n drawn from $N(\mu, \sigma^2)$ by maximum likelihood method.

4. Define pole and zero of a Z-transform, and determine the zero and pole of the signal $x(n) = (1/2)^n u(n)$, where u(n) is the unit step signal.

5. (a) Explain how the Fourier transform is relatedtothe Z-transform.

   (b) State and explain the Dirichlet conditions.

   (c) Explain band width of signal, low frequency and high frequency signal.

6. (a) (i) How structure differ from each other

   (ii) How structure, union, and bit filed differ from each

   (b) Define a structure called **cricket** that will describe the information player name, team name, batting average. Using structure **cricket**, declare an array player with 5 elements.

   (c) In what ways does a switch statement is differ from if statement, Write a C program for marks range to grade conversion using switch and case, for following data :

Marks Range	Grade
< 40	E
40-54	D
55-69	C
70-85	B
> 85	A

7. Show thae $(\exists x)\, M(x)$ follows logically from the premises $(\forall x)\,(H(x) \rightarrow M(x)$ and $(\exists x)\, H(x)$

8. Define fuzzy set. Generalize the intersection, union operation on crisp set using maximum and minimum for fuzzy set.

9. Density function of a random variable X is defined as follows :

$$f_x(x) = \begin{cases} kx; 0 \le x \le 2 \\ 0; \text{otherwise} \end{cases}$$

Find the following :

(i) Value of k

(ii) Find the probability $P(0 < X < 0.75)$

(iii) Determine c so that $P(X \le c) = 0.81$

10. (a) Suppose the following number is stored in array A :

32, 51, 27, 85, 66, 23, 13, 57

Simulate the bubble sort procedure

(b) Supose A is stored array with 200 elements, and suppose a given element x appears with the same probability in any place in A. Find the worst-case running time f(n) the average case running time g(n) to find x in A using the Binary search.

F/of Arts

Set No. : 1

Question Booklet No. 00056

RET/15/TEST-B

46

904 Mathematics

(To be filled up by the candidate by blue/black ball point pen)

Roll No. | | | | | | | | | |

Roll No. (Write the digits in words) .....

Serial No. of OMR Answer Sheet .....

Day and Date .....

(Signature of Invigilator)

INSTRUCTIONS TO CANDIDATES

(Use only blue/black ball-point pen in the space above and on both sides of the Answer Sheet)

1. Within 10 minutes of the issue of the Question Booklet, Please ensure that you have got the correct booklet and it contains all the pages in correct sequence and no page/question is missing. In case of faulty Question Booklet, Bring it to the notice of the Superintendent/Invigilators immediately to obtain a fesh Question Booklet.
2. Do not bring any loose paper, written or blank, inside the Examination Hall ***except the Admit Card without its envelope.***
3. *A separate Answer Sheet is given. It should not be folded or mutilated. A second Answer Sheet* ***shall not be provided.***
4. Write your Roll Number and Serial Number of the Answer Sheet by pen in the space provided above.
5. ***On the front page of the Answer Sheet, write by pen your Roll Number in the space provided at the top, and by darkening the circles at the bottom. Also, wherever applicable, write the Question Booklet Number and the Set Number in appropriate places.***
6. ***No overwriting is allowed in the entries of Roll No., Question Booklet No. and Set No. (if any) on OMR sheet and Roll No. and OMR sheet no. on the Question Booklet.***
7. ***Any change in the aforesaid entries is to be verified by the invigilator, otherwise it will be taken as unfair means.***
8. ***This Booklet contains 40 multiple choice questions followed by 10 short answer questions. For each MCQ, you are to record the correct option on the Answer Sheet by darkening the appropriate circle in the corresponding row of the Answer Sheet, by pen as mentioned in the guidelines given on the first page of the Answer Sheet. For answering any five short Answer Questions use five Blank pages attached at the end of this Question Booklet.***
9. For each question, darken only **one** circle on the Answer Sheet. If you darken more than one circle or darken a circle partially, the answer will be treated as incorrect.
10. ***Note that the answer once filled in ink cannot be changed. If you do not wish to attempt a question, leave all the circles in the corresponding row blank (such question will be awarded zero marks).***
11. For rough work, use the inner back pages of the title cover and the blank page at the end of this Booklet.
12. ***Deposit both OMR Answer Sheet and Question Booklet at the end of the Test.***
13. You are not permitted to leave the Examination Hall until the end of the Test.
14. If a candidate attempts to use any form of unfair means, he/she shall be liable to such punishment as the University may determine and impose on him/her.

Total No. of Printed Pages : 28

No. of Questions : 50
प्रश्नों की संख्या : 50

Time : 2 Hours ***Full Marks : 200***
समय : 2 घण्टे **पूर्णांक : 200**

Note: (1) This Question Booklet contains **40** Multiple Choice Questions followed by **10** Short Answer Questions.

इस प्रश्न पुस्तिका में **40** वस्तुनिष्ठ व **10** लघु उत्तरीय प्रश्न हैं।

(2) Attempt as many MCQs as you can. Each MCQ carries **3 (Three)** marks. **1 (One)** mark will be deducted for each incorrect answer. **Zero** mark will be awarded for each unattempted question. If more than one alternative answers of MCQs seem to be approximate to the correct answer, choose the closest one.

अधिकाधिक वस्तुनिष्ठ प्रश्नों को हल करने का प्रयत्न करें। प्रत्येक वस्तुनिष्ठ प्रश्न **3 (तीन)** अंकों का है। प्रत्येक गलत उत्तर के लिए **1 (एक)** अंक काटा जायेगा। प्रत्येक अनुत्तरित प्रश्न का प्राप्तांक **शून्य** होगा। यदि वस्तुनिष्ठ प्रश्नों के एकाधिक वैकल्पिक उत्तर सही उत्तर के निकट प्रतीत हों, तो निकटतम सही उत्तर दें।

(3) Answer only **5** Short Answer Questions. Each question carries **16 (Sixteen)** marks and should be answered in **150-200** words. Blank **5 (Five)** pages attached with this booklet shall only be used for the purpose. Answer each question on separate page after writing Question No.

केवल **5 (पाँच)** लघुउत्तरीय प्रश्नों के उत्तर दें। प्रत्येक प्रश्न **16 (सोलह)** अंकों का है तथा उनका उत्तर **150-200** शब्दों के बीच होना चाहिए। इसके लिए इस पुस्तिका में लगे हुए सादे **5 (पाँच)** पृष्ठों का ही उपयोग आवश्यक है। प्रत्येक प्रश्न का उत्तर एक नए पृष्ठ से, प्रश्न संख्या लिखकर शुरू करें।

P.T.O.

01. The newspaper 'Commonweal' was associated with :

(1) Annie Besant (2) Motilal Nehru

(3) Pt. Madan Mohan Malviya (4) Dinshaw Wacha

'कॉमनवील' समाचार-पत्र किससे सम्बद्ध था ?

(1) एनी बेसेंट (2) मोतीलाल नेहरू

(3) पं० मदनमोहन मालवीय (4) दिनशा वाचा

02. Who presided over the fourth Buddhist council ?

(1) Ashwaghosha (2) Vasumitra (3) Raivata (4) Nagarjuna

चतुर्थ बौद्ध संगीति की अध्यक्षता किसने की थी ?

(1) अश्वघोष (2) वसुमित्र (3) रैवत (4) नागार्जुन

03. Who among the following has been considered a lady Tirthankara in the Shevtambara tradition ?

(1) Kunthunatha (2) Mallinatha

(3) Sumatinatha (4) Shantinatha

श्वेताम्बर परम्परा में निम्नलिखित में से किसे स्त्री तीर्थंकर माना गया है ?

(1) कुन्थुनाथ (2) मल्लिनाथ (3) सुमतिनाथ (4) शान्तिनाथ

04. Who of the following Jain saints is associated with Chandra Gupta Maurya ?

(1) Umasvati (2) Haribhadra (3) Hemchandra (4) Bhadrabahu

चन्द्रगुप्त मौर्य के साथ किस जैन साधु का नाम जुड़ा है ?

(1) उमास्वाति (2) हरिभद्र (3) हेमचन्द्र (4) भद्रबाहु

05. Which one of the following pairs is **not** correctly matched ?

(1) Chand Bardai - Prithviraj Raso

(2) Narpati Nalha - Bisaldev Raso

(3) Jagnik - Alha Khand

(4) Abdur Rahman - Hamir Raso

निम्नलिखित युग्मों में से कौन सही सुमेलित नहीं है ?

(1) चंद बरदाई - पृथ्वीराज रासो (2) नरपति नाल्ह - बीसलदेव रासो

(3) जगनिक - आल्हा खण्ड (4) अब्दुर्रहमान - हम्मीर रासो

06. Who among the following was a noted Jehangiri painter ?

(1) Mir sayyad Ali (2) Abdus Samad

(3) Govardhan (4) Abdus Hasan

निम्नलिखित में से कौन प्रख्यात जहाँगीरी चित्रकार था ?

(1) मीर सैय्यद अली (2) अब्दुस समद (3) गोवर्धन (4) अब्दुस हसन

07. Who among the following was a disciple of Hazarat Nizamuddin Auliya ?

(1) Ibrahim Lodi (2) Nasiruddin

(3) Amir Khusrau (4) Alauddin Sabir

निम्नलिखित में से कौन हजरत निजामुद्दीन औलिया का शिष्य था ?

(1) इब्राहीम लोदी (2) नसीरुद्दीन

(3) अमीर खुसरो (4) अलाउद्दीन साबिर

08. Who of the following was responsible for the compilation of few upanishads ?

(1) Elphinston (2) Briggs (3) Dara Shikoh (4) Baveridge

निम्नलिखित में से किसने कतिपय उपनिषदों का संकलन करवाया था ?

(1) एलफिन्स्टन (2) ब्रिग्स (3) दाराशिकोह (4) बेवरिज

09. The scholar who deciphered the ancient Brahmi script for the first time was :

(1) William carry (2) William Jones

(3) Nathanial wallich (4) James Princep

प्राचीन ब्राह्मी लिपि को पढ़ने में सफल होने वाला प्रथम विद्वान कौन था ?

(1) विलियम केरी (2) विलियम जोन्स

(3) नथैनिएल वालिच (4) जेम्स प्रिन्सेप

10. In which of following caves the mural of Padmapani was found ?

(1) Ajanta Cave (2) Ellora Cave

(3) Armamalai Cave (4) Bhimbhetka Rock

निम्नलिखित में से किस गुफा में पद्मपाणि का भित्तिचित्र पाया गया ?

(1) अजंता गुफा (2) एलोरा गुफा

(3) अरमामलई गुफा (4) भीमभेटका रॉक

11. A rod is set rotating about its one end in space in any manner. Then,

(1) The number of Euler's angle and degree of freedom both will be two.

(2) The number of Euler's angle will be three and degree of freedom will be two.

(3) The number of Euler's angle and degree of freedom will be three.

(4) The number of Euler's angle will be one and degree of freedom will be infinite.

एक छड़, अन्तरिक्ष में अपने एक सिरे के चारों ओर घूमती है, तब

(1) आयलर कोणों व स्वातंत्र्य कोटि दोनों की संख्या दो होगी।

(2) आयलर कोण तीन और स्वातंत्र्य कोटि की संख्या दो होगी।

(3) आयलर कोण व स्वातंत्र्य कोटि दोनों की संख्या तीन होगी।

(4) आयलर कोण की संख्या एक और स्वातंत्र्य कोटि की संख्या अनंत होगी।

12. If kinetic energy and potential energy of a system are given by $T = \frac{1}{2}\left(\dot{q}_1^2 + \dot{q}_2^2 + q_1\dot{q}_2 + 6\dot{q}_1\right)$ and $V = C + \frac{1}{2}q_1$, then

(1) q_1 and q_2 are cyclic coordinates

(2) q_1 is cyclic coordinate

(3) q_2 is cyclic coordinate

(4) neither q_1 nor q_2 is cyclic coordinate

यदि किसी निकाय की गतिज ऊर्जा व स्थितिज ऊर्जा को क्रमशः $T = \frac{1}{2}\left(\dot{q}_1^2 + \dot{q}_2^2 + q_1\dot{q}_2 + 6\dot{q}_1\right)$ तथा $V = C + \frac{1}{2}q_1$ से व्यक्त किया गया हो तो

(1) q_1 और q_2 चक्रीय निर्देशांक है

(2) q_1 चक्रीय निर्देशांक है

(3) q_2 चक्रीय निर्देशांक है

(4) न तो q_1 और न ही q_2 ही चक्रीय निर्देशांक है

13. For the expressions of kinetic energy, $T=\frac{1}{2}\left(\dot{\theta}^2 + \dot{\phi}^2 + \dot{\theta}\dot{\phi}\right)$ and potential energy, $V=\frac{1}{2}(\theta + \phi)+C$, Hamiltonian function may be written as :

गतिज ऊर्जा व स्थितिज ऊर्जा के व्यंजकों क्रमशः $T=\frac{1}{2}\left(\dot{\theta}^2 + \dot{\phi}^2 + \dot{\theta}\dot{\phi}\right)$ व $V=\frac{1}{2}(\theta + \phi)+C$ के लिए हैमिल्टन फलन होगा :

(1) $H = T - V$ (2) $H = T + V$ (3) $H = T/V$ (4) $H = TV$

14. The flow formed by the velocity vector $\bar{q}=(-ay, ax, 0)$, where a is a constant, is :

(1) not a possible flow (2) a possible rotational flow

(3) an irrotational flow (4) a possible irrotational flow

वेग सदिश $\bar{q}=(-ay, ax, 0)$ जहाँ a स्थिरांक है, द्वारा निर्मित प्रवाह होगा :

(1) प्रवाह संभव नहीं है (2) एक घूर्णीय प्रवाह संभव है

(3) एक अघूर्णीय प्रवाह संभव है (4) एक अघूर्णीय प्रवाह है

15. The Reynolds number is ratio of :

(1) inertia force to viscous force (2) inertia force to gravity force

(3) viscous force to thermal force (4) inertia force to thermal force

रेनाल्ड संख्या, अनुपात है :

(1) जड़त्व बल और श्यान बल का (2) जड़त्व बल और गुरुत्व बल का

(3) श्यान बल और ऊष्मीय बल का (4) जड़त्व बल और ऊष्मीय बल का

16. The boundary value problem corresponding to the integral equation $y(x) = \lambda \int_0^x (x - t)\, y(t)\, dt - \lambda x \int_0^1 (1-t)\, y(t)\, dt$ is :

समाकलन समीकरण $y(x) = \lambda \int_0^x (x - t)\, y(t)\, dt - \lambda x \int_0^1 (1-t)\, y(t)\, dt$ से सम्बन्धित सीमा मूल्य समस्या है :

(1) $y'' - \lambda y = 0, \quad y(0) = y(1) = 0$

(2) $y'' + \lambda y = 0, \quad y(0) = y(1) = 0$

(3) $y'' - \lambda y = 0, \quad y(0) = 0, y(1) = 1$

(4) $y'' + \lambda y = 0, \quad y(0) = 1, y(1) = 0$

17. Using the central difference schemes, the finite difference equation corresponding to the differential equation $y'' - 2y' + y = x^2$ at the grid x_i when $x_i - x_{i-1} = h$ is :

अवकल समीकरण $y'' - 2y' + y = x^2$ के लिए, केन्द्रीय अन्तराल पद्धति का प्रयोग करते हुए, ग्रिड x_i पर, जबकि $x_i - x_{i-1} = h$ नियत अन्तराल समीकरण होगा :

(1) $y_{i-1} - hy_i + y_{i-1} = x_i^2$

(2) $(1-h)\,y_{i-1} + (h^2 - 2)\,y_i + (1+h)y_{i-1} = h^2 x_i^2$

(3) $(1+h)\,y_{i-1} - (2-h^2)\,y_i + (1-h)y_{i+1} = h^2 x_i$

(4) $(1+h)\,y_{i-1} - (2-h^2)y_i + (1-h)y_{i+1} = h^2 x_i^2$

18. The partial differential equation $\sin^2 x\, U_{xx} + \sin 2x\, U_{xy} + \cos^2 x\, U_{yy} = x$ is :

(1) elliptic (2) parabolic (3) hyperbolic (4) circular

आंशिक अवकल समीकरण $\sin^2 x\, U_{xx} + \sin 2x\, U_{xy} + \cos^2 x\, U_{yy} = x$ है :

(1) दीर्घ वृत्तीय (2) परवलयीय

(3) अतिपरवलयीय (4) चक्रीय (वृत्तीय)

19. The solution of the partial differential equation $\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$ subject to $u(o, x, y, z) = e^z \sin x \cos y$ and $t \geq 0$ is :

आंशिक अवकल समीकरण $\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$

जबकि $u(o, x, y, z) = e^z \sin x \cos y$ और $t \geq 0$ का हल होगा :

(1) $u(t, x, y, z) = e^{z+t} \sin (x + t) \cos (y - t)$

(2) $u(t, x, y, z) = e^{z-t} \sin (x - t) \cos (y - t)$

(3) $u(t, x, y, z) = e^{z+t} \sin (x + t) \cos (y + t)$

(4) $u(t, x, y, z) = e^{z+t} \sin (x - t) \cos (y + t)$

20. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x) = x_1^2 - x_2^2$. Then :

(1) $[0, 0]^T$ satisfies the first order necessary condition

(2) $[0, 0]^T$ satisfies the second order necessary condition

(3) $[0, 0]^T$ is a minimizer of f

(4) $[0, 0]^T$ satisfies the second order sufficient condition

यदि $f : \mathbb{R}^2 \to \mathbb{R}$ को $f(x) = x_1^2 - x_2^2$ द्वारा परिभाषित किया गया हो तब :

(1) $[0, 0]^T$ प्रथम कोटि के आवश्यक शर्त को संतुष्ट करता है

(2) $[0, 0]^T$ द्वितीय कोटि के आवश्यक शर्त को पूर्ण करता है

(3) $[0, 0]^T$ f का मिनिमाइजर है

(4) $[0, 0]^T$ द्वितीय कोटि की पर्याप्त शर्त को पूरा करता है

21. Consider the minimization of $f(x_1, x_2) = \frac{1}{2} x^T \begin{bmatrix} 4 & 2 \\ 2 & 2 \end{bmatrix} x - x^T \begin{bmatrix} -1 \\ 1 \end{bmatrix}$, $x \in \mathbb{R}^2$ using the conjugate direction method with $x^0 = [0, 0]^T$ and Q-conjugate directions $d^0 = [1, 0]^T$ and $d^1 = \left[-\frac{3}{8}, \frac{3}{4}\right]^T$. Then -

$x^0 = [0, 0]^T$ और Q-युग्मित दिशाएं $d^0 = [1, 0]^T$, $d^1 = \left[-\frac{3}{8}, \frac{3}{4}\right]^T$ के साथ युग्मित दिशा विधि का प्रयोग करते हुए $f(x_1, x_2) = \frac{1}{2} x^T \begin{bmatrix} 4 & 2 \\ 2 & 2 \end{bmatrix} x - x^T \begin{bmatrix} -1 \\ 1 \end{bmatrix}$, $x \in \mathbb{R}^2$ के निम्नीकरण पर विचार करें तब :

(1) $g^0 = [-1, 1]^T$ (2) $\alpha_0 = \frac{1}{4}$ (3) $x^1 = \left[-\frac{1}{4}, 0\right]^T$ (4) $g^1 = \left[0, \frac{3}{2}\right]^T$

22. Let $f(x_1, x_2) = x_1^2 + \frac{1}{2}x_2^2 + 3$. By taking $x^0 = [1, 2]^T$, $H_0 = I_2$ and applying the rank one correction algorithm to minimize f, we get :

यदि $f(x_1, x_2) = x_1^2 + \frac{1}{2}x_2^2 + 3$ रैंक वन करेक्शन एल्गोरिथ्म का प्रयोग करते हुए और $x^0 = [1, 2]^T$ तथा $H_0 = I_2$ मानते हुए f को निम्न करें तो हमें प्राप्त होगा :

(1) $d^0 = [2, 2]^T$ (2) $\alpha_0 = \frac{3}{2}$ (3) $\alpha_0 = \frac{2}{3}$ (4) $x^1 = \left[\frac{1}{3}, \frac{1}{3}\right]^T$

23. Total number of group homomorphisms from groups $\not{c}_{16} \to \not{c}_{24}$ are :

समूहों $\not{c}_{16} \to \not{c}_{24}$ से बने समूह समरूपता की कुल संख्या होगी :

(1) 2 (2) 6 (3) 8 (4) 48

24. If W_1 and W_2 are subspaces of a finite dimensional vector space U, then annihilator $(W_1 \text{ I } W_2)^0$ is equal to :

नियत आयामी सदिश स्पेस U के सबस्पेस W_1 तथा W_2 है तो उनका उन्मूलक $(W_1 \text{ I } W_2)^0$ बराबर होगा :

(1) $W_1^0 \text{ I } W_2^0$ (2) $W_1^0 \text{ U } W_2^0$ (3) $W_1^0 + W_2^0$ (4) $W_1 \oplus W_2$

25. In group U(24) = {1, 5, 7, 11, 13, 17, 19, 23}
if H = {1, 13} and K = {1, 17}, then HK is given by :

समूह U(24) = {1, 5, 7, 11, 13, 17, 19, 23} में यदि H = {1, 13} और K = {1, 17}, हो तब HK होगा -

(1) {1, 13, 1, 17}
(2) {1, 13, 17}
(3) {1, 5, 13, 17}
(4) {1, 13, 17, 221}

26. Let U be the vector space of all m×m matrices over the field F and V be the vector space of all n×n matrices over the same field F. Then Hom (U, V), the vector space of all linear transformations from U to V is of dimension :

माना कि क्षेत्र F के सभी m×m आव्यूह का सदिश स्पेस U है तथा उसी क्षेत्र F के सभी n×n आव्यूह का सदिश स्पेस V है। तब U से V के सभी रैखिक रूपांतरण के सदिश स्पेस Hom (U, V) का आयाम होगा :

(1) $m + n$ (2) $(m + n)^2$ (3) mn (4) m^2n^2

27. Let $R = \left\{ \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} \middle| a_1, a_2, a_3, a_4 \in \mathbb{C} \right\}$ and I be the ideal of R consisting of matrices with even integers. How many elements are in the quotient ring R/I ?

(1) 4 (2) 8 (3) 16 (4) infinite

यदि $R = \left\{ \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} \middle| a_1, a_2, a_3, a_4 \in \mathbb{C} \right\}$ और I, R का आइडियल है जबकि R के आव्यूहों में सम पूर्णांक संख्या है। क्वोरोंट वलय R/I में अवयवों की संख्या कितनी होगी :

(1) 4 (2) 8 (3) 16 (4) अनंत

28. Let X and Y be topological spaces and $F : X \to Y$ a continuous function. Then :

(1) if X is Housdorff space then Y is also Housdorff

(2) both X and Y are Housdorff or none of them is Housdorff

(3) if Y is Housdorff then X is Housdorff

(4) either X or Y is Housdorff space

माना X और Y, टोपोलॉजिकल स्पेस है तथा $F : X \to Y$, एक सतत फलन है तब :

(1) यदि X हाउसड्राफ स्पेस है तो Y भी हाउसड्राफ स्पेस होगा

(2) दोनों ही X और Y हाउसड्राफ स्पेस होगें अन्यथा कोई नहीं होगा

(3) यदि Y हाउसड्राफ स्पेस है तो X हाउसड्राफ स्पेस होगा

(4) या तो X या Y हाउसड्राफ स्पेस है

29. Consider the following statements :

(A) The property of "compactness" is a hereditary property.

(B) The property of "compactness" is a topological property.

Then :

(1) A is true and B is false (2) B is true and A is false

(3) both A and B are true (4) both A and B are false

निम्न कथनों पर विचार कीजिए :

(क) 'कम्पैक्टनेस' का गुण आनुवांशिक गुण है।

(ख) 'कम्पैक्टनेस' का गुण टोपोलॉजिकल गुण है।

तब :

(1) क सत्य है और ख असत्य (2) ख सत्य है और क असत्य

(3) दानों क और ख, सत्य है (4) दोनों क और ख, असत्य है

30. Let X be an uncountable set with cofinite topology. Then :

(1) X is first countable but not second countable

(2) X is second countable

(3) X is not first countable

(4) X is separable

माना X कोई अपरिमेय समुच्चय है जिसकी टोपोलाजी कोफाइनाइट है, तब

(1) X प्रथम परिमेय है पर द्वितीय परिमेय नहीं

(2) X द्वितीय परिमेय है

(3) X प्रथम परिमेय नहीं है

(4) X पृथक्करणीय है

31. Let $x_1 = \sqrt{2}$ and for any natural number $n \geq 1, x_{n+1} = \sqrt{2+x_n}$ Then :

(1) the sequence (x_n) is monotonically decreasing and $\lim_{n\to\infty} x_n = 0$

(2) the sequence (x_n) is monotonically increasing and $\lim_{n\to\infty} x_n = \sqrt{2}$

(3) the sequence (x_n) is not monotonically increasing

(4) $\lim_{n\to\infty} x_n = 2$

माना $x_1 = \sqrt{2}$ और किसी भी प्राकृत संख्या $n \geq 1$, के लिए $x_{n+1} = \sqrt{2+x_n}$ है तब :

(1) अनुक्रम (x_n) एक समान रूप से घट रहा है और $\lim_{n\to\infty} x_n = 0$

(2) अनुक्रम (x_n) एक समान रूप से बढ़ रहा है और $\lim_{n\to\infty} x_n = \sqrt{2}$

(3) अनुक्रम (x_n) एक समान रूप से बढ़ नहीं रहा है

(4) $\lim_{n\to\infty} x_n = 2$

32. Let $f : \mathbb{R} \to \mathbb{R}$ be a monotonie function and S denote the set of points where f is discontinuous. Then S is :

(1) a finite set (2) a countable set

(3) a countably infinite set (4) an uncountable set

माना $f:\mathbb{R} \to \mathbb{R}$ मोनोटोनिक फलन है तथा S उन बिन्दुओं का समुच्चय है जहाँ F असतत् है तब S होगा :

(1) निश्चित समुच्चय (2) परिमेय समुच्चय

(3) परिमेय पर अनंत समुच्चय (4) अपरिमेय समुच्चय

33. Let E be a subset of $\mathbb{R}$ .Then :

(1) if E is Lebesgne measurable then E is a Borel set

(2) if E is not a Borel set then E is Lebesgne measurable

(3) if E is a Borel set then E is Lebesgne measurable

(4) none of the above

माना E, $\mathbb{R}$ का एक उप समुच्चय है। तब :

(1) यदि E लेबेस्ने मेजरेबल है तो E एक बोरेल समुच्चय होगा।

(2) यदि E एक बोरेल समुच्चय नहीं होगा तब E लेबेस्ने मेजरेबल होगा।

(3) यदि E बोरेल समुच्चय है तब E लेबेस्ने मेजरेबल होगा।

(4) उपरोक्त में से कोई नहीं

34. Let $f(z) = \begin{cases} ze^{1/z} & z \neq 0 \\ 0 & z = 0 \end{cases}$

then z = 0 is a :

(1) pole of f(z)

(2) removable singular point of f(z)

(3) non-isolated singular point of f(z)

(4) essential singularity of f(z)

यदि $f(z) = \begin{cases} ze^{1/z} & z \neq 0 \\ 0 & z = 0 \end{cases}$

तब $z = 0$ है :

(1) f(z) का ध्रुव

(2) f(z) का निराकरणीय एकल बिन्दु

(3) f(z) का असंपृक्त एकल बिन्दु

(4) f(z) की आवश्यक एकलता

35. Let (X, II, II) be a normed linear space. Then the "norm" is :

(1) Uniformly continuous function on X

(2) Continuous on X but not uniformly continuous

(3) Bounded function on X

(4) None of the above

माना (X, II, II) एक नार्म्ड रैखिक स्पेस है। तब "नार्म" है :

(1) X पर एक समान रूप से सतत फलन है

(2) X पर सतत है पर समान रूप से नहीं

(3) X पर आबद्ध फलन है

(4) उपरोक्त में से कोई नहीं

36. By contour integration, the value of $\int_{-\infty}^{\infty} \frac{e^{x/2}}{1+e^x} dx$ is :

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{2}$ (3) π (4) None of these

कन्टूर समाकलन से $\int_{-\infty}^{\infty} \frac{e^{x/2}}{1+e^x} dx$

का मान है :

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{2}$

(3) π (4) उपरोक्त में से कोई नहीं

37. Let H be a Hilbert space over a field C. If T_1 an T_2 are normal operators on H into itself such that either commutes with adjoint of the other, then :

(1) $T_1 + T_2$ is normal but T_1T_2 is not normal

(2) T_1T_2 is normal but $T_1 + T_2$ is not normal

(3) neither $T_1 + T_2$ is normal nor T_1T_2 is normal

(4) $T_1 + T_2$ and T_1T_2 both are normal.

यदि क्षेत्र C पर कोई हिल्बर्ट स्पेस H है। यदि T_1 और T_2 स्वयं ही H पर नार्मल ऑपरेटर है तथा दोनों ही एक दूसरे के संयोजन से विनियोज्य है तब :

(1) $T_1 + T_2$ नार्मल है परन्तु T_1T_2 नार्मल नहीं है

(2) T_1T_2 नार्मल है परन्तु $T_1 + T_2$ नार्मल नहीं है

(3) न तो $T_1 + T_2$ और न ही T_1T_2 नार्मल है

(4) $T_1 + T_2$ और T_1T_2 दोनों नार्मल हैं

38. If (X, T_1) and (Y, T_2) are two topological spaces and $f : X \to Y$ is a homeomorphism on X onto Y, then f is :

(1) open but not closed (2) closed but not open

(3) neither closed nor open (4) closed as well as open

यदि (X, T_1) और (Y, T_2) दो टोपोलॉजिकल स्पेस है तथा $f : X \to Y$, X का Y पर होमियोमार्फिज्म है तब f है :

(1) खुला पर बंद नहीं (2) बंद पर खुला नहीं

(3) न तो खुला और न ही बंद (4) खुला और बंद दोनों ही

39. The series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n + x^{10}}, \forall x \in \mathbb{R}$ is :

(1) uniformly as well as absolutely convergent

(2) uniformly but not absolutely convergent

(3) absolutely but not uniformly convergent

(4) neither absolutely nor uniformly convergent

श्रेणी $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n + x^{10}}, \forall x \in \mathbb{R}$ है :

(1) समान रूप से और पूर्णतः अभिसारी है

(2) समान रूप से पर पूर्णतः अभिसारी नहीं है

(3) पूर्णतः पर समान रूप से अभिसारी नहीं है

(4) न तो पूर्णतः और न ही समान रूप से अभिसारी है

40. Let (X, II, II) be a Banach space are C. Then :

(1) every series in X is convergent

(2) every convergent series in X is absolutely convergent

(3) every absolutely convergent series in X is convergent

(4) every absolutely convergent series in X is not convergent

माना (X, II, II) एक बैनेच स्पेस, C पर है। तब :

(1) X में प्रत्येक श्रेणी अभिसारी है

(2) X में प्रत्येक अभिसारी श्रेणी पूर्णतः अभिसारी है

(3) X में प्रत्येक पूर्णतः अभिसारी श्रेणी अभिसारी है

(4) X में प्रत्येक पूर्ण अभिसारी श्रेणी अभिसारी नहीं है

Short Answer Questions

लघु उत्तरीय प्रश्न

Note: Attempt any **five** questions. Write answer in **150-200** words. Each questions carries **16** marks. Answer each question on separate page, after writing Question Number.

किन्हीं पाँच प्रश्नों के उत्तर दीजिए। प्रत्येक का उत्तर **150-200** शब्दों में दीजिए। प्रत्येक प्रश्न **16** अंकों का है। प्रत्येक प्रश्न का उत्तर अलग पृष्ठ पर प्रश्न संख्या लिखकर शुरु करें।

01. If q_r and p_r , r = 1, 2, --------- , n are generalized coordinates and momenta variables respectively of a rigid body, then show that $\sum_{r=1}^{n} p_r\, q_r = 2T$,

Where T is the kinetic energy.

यदि q_r तथा p_r , r = 1, 2, --------- , n क्रमशः सामान्यीकृत और मोमेन्टा चर है एक दृढ़ पिण्ड के तो दिखाएँ कि

$\sum_{r=1}^{n} p_r\, q_r = 2T$,

जहाँ T गतिज ऊर्जा है।

02. Derive Bernoulli's equation in its general form.

बरनौली समीकरण के सामान्य रूप का निगमन करें।

03. Use revised simplese method to minimize

$6x_1 + 4x_2 + 7x_3 + 5x_4$, subject to :

$x_1 + 2x_2 + x_3 + 2x_4 \leq 20$,

$6x_1 + 5x_2 + 3x_3 + 2x_4 \leq 100$,

$3x_1 + 4x_2 + 9x_3 + 12x_4 \leq 75$.

$x_1, x_2, x_3, x_4 \geq 0$

$6x_1 + 4x_2 + 7x_3 + 5x_4$ को निम्न करने के लिए रिवाइज्ड सिम्पल मेथड का प्रयोग करें। दिया है :

$x_1 + 2x_2 + x_3 + 2x_4 \leq 20$,

$6x_1 + 5x_2 + 3x_3 + 2x_4 \leq 100$,

$3x_1 + 4x_2 + 9x_3 + 12x_4 \leq 75$,

$x_1, x_2, x_3, x_4 \geq 0$

04. Let G be a finite obelian group and p be a prime such that $p|0(G)$, then show that there exists an element $a \in G$ such that $a^p = e$.

माना G एक परिमेय ओबेलियन समूह है तथा p एक ऐसा प्राइम है कि $p|0(G)$, तब दर्शाइये कि $a \in G$ ऐसा है कि $a^p = e$

05. Show that no group of order 108 is simple.

सिद्ध करें कि 108 कोटि का कोई भी समूह साधारण नहीं है।

06. Define a locally connected topological space. Give an example of a topological space which is connected but is not locally connected. Also, prove that every component of a locally connected space is open.

'स्थानीय बद्ध' टोपोलॉजिकल स्पेस की परिभाषा दीजिए। एक ऐसे टोपोलॉजिकल स्पेस का उदाहरण दीजिए जो 'बद्ध' तो हो पर 'स्थानीय बद्ध' न हो। यह भी सिद्ध करें कि स्थानीय बद्ध स्पेस के सभी घटक खुले होते हैं।

07. Define Cantor set. Show that Cantor set has Lebesgne measure zero. Is it countable ?

कन्टूर समुच्चय को परिभाषित करें। सिद्ध करें कि कन्टूर समुच्चय का लेबेस्ने मापन शून्य होता है। क्या यह परिमेय होता है ?

08. Prove that a normed linear space is a Banach space if and only if every absolutely summable series is summable. Using the above criterion, give an example of a normed linear space and show that it is not a Banach space.

सिद्ध करें कि एक नार्म्ड रैखिक स्पेस एक बैनेच स्पेस होगा यदि और केवल यदि प्रत्येक संयोजनीय श्रेणी संयोजनीय हो। उपरोक्त शर्त का प्रयोग करते हुए, एक नार्म्ड रैखिक स्पेस का उदाहरण दीजिए और दिखाइए कि यह बैनेच स्पेस नहीं है।

09. Define the radius of convergence R of the power series $\sum_{n=0}^{\infty} a_n z^n$ and give one example.

Show that :

$$\frac{1}{R} = \overline{\lim_{n\to\infty}} |a_n|^{1/n}$$

घातांक श्रेणी $\sum_{n=0}^{\infty} a_n z^n$ के अभिसारण त्रिज्या R को परिभाषित कीजिए और एक उदाहरण दीजिए, दिखाइए कि :

$$\frac{1}{R} = \overline{\lim_{n\to\infty}} |a_n|^{1/n}$$

10. *Solve the boundary value problem* :

$y'' + xy' - y = 2x^2$, $y(0) = 0$, $y(1) = 1$

by using the Ritz's method and taking the approximating function as $y(x) = x + c_1 x(1 - x)$.

सीमा मूल्य समस्या को हल कीजिए :

$y'' + xy' - y = 2x^2$, $y(0) = 0$, $y(1) = 1$

जबकि एप्राक्सिमेटिंग फलन $y(x) = x + c_1 x(1 - x)$ है

तथा रिट्ज विधि का प्रयोग करना है।

RET/14/Test B | **895** | **Mathematics**

Question Booklet No. .....

*(To be filled up by the candidate by **blue/black ball-point pen**)*

Roll No

Roll No. (Write the digits in words) .....

Serial No. of OMR Answer Sheet .....

Day and Date ....

(Signature of Invigilator)

INSTRUCTIONS TO CANDIDATES

Use only ***blue/black ball-point pen*** in the space above and on both sides of the ***Answer Sheet***.

1. Within 10 minutes of the issue of the Question Booklet, Please ensure that you have got the correct booklet and it contains all the pages in correct sequence and no page/question is missing. In case of faulty Question Booklet, bring it to the notice of the Superintendent/Invigilators immediately to obtain a fresh Question Booklet.
2. Do not bring any loose paper, written or blank, inside the Examination Hall ***except the Admit Card without its envelope.***
3. ***A separate Answer Sheet is given. It should not be folded or mutilated. A second Answer Sheet shall not be provided.***
4. Write your Roll Number and Serial Number of the Answer Sheet by pen in the space provided above.
5. ***On the front page of the Answer Sheet, write by pen your Roll Number in the space provided at the top, and by darkening the circles at the bottom. Also, wherever applicable, write the Question Booklet Number and the Set Number in appropriate places.***
6. ***No overwriting is allowed in the entries of Roll No., Question Booklet No. and Set No. (if any) on OMR sheet and Roll No. and OMR sheet No. on the Question Booklet.***
7. ***Any changes in the aforesaid-entries is to be verified by the invigilator, otherwise it will be taken as unfair means.***
8. ***This Booklet contains 40 multiple choice questions followed by 10 short answer questions. For each MCQ, you are to record the correct option on the Answer Sheet by darkening the appropriate circle in the corresponding row of the Answer Sheet, by pen as mentioned in the guidelines given on the first page of the Answer Sheet. For answering any five short Answer Questions use five Blank pages attached at the end of this Question Booklet.***
9. For each question, darken only one circle on the Answer Sheet. If you darken more than one circle or darken a circle partially, the answer will be treated as incorrect.
10. ***Note that the answer once filled in ink cannot be changed.*** If you ***do not wish to attempt*** a question, leave all the circles in the corresponding row blank (such question will be awarded zero marks).
11. For rough work, use the inner back page of the title cover and the blank page at the end of this Booklet.
12. Deposit ***both OMR Answer Sheet and Question Booklet*** at the end of the Test.
13. You are not permitted to leave the Examination Hall until the end of the Test.
14. If a candidate attempts to use any form of unfair means, he/she shall be liable to such punishment as the University may determine and impose on him/her.

Total No. of Printed Pages : 15

Research Entrance Test – 2014

No. of Questions : 50

Time : 1 Hours *Full Marks : 200*

Note : (i) This Question Booklet contains **40** Multiple Choice Questions followed by **10** Short Answer Questions.

(ii) Attempt as many MCQs as you can. Each MCQ carries **3 (Three)** marks. **1 (One)** mark will be deducted for each incorrect answer. Zero mark will be awarded for each unattempted question. If more than **one** alternative answers of MCQs seem to be approximate to the correct answer, choose the closest one.

(iii) Answer only 5 Short Answer Questions. Each question carries **16 (Sixteen)** marks and should be answered in **150-200** words. Blank **5 (Five)** pages attached with this booklet shall only be used for the purpose. Answer each question on separate page, after writing Question No.

Mathematics Code No. : **895**

1. Which of the following is ***not*** a greenhouse gas ?
(1) Carbon dioxide (2) Methane (3) Sulphur dioxide (4) Nitrogen

2. The saliva of mammals contains starch splitting enzyme. The name of that enzyme is :
(1) Amylase (Ptyalin) (2) Secretin (3) Lysozyme (4) Mucin

3. Cytosine in DNA combines with :
(1) Adinosine (2) Uracil (3) Guanine (4) Thiamine

4. If Vectors $2i-j+k,\ i+2j-3k,\ 3i+\lambda j+5k$ are coplanar, then the value of λ is :
(1) -2 (2) -3 (3) -4 (4) -5

5. The value of $\left(-1+i\sqrt{3}\right)^{3/2}$ is :
(1) $\sqrt{2}$ (2) $2\sqrt{2}$ (3) $2+\sqrt{2}$ (4) $2-\sqrt{2}$

6. The number of electrons contained in 1 Coulomb of charge equals to :
(1) 6.25×10^{17} (2) 6.25×10^{18} (3) 6.25×10^{19} (4) 1.6×10^{19}

7. A unit mass of solid is converted to liquid at its melting ; the heat required for this process is the :
(1) Specific heat (2) Latent heat of vaporization
(3) Latent heat of fusion (4) External latent heat

8. Granite is :
(1) a sedimentary rock (2) a metamorphic rock
(3) a volcanic rock (4) a plutonic igneous rock

9. Coal is a :
(1) Sedimentary rock (2) Hydrothermal deposit
(3) Low-grade metamorphic rock (4) High-grade metamorphic rock

10. Which one of the following gases is present in the stratosphere that filters out some of the sun's ultraviolet light and provides an effective shield against radiation damage to living things ?
(1) Oxygen (2) Methane (3) Ozone (4) Helium

11. A cube is set rotating under no forces about its centre with uniform angular velocity. After certain time,
(1) only angular velocity will be changed
(2) only angular momentum will be changed
(3) both angular velocity and angular momentum will be changed
(4) neither angular velocity nor angular momentum will be changed

12. The geometrical equations of a rigid body having n generalized co-ordinates do not contain time variable explicitly, then :
(1) Hamilton function will be H = T – V
(2) Hamilton function will be H = T + V
(3) Lagrange function will be L = T + V
(4) Hamilton characteristic function will be A = 2T, where T and V are kinetic and potential energy respectively.

13. If p_r, q_r are momentum variables and generalized co-ordinates of a system, then :
(1) $p_r = \frac{\partial H}{\partial q_r}$, H is Hamilton function
(2) $p_r = \frac{\partial T}{\partial q_r}$, T is kinetic energy
(3) $p_r = \frac{\partial S}{\partial q_r}$, S is Hamilton principal function
(4) $p_r = \frac{\partial L}{\partial q_r}$, L is Lagrange function

14. The components of velocity of an incompressible fluid in the case of a two-dimensional flow at the point (x, y) are (ax, – ay), where a is a constant. The equation of the stream line passing through the point (2, 2) is :
(1) xy = 1 (2) xy = 2 (3) xy = 3 (4) xy = 4

15. The image system for a source outside a circle consists of :
(1) an equal source at the inverse point and an equal source at the centre of the circle
(2) an equal source at the inverse point and an equal sink at the centre of the circle
(3) an equal sink at the inverse point and an equal source at the centre of the circle
(4) an equal sink at the inverse point and an equal sink at the centre of the circle

16. If σ_1, σ_2 and σ_3 are principal stresses at a point, then the first stress invariant is :
(1) $\sigma_1 + \sigma_2 + \sigma_3$
(2) $\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1$
(3) $\sigma_1\sigma_2\sigma_3$
(4) $\sigma_1\sigma_2^2 + \sigma_2\sigma_3^2 + \sigma_3\sigma_1^2$

17. If a function $f : \mathbb{C} \longrightarrow \mathbb{C}$ is defined by :
$f(z) = f(x + iy) = u(x, y) + iv(x, y) = x \; \forall z \in \mathbb{C}$,
where $\mathbb{C}$ is the set of complex numbers, then :
(1) $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$, $\frac{\partial v}{\partial x}$ and $\frac{\partial v}{\partial y}$ do not exist at z = 0
(2) Cauchy - Riemann equations are satisfied but f is not differentiable at z = 0
(3) f is differentiable at z = 0
(4) Cauchy - Riemann equations are not satisfied for any value of z

18. For the power series $f(z)=\sum_{n=0} a_n z^n, \frac{1}{R}=\overline{\lim_{n\to\infty}}|a_n|^{\frac{1}{n}}, R\neq 1$, then :

(1) the radius of convergence of this series is R^2

(2) f(z) has no singularity on $|z|=R$

(3) f(z) has at least one singularity on $|z| = R$

(4) f(z) has at least one singularity on $|z| = \frac{R}{2}$

19. Let $f(x)=x, g(x)=x^2 \forall x\in[0,1]$

If $\int_0^1 f\,dg=\mu(g(1)-g(0))$ then the value of μ is :

(1) $\frac{2}{3}$ (2) $\frac{1}{3}$ (3) $\frac{3}{2}$ (4) $\frac{1}{2}$

20. A function $f:[0,1]\to\mathbb{R}$ is defined by :

$$f(x)=\begin{cases}1, & \text{when } x \text{ is irrational in } [0,1]\\ 0, & \text{when } x \text{ is rational in } [0,1]\end{cases}$$

Then f is :

(1) Riemann as well as Lebesgue integrable over [0, 1]

(2) Lebesgue integrable but not Riemann integrable over [0, 1]

(3) Neither Riemann nor Lebesgue integrable over [0, 1]

(4) Riemann integrable and Riemann integral of f over [0, 1] is 1

21. Let (X, s, μ) be a measure space and f be an extended real-valued measurable function on X such that $\int_x f\,d\mu$ exists. Define V_f on s by :

$$V_f(E)=\int_E f\,d\mu \;\forall E\in s$$

Then V_f is :

(1) measure as well as signed measure on X

(2) measure but not signed measure on X

(3) signed measure but not measure on X

(4) neither measure nor signed measure on X

22. The series :

$(1-x)^2+x(1-x)^2+x^2(1-x)^2+\ldots\ldots\ldots, \forall x\in[0,1]$ is :

(1) point-wise as well as uniformly convergent in [0, 1]

(2) not point-wise convergent in [0,1]

(3) point-wise but not uniformly convergent in [0,1]

(4) point-wise as well as uniformly convergent in $\left[\frac{1}{2},1\right]$

23. Let G be a group of order p^n, p = prime integer, n = positive integer, then :
(1) $O(Z(G)) < 1$ (2) $O(Z(G)) = 1$ (3) $O(Z(G)) > 1$ (4) $O(Z(G)) = p^n$

24. Tell which is the ***correct*** statement for the following groups :
(1) $\mathbb{Z}_2 \times \mathbb{Z}_2 \cong \mathbb{Z}_4$ (2) $\mathbb{Z} \times \mathbb{Z}$ is cyclic (3) $\mathbb{Z}_2 \times \mathbb{Z}_3 \cong \mathbb{Z}_6$ (4) $\mathbb{Z}_2 \times \mathbb{Z}_6 \cong \mathbb{Z}_{12}$

25. Non abelian groups of order 6 are :
(1) 1 (2) 3 (3) 2 (4) $\lfloor 6$

26. Total number of composition series of the group $\mathbb{Z}_{30}$ are :
(1) 4 (2) 5 (3) 8 (4) 6

27. Let Q be the field of rational numbers. Then the degree of $Q(\sqrt{3})$ over $Q(\sqrt{2})$ is :
(1) 4 (2) 10 (3) 6 (4) 2

28. If E_1 and E_2 are the splitting fields of the polynomials x^2+3 and x^2+x+1 over the field of rationales Q, then :
(1) $E_1 \not\subset E_2$ (2) $E_2 \not\subset E_1$ (3) $E_1 = E_2$ (4) $E_1 \neq E_2$

29. A C^2 function u satisfying $\nabla^2 u = 0$ is called a :
(1) Poisson function (2) Gauss function
(3) Green's function (4) Harmonic function

30. If $\Phi(x,t)$ is a fundamental solution of heat equation, then which of the following is ***true*** ?
(1) $\int_{R^n} \Phi(x,t)dx = 0$ (2) $\int_{R^n} \Phi(x,t)dx = 1$ (3) $\int_{R^n} \Phi(x,t)dx = e^x$ (4) $\int_{R^n} \Phi(x,t)dx = \log x$

31. Let a function $f : R^2 \to R$ is given by $f(x) = x_1^2 + x_2^2$, then its Hessian matrix is given by :
(1) $\begin{bmatrix} 0 & 2 \\ 1 & -2 \end{bmatrix}$ (2) $\begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}$ (3) $\begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$ (4) $\begin{bmatrix} -2 & 0 \\ 0 & 2 \end{bmatrix}$

32. Which of the following points satisfy first order necessary condition for the optimization problem ?
Minimize $x_1^2 + \frac{1}{2}x_2^2 + 3x_2$
subject to $x_1, x_2 > 0$
(1) $[0 \ 3]^T$ (2) $[1 \ 2]^T$ (3) $[1 \ 1]^T$ (4) $[0 \ 0]^T$

33. Let d be feasible direction of $f : R^n \to R$ then directional derivative $\frac{\partial}{\partial d} f(x)$ at x is defined as :
(1) $\lim_{t \to 0} \left[\frac{f(x+td) - f(x)}{t} \right]$ (2) $\lim_{t \to 0} \left[\frac{f(x-td) - f(x)}{t} \right]$
(3) $\lim_{d \to 0} \left[\frac{f(x+td) - f(x)}{d} \right]$ (4) $\lim_{d \to 0} \left[\frac{f(x-td) - f(x)}{d} \right]$

34. If primal of a linear programming problem is :
Minimize $C^T x$
subject to $Ax = b, x \geq 0$
then its dual is given by :
(1) max $\lambda^T b$, subject to $\lambda^T A \leq C^T$. (2) min $\lambda^T b$, subject to $\lambda^T A = C^T$.
(3) max $\lambda^T b$, subject to $\lambda^T A = C^T$. (4) min $\lambda^T b$, subject to $\lambda^T A \geq C^T$.

35. Euler's equations of motion of a rigid body are used for :
(1) rotation under finite forces about frame of reference fixed in the space
(2) rotation under finite forces about the frame of reference fixed with the body
(3) linear motion under impulsive forces
(4) linear motion under finite forces

36. The radius of convergence of the power series $\sum_{n=1}^{\infty} n^{n^2} z^{n^3}$ is :

(1) 1 (2) e (3) $\frac{1}{e}$ (4) e^2

37. The solution of the equation $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = 0$, where $z = f(x)$ and $\frac{\partial z}{\partial y} = g(x)$ on $y = 0$, is given by :

(1) $z(x,y) = \frac{1}{2}\big(f(x-y) + f(x+y)\big) + \frac{1}{2}\int_{x-y}^{x+y} g(u)\,du$

(2) $z(x,y) = \frac{1}{2}\big(f(x-y) + f(x+y)\big) + \frac{1}{2}\int_{x}^{x+y} g(u)\,du$

(3) $z(x,y) = \frac{1}{2} f(x-y) + \frac{1}{2}\int_{x-y}^{x+y} g(u)\,du$

(4) $z(x,y) = \frac{1}{2}\big(f(x-y) + f(x+y)\big) + \frac{1}{2}\int_{y}^{x+y} g(u)\,du$

38. If the series $\sum_{n=0}^{\infty} C_n$ is convergent and $f(x) = \sum_{n=0}^{\infty} C_n x^n, x \in (-1, 1)$
then $\lim_{x \to 1} f(x)$ is :

(1) $1 + \sum_{n=0}^{\infty} C_n^2$ (2) $\sum_{n=0}^{\infty} C_n$ (3) $2 + \sum_{n=0}^{\infty} C_n$ (4) $3 + \sum_{n=0}^{\infty} C_n^3$

39. A set expressed as the intersection of a finite number of half spaces is called a :
(1) polyhedron (2) non-convex polytope
(3) convex polytope (4) concave polytope

40. The number of cyclic subgroups of order 10 of the group $\mathbb{Z}_{100} \times \mathbb{Z}_{25}$ are :

(1) 10 (2) 14 (3) 20 (4) 24

Attempt any five questions. Write answer in 150-200 words. Each question carries 16 marks. Answer each question on separate page, after writing Question Number.

1. State and prove conservation law of energy using Lagrangian approach.
2. Find the equation of continuity in the spherical polar co-ordinates.
3. Prove that all possible norms defined on a finite dimensional vector space X over K (= $\mathbb{R}$ or $\mathbb{C}$) are equivalent.
4. Let (X, ζ) be a topological space and B be a sub collection of ζ. Prove that B is a base of ζ if and only if every ζ-open set is expressed as a union of members of B.
5. Let G be a group and let O(G) = pq, where p, q are distinct primes, p < q and $p \times (q-1)$. Show that G is cyclic.
6. Let $f(x) \in F[x]$ be of degree $n \geq 1$. Then show that there is a finite extension E of F of degree atmost $\lfloor n$ in which $f(x)$ has n roots.
7. Show that two dimensional Laplace equation $\nabla^2 u = 0$ in polar co-ordinates takes the form :

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$$

8. Consider the conjugate direction algorithm to find minimizer of :

$f(x_1, x_2) = \frac{1}{2} x^T \begin{bmatrix} 4 & 2 \\ 2 & 2 \end{bmatrix} x - x^T \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ with initial point $x^0 = [0\ 0]^T$ and a conjugate direction $d^0 = [1, 0]^T$, then find the minimizer at one iterate.

9. If a function $f : [a, b] \to \mathbb{R}^n, n \geq 1$, is continuous on [a, b] and differentiable on (a, b) then prove that there exists c $\in$ (a, b) such that :

$$||f(b) - f(a)|| \leq (b-a)||f'(c)||$$

10. If (X, || ||) is a normal linear space over a field K (= $\mathbb{R}$ or $\mathbb{C}$) and x be a non-zero vector in X then prove that there is a bounded linear functional F on X such that :

$$F(x) = ||x|| \text{ and } ||F|| = 1$$

Roll No. : .....

Q. No. :

Some Useful Links:

1. **Free Maths Study Materials (**https://pkalika.in/2020/04/06/free-maths-study-materials/**)**

2. **BSc/MSc Free Study Materials (**https://pkalika.in/2019/10/14/study-material/**)**

3. **MSc Entrance Exam Que. Paper: (**https://pkalika.in/2020/04/03/msc-entrance-exam-paper/**)**
   [JAM(MA), JAM(MS), BHU, CUCET, ...etc]

4. **PhD Entrance Exam Que. Paper: (**https://pkalika.in/que-papers-collection/**)**
   [CSIR-NET, GATE(MA), BHU, CUCET,IIT, NBHM, ...etc]

5. **CSIR-NET Maths Que. Paper: (**https://pkalika.in/2020/03/30/csir-net-previous-yr-papers/**)**
   [Upto 2019 Dec]

6. **Practice Que. Paper: (**https://pkalika.in/2019/02/10/practice-set-for-net-gate-set-jam/**)**
   [Topic-wise/Subject-wise]

7. **List of Maths Suggested Books** (https://pkalika.in/suggested-books-for-mathematics/)

8. **CSIR-NET Mathematics Details Syllabus (**https://wp.me/p6gYUB-Fc)