DU Mathematics

M.Sc Entrance Test Papers

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DU MA MSc Mathematics

Topic:- DU_J19_MA_MATHS

1) The order of Sylow subgroups of a finite group G of order 56 are [Question ID = 24519]

1. 2 and 28 [Option ID = 38076]
2. 7 and 8 [Option ID = 38074]
3. 8 and 14 [Option ID = 38077]
4. 4 and 14 [Option ID = 38075]

Correct Answer :-

- 7 and 8 [Option ID = 38074]

2) The remainder when 5^{2019} is divided by 11 is [Question ID = 24520]

1. 6 [Option ID = 38080]
2. 9 [Option ID = 38081]
3. 1 [Option ID = 38078]
4. 4 [Option ID = 38079]

Correct Answer :-

- 1 [Option ID = 38078]

3) The smallest positive integer n, which leaves remainders 2,3 and 4 when divided by 5,7 and 11 respectively, is [Question ID = 24521]

1. 751 [Option ID = 38083]
2. 1136 [Option ID = 38085]
3. 176 [Option ID = 38082]
4. 367 [Option ID = 38084]

Correct Answer :-

- 176 [Option ID = 38082]

4) Suppose that the equation $x^2 \cdot a \cdot x = a^{-1}$ is solvable for a in a group G. Then, there exists b in G such that

[Question ID = 24515]

1. $a = b^3$ [Option ID = 38059]
2. $a = b^5$ [Option ID = 38061]
3. $a = b^4$ [Option ID = 38060]
4. $a = b^2$ [Option ID = 38058]

Correct Answer :-

- $a = b^2$ [Option ID = 38058]

5) Consider the following statements:

(i) Every metric space is totally bounded.
(ii) A totally bounded metric space is bounded.

Then

[Question ID = 24536]

1. neither (i) nor (ii) is true [Option ID = 38145]
2. only (ii) is true [Option ID = 38143]
3. only (i) is true [Option ID = 38142]
4. both (i) and (ii) are true [Option ID = 38144]

Correct Answer :-

- only (i) is true [Option ID = 38142]

6)

Consider the following statements:

(i) Every minimal generating set of a vector space is a basis.
(ii) Every maximal linearly independent subset of a vector space is a basis.
(iii) Every vector space admits a basis.

Then

[Question ID = 24510]

1. all of (i), (ii) and (iii) are true [Option ID = 38041]
2. only (i) and (ii) are true [Option ID = 38038]
3. only (i) and (iii) are true [Option ID = 38040]
4. only (ii) and (iii) are true [Option ID = 38039]

Correct Answer :-

- only (i) and (ii) are true [Option ID = 38038]

7) The differential equation of a family of parabolas with foci at origin and axis along x-axis is

[Question ID = 24506]

1. $y\left(\frac{dy}{dx}\right)^2 + 2x^2\frac{dy}{dx} + y = 0$ [Option ID = 38023]
2. $y\left(\frac{dy}{dx}\right)^2 + 2x\frac{dy}{dx} - y = 0$ [Option ID = 38024]
3. $y\left(\frac{dy}{dx}\right)^2 + 2x\frac{dy}{dx} + y = 0$ [Option ID = 38025]
4. $y^2\left(\frac{dy}{dx}\right)^2 + 2x\frac{dy}{dx} - y^2 = 0$ [Option ID = 38022]

Correct Answer :-

- $y^2\left(\frac{dy}{dx}\right)^2 + 2x\frac{dy}{dx} - y^2 = 0$ [Option ID = 38022]

8) Number of iterations required to solve $x^3 + 4x^2 - 10 = 0$ using bisection method with accuracy 10^{-3} (with initial bracket $[1, 2]$) are

[Question ID = 24495]

1. 7 [Option ID = 37978]
2. 12 [Option ID = 37981]
3. 10 [Option ID = 37980]
4. 8 [Option ID = 37979]

Correct Answer :-

- 7 [Option ID = 37978]

9) Let $P_2(t)$ denote the set of all polynomials over $\mathbb{R}$ of degree at most 2. With respect to the inner product

$$\langle p, q\rangle = \int_{-1}^{1} p(t)q(t)dt,$$

the set of vectors $\{1, t, t^2 - \frac{1}{3}\}$ is

[Question ID = 24513]

1. not a linearly independent set [Option ID = 38053]
2. orthogonal basis of $P_2(t)$ [Option ID = 38050]
3. basis of $P_2(t)$ but not orthogonal [Option ID = 38052]

4. orthogonal but not a basis of $P_2(t)$ [Option ID = 38051]

Correct Answer :-

- orthogonal basis of $P_2(t)$ [Option ID = 38050]

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10) A function $f : \mathbb{R} \to \mathbb{R}$ is said to be periodic if there exists $p > 0$ such that $f(x+p) = f(x)$, for all $x \in \mathbb{R}$. If f is a continuous periodic function on $\mathbb{R}$, then

[Question ID = 24543]

1. f^2 is unbounded [Option ID = 38173]
2. $|f|$ is unbounded [Option ID = 38170]
3. $|f|$ is not uniformly continuous [Option ID = 38172]
4. f^2 is uniformly continuous and bounded on $\mathbb{R}$ [Option ID = 38171]

Correct Answer :-

- $|f|$ is unbounded [Option ID = 38170]

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11) Consider the following statements:

(i) Every separable metric space is compact.

(ii) Every compact metric space is separable.

Then

[Question ID = 24534]

1. only (i) is true [Option ID = 38134]
2. only (ii) is true [Option ID = 38135]
3. both (i) and (ii) are true [Option ID = 38136]
4. neither (i) nor (ii) is true [Option ID = 38137]

Correct Answer :-

- only (i) is true [Option ID = 38134]

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12) The partial differential equation $x^3 u_{xx} - (y^2 - 1)u_{yy} = u_x$ is

[Question ID = 24502]

1. parabolic in $\{(x, y) \mid y < 0\}$ [Option ID = 38006]
2. elliptic in $\mathbb{R}^2$ [Option ID = 38008]
3. hyperbolic in $\{(x, y) \mid x > 0\}$ [Option ID = 38007]
4. parabolic in $\{(x, y) \mid y > 0\}$ [Option ID = 38009]

Correct Answer :-

- parabolic in $\{(x, y) \mid y < 0\}$ [Option ID = 38006]

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13) Consider the following statements

(i) $\mathbb{Z}[x]$ is a principal ideal domain.

(ii) If R is a principal ideal domain, then every subring of R containing 1 is also a principal ideal domain.

Then

[Question ID = 24522]

1. only (i) is true [Option ID = 38086]
2. both (i) and (ii) are true [Option ID = 38088]
3. only (ii) is true [Option ID = 38087]
4. neither (i) nor (ii) is true [Option ID = 38089]

Correct Answer :-

- only (i) is true [Option ID = 38086]

14) Let $N \neq \{e\}$ be a normal subgroup of a non-abelian group G such that $N \cap G' = \{e\}$, where G' is the commutator subgroup of G. Then

[Question ID = 24517]

1. None of these [Option ID = 38069]
2. N is not abelian [Option ID = 38067]
3. $N \subseteq Z(G)$, the centre of G [Option ID = 38068]
4. G/N is abelian [Option ID = 38066]

Correct Answer :-

- G/N is abelian [Option ID = 38066]

15) Let $f(t) = t^2 e^t \log t;\ 1 \leq t \leq 3$. Then there exists some $c \in (1, 3)$ such that $\int_1^3 f(t)dt$ is equal to

[Question ID = 24525]

1. $\frac{1}{3} e^c \log c^{26}$ [Option ID = 38098]
2. $c^2 e^c \log 3$ [Option ID = 38101]
3. $2^2 c^2 \log c$ [Option ID = 38099]
4. $26 e^c \log c$ [Option ID = 38100]

Correct Answer :-

- $\frac{1}{3} e^c \log c^{26}$ [Option ID = 38098]

16) For two ideals I and J of a commutative ring R define $(I : J) = \{r \in R \mid rI \subseteq J\}$. Then for the ring $\mathbb{Z}$ of integers what is $(8\mathbb{Z} : 12\mathbb{Z})$

[Question ID = 24523]

1. $4\mathbb{Z}$ [Option ID = 38093]
2. $\mathbb{Z}$ [Option ID = 38090]
3. $2\mathbb{Z}$ [Option ID = 38091]
4. $3\mathbb{Z}$ [Option ID = 38092]

Correct Answer :-

- $\mathbb{Z}$ [Option ID = 38090]

17) Consider the set $\mathbb{R}^2$ with metric defined by

$$d(x, y) = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2}; \quad x = (x_1, x_2),\ y = (y_1, y_2).$$

Then which of the following set is not connected

[Question ID = 24535]

1. $\{(x,y) \in \mathbb{R}^2 \mid y^2 = x\}$ [Option ID = 38138]
2. $\{(x,y) \in \mathbb{R}^2 \mid x^2 - y^2 = 1\}$ [Option ID = 38141]
3. $\{(x,y) \in \mathbb{R}^2 \mid \frac{x^2}{4} + \frac{y^2}{9} = 1\}$ [Option ID = 38140]
4. $\{(x,y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1\}$ [Option ID = 38139]

Correct Answer :-

- $\{(x,y) \in \mathbb{R}^2 \mid y^2 = x\}$ [Option ID = 38138]

18) Let $f(x) = \lim_{n\to\infty} \frac{n^x - n^{-x}}{n^x + n^{-x}}, x \in \mathbb{R}$. Then

[Question ID = 24542]

1. f is continuous at $(1, \infty)$ [Option ID = 38169]
2. f is not differentiable at $x = 1$ [Option ID = 38168]
3. f is not continuous at $x = -1$ [Option ID = 38167]
4. f is continuous at $x = 0$ [Option ID = 38166]

Correct Answer :-

- f is continuous at $x = 0$ [Option ID = 38166]

19) For $x \in [-1, 1]$, let

$$f(x) = \begin{cases} x \operatorname{sgn}(\sin \frac{1}{x}), & \text{if } x \neq 0 \\ 0, & \text{if } x = 0, \end{cases}$$

where sgn denotes the signum function. Then

[Question ID = 24526]

1. f is continuous on $[-1, 1]$ [Option ID = 38104]
2. f is not differentiable at any point of $[-1, 1]$ [Option ID = 38103]
3. f is Riemann integrable on $[-1, 1]$ [Option ID = 38102]
4. the set of points of discontinuity of f in $[-1, 1]$ is finite [Option ID = 38105]

Correct Answer :-

- f is Riemann integrable on $[-1, 1]$ [Option ID = 38102]

20) The integral surface of the partial differential equation $p^2 + q^2 = 2$ which passess through $x = 0,\ z = y$ is

[Question ID = 24503]

1. $x^2 + y^2 + z^2 = 1$ [Option ID = 38013]
2. $z = y \pm x$ [Option ID = 38010]
3. $z^2 = x \pm y^2$ [Option ID = 38011]
4. $z^3 = x \pm y$ [Option ID = 38012]

Correct Answer :-

- $z = y \pm x$ [Option ID = 38010]

21) Does the sequence $a_n = n^2 \cos\left(\frac{2}{n^2} + \frac{\pi}{2}\right)$ has a limit?

[Question ID = 24529]

1. No, it oscillates [Option ID = 38115]
2. No, it diverges [Option ID = 38114]
3. Yes, −2 is the limit [Option ID = 38117]
4. Yes, −1 is the limit [Option ID = 38116]

Correct Answer :-

- No, it diverges [Option ID = 38114]

22) The orthogonal trajectory of the family of curves $ay^2 = x^3$, where a is an arbitrary constant, is

[Question ID = 26021]

1. $3y^2 + 2x^2 = \text{constant}$ [Option ID = 44082]
2. $2y^2 - 3x^2 = \text{constant}$ [Option ID = 44080]
3. $3y^2 - 2x^2 = \text{constant}$ [Option ID = 44079]
4. $2y^2 + 3x^2 = \text{constant}$ [Option ID = 44081]

Correct Answer :-

- $3y^2 - 2x^2 = \text{constant}$ [Option ID = 44079]

23) The integral surface of the linear partial differential equation

$$xp + yq = z$$

which contains the circle defined by $x^2 + y^2 + z^2 = 4$, $x + y + z = 2$, is

[Question ID = 24504]

1. $\frac{x}{y} + \frac{z}{x} + \frac{y}{z} + 1 = 0$ [Option ID = 38015]
2. $xy + xz + yz = 0$ [Option ID = 38016]
3. $xy^2 + xz^2 = 0$ [Option ID = 38014]
4. $xyz = 1$ [Option ID = 38017]

Correct Answer :-

- $xy^2 + xz^2 = 0$ [Option ID = 38014]

24) Initial estimate for the root of the equation $f(x) = 0$ is $x_0 = 2$ and $f(2) = 4$. The tangent line to $f(x)$ at $x_0 = 2$ makes an angle of 42^0 with the x axis. The next estimate of the root by Newton-Raphson method is approximately

[Question ID = 24499]

1. 2.0102 [Option ID = 37995]
2. 4.4424 [Option ID = 37997]
3. 0.2412 [Option ID = 37994]
4. −2.4424 [Option ID = 37996]

Correct Answer :-

- 0.2412 [Option ID = 37994]

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25) The numerical scheme using the first three terms of the Taylor series for solving the differential equation

$$\frac{dy}{dx} + y = e^{-3x}, \quad y(0) = 5,$$

with $h = x_{i+1} - x_i$, is given by

[Question ID = 24497]

1. $y_{i+1} = y_i + h(e^{-3x_i} - y_i) + \frac{h^2}{2}(-3e^{-3x_i} - y_i)$ [Option ID = 37988]
2. $y_{i+1} = y_i + h(e^{-3x_i} - y_i) + \frac{h^2}{2}(-4e^{-3x_i} + y_i)$ [Option ID = 37987]
3. $y_{i+1} = y_i - h(e^{-3x_i} - y_i) + \frac{h^2}{2}(y_i - e^{-3x_i})$ [Option ID = 37989]
4. $y_{i+1} = y_i + h(e^{-3x_i} - y_i) + \frac{h^2}{2}y_i$ [Option ID = 37986]

Correct Answer :-

- $y_{i+1} = y_i + h(e^{-3x_i} - y_i) + \frac{h^2}{2}y_i$ [Option ID = 37986]

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26) Let $X = \mathbb{C}^n$, $0 < p < 1$ and $q = 1/p$. For $x = (x_1, \ldots, x_n)$ and $y = (y_1, \ldots, y_n)$ in X define

$$d_p(x,y) = \Big(\sum_{i=1}^{n} |x_i - y_i|^p\Big)^{1/p}$$

and

$$d_q(x,y) = \Big(\sum_{i=1}^{n} |x_i - y_i|^q\Big)^{1/q}.$$

Then

[Question ID = 24533]

1. neither $d_p(x,y)$ nor $d_q(x,y)$ is a metric on X [Option ID = 38133]
2. both $d_p(x,y)$ and $d_q(x,y)$ are metrics on X [Option ID = 38130]
3. only $d_q(x,y)$ is a metric on X [Option ID = 38132]
4. only $d_p(x,y)$ is a metric on X [Option ID = 38131]

Correct Answer :-

- both $d_p(x,y)$ and $d_q(x,y)$ are metrics on X [Option ID = 38130]

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27) Let $f(x) = x \sin x$, $x \in \mathbb{R}$. Then $|f|$ is

[Question ID = 26030]

1. differentiable at $x = \pi$ [Option ID = 44117]
2. differentiable at $x = 0$ [Option ID = 44115]
3. uniformly continuous on $\mathbb{R}$ [Option ID = 44118]
4. differentiable at $x = -\pi$ [Option ID = 44116]

Correct Answer :-

- differentiable at $x = 0$ [Option ID = 44115]

28) Which of the following function f is not uniformly continuous on $\mathbb{R}$

[Question ID = 24541]

1. $f(x) = x + \sin x$ [Option ID = 38163]
2. $f(x) = x + \sin^3 x$ [Option ID = 38165]
3. $f(x) = x^2 + \sin x$ [Option ID = 38164]
4. $f(x) = \sin^2 x$ [Option ID = 38162]

Correct Answer :-

- $f(x) = \sin^2 x$ [Option ID = 38162]

29) Let

$$W = \{(x, y) \in \mathbb{R}^2 \mid xy = 0\},$$
$$X = \{(x, y) \in \mathbb{R}^2 \mid y = 3x\},$$
$$Y = \{(x, y) \in \mathbb{R}^2 \mid x^2 - y^2 = 0\},$$
$$Z = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 = 0\}.$$

Then the subspaces of $\mathbb{R}^2$ are

[Question ID = 24512]

1. X and Z [Option ID = 38047]
2. Y and Z [Option ID = 38049]
3. W and Y [Option ID = 38046]
4. W and X [Option ID = 38048]

Correct Answer :-

- W and Y [Option ID = 38046]

30) The solution of the Sturm-Liouville problem $\frac{d^2y}{dx^2} + \lambda y = 0, y(0) = 0, y(\pi) = 0$, where λ is a constant, is non-trivial for

[Question ID = 24509]

1. all $\lambda > 0$ [Option ID = 38034]
2. all $\lambda < 0$ [Option ID = 38037]
3. $\lambda = 0$ [Option ID = 38036]
4. $\lambda = 1, 4, 9, \ldots$ [Option ID = 38035]

Correct Answer :-

- all $\lambda > 0$ [Option ID = 38034]

31) The maximum and minimum values of the function $f(x, y) = 5x^2 + 2xy + 5y^2$ on the circle $x^2 + y^2 = 1$ denoted by $\max f$ and $\min f$, respectively are

[Question ID = 24539]

1. $\max f = 6, \ \min f = 0$ [Option ID = 38156]
2. $\max f = 6, \ \min f = 4$ [Option ID = 38155]
3. $\max f = \infty, \ \min f = -\infty$ [Option ID = 38157]

4. $\max f = \min f = 5$ [Option ID = 38154]

Correct Answer :-

- $\max f = \min f = 5$ [Option ID = 38154]

32) The general solution of the differential equation $x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 4y = 0$ is

[Question ID = 24507]

1. $y = (c_1 + c_2x^2)e^x$ [Option ID = 38027]
2. $y = (c_1 + c_2x)e^{2x}$ [Option ID = 38026]
3. $y = (c_1 + c_2\log\, x)x$ [Option ID = 38028]
4. $y = (c_1 + c_2\log\, x)x^2$ [Option ID = 38029]

Correct Answer :-

- $y = (c_1 + c_2x)e^{2x}$ [Option ID = 38026]

33) Let A be a 3×3 matrix over $\mathbb{R}$ with characteristic polynomial $p(\lambda) = \lambda(\lambda - 1)(\lambda - 3)$. Consider the following statements:

(i) The matrix A is not invertible.

(ii) There are three eigen vectors v_1, v_2, v_3 which forms a basis of $\mathbb{R}^3$.

(iii) Each eigen space of A is one dimensional.

(iv) The linear system $(A - 3I)X = B$ has a unique solution for each $B \in \mathbb{R}^3$.

Then

[Question ID = 24511]

1. only (ii) and (iii) are true [Option ID = 38044]
2. only (ii), (iii) and (iv) are true [Option ID = 38043]
3. only (i) and (ii) are true [Option ID = 38045]
4. only (i), (ii) and (iii) are true [Option ID = 38042]

Correct Answer :-

- only (i), (ii) and (iii) are true [Option ID = 38042]

34) The solution of the wave equation

$$u_{tt} - c^2u_{xx} = 0, \quad 0 \le x \le L,\ t > 0$$

with

$$u(0,t) = 0,\ t > 0; \quad u(L,t) = 0,\ t > 0$$

by the method of separation of variables is given by

[Question ID = 24500]

1. $\sum_{n=1}^{\infty} \cos\frac{n\pi x}{L}\left(A_n \sin\frac{n\pi ct}{L} + B_n \sin\frac{n\pi ct}{L}\right)$ [Option ID = 38000]
2. $\sum_{n=1}^{\infty} \sin\frac{n\pi x}{L}\left(A_n \cos\frac{n\pi ct}{L} + B_n \sin\frac{n\pi ct}{L}\right)$ [Option ID = 37998]
3. $\sum_{n=1}^{\infty} \sin\frac{n\pi x}{L}\left(A_n \sin n\pi ct + B_n \cos n\pi ct\right)$ [Option ID = 38001]

4. $\sum_{n=1}^{\infty} \cos \frac{n\pi x}{L} \left(A_n \cos n\pi ct + B_n \sin n\pi ct \right)$ [Option ID = 37999]

Correct Answer :-

- $\sum_{n=1}^{\infty} \sin \frac{n\pi x}{L} \left(A_n \cos \frac{n\pi ct}{L} + B_n \sin \frac{n\pi ct}{L} \right)$ [Option ID = 37998]

35) The values of c_0, c_1 and c_2 so that the formula $\int_{-1}^{1} f(x)dx = c_0 f(-1) + c_1 f(0) + c_2 f(1)$ is exact for all polynomials of degree less than or equal to 2 are

[Question ID = 24496]

1. $c_0 = 1,\ c_1 = 1,\ c_2 = 0$ [Option ID = 37983]
2. $c_0 = 1/3,\ c_1 = 4/3,\ c_2 = 1/3$ [Option ID = 37982]
3. $c_0 = 0,\ c_1 = 0,\ c_2 = 1$ [Option ID = 37985]
4. $c_0 = 2/3,\ c_1 = 2/3,\ c_2 = 2/3$ [Option ID = 37984]

Correct Answer :-

- $c_0 = 1/3,\ c_1 = 4/3,\ c_2 = 1/3$ [Option ID = 37982]

36) Let S, T be linear transformations from $\mathbb{R}^n$ to $\mathbb{R}^n$ such that $ST = I$, the identity map. Then

[Question ID = 24514]

1. S is one-one but T is not [Option ID = 38055]
2. T is one-one but S is not [Option ID = 38054]
3. Both S and T are one-one [Option ID = 38056]
4. Neither S nor T is one-one [Option ID = 38057]

Correct Answer :-

- T is one-one but S is not [Option ID = 38054]

37) In cylindrical coordinates (r, θ, z), the Laplace equation $\nabla^2 u = 0$ takes the form

[Question ID = 24501]

1. $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial^2 u}{\partial \theta^2} + \frac{\partial^2 u}{\partial z^2} = 0$ [Option ID = 38002]
2. $\frac{\partial}{\partial r}\left(\frac{1}{r^2}\frac{\partial u}{\partial r}\right) + \frac{\partial^2 u}{\partial \theta^2} + \frac{\partial^2 u}{\partial z^2} = 0$ [Option ID = 38004]
3. $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} + \frac{\partial^2 u}{\partial z^2} = 0$ [Option ID = 38005]
4. $\frac{\partial^2 u}{\partial r^2} + \frac{\partial^2 u}{\partial \theta^2} + \frac{\partial^2 u}{\partial z^2} = 0$ [Option ID = 38003]

Correct Answer :-

- $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial^2 u}{\partial \theta^2} + \frac{\partial^2 u}{\partial z^2} = 0$ [Option ID = 38002]

38) Let a and b be any two permutations in S_5, the symmetric group on 5 letters. Let $c = a^{-1}(12)a$ and $d = b^{-1}(12)(34)b$. Then

[Question ID = 24518]

1. both c and d are even [Option ID = 38072]

2. both c and d are odd [Option ID = 38073]
3. c is even and d is odd [Option ID = 38071]
4. c is odd and d is even [Option ID = 38070]

Correct Answer :-
- c is odd and d is even [Option ID = 38070]

39) For a commutative ring R with identity consider the following statements

(i) Let I be an ideal of R such that every element of R not in I is a unit (invertible). Then R/I is a field.

(ii) An ideal I of R is prime if and only if R/I is an integral domain.

(iii) Every non-zero prime ideal of R is maximal.

Then

[Question ID = 24524]

1. only (ii) and (iii) are true [Option ID = 38095]
2. only (i) and (iii) are true [Option ID = 38096]
3. only (i) and (ii) are true [Option ID = 38094]
4. all of (i), (ii) and (iii) are true [Option ID = 38097]

Correct Answer :-
- only (i) and (ii) are true [Option ID = 38094]

40) Let $\mathcal{A}$ denote the subset $\mathbb{Q} \times \mathbb{Q}$ of $\mathbb{R}^2$ and $\mathcal{U}$ denote the set of all lines in $\mathbb{R}^2$ that intersect with $\mathcal{A}$ in at least two points. Then

[Question ID = 24537]

1. both $\mathcal{A}$ and $\mathcal{U}$ are uncountable [Option ID = 38146]
2. both $\mathcal{A}$ and $\mathcal{U}$ are countable [Option ID = 38147]
3. $\mathcal{A}$ is countable but $\mathcal{U}$ is uncountable [Option ID = 38148]
4. $\mathcal{U}$ is countable but $\mathcal{A}$ is uncountable [Option ID = 38149]

Correct Answer :-
- both $\mathcal{A}$ and $\mathcal{U}$ are uncountable [Option ID = 38146]

41) A set $X \subseteq \mathbb{R}$ is said to be a null set if for every $\epsilon > 0$ there exists a countable collection $\{(a_k, b_k)\}_{k=1}^{\infty}$ of open intervals such that $X \subseteq \bigcup_{k=1}^{\infty}(a_k, b_k)$ and $\sum_{k=1}^{\infty}(b_k - a_k) \leq \epsilon$. Which of the following set is not a null set?

[Question ID = 24527]

1. Every finite set [Option ID = 38109]
2. $\mathbb{Q}^c$, the set of irrational numbers [Option ID = 38108]
3. $\mathbb{N}$, the set of natural numbers [Option ID = 38106]
4. $\mathbb{Q}$, the set of rational numbers [Option ID = 38107]

Correct Answer :-
- $\mathbb{N}$, the set of natural numbers [Option ID = 38106]

42) Let f be a bounded Riemann integrable function on $[a, b]$ and F be its indefinite integral. Which of the following is not true?

[Question ID = 24528]

1. F is continuous on $[a, b]$ [Option ID = 38111]
2. F need not be differentiable on $[a, b]$ [Option ID = 38113]
3. F is differentiable on $[a, b]$ and $F'(x) = f(x)$ for every $x \in [a, b]$ [Option ID = 38112]
4. F satisfies Lipschitz's condition [Option ID = 38110]

Correct Answer :-

- F satisfies Lipschitz's condition [Option ID = 38110]

43) Let $\langle a_n \rangle$ be a bounded sequence of real numbers with $\limsup a_n \neq \liminf a_n$. Consider the following statements

(i) $\lim a_n$ does not exist.

(ii) $\liminf a_n < \limsup a_n$.

(iii) There is a convergent subsequence of $\langle a_n \rangle$.

Then

[Question ID = 24530]

1. all of (i), (ii) and (iii) are true [Option ID = 38121]
2. only (ii) is true [Option ID = 38119]
3. only (i) and (ii) are true [Option ID = 38118]
4. only (ii) and (iii) are ture [Option ID = 38120]

Correct Answer :-

- only (i) and (ii) are true [Option ID = 38118]

44) The area bounded by the curve and x axis with data

x	7.47	7.48	7.49	7.50	7.51	7.52
$f(x)$	1.93	1.95	1.98	2.01	2.03	2.06

using trapezoidal rule is

[Question ID = 24498]

1. 0.0996 [Option ID = 37991]
2. 0.0876 [Option ID = 37990]
3. 0.0745 [Option ID = 37992]
4. 0.0912 [Option ID = 37993]

Correct Answer :-

- 0.0876 [Option ID = 37990]

45) The series $\sum \frac{(-1)^n}{n^p}$

[Question ID = 24531]

1. converges for all values of p [Option ID = 38124]
2. converges for p > 0, diverges for p ≤ 0 [Option ID = 38122]
3. does not converges for any value of p [Option ID = 38125]
4. converges for p > 1, diverges for p ≤ 1 [Option ID = 38123]

Correct Answer :-

- converges for p > 0, diverges for p ≤ 0 [Option ID = 38122]

46)

The solution of the differential equations

$$x'(t) = -y + t,$$

$$y'(t) = x - t$$

with c_1 and c_2 as arbitrary constants, is

[Question ID = 26022]

1. $x = c_1 \cos t - c_2 \sin t + t + 1; \quad y = c_1 \sin t + c_2 \cos t - t + 1$ [Option ID = 44086]
2. $x = c_1 \cos t + c_2 \sin t + t + 1; \quad y = c_1 \sin t - c_2 \cos t + t - 1$ [Option ID = 44083]
3. $x = c_1 \cos t - c_2 \sin t + t + 1; \quad y = c_1 \sin t + c_2 \cos t + t + 1$ [Option ID = 44084]
4. $x = c_1 \cos t + c_2 \sin t + t + 1; \quad y = c_1 \sin t - c_2 \cos t + t + 1$ [Option ID = 44085]

Correct Answer :-

- $x = c_1 \cos t + c_2 \sin t + t + 1; \quad y = c_1 \sin t - c_2 \cos t + t - 1$ [Option ID = 44083]

---

47) The proof of the fact that the sequence $\left\langle \frac{1}{n} \right\rangle$ converges to zero relies on

[Question ID = 24538]

1. None of these [Option ID = 38153]
2. both completeness and the archimedian properties of $\mathbb{R}$. [Option ID = 38152]
3. only the completeness property of $\mathbb{R}$. [Option ID = 38151]
4. only the archimedian property of $\mathbb{R}$. [Option ID = 38150]

Correct Answer :-

- only the archimedian property of $\mathbb{R}$. [Option ID = 38150]

---

48) Which sets are compact?

$X = \{x^{-1} \mid x \geq 2\} \subseteq \mathbb{R}$

$Y = \left\{(x, y) \in \mathbb{R}^2 \mid x^3 + y^3 = 1\right\}$

$Z = \left\{(x, y) \in \mathbb{R}^2 \mid \frac{x^2}{4} + \frac{y^2}{9} = 1\right\}$

[Question ID = 24532]

1. All of X, Y and Z [Option ID = 38126]
2. Only Y and Z [Option ID = 38127]
3. Only X and Z [Option ID = 38128]
4. Only Z [Option ID = 38129]

Correct Answer :-

- All of X, Y and Z [Option ID = 38126]

---

49) Let $\mathbf{F}(x, y, z) = \frac{y}{x^2 + y^2}\mathbf{i} - \frac{x}{x^2 + y^2}\mathbf{j} + \mathbf{k}$ be defined on $D = \{(x, y) \in \mathbb{R}^2 \; : \; x^2 + y^2 > 0\}$. If C denotes the unit circle in xy plane, then

[Question ID = 24540]

1. curl $\mathbf{F} = \mathbf{0}$ in D and $\int_C \mathbf{F}.\mathbf{ds} = 0$ [Option ID = 38161]
2. curl $\mathbf{F} \neq \mathbf{0}$ in D but $\int_C \mathbf{F}.\mathbf{ds} = 0$ [Option ID = 38159]

3. $\text{curl } \mathbf{F} \neq \mathbf{0}$ in D and $\int_C \mathbf{F}.\mathbf{ds} \neq 0$ [Option ID = 38160]
4. $\text{curl } \mathbf{F} = \mathbf{0}$ in D but $\int_C \mathbf{F}.\mathbf{ds} \neq 0$ [Option ID = 38158]

Correct Answer :-

- $\text{curl } \mathbf{F} = \mathbf{0}$ in D but $\int_C \mathbf{F}.\mathbf{ds} \neq 0$ [Option ID = 38158]

50) Let *K* be any subgroup of a group *G* and *H* be the only subgroup of order *m* in *G*. Which of the following is not true?

[Question ID = 24516]

1. *H* is a normal subgroup of *G* [Option ID = 38062]
2. *G = N(H)*, where *N(H)* is the normalizer of *H in G*. [Option ID = 38065]
3. $ab \in H$ implies that $ba \in H$ [Option ID = 38064]
4. *HK* is not a subgroup of *G* [Option ID = 38063]

Correct Answer :-

- *H* is a normal subgroup of *G* [Option ID = 38062]

DU MA MSc Mathematics

Topic:- DU_J18_MA_MATHS_Topic01

1)
The complete integral of the partial differential equation $xpq + yq^2 - 1 = 0$ where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$ is

[Question ID = 2159]

1. $(z+b)^2 = 4(ax+y)$. [Option ID = 8635]
2. $z+b = 2(ax+y)$. [Option ID = 8633]
3. $z+b = 4(ax+y)^2$. [Option ID = 8636]
4. $z+b = 2(ax+y)^2$. [Option ID = 8634]

Correct Answer :-

- $(z+b)^2 = 4(ax+y)$. [Option ID = 8635]

2)
Let P be the set of all the polynomials with rational coefficients and S be the set of all sequences of natural numbers. Then which one of the following statements is true?

[Question ID = 2139]

1. S is countable but P is not. [Option ID = 8555]
2. Both the sets P and S are uncountable. [Option ID = 8556]
3. Both the sets P and S are countable. [Option ID = 8553]
4. P is countable but S is not. [Option ID = 8554]

Correct Answer :-

- P is countable but S is not. [Option ID = 8554]

3)
For the differential equation

$$x\frac{dy}{dx} + 6y = 3xy^{4/3}$$

consider the following statements:

(i) The given differential equation is a linear equation.

(ii) The differential equation can be reduced to linear equation by the transformation $V = y^{-1/3}$.

(iii) The differential equation can be reduced to linear equation by the transformation $V = x^{-1/3}$.

Which of the above statements are true?

are [Question ID = 2156]

1. Only (i). [Option ID = 8622]
2. Only (iii). [Option ID = 8624]
3. Only (ii). [Option ID = 8623]
4. Both (i) and (ii). [Option ID = 8621]

Correct Answer :-

- Only (ii). [Option ID = 8623]

---

4)

Which one of the following statements is not true for Simpson's 1/3 rule to find approximate value of the definite integral $I = \int_0^1 f(x)dx$?

[Question ID = 2151]

1. If $y_0 = f(0), y_1 = f(0.5), y_2 = f(1)$, the approximate value of I is $\frac{1}{6}[y_0 + 3y_1 + y_2]$. [Option ID = 8603]
2. The approximating function has odd number of points common with the function $f(x)$. [Option ID = 8604]
3. Simpson's 1/3 rule improves trapezoidal rule. [Option ID = 8602]
4. The function $f(x)$ is approximated by a parabola. [Option ID = 8601]

Correct Answer :-

- If $y_0 = f(0), y_1 = f(0.5), y_2 = f(1)$, the approximate value of I is $\frac{1}{6}[y_0 + 3y_1 + y_2]$. [Option ID = 8603]

---

5) The equation of the tangent plane to the surface $z = 2x^2 - y^2$ at the point $(1, 1, 1)$ is

[Question ID = 2133]

1. $x - y - 2z = 2.$ [Option ID = 8531]
2. $4x - y - 3z = 1.$ [Option ID = 8532]
3. $2x - y - 2z = 1.$ [Option ID = 8529]
4. $4x - 2y - z = 1.$ [Option ID = 8530]

Correct Answer :-

- $4x - 2y - z = 1.$ [Option ID = 8530]

---

6)

If $\{x, y\}$ is an orthonormal set in an inner product space then the value of $\|x - y\| + \|x + y\|$ is

[Question ID = 2128]

1. $2\sqrt{2}.$ [Option ID = 8510]
2. $2 + \sqrt{2}.$ [Option ID = 8512]
3. $\sqrt{2}.$ [Option ID = 8511]

4. $2.$ [Option ID = 8509]

Correct Answer :-

- $2\sqrt{2}.$ [Option ID = 8510]

---

7) Which one of the following spaces, with the usual metric, is not separable?

[Question ID = 2147]

1. The space $C[a,b]$ of the set of all real valued continuous functions defined on $[a,b]$. [Option ID = 8586]
2. The space l^∞ of all bounded real sequences with supremum metric. [Option ID = 8588]
3. The Euclidean space $\mathbb{R}^n$. [Option ID = 8585]
4. The space l^1 of all absolutely convergent real sequences. [Option ID = 8587]

Correct Answer :-

- The space l^∞ of all bounded real sequences with supremum metric. [Option ID = 8588]

---

8) Let G be an abelian group of order 2018 and $f: G \to G$ be defined as $f(x) = x^5$. Then

[Question ID = 2118]

1. f is not injective. [Option ID = 8470]
2. f is not surjective. [Option ID = 8471]
3. there exists $e \neq x \in G$ such that $f(x) = x^{-1}$. [Option ID = 8472]
4. f is an automorphism of G. [Option ID = 8469]

Correct Answer :-

- f is an automorphism of G. [Option ID = 8469]

---

9) If $f: \mathbb{R} \to \mathbb{R}$ is a continuous function such that

$$f(x+y) = f(x) + f(y), \text{ for all } x, y \in \mathbb{R},$$

then

[Question ID = 2138]

1. f is increasing if $f(1) \geq 0$ and decreasing if $f(1) \leq 0$. [Option ID = 8551]
2. f is increasing if $f(1) \leq 0$ and decreasing if $f(1) \geq 0$. [Option ID = 8552]
3. f is a not an increasing function. [Option ID = 8549]
4. f is neither an increasing nor a decreasing function. [Option ID = 8550]

Correct Answer :-

- f is increasing if $f(1) \geq 0$ and decreasing if $f(1) \leq 0$. [Option ID = 8551]

---

10) The central difference operator δ and backward difference operator ∇ are related as

[Question ID = 2154]

1. $\delta = \nabla(1-\nabla)^{\frac{1}{2}}.$ [Option ID = 8615]
2. $\delta = \nabla(1+\nabla)^{-\frac{1}{2}}.$ [Option ID = 8614]
3. $\delta = \nabla(1-\nabla)^{-\frac{1}{2}}.$ [Option ID = 8616]
4. $\delta = \nabla(1+\nabla)^{\frac{1}{2}}.$ [Option ID = 8613]

Correct Answer :-

- $\delta = \nabla(1-\nabla)^{-\frac{1}{2}}.$ [Option ID = 8616]

11)
How many continuous real functions f can be defined on $\mathbb{R}$ such that $(f(x))^2 = x^2$ for every $x \in \mathbb{R}$?

[Question ID = 2144]

1. Infinitely many. [Option ID = 8576]
2. None. [Option ID = 8575]
3. 4. [Option ID = 8574]
4. 2. [Option ID = 8573]

Correct Answer :-

- 4. [Option ID = 8574]

12) The greatest common divisor of $11+7i$ and $18-i$ in the ring of Gaussian integers $\mathbb{Z}[i]$ is

[Question ID = 2122]

1. $3i.$ [Option ID = 8485]
2. $1.$ [Option ID = 8488]
3. $1+i.$ [Option ID = 8487]
4. $2+i.$ [Option ID = 8486]

Correct Answer :-

- $1.$ [Option ID = 8488]

13) The complete integral of the partial differential equation

$$\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = e^{x+2y}$$

is

[Question ID = 2161]

1. $\phi_1(y-x) + x\phi_2(y+x) + e^{x+2y}.$ [Option ID = 8643]

2. $\phi_1(y+x)+x\phi_2(y+x)+xe^{x+2y}$. [Option ID = 8644]
3. $\phi_1(y-x)+\phi_2(y+x)+e^{x+2y}$. [Option ID = 8641]
4. $\phi_1(y+x)+x\phi_2(y+x)+e^{x+2y}$. [Option ID = 8642]

Correct Answer :-

- $\phi_1(y+x)+x\phi_2(y+x)+e^{x+2y}$. [Option ID = 8642]

14) If $S=\{(1,0,i),(1,2,1)\}\subseteq\mathbb{C}^3$ then $S^\perp$ is

[Question ID = 2127]

1. $\text{span}\,\{(i,-\frac{1}{2}(i+1),-1)\}$. [Option ID = 8506]
2. $\text{span}\,\{(-i,\frac{1}{2}(i+1),1)\}$. [Option ID = 8505]
3. $\text{span}\,\{(i,-\frac{1}{2}(i+1),1)\}$. [Option ID = 8507]
4. $\text{span}\,\{(i,\frac{1}{2}(i+1),-1)\}$. [Option ID = 8508]

Correct Answer :-

- $\text{span}\,\{(i,-\frac{1}{2}(i+1),1)\}$. [Option ID = 8507]

15) The improper integral $\int_{-\infty}^{0} 2^x dx$ is

[Question ID = 2135]

1. convergent and converges to 2. [Option ID = 8540]
2. divergent. [Option ID = 8539]
3. convergent and converges to $\frac{1}{\ln 2}$. [Option ID = 8538]
4. convergent and converges to $-\ln 2$. [Option ID = 8537]

Correct Answer :-

- convergent and converges to $\frac{1}{\ln 2}$. [Option ID = 8538]

16)
Let $f:\mathbb{R}\to\mathbb{R}$ be a continuous function which takes irrational values at rational points and rational values at irrational points. Then which one of the following statements is true?

[Question ID = 2145]

1. f is uniformly continuous on $\mathbb{Q}$. [Option ID = 8578]
2. f is uniformly continuous on $\mathbb{R}$. [Option ID = 8577]

3. f is uniformly continuous on $\mathbb{Q}^c$. [Option ID = 8579]
4. No such function exists. [Option ID = 8580]

Correct Answer :-

- No such function exists. [Option ID = 8580]

---

17) If $f:[0,10] \to \mathbb{R}$ is defined as

$$f(x) = \begin{cases} 0, & 0 \le x < 2, \\ 1, & 2 \le x \le 5 \\ 0, & 5 < x \le 10, \end{cases}$$

and $F(x) = \int_0^x f(t)dt$ then

[Question ID = 2134]

1. $F(x) = 3$ for $x \le 5$. [Option ID = 8536]
2. $F'(x) = f(x)$ for every x. [Option ID = 8534]
3. F is not differentiable at $x = 2$ and $x = 5$. [Option ID = 8535]
4. F is differentiable everywhere on $[0, 10]$. [Option ID = 8533]

Correct Answer :-

- F is not differentiable at $x = 2$ and $x = 5$. [Option ID = 8535]

---

18) The Maclaurin series expansion

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots$$

is valid

[Question ID = 2136]

1. only if $x \in [-1,1]$. [Option ID = 8543]
2. if $x > -1$. [Option ID = 8541]
3. only if $x \in (-1,1]$. [Option ID = 8542]
4. for every $x \in \mathbb{R}$. [Option ID = 8544]

Correct Answer :-

- only if $x \in (-1,1]$. [Option ID = 8542]

---

19) If $4x \equiv 2 \pmod 6$ and $3x \equiv 5 \pmod 8$ then one of the value of x is

[Question ID = 2115]

1. 32 [Option ID = 8460]
2. 34 [Option ID = 8457]
3. 26 [Option ID = 8459]
4. 23 [Option ID = 8458]

Correct Answer :-

- 23 [Option ID = 8458]

20)

If $f(x) = \lim_{n\to\infty} S_n(x)$, where

$$S_n(x) = \frac{x}{(x+1)(2x+1)} + \frac{x}{(2x+1)(3x+1)} + \cdots + \frac{x}{(nx+1)((n+1)x+1)}$$

then the function f is

[Question ID = 2131]

1. continuous nowhere. [Option ID = 8524]
2. continuous everywhere. [Option ID = 8521]
3. continuous everywhere except at countably many points. [Option ID = 8522]
4. continuous everywhere except at one point. [Option ID = 8523]

Correct Answer :-

- continuous everywhere except at one point. [Option ID = 8523]

21)

The rate of change of $f(x,y) = 4y - x^2$ at the point $(1, 5)$ in the direction from $(1, 5)$ to the point $(4, 3)$ is

[Question ID = 2130]

1. $\frac{-6}{\sqrt{5}}$. [Option ID = 8519]
2. $\frac{-14}{\sqrt{13}}$. [Option ID = 8518]
3. $\frac{-12}{\sqrt{5}}$. [Option ID = 8520]
4. $\frac{-19}{\sqrt{13}}$. [Option ID = 8517]

Correct Answer :-

- $\frac{-14}{\sqrt{13}}$. [Option ID = 8518]

22) Let $G = \{a_1, a_2, \ldots, a_{25}\}$ be a group of order 25. For $b, c \in G$ let

$$bG = \{ba_1, ba_2, \ldots, ba_{25}\},\ Gc = \{a_1c, a_2c, \ldots, a_{25}c\}.$$

Then

[Question ID = 2119]

1. $bG = Gc$ only if $b = c$. [Option ID = 8475]
2. $bG = Gc\ \forall b, c \in G$. [Option ID = 8473]
3. $bG = Gc$ only if $b^{-1} = c$. [Option ID = 8476]
4. $bG \neq Gc$, if $b \neq c$. [Option ID = 8474]

Correct Answer :-

- $bG = Gc\ \forall b, c \in G$. [Option ID = 8473]

23)

If $\langle x_n \rangle$ is a sequence such that $x_n \geq 0$, for every $n \in \mathbb{N}$ and if $\lim_{n\to\infty}((-1)^n x_n)$ exists then which one of the following statements is true?

[Question ID = 2141]

1. The sequence $\langle x_n \rangle$ is a Cauchy sequence. [Option ID = 8562]
2. The sequence $\langle x_n \rangle$ is not a Cauchy sequence. [Option ID = 8564]
3. The sequence $\langle x_n \rangle$ is unbounded. [Option ID = 8563]
4. The sequence $\langle x_n \rangle$ is divergent. [Option ID = 8561]

Correct Answer :-

- The sequence $\langle x_n \rangle$ is a Cauchy sequence. [Option ID = 8562]

---

24) If $n > 2$, then $n^5 - 5n^3 + 4n$ is divisible by

[Question ID = 2113]

1. 80 [Option ID = 8449]
2. 120 [Option ID = 8451]
3. 100 [Option ID = 8450]
4. 125 [Option ID = 8452]

Correct Answer :-

- 120 [Option ID = 8451]

---

25) Let

$$S = \cap_{n=1}^{\infty}\left[2 - \frac{1}{n}, 3 + \frac{1}{n}\right].$$

Then S equals

[Question ID = 2140]

1. $(2, 3]$. [Option ID = 8558]
2. $[2, 3]$. [Option ID = 8560]
3. $[2, 3)$. [Option ID = 8557]
4. $(2, 3)$. [Option ID = 8559]

Correct Answer :-

- $[2, 3]$. [Option ID = 8560]

---

26) If $a_n = n^{\sin(\frac{n\pi}{2})}$ then

[Question ID = 2137]

1. $\limsup a_n = +\infty, \liminf a_n = -1$. [Option ID = 8547]
2. $\limsup a_n = +\infty, \liminf a_n = 0$. [Option ID = 8548]

3. $\limsup a_n = +\infty, \liminf a_n = -\infty.$ [Option ID = 8546]
4. $\limsup a_n = 1, \liminf a_n = -1.$ [Option ID = 8545]

Correct Answer :-

- $\limsup a_n = +\infty, \liminf a_n = 0.$ [Option ID = 8548]

---

27)

Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined as $f(x, y) = |x| + |y|$. Then which one of the following statements is true?

[Question ID = 2129]

1. f is continuous at (0, 0) and $f_x(0,0) \neq f_y(0,0)$. [Option ID = 8515]
2. f is continuous at (0, 0) and $f_x(0,0) = f_y(0,0)$. [Option ID = 8514]
3. f is discontinuous at (0, 0) and $f_x(0,0) = f_y(0,0)$. [Option ID = 8516]
4. f is continuous at (0, 0) but f_x and f_y does not exist at (0, 0). [Option ID = 8513]

Correct Answer :-

- f is continuous at (0, 0) but f_x and f_y does not exist at (0, 0). [Option ID = 8513]

---

28)

Let A and B be two subsets of a metric space X. If int$A$ denotes the interior A of then which one of the following statements is not true?

[Question ID = 2146]

1. $A \subseteq B \Rightarrow \text{int}A \subseteq \text{int}B.$ [Option ID = 8584]
2. $\text{int}(A \cup B) = \text{int}A \cup \text{int}B.$ [Option ID = 8581]
3. $\text{int}(A \cap B) = \text{int}A \cap \text{int}B.$ [Option ID = 8583]
4. $\text{int}(A \cup B) \supseteq \text{int}A \cup \text{int}B.$ [Option ID = 8582]

Correct Answer :-

- $\text{int}(A \cup B) = \text{int}A \cup \text{int}B.$ [Option ID = 8581]

---

29) Which one of the following statements is false?

[Question ID = 2123]

1. A subring of a field is a subfield. [Option ID = 8490]
2. A subring of the ring of integers $\mathbb{Z}$, is an ideal of $\mathbb{Z}$. [Option ID = 8489]
3. A commutative ring with unity is a field if it has no proper ideals. [Option ID = 8492]
4. A field has no proper ideals. [Option ID = 8491]

Correct Answer :-

- A subring of a field is a subfield. [Option ID = 8490]

---

30) Let $\sigma = (37125)(43216) \in S_7$, the symmetric group of degree 7. The order of σ is

[Question ID = 2120]

1. 7 [Option ID = 8480]
2. 4 [Option ID = 8478]
3. 5 [Option ID = 8479]
4. 2 [Option ID = 8477]

Correct Answer :-

- 4 [Option ID = 8478]

---

31) Let

$$S = \cap_{n=1}^{\infty} \left[0, \frac{1}{n}\right].$$

Then which one of the following statements is true?

[Question ID = 2143]

1. $\inf S > 0.$ [Option ID = 8571]
2. $\sup S = 1$ and $\inf S = 0.$ [Option ID = 8572]
3. $\sup S > 0.$ [Option ID = 8569]
4. $\sup S = \inf S = 0.$ [Option ID = 8570]

Correct Answer :-

- $\sup S = \inf S = 0.$ [Option ID = 8570]

---

32) The characteristics of the partial differential equation

$$36\frac{\partial^2 z}{\partial x^2} - y^{14}\frac{\partial^2 z}{\partial y^2} - 8x^{12}\frac{\partial z}{\partial x} = 0$$

when it is of hyperbolic type are given by

[Question ID = 2160]

1. $x + \frac{36}{y^6} = c_1, x - \frac{36}{y^6} = c_2.$ [Option ID = 8638]
2. $x + \frac{1}{y^6} = c_1, x - \frac{1}{y^6} = c_2.$ [Option ID = 8637]
3. $x + \frac{1}{y^7} = c_1, x - \frac{1}{y^7} = c_2.$ [Option ID = 8639]
4. $x + \frac{36}{y^7} = c_1, x - \frac{36}{y^7} = c_2.$ [Option ID = 8640]

Correct Answer :-

- $x + \frac{1}{y^6} = c_1, x - \frac{1}{y^6} = c_2.$ [Option ID = 8637]

---

33) A bound for the error for the trapezoidal rule for the definite integral $\int_0^1 \frac{1}{1+x}dx$ is

[Question ID = 2150]

1. $\frac{1}{6}$. [Option ID = 8600]
2. $\frac{2}{25}$. [Option ID = 8597]
3. $\frac{1}{15}$. [Option ID = 8598]
4. $\frac{1}{20}$. [Option ID = 8599]

Correct Answer :-

- $\frac{1}{6}$. [Option ID = 8600]

34) Exact value of the definite integral $\int_a^b f(x)dx$ using Simpson's rule

[Question ID = 2152]

1. cannot be given for any polynomial. [Option ID = 8608]
2. is given when $f(x)$ is a polynomial of degree 4. [Option ID = 8605]
3. is given when $f(x)$ is a polynomial of degree 5. [Option ID = 8607]
4. is given when $f(x)$ is a polynomial of degree 3. [Option ID = 8606]

Correct Answer :-

- is given when $f(x)$ is a polynomial of degree 3. [Option ID = 8606]

35) Let p be a prime and let G be a non-abelian p-group. The least value of m such that $p^m \backslash o\left(\frac{G}{Z(G)}\right)$ is

[Question ID = 2121]

1. 0 [Option ID = 8481]
2. 1 [Option ID = 8482]
3. 3 [Option ID = 8484]
4. 2 [Option ID = 8483]

Correct Answer :-

- 2 [Option ID = 8483]

36) If φ is Euler's Phi function then the value of $\varphi(720)$ is

[Question ID = 2114]

1. 248 [Option ID = 8456]
2. 144 [Option ID = 8453]
3. 192 [Option ID = 8454]
4. 72 [Option ID = 8455]

Correct Answer :-

- 192 [Option ID = 8454]

37)

The total number of arithmetic operations required to find the solution of a system of n linear equations in n unknowns by Gauss elimination method is

[Question ID = 2153]

1. $\frac{2}{3}n^3 + \frac{1}{2}n^2 - \frac{5}{6}n.$ [Option ID = 8609]
2. $n^3 - \frac{1}{6}n.$ [Option ID = 8610]
3. $\frac{2}{3}n^3 + \frac{3}{2}n^2 - \frac{7}{6}n.$ [Option ID = 8611]
4. $\frac{1}{3}n^3 + \frac{1}{2}n^2 - \frac{5}{6}n.$ [Option ID = 8612]

Correct Answer :-

- $\frac{2}{3}n^3 + \frac{3}{2}n^2 - \frac{7}{6}n.$ [Option ID = 8611]

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38) If $\langle x_n \rangle$ is a sequence defined as

$$x_n = \left[\frac{5+n}{2n}\right], \text{ for every } n \in \mathbb{N}$$

where [.] denotes the greatest integer function then $\lim_{n\to\infty} x_n$

[Question ID = 2142]

1. $1.$ [Option ID = 8568]
2. $\frac{1}{2}.$ [Option ID = 8566]
3. does not exist. [Option ID = 8565]
4. $0.$ [Option ID = 8567]

Correct Answer :-

- $0.$ [Option ID = 8567]

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39)

Let R be a ring with characteristic n where $n \geq 2$. If M is the ring of 2×2 matrices over R then the characteristic of M is

[Question ID = 2125]

1. $1.$ [Option ID = 8500]
2. $0.$ [Option ID = 8498]
3. $n - 1.$ [Option ID = 8499]
4. $n.$ [Option ID = 8497]

Correct Answer :-

- $n.$ [Option ID = 8497]

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40)

If $A = \begin{bmatrix} a & 2 \\ 1 & b \end{bmatrix}$ is a matrix with eigen values $\sqrt{6}$ and $-\sqrt{6}$, then the values of a and b are respectively,

[Question ID = 2116]

1. 2 and –1. [Option ID = 8463]
2. 2 and –2. [Option ID = 8464]
3. 2 and 1. [Option ID = 8461]
4. –2 and 1. [Option ID = 8462]

Correct Answer :-

- 2 and –2. [Option ID = 8464]

41) The dimension of the vector space of all 6×6 real skew-symmetric matrices is

[Question ID = 2126]

1. 36 [Option ID = 8504]
2. 21 [Option ID = 8502]
3. 30 [Option ID = 8503]
4. 15 [Option ID = 8501]

Correct Answer :-

- 21 [Option ID = 8502]

42)

Let $(x_0, f(x_0)) = (0, -1), (x_1, f(x_1)) = (1, a)$ and $(x_2, f(x_2)) = (2, b)$. If the first order divided differences $f[x_0, x_1] = 5$ and $f[x_1, x_2] = c$ and the second order divided difference $f[x_0, x_1, x_2] = -\frac{3}{2}$, then the values of a, b and c are

[Question ID = 2148]

1. 4, 2, 4. [Option ID = 8592]
2. 2, 4, 6. [Option ID = 8590]
3. 4, 6, 2. [Option ID = 8589]
4. 6, 2, 4. [Option ID = 8591]

Correct Answer :-

- 4, 6, 2. [Option ID = 8589]

43)

Let the polynomial $f(x) = 3x^5 + 15x^4 - 20x^3 + 10x + 20 \in \mathbb{Z}[x]$, and $f_0(x)$ be the polynomial in $\mathbb{Z}_3[x]$ obtained by reducing the coefficients of $f(x)$ modulo 3. Which one of the following statements is true?

[Question ID = 2124]

1. $f(x)$ is reducible over $\mathbb{Q}$, $f_0(x)$ is reducible over $\mathbb{Z}_3$. [Option ID = 8496]
2. $f(x)$ is irreducible over $\mathbb{Q}$, $f_0(x)$ is reducible over $\mathbb{Z}_3$. [Option ID = 8495]
3. $f(x)$ is reducible over $\mathbb{Q}$, $f_0(x)$ is irreducible over $\mathbb{Z}_3$. [Option ID = 8494]
4. $f(x)$ is irreducible over $\mathbb{Q}$, $f_0(x)$ is irreducible over $\mathbb{Z}_3$. [Option ID = 8493]

Correct Answer :-

- $f(x)$ is irreducible over $\mathbb{Q}$, $f_0(x)$ is reducible over $\mathbb{Z}_3$. [Option ID = 8495]

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44) The general solution of the system of the differential equations

$$x_1' = 3x_1 - 2x_2$$
$$x_2' = 2x_1 - 2x_2$$

is given by

[Question ID = 2158]

1. $\begin{pmatrix} c_1e^{-t} + 2c_2e^{2t} \\ 2c_1e^{-t} + c_2e^{2t} \end{pmatrix}$. [Option ID = 8632]
2. $\begin{pmatrix} c_1e^{t} + 2c_2e^{-2t} \\ 2c_1e^{t} + 2c_2e^{-2t} \end{pmatrix}$. [Option ID = 8631]
3. $\begin{pmatrix} c_1e^{t} + 2c_2e^{-2t} \\ c_1e^{t} + c_2e^{-2t} \end{pmatrix}$. [Option ID = 8629]
4. $\begin{pmatrix} c_1e^{-t} + c_2e^{2t} \\ c_1e^{-t} - c_2e^{2t} \end{pmatrix}$. [Option ID = 8630]

Correct Answer :-

- $\begin{pmatrix} c_1e^{-t} + 2c_2e^{2t} \\ 2c_1e^{-t} + c_2e^{2t} \end{pmatrix}$. [Option ID = 8632]

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45) The eigenvalues for the Sturm–Liouville problem

$$y'' + \lambda y = 0, 0 \le x \le \pi,$$
$$y(0) = 0, y'(\pi) = 0$$

are [Question ID = 2155]

1. $\lambda_n = n^2\pi^2, n = 1, 2, \ldots ..$ [Option ID = 8619]
2. $\lambda_n = n^2, n = 1, 2, \ldots ..$ [Option ID = 8618]
3. $\lambda_n = n\pi, n = 1, 2, \ldots ..$ [Option ID = 8617]
4. $\lambda_n = \frac{(2n-1)^2}{4}, n = 1, 2, \ldots ..$ [Option ID = 8620]

Correct Answer :-

- $\lambda_n = \frac{(2n-1)^2}{4}, n = 1, 2, \ldots ..$ [Option ID = 8620]

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46)

The initial value problem

$$x\frac{dy}{dx} - 2y = 0,$$
$$x > 0, y(0) = 0$$

has

[Question ID = 2157]

1. exactly two solutions [Option ID = 8626]
2. a unique solution. [Option ID = 8627]
3. no solution. [Option ID = 8628]
4. infinitely many solutions. [Option ID = 8625]

Correct Answer :-

- infinitely many solutions. [Option ID = 8625]

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47) The partial differential equation

$$(x^2 - 1)\frac{\partial^2 z}{\partial x^2} + 2y\frac{\partial^2 z}{\partial x \partial y} - \frac{\partial^2 z}{\partial y^2} = 0$$

is

[Question ID = 2162]

1. hyperbolic for $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 < 1\}$. [Option ID = 8645]
2. parabolic for $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 < 1\}$. [Option ID = 8646]
3. hyperbolic for $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 > 1\}$. [Option ID = 8648]
4. elliptic for $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 > 1\}$. [Option ID = 8647]

Correct Answer :-

- hyperbolic for $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 > 1\}$. [Option ID = 8648]

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48)

Let f be a convex function with $f(0) = 0$. Then the function g defined on $(0, +\infty)$ as $g(x) = \frac{f(x)}{x}$

[Question ID = 2132]

1. is an increasing function. [Option ID = 8525]
2. is such that its monotonicity cannot be determined. [Option ID = 8528]
3. is neither increasing nor decreasing function. [Option ID = 8527]
4. is a decreasing function. [Option ID = 8526]

Correct Answer :-

- is an increasing function. [Option ID = 8525]

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49) Which one of the statements is false? [Question ID = 2117]

1. Every quotient group of a cyclic group is cyclic. [Option ID = 8465]
2. If G and H are groups and $f: G \to H$ is a homomorphism then f induces an isomorphism of $\frac{G}{\text{Ker}(f)}$ with H. [Option ID = 8467]
3. Every quotient group of an abelian group is abelian. [Option ID = 8468]

4. If G is a group and $Z(G)$ is its centre such that the quotient group of G by $Z(G)$ is cyclic, then G is abelian. [Option ID = 8466]

Correct Answer :-

- If G and H are groups and $f : G \to H$ is a homomorphism then f induces an isomorphism of $\frac{G}{\text{Ker}(f)}$ with H. [Option ID = 8467]

50) For cubic spline interpolation which one of the following statements is true? [Question ID = 2149]

1. The second derivatives of the splines are continuous at the interior data points but not the first derivatives. [Option ID = 8594]
2. The third derivatives of the splines are continuous at the interior data points. [Option ID = 8596]
3. The first derivatives of the splines are continuous at the interior data points but not the second derivatives. [Option ID = 8593]
4. The first and the second derivatives of the splines are continuous at the interior data points. [Option ID = 8595]

Correct Answer :-

- The first and the second derivatives of the splines are continuous at the interior data points. [Option ID = 8595]

SEAL

M A / M Sc

11029

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SEAL

SEAL

Note : All symbols carry their usual, unless specified otherwise.

1. The sequence $(n^{1/n})$ is

(A) monotonically decreasing

(B) monotonically increasing

(C) convergent and converges to zero

(D) neither monotonically increasing nor monotonically decreasing

2. Let

$$S = \prod_{n=1}^{\infty} \left[-\frac{1}{n}, 1+\frac{1}{n}\right].$$

then S equals

(A) $[0, 1]$

(B) $(0, 1]$

(C) $(0, 1)$

(D) $[0, 1)$

3. Consider the series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}\left(\sqrt{n+1}-\sqrt{n-1}\right).$$

Then

(A) the series is convergent but not absolutely convergent

(B) the series is divergent

(C) the nth term of seires does not converge to zero

(D) the series is absolutely convergent

4. Consider the sets

$$S = \left\{\frac{1}{n} : n \in \mathbb{N} \text{ and } n \text{ is prime}\right\}$$

$$T = \left\{x^2 : x \in \mathbb{R}\right\}.$$

Then

(A) $\sup (S \cap T) = 1$

(B) $\sup S = 1$ and $\inf T = 0$

(C) $\sup S = \frac{1}{2}$ and $\inf T = 0$

(D) $\inf (S \cup T) = \frac{1}{2}$

5. Consider the following functions from $\mathbb{R} \times \mathbb{R} \to \mathbb{R}$ defined by

$$d_1(x, y) = |x| + |y|,$$

$$d_2(x, y) = \begin{cases} 2, & x \neq y \\ 0, & x = 0 \end{cases}$$

$$d_3(x, y) = \sqrt{|x - y|}.$$

Which of the following statements is true ?

(A) Only d_2 and d_3 are metrics on $\mathbb{R}$

(B) Only d_3 is a metric on $\mathbb{R}$

(C) Only d_1 and d_2 are metrics on $\mathbb{R}$

(D) All are metrics on $\mathbb{R}$

6. $S = \{(x, y) \in \mathbb{R}^2 : xy < 0\}$ is

(A) neither connected nor compact subset of $\mathbb{R}^2$

(B) not connected but is compact subset of $\mathbb{R}^2$

(C) is both connected and compact subset of $\mathbb{R}^2$

(D) is not compact subset of $\mathbb{R}^2$ but connected

7. Let (x_n) be a sequence defined by :

$$x_1 = 3 \text{ and } x_{n+1} = \frac{1}{4 - x_n}.$$

Then

(A) (x_n) is a monotonically decreasing sequence that is not bounded below

(B) (x_n) converges to $2+\sqrt{3}$

(C) (x_n) converges to $2-\sqrt{3}$

(D) (x_n) diverges

8. The value of the series

$$\sum_{n=1}^{\infty} \frac{n}{2^n}$$

is given by

(A) 2

(B) 4

(C) 6

(D) 8

9. Let f be a continuous function on $\mathbb{R}$. Define

$$G(x) = \int_0^{\sin x} f(t)\,dt \quad \forall\, x \in \mathbb{R}.$$

Then

(A) $G'(x) = f(\cos x)\sin x$

(B) $G'(x) = -f(\sin x)\cos x$

(C) $G'(x) = f(\sin x)\cos x$

(D) $G'(x) = f(\sin x)\sin x$

10. Let (X, d) be a metric space where X is an infinite set and d is the discrete metric. Then

(A) Heine-Borel theorem holds for (X, d)

(B) Heine-Borel theorem does not hold for (X, d)

(C) X is not bounded

(D) X is compact

11. Let

$$f_n(x) = \frac{1}{1+(nx-1)^2},\ x \in [0, 1].$$

Then the sequence (f_n) is

(A) pointwise convergent but not uniformly convergent on [0, 1]

(B) uniformly convergent but not pointwise convergent on [0, 1]

(C) both pointwise and uniformly convergent on [0, 1]

(D) neither pointwise nor uniformly convergent on [0, 1]

12. The limit inferior of the sequence (x_n) where

$$x_n = 1 + (-1)^n + \frac{1}{3^n}$$

is

(A) 1

(B) 3

(C) 2

(D) 0

13. Which of the following sets is in one-to-one correspondence with $\mathbb{N}$

(I) $\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots\ldots\right\}$

(II) $\{\ldots\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\ldots\}$

(III) $\left\{\frac{p}{q} : p, q \in \mathbb{Z}, q \neq 0\right\}$

(IV) $\left\{\frac{p}{q} : p, q \in \mathbb{N}\right\}$

(A) (I) and (II)

(B) (I), (II) and (III)

(C) (I) and (IV)

(D) All of the above

14. Suppose f and g are differentiable on the interval $[a, \infty)$ such that $f(a) \leq g(a)$ and $f'(x) < g'(x)\ \forall\ x > a$. Then which of the following statements is true ?

(A) $f(x) = g(x)\ \ \forall\ x \in [a, \infty)$

(B) $f(x) > g(x)$

(C) $f(x) < g(x)$

(D) None of the above

15. Which of the following statements are true ?

(I) There exists a continuous function from $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ onto $(0, 1)$

(II) There exists a continuous function from $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ onto $\mathbb{R}$

(III) There exists a continuous function from $[0, \pi] \cup [2\pi, 3\pi]$ onto $[0, 1]$

(IV) There exists a continuous function from $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ onto $\left[0, \frac{1}{3}\right] \cup \left[\frac{2}{3}, 1\right]$

(A) (I) and (II)

(B) (II) and (III)

(C) (III) and (IV)

(D) (I) and (IV)

16. For

$$x = (x_1, x_2, x_3),\ y = (y_1, y_2, y_3) \in \mathbb{R}^3$$

define

$$d_1(x, y) = \max_{1 \le j \le 3} |x_j - y_j|$$

$$d_2(x, y) = \left[\sum_{j=1}^{3} (x_j - y_j)^2\right]^{1/2}.$$

Consider the metric spaces $(\mathbb{R}^3, d_1)$ and $(\mathbb{R}^3, d_2)$. Then

(A) $(\mathbb{R}^3, d_1)$ is complete, but $(\mathbb{R}^3, d_2)$ is not complete

(B) $(\mathbb{R}^3, d_2)$ is complete, but $(\mathbb{R}^3, d_1)$ is not complete

(C) Both $(\mathbb{R}^3, d_1)$ and $(\mathbb{R}^3, d_2)$ are complete

(D) Neither $(\mathbb{R}^3, d_1)$ nor $(\mathbb{R}^3, d_2)$ is complete

17. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x, y) = \begin{cases} \dfrac{x^2 y}{x^4 + y^2}, & (x, y) \ne (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

Then

(A) f is not continuous at (0, 0) but all directional derivatives of f at (0, 0) exist.

(B) f is continuous in $\mathbb{R}^2$ and all directional derivatives of f at (0, 0) exist.

(C) f is continuous in $\mathbb{R}^2$ but not all directional derivatives at (0, 0) exist.

(D) f is not continuous at (0, 0) and not all directional derivatives at (0, 0) exist.

18. Let

$$X = \{(x, y) \in \mathbb{R}^2 : x \in \mathbb{Q}, y \in \mathbb{R} \setminus \mathbb{Q}\}$$

where $\mathbb{Q}$ is the set of rationals. Then

(A) X is an open and dense subset of $\mathbb{R}^2$

(B) X is an open but not dense subset of $\mathbb{R}^2$

(C) X is not an open but a dense subset of $\mathbb{R}^2$

(D) X is neither an open nor a dense subset of $\mathbb{R}^2$

19. Let $n \in \mathbb{N}, n \geq 3$ be fixed and let $f : [0, 1] \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} x, & 0 \leq x \leq 1/n \\ x - \dfrac{(2k-1)}{2n}, & \dfrac{k-1}{n} < x \leq \dfrac{k}{n} \\ & k = 2, 3, \ldots\ldots, n. \end{cases}$$

Then

(A) f is continuous and Riemann integrable on [0, 1].

(B) f is not continuous but is Riemann integrable on [0, 1].

(C) f is continuous but not Riemann integrable on [0, 1].

(D) f is neither continuous nor Riemann integrable on [0, 1].

20. Let

$$S = \left\{x \in \mathbb{R} : 3 - x^2 > 0\right\}.$$

Then

(A) S is bounded above and 3 is the least upper bound of S.

(B) S is bounded above and does not have a least upper bound in $\mathbb{R}$.

(C) S is bounded above and does not have a least upper bound in $\mathbb{Q}$, the set of rational numbers.

(D) S is not bounded above.

21. Let p and q be distinct primes and let G and H be two groups such that $o(G) = p$ and $o(H) = q$. The number of distinct homomorphims from G to H is/are

(A) 1

(B) $p - 1$

(C) $q - 1$

(D) pq

22. Let G be a cyclic group such that G has an element of infinite order. Then the number of elements of finite order in G is/are

(A) 0

(B) 1

(C) infinite

(D) none of these

23. Let G be a non-abelian group of order p^3 where p is a prime. Let $Z(G) \neq \{e\}$. Then

(A) $o(Z(G)) = p$

(B) $o(Z(G)) = p^2$

(C) $\frac{G}{Z(G)}$ is cyclic

(D) none of the above

24. Let G be a group of order pqr, where p, q, r are primes and $p < q < r$. Which of the following statements are true ?

(*i*) G has a normal subgroup of order qr

(*ii*) Sylow r-subgroup of G is normal

(*iii*) G is abelian

(A) only (*i*) and (*ii*)

(B) only (*ii*) and (*iii*)

(C) only (*i*) and (*iii*)

(D) (*i*), (*ii*) and (*iii*)

25. Let R be a ring with unity such that each element of R is an idempotent. Then the characteristic of R is

(A) 0

(B) 2

(C) an odd prime

(D) none of the above

26. Let

$$F = \mathbb{Q}\left(\sqrt{2i}\right).$$

Which one of the following is *not* true ?

(A) $\sqrt{2} \in F$

(B) $i \in F$

(C) $x^8 - 16 = 0$ has a solution in F

(D) $\dim_{\mathbb{Q}}(F) = 2$

27. The ideal $\langle x \rangle$ of the ring $\mathbb{Z}[x]$ is

(A) maximal but not prime

(B) prime but not maximal

(C) both prime and maximal

(D) neither prime nor maximal

28. The smallest subring of $\mathbb{Q}$ containing $\frac{2}{3}$ is

(A) $S = \left\{a + b\frac{2}{3} \mid a, b \in \mathbb{Z}\right\}$

(B) $S = \mathbb{Q}$

(C) $S = \left\{a\left(\frac{2}{3}\right)^k \mid k \in \mathbb{N},\ a \in \mathbb{Z}\right\}$

(D) $S = \left\{a_0 + a_1\frac{2}{3} + a_2\left(\frac{2}{3}\right)^2 + \ldots\ldots\ldots + a_n\left(\frac{2}{3}\right)^n \mid n \in \mathbb{N},\ a_0, a_1 \ldots\ldots, a_n \in \mathbb{Z}\right\}$

29. If p is an odd prime, then

$$\phi(p) + \phi(2p) + \phi(2^2 p) + \ldots\ldots\ldots + \phi(2^m p)$$

is equal to

(A) $(2^m - 1)(p-1)$

(B) $2^m (p-1)$

(C) $(2^m + 1)(p-1)$

(D) $2^{m+1}(p-1)$

30. Let

$$A(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}, \ \theta \in (0, 2\pi).$$

Which of the following statements is true ?

(A) $A(\theta)$ has eigenvectors in $\mathbb{R}^2$ for every $\theta \in (0, 2\pi)$

(B) $A(\theta)$ does not have eigenvectors in $\mathbb{R}^2$ for any $\theta \in (0, 2\pi)$

(C) $A(\theta)$ has eigenvectors in $\mathbb{R}^2$ for exactly one value of $\theta \in (0, 2\pi)$

(D) $A(\theta)$ has eigenvectors in $\mathbb{R}^2$ for exactly two values of $\theta \in (0, 2\pi)$

31. Let $M(n, \mathbb{R})$ be the vector space of $n \times n$ matrices with real entries and U be the subset of $M(n, \mathbb{R})$ given by

$$\{(a_{ij}) \mid a_{11} + a_{22} + \ldots.. + a_{nn} = 0\}.$$

Which one of the following statements is true ?

(A) U is a subspace of dimension $n^2 - 1$

(B) U is a subspace of dimension $n^2 - n$

(C) U is not a subspace

(D) None of the above

32. Let

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Then det $(A^3 - 6A^2 + 5A + 3I)$ is

(A) 24 (B) 15

(C) 3 (D) 0

33. Let

$$V = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \middle| \, a, b, c, d \in \mathbb{R} \right\}$$

and $W = \left\{ a + bx + cx^2 \,\middle|\, a, b, c \in \mathbb{R} \right\}$.

Define $T : V \to W$ by

$$T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix} \right) = (a+b) + (b-c)\, x + (c+d)\, x^2$$

The null space of T is

(A) $\left\{ a \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \middle| \, a \in \mathbb{R} \right\}$

(B) $\left\{ a \begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix} \middle| \, a \in \mathbb{R} \right\}$

(C) $\left\{ a \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \middle| \, a \in \mathbb{R} \right\}$

(D) $\left\{ a \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} \middle| \, a \in \mathbb{R} \right\}$

34. Let

$$W_1 = \{(a, 2a, 0) \mid a \in \mathbb{R}\},$$

$$W_2 = \{(a, 0, -a) \mid a \in \mathbb{R}\}.$$

Then

(A) $W_1 + W_2$ is a subspace of $\mathbb{R}^3$ but $W_1 \cup W_2$ is not

(B) $W_1 + W_2$, $W_1 \cup W_2$ are both subspaces of $\mathbb{R}^3$

(C) neither $W_1 + W_2$ nor $W_1 \cup W_2$ is a subspace of $\mathbb{R}^3$

(D) $W_1 \cup W_2$ is a subspace of $\mathbb{R}^3$ but $W_1 + W_2$ is not

35. Let $V = C[0, \pi]$ be an inner product space with inner product

$$\langle f, g \rangle = \int_0^{\pi} f(x)\, g(x)\, dx.$$

Let $f(x) = \cos x$, $g(x) = \sin x$. Then

(A) f, g are orthogonal but not linearly independent

(B) f, g are orthogonal and linearly independent

(C) f, g are linearly independent but not orthogonal

(D) neither f, g are linearly independent nor orthogonal

36. If the partial differential equation

$$(x-2)^2 \frac{\partial^2 u}{\partial x^2} - (y-3)^2 \frac{\partial^2 u}{\partial y^2} + 2x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = u$$

is parabolic in the region $S \subseteq \mathbb{R}^2$ but not in $\mathbb{R}^2 \setminus S$, then S is

(A) $\{(x, y) \in \mathbb{R}^2 : x = 2 \text{ or } y = 3\}$

(B) $\{(x, y) \in \mathbb{R}^2 : x = 2 \text{ and } y = 3\}$

(C) $\{(x, y) \in \mathbb{R}^2 : x = 2\}$

(D) $\{(x, y) \in \mathbb{R}^2 : y = 3\}$

37. Let $u(x, y)$ be the solution of the Cauchy problem

$$x^2 \frac{\partial u}{\partial x} - y^2 \frac{\partial u}{\partial y} = 0,$$

$$u \to e^x \text{ as } y \to \infty.$$

Then $u(1, 1)$

(A) -1 (B) 0

(C) 1 (D) e^{-2}

38. The initial value problem

$$x\frac{dy}{dx} = 2y,$$

$$y(a) = b$$

has

(A) infinitely many solutions through $(0, b)$ if $b \neq 0$

(B) unique solution for all a and b

(C) no solution if $a = b = 0$

(D) infinitely many solutions if $a = b = 0$

39. The solution of the differential equation

$$\frac{d^2y}{dx^2} + 4y = \cos 2x,$$

is given by

(A) $c_1 \cos 2x + c_2 \sin 2x + \frac{x}{4} \sin 2x$

(B) $c_1 \cos 2x + c_2 \sin 2x + \frac{x}{2} \sin 2x$

(C) $c_1 \cos 2x + c_2 \sin 2x + \frac{x}{4} \cos 2x$

(D) $c_1 \cos 2x + c_2 \sin 2x + x \cos 2x$

40. The following initial value problem of a first order linear system

$$x' = 3x - 2y, \ \ x(0) = 1$$

$$y' = -3x + 4y, \ \ y(0) = -2$$

can be converted into an initial value problem of a 2nd order differential equation for $x(t)$. It is

(A) $x'' - 7x' + 6x = 0; \ x(0) = 1, \ x'(0) = -2$

(B) $x'' - 7x' + 6x = 0; \ x(0) = 1, \ x'(0) = 0$

(C) $x'' - 7x' + 6x = 0; \ x(0) = 1, \ x'(0) = 7$

(D) $x'' - x' + 6x = 0; \ \ x(0) = 1, \ x'(0) = -2$

41. The characteristic values of the Sturm-Liouville problem

$$\frac{d^2y}{dx^2} + \lambda y = 0;\ y(0) = 0,\ y(\pi) - y'(\pi) = 0,$$

are

(A) $\lambda = \alpha_n^2$ where $\alpha_n (n = 1, 2, 3, ...)$ are the positive roots of equation $\alpha = \cot \pi\alpha$

(B) $\lambda = \alpha_n^2$ where $\alpha_n (n = 1, 2, 3, ...)$ are roots of the equation $\alpha = \tan \pi\alpha$

(C) 0, 1

(D) negative real numbers

42. Determine an interval in which the solution of the following initial value problem is certain to exist

$$y' + (\tan t)y = \sin t,\ y(\pi) = 0.$$

(A) $\frac{\pi}{2} < t < \frac{3\pi}{2}$

(B) $0 < t < \frac{3\pi}{2}$

(C) $\frac{\pi}{2} < t < 6$

(D) $0 < t < 3\pi$

43. The derivative $\frac{du}{dx}$ can be approximated most accurately by which finite difference ?

(A) $\frac{v_{k+1}^n - v_k^n}{\Delta x}$

(B) $\frac{v_k^n - v_{k-1}^n}{\Delta x}$

(C) $\frac{v_{k+1}^n - v_{k-1}^n}{2\Delta x}$

(D) All are equally accurate

44. What are the solutions α if any, of the equation $x = \sqrt{1+x}$? Does the iteration $x_{n+1} = \sqrt{1+x_n}$ converge to any of these solutions ?

(A) Root $= \frac{1+\sqrt{5}}{2}$, iterations converge with $x_0 = 1$

(B) Root $= \frac{1-\sqrt{5}}{2}$, iterations converge with $x_0 = -1$

(C) Both (A) and (B)

(D) Roots $= \frac{1\pm\sqrt{5}}{2}$ but the iterations do not converge to any root

45. Is the following function a cubic spline on the interval $0 \le x \le 2$

$$s(x) = \begin{cases} (x-1)^3 & , \quad 0 \le x \le 1 \\ 2(x-1)^3 & , \quad 1 \le x \le 2 \end{cases}$$

(A) Yes, it is a cubic spline on [0, 2]

(B) It is a cubic spline only on [0, 1]

(C) It is a cubic spline only on [1, 2]

(D) It is not a cubic spline

46. Consider the second order differential equation

$$x^2 y''(x) + x\,y'(x) - 9y(x) = 0 \text{ for } x > 0.$$

If the solution satisfies the intial conditions $y(1) = 0$, $y'(1) = 2$, then $y(2)$ is

(A) $\frac{21}{8}$ (B) $\frac{63}{8}$

(C) $\frac{7}{16}$ (D) $\frac{63}{4}$

47. The eigenvalues associated with the BVP

$$y''(x) - 2y'(x) + (1-\lambda)\,y(x) = 0$$

$$y(0) = 0, \quad y(1) = 0$$

is/are

(A) $\lambda = 0$

(B) $\lambda = \pi^2 n^2,\ n = 1, 2, 3, \ldots\ldots$

(C) $\lambda = -\pi^2 n^2,\ n = 1, 2, 3, \ldots\ldots$

(D) $\lambda = \pi n,\ n = 1, 2, 3, \ldots\ldots$

48. The value of

$$I = \int_0^{\sqrt{\pi}} \sin x^2 \, dx$$

using the trapezium rule with two subintervals is

(A) $\frac{\pi}{4}$ (B) $\frac{\sqrt{\pi}}{4}$

(C) $\frac{\sqrt{\pi}}{2}$ (D) $\frac{\sqrt{2\pi}}{4}$

49. Consider the system of equations

$$\begin{bmatrix} 1 & -a \\ -a & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix},$$

where 'a' is a real constant. Then Gauss-Seidel method for the solution of the above system converges for

(A) all values of a

(B) $|a| < 1$

(C) $|a| > 1$

(D) $a > 2$

50. The error in the value of y at 0.2 when modified Euler's method is used to solve the problem

$$\frac{dy}{dx} = x - y, \quad y(0) = 1, \quad h = 0.2$$

is of the order

(A) 10^{-1}

(B) 10^{-2}

(C) 10^{-3}

(D) 0

DEPARTMENT OF MATHEMATICS
Answer key for M.A./M.Sc. Mathematics Entrance Test (SERIES– A)

Question No.	Answer	Question No.	Answer
1.	D	26.	A
2. *	----	27.	B
3.	D	28.	D
4.	C	29.	B
5. *	----	30.	C
6.	A	31.	A
7.	C	32.	C
8.	A	33.	A
9.	C	34.	A
10.	B	35.	B
11.	A	36.	A
12.	D	37. *	----
13.	D	38.	D
14. *	----	39.	A
15. *	----	40.	C
16.	C	41.	B
17.	A	42.	A
18.	C	43.	C
19.	B	44.	A
20.	C	45.	A
21.	A	46.	A
22.	B	47.	C
23.	A	48.	D
24.	A	49.	B
25.	B	50.	C

* This question shall not be considered for evaluation.

M.A./M.Sc. (Mathematics) Entrance Examination 2016-17

Max Time: 2 hours Max Marks: 150

Instructions: There are 50 questions. Every question has four choices of which exactly one is correct. *For correct answer, 3 marks will be given. For wrong answer, 1 mark will be deducted.* Scientific calculators are allowed.

In the following $\mathbb{R}$, $\mathbb{N}$, $\mathbb{Q}$, $\mathbb{Z}$ and $\mathbb{C}$ denote the set of all real numbers, natural numbers, rational numbers, integers and complex numbers respectively.

(1) Let X be a countably infinite subset of $\mathbb{R}$ and A be a countably infinite subset of X. Then the set $X \setminus A = \{x \in X \mid x \notin A\}$

A) is empty. B) is a finite set .
C) can be a countably infinite set. D) can be an uncountable set.

(2) The subset $A = \{x \in \mathbb{Q} : x^2 < 4\}$ of $\mathbb{R}$ is

A) bounded above but not bounded below.
B) bounded above and $\sup A = 2$.
C) bounded above but does not have a supremum .
D) not bounded above .

(3) Let f be a function defined on $[0, \infty)$ by $f(x) = [x]$, the greatest integer less than or equal to x. Then

A) f is continuous at each point of $\mathbb{N}$.
B) f is continuous on $[0, \infty)$.
C) f is discontinuous at $x = 1, 2, 3, \ldots$.
D) f is continuous on $[0, 7]$.

(4) The series $x + \dfrac{2^2x^2}{2!} + \dfrac{3^3x^3}{3!} + \cdots$ is convergent if x belongs to the interval

A) $(0, 1/e)$. B) $(1/e, \infty)$. C) $(2/e, 3/e)$. D) $(3/e, 4/e)$.

(5) The subset $A = \{x \in \mathbb{Q} : -1 < x < 0\} \cup \mathbb{N}$ of $\mathbb{R}$ is

A) bounded, infinite set and has a limit point in $\mathbb{R}$.
B) unbounded, infinite set and has a limit point in $\mathbb{R}$.
C) unbounded, infinite set and does not have a limit point in $\mathbb{R}$.
D) bounded, infinite set and does not have a limit point in $\mathbb{R}$.

(6) Let f be a real-valued monotone non-decreasing function on $\mathbb{R}$. Then

A) for $a \in \mathbb{R}$, $\lim_{x \to a} f(x)$ exists .
B) f is an unbounded function.

C) $h(x) = e^{-f(x)}$ is a bounded function.
D) if $a < b$, then $\lim_{x \to a^+} f(x) \leq \lim_{x \to b^-} f(x)$.

(7) Let $X = C[0,1]$ be the space of all real-valued continuous functions on $[0,1]$. Then (X, d) is not a complete metric space if

A) $d(f,g) = \int_0^1 |f(x) - g(x)|\, dx$. B) $d(f,g) = \max_{x \in [0,1]} |f(x) - g(x)|$.

C) $d(f,g) = \sup_{x \in [0,1]} |f(x) - g(x)|$. D) $d(f,g) = \begin{cases} 0, & \text{if } f = g \\ 1, & \text{if } f \neq g \end{cases}$.

(8) The series $\sum_{k=0}^{\infty} \frac{k^2 + 3k + 1}{(k+2)!}$ converges to

A) 1. B) 1/2. C) 2. D) 3.

(9) We know that $xe^x = \sum_{n=1}^{\infty} \frac{x^n}{(n-1)!}$. The series $\sum_{n=1}^{\infty} \frac{2^n n^2}{n!}$ converges to

A) e^2. B) $2\,e^2$. C) $4\,e^2$. D) $6\,e^2$.

(10) Let $X = \mathbb{R}^2$ with metric defined by $d(x,y) = 1$ if $x \neq y$ and $d(x,x) = 0$. Then

A) every subset of X is dense in (X, d).
B) (X, d) is separable .
C) (X, d) is compact but not connected.
D) every subspace of (X, d) is complete.

(11) Let d_1 and d_2 be metrics on a non-empty set X. Which of the following is not a metric on X?

A) $\max(d_1, d_2)$. B) $\sqrt{d_1^2 + d_2^2}$. C) $1 + d_1 + d_2$. D) $\frac{1}{4}d_1 + \frac{3}{4}d_2$.

(12) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be given by $f(x,y) = \sqrt{|xy|}$. Then at origin

A) f is continuous and $\frac{\partial f}{\partial x}$ exists .
B) f is discontinuous and $\frac{\partial f}{\partial x}$ exists.
C) f is continuous but $\frac{\partial f}{\partial x}$ does not exist .
D) f is discontinuous but $\frac{\partial f}{\partial x}$ exists.

(13) The sequence of real-valued functions $f_n(x) = x^n$, $x \in [0,1] \cup \{2\}$, is

A) pointwise convergent but not uniformly convergent.
B) uniformly convergent.

C) bounded but not pointwise convergent.
D) not bounded.

(14) The integral $\int_0^\infty \sin x \, dx$

A) exists and equals 0.
B) exists and equals 1.
C) exists and equals -1.
D) does not exist.

(15) If $\{a_n\}$ is a bounded sequence of real numbers, then

A) $\inf_n a_n \leq \liminf_{n\to\infty} a_n$ and $\sup_n a_n \leq \limsup_{n\to\infty} a_n$.
B) $\liminf_{n\to\infty} a_n \leq \inf_n a_n$.
C) $\liminf_{n\to\infty} a_n \leq \inf_n a_n$ and $\sup_n a_n \leq \limsup_{n\to\infty} a_n$.
D) $\inf_n a_n \leq \liminf_{n\to\infty} a_n$ and $\limsup_{n\to\infty} a_n \leq \sup_n a_n$.

(16) The series $\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2}$

A) diverges .
B) converges to 1.
C) converges to $\frac{1}{2}$.
D) converges to $\frac{1}{7}$.

(17) Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by

$$f(x,y) = \begin{cases} \dfrac{xy}{x^2+y}, & x^2 \neq -y \\ 0, & x^2 = -y. \end{cases}$$

Then

A) directional derivative does not exist at $(0,0)$.
B) f is continuous at $(0,0)$.
C) f is differentiable at $(0,0)$.
D) each directional derivative exists at $(0,0)$ but f is not continuous.

(18) Let $f : [-1,1] \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} \dfrac{x}{|x|}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

and F be its indefinite integral. Which of the following is not true?

A) $F'(0)$ does not exist.
B) F is an anti-derivative of f on $[-1,1]$.
C) F is Riemann integrable on $[-1,1]$.

D) F is continuous on $[-1, 1]$.

(19) Let $f(x) = x^2$, $x \in [0, 1]$. For each $n \in \mathbb{N}$, let P_n be the partition of $[0, 1]$ given by $P_n = \{0, \frac{1}{n}, \frac{2}{n}, \ldots, \frac{n}{n}\}$. If $\alpha_n = U(f, P_n)$ (upper sum) and $\beta_n = L(f, P_n)$ (lower sum) then

A) $\alpha_n = (n+2)(2n+1)/(6n^2)$.
B) $\beta_n = (n-2)(2n+1)/(6n^2)$.
C) $\beta_n = (n-1)(2n-1)/(6n^2)$.
D) $\lim_{n\to\infty} \alpha_n \neq \lim_{n\to\infty} \beta_n$.

(20) Let $I = \int_0^{\pi/2} \log \sin x \, dx$. Then

A) I diverges at $x = 0$.
B) I converges and is equal to $-\pi \log 2$.
C) I converges and is equal to $-\frac{\pi}{2} \log 2$.
D) I diverges at $x = \frac{\pi}{4}$.

(21) Which of the following polynomials is not irreducible over $\mathbb{Z}$?

A) $x^4 + 125x^2 + 25x + 5$.
B) $2x^3 + 6x + 12$.
C) $x^3 + 2x + 1$.
D) $x^4 + x^3 + x^2 + x + 1$.

(22) A complex number α is said to be algebraic integer if it satisfies a monic polynomial equation with integer coefficients. Which of the following is not an algebraic integer?

A) $\sqrt{2}$.
B) $\frac{1}{\sqrt{2}}$.
C) $\frac{1-\sqrt{5}}{2}$.
D) $\sqrt{\alpha}$, α is an algebraic integer.

(23) If $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & -1 \\ 0 & 0 & -2 \end{bmatrix}$, then the value of $A^4 - A^3 - 4A^2 + 4I$ is

A) $4 \begin{bmatrix} -2 & 1 & 0 \\ 0 & -3 & 1 \\ 0 & 0 & 1 \end{bmatrix}$.
B) $4 \begin{bmatrix} 0 & -1 & -1 \\ 0 & -1 & 1 \\ 0 & 0 & 3 \end{bmatrix}$.
C) $4 \begin{bmatrix} 2 & 1 & 1 \\ 0 & 3 & -1 \\ 0 & 0 & -1 \end{bmatrix}$.
D) $4 \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & -3 \end{bmatrix}$.

(24) Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation defined by $T(x,y) = (x+y, x-y, 2y)$. If $\{(1,1),(1,0)\}$ and $\{(1,1,1),(1,0,1),(0,0,1)\}$ are ordered bases of $\mathbb{R}^2$ and $\mathbb{R}^3$ respectively, then the matrix representation of T with respect to the ordered bases is

A) $\begin{bmatrix} 0 & 1 \\ 2 & 1 \\ 1 & -1 \end{bmatrix}$.
B) $\begin{bmatrix} 1 & 0 \\ -1 & 0 \\ 2 & -1 \end{bmatrix}$.
C) $\begin{bmatrix} 2 & 1 \\ -2 & 0 \\ 0 & -1 \end{bmatrix}$.
D) $\begin{bmatrix} 0 & 1 \\ 2 & 0 \\ 0 & -1 \end{bmatrix}$.

(25) Let P_4 be real vector space of real polynomials of degree ≤ 4. Define an inner product on P_4 by

$$\left\langle \sum_{i=0}^{4} a_i x^i, \sum_{i=0}^{4} b_i x^i \right\rangle = \sum_{i=0}^{4} a_i b_i.$$

Then the set $\{1, x, x^2, x^3, x^4\}$ is

A) orthogonal but not orthonormal .
B) orthonormal .
C) not orthogonal.
D) none of these.

(26) If $\{a+ib, c+id\}$ is a basis of $\mathbb{C}$ over $\mathbb{R}$, then

A) $ac - bd = 0$.
B) $ac - bd \neq 0$.
C) $ad - bc = 0$.
D) $ad - bc \neq 0$.

(27) Consider $M_1 = \begin{pmatrix} 1 & 0 \\ 3 & 2 \end{pmatrix}$, $M_2 = \begin{pmatrix} -1 & 2 \\ 3 & 2 \end{pmatrix}$, $M_3 = \begin{pmatrix} 5 & -6 \\ -3 & -2 \end{pmatrix}$ and $M_4 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ of $M_{2\times 2}(\mathbb{R})$. Then

A) $\{M_2, M_3, M_4\}$ is linearly independent.
B) $\{M_1, M_2, M_4\}$ is linearly independent.
C) $\{M_1, M_3, M_4\}$ is linearly independent.
D) $\{M_1, M_2, M_3\}$ is linearly dependent.

(28) If $A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} = \alpha M_1 + \beta M_2 + \gamma M_3$, where $M_1 = I_{2\times 2}$, $M_2 = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$ and $M_3 = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$, then

A) $\alpha = \beta = 1,\ \gamma = 2$.
B) $\alpha = \beta = -1,\ \gamma = 2$.
C) $\alpha = 1,\ \beta = -1,\ \gamma = 2$.
D) $\alpha = -1,\ \beta = 1,\ \gamma = 2$.

(29) Let W be the subset of the vector space $V = M_{n\times n}(\mathbb{R})$ consisting of symmetric matrices. Then

A) W is not a subspace of V.
B) W is a subspace of V of dimension $\frac{n(n-1)}{2}$.
C) W is a subspace of V of dimension $\frac{n(n+1)}{2}$.
D) W is a subspace of V of dimension $n^2 - n$.

(30) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation and B be a basis of $\mathbb{R}^3$ given by $B = \{(1,1,1)^t, (1,2,3)^t, (1,1,2)^t\}$. If $T\big((1,1,1)^t\big) = (1,1,1)^t$, $T\big((1,2,3)^t\big) = (-1,-2,-3)^t$ and $T\big((1,1,2)^t\big) = (2,2,4)^t$ (A^t being the transpose of A), then $T\big((2,3,6)^t\big)$ is

A) $(2,1,4)^t$.
B) $(1,2,4)^t$.
C) $(3,2,1)^t$.
D) $(2,3,4)^t$.

(31) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation and $B = \{v_1, v_2, v_3\}$ be a basis for $\mathbb{R}^3$. Suppose that $T(v_1) = (1,1,0)^t$, $T(v_2) = (1,0,-1)^t$ and $T(v_3) = (2,1,-1)^t$ then

A) $w = (1,2,1)^t \notin$ Range of T .
B) dim (Range of T) = 1.
C) dim (Null space of T) = 2.
D) Range of T is a plane in $\mathbb{R}^3$.

(32) The last two digits of the number $9^{(9^9)}$ is

A) 29. B) 89. C) 49. D) 69.

(33) Let G be the group of all 2×2 matrices $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ under matrix multiplication, where $ad - bc \neq 0$ and a, b, c, d are integers modulo 3. The order of G is

A) 24. B) 16. C) 48. D) 81.

(34) For the ideal $I = \langle x^2 + 1\rangle$ of $\mathbb{Z}[x]$, which of the following is true?

A) I is a maximal ideal but not a prime ideal.
B) I is a prime ideal but not a maximal ideal.
C) I is neither a prime ideal nor a maximal ideal.
D) I is both prime and maximal ideal.

(35) Consider the following statements:

1. Every Euclidean domain is a principal ideal domain;
2. Every principal ideal domain is a unique factorization domain;
3. Every unique factorization domain is a Euclidean domain.

Then

A) statements 1 and 2 are true.
B) statements 2 and 3 are true.
C) statements 1 and 3 are true.
D) statements 1, 2 and 3 are true.

(36) The ordinary differential equation:

$$\frac{dy}{dx} = \frac{2y}{x}$$

with the initial condition $y(0) = 0$, has

A) infinitely many solutions.
B) no solution.
C) more than one but only finitely many solutions.
D) unique solution.

(37) Consider the partial differential equation:

$$4\frac{\partial^2 u}{\partial x^2} + 12\frac{\partial^2 u}{\partial x \partial y} + 9\frac{\partial^2 u}{\partial y^2} - 9u = 9.$$

Which of the following is not correct?

A) It is a second order parabolic equation.
B) The characteristic curves are given by $\zeta = 2y - 3x$ and $\eta = y$.
C) The canonical form is given by $\dfrac{\partial^2 u}{\partial \eta^2} - u = 1$, where η is a characteristic variable.
D) The canonical form is $\dfrac{\partial^2 u}{\partial \eta^2} + u = 1$, where η is a characteristic variable.

(38) Consider the one dimensional wave equation:

$$\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2},\ x > 0,\ t > 0$$

subject to the initial conditions:

$$u(x,0) = |\sin x|,\ x \geq 0$$
$$u_t(x,0) = 0,\ x \geq 0$$

and the boundary condition:

$$u(0,t) = 0,\ t \geq 0.$$

Then $u\left(\pi, \frac{\pi}{4}\right)$ is equal to

A) 1. B) 0. C) $\frac{1}{2}$. D) $-\frac{1}{2}$.

(39) The initial value problem

$$x\frac{dy}{dx} = y + x^2,\ x > 0,\ y(0) = 0$$

has

A) infinitely many solutions. B) exactly two solutions.
C) a unique solution. D) no solution.

(40) In a tank there is 120 litres of brine (salted water) containing a total of 50 kg of dissolved salt. Pure water is allowed to run into the tank at the rate of 3 litres per minute. Brine runs out of the tank at the rate of 2 litres per minute. The instantaneous concentration in the tank is kept uniform by stirring. How much salt is in the tank at the end of one hour?

A) 15.45 kg. B) 19.53 kg. C) 14.81 kg. D) 18.39 kg.

(41) If the differential equation

$$2t^2\frac{d^2y}{dt^2} + t\frac{dy}{dt} - 3y = 0$$

is associated with the boundary conditions $y(1) = 5$, $y(4) = 9$, then $y(9) =$

A) 27.44. B) 13.2. C) 19. D) 11.35.

(42) The third degree hermite polynomial approximation for the function $y = y(x)$ such that $y(0) = 1$, $y'(0) = 0$, $y(1) = 3$ and $y'(1) = 5$ is given by

A) $1 + x^2 + x^3$. B) $1 + x^3 + x$.
C) $x^2 + x^3$. D) none of the above.

(43) Let y be the solution of the initial value problem

$$\frac{dy}{dx} = y - x,\ y(0) = 2.$$

Using Runge-Kutta second order method with step size $h = 0.1$, the approximate value of $y(0.1)$ correct to four decimal places is given by

A) 2.8909. B) 2.7142. C) 2.6714. D) 2.7716.

(44) Consider the system of linear equations

$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7 \\ 1 \\ 1 \end{bmatrix}.$$

With the initial approximation $[x_1^{(0)}, x_2^{(0)}, x_3^{(0)}]^T = [0,0,0]^T$, the approximate value of the solution $[x_1^{(1)}, x_2^{(1)}, x_3^{(1)}]^T$ after one iteration by Gauss Seidel method is

A) $[3.2, 2.25, 1.5]^T$. B) $[3.5, 2.25, 1.625]^T$.
C) $[2.25, 3.5, 1.625]^T$. D) $[2.5, 3.5, 1.6]^T$.

(45) For the wave equation

$$u_{tt} = 16\, u_{xx},$$

the characteristic coordinates are

A) $\xi = x + 16\,t$, $\eta = x - 16\,t$. B) $\xi = x + 4\,t$, $\eta = x - 4\,t$.
C) $\xi = x + 256\,t$, $\eta = x - 256\,t$. D) $\xi = x + 2\,t$, $\eta = x - 2\,t$.

(46) Let f_1 and f_2 be two solutions of

$$a_0(x)\frac{d^2y}{dx^2} + a_1(x)\frac{dy}{dx} + a_2(x)y = 0,$$

where a_0, a_1 and a_2 are continuous on $[0,1]$ and $a_0(x) \neq 0$ for all $x \in [0,1]$. Moreover, let $f_1\left(\frac{1}{2}\right) = f_2\left(\frac{1}{2}\right) = 0$. Then

A) one of f_1 and f_2 must be identically zero.
B) $f_1(x) = f_2(x)$ for all $x \in [0,1]$.
C) $f_1(x) = c\,f_2(x)$ for some constant c.
D) none of the above.

(47) The Laplace transform of $e^{4\,t}$ is

A) $1/(s+2)$. B) $1/(s-2)$.
C) $1/(s+4)$. D) $1/(s-4)$.

(48) Let $f(t) = 4\sin^2 t$ and let $\sum_{n=0}^{\infty} a_n \cos n\,t$ be the Fourier cosine series of $f(t)$. Which one is true?

A) $a_0 = 0$, $a_2 = 1$, $a_4 = 2$. B) $a_0 = 2$, $a_2 = 0$, $a_4 = -2$.
C) $a_0 = 2$, $a_2 = -2$, $a_4 = 0$. D) $a_0 = 0$, $a_2 = -2$, $a_4 = 2$.

(49) For a, b, $c \in \mathbb{R}$, if the differential equation

$$(ax^2 + bxy + y^2)dx + (2x^2 + cxy + y^2)dy = 0$$

is exact, then

A) $b = 2,\ c = 2\,a$.
B) $b = 4,\ c = 2$.
C) $b = 2,\ c = 4$.
D) $b = 2,\ a = 2\,c$.

(50) Let $u(x,t)$ be the solution of the wave equation

$$u_{xx} = u_{tt}, \quad u(x,0) = \cos(5\pi x), \quad u_t(x,0) = 0.$$

Then the value of $u(1,1)$ is

A) -1.
B) 0.
C) 2.
D) 1.

(1) C
(2) B
(3) C
(4) A
(5) B
(6) D
(7) A
(8) C
(9) D
(10) D
(11) C
(12) A
(13) C
(14) D
(15) D
(16) A
(17) D
(18) B
(19) C
(20) C
(21) B
(22) B
(23) B
(24) D
(25) B
(26) D
(27) D
(28) B
(29) C
(30) A
(31) D
(32) B
(33) C
(34) B
(35) A
(36) A
(37) D
(38) B
(39) A
(40) D
(41) A
(42) A

(43) A
(44) B
(45) B
(46) D
(47) D
(48) C
(49) B
(50) D

(1) Consider $A = \{q \in \mathbb{Q} : q^2 \geq 2\}$ as a subset of the metric space $(\mathbb{Q}, d)$, where $d(x, y) = |x - y|$. Then A is

A) closed but not open in $\mathbb{Q}$
B) open but not closed in $\mathbb{Q}$
C) neither open nor closed in $\mathbb{Q}$
D) both open and closed in $\mathbb{Q}$.

(2) The set $\mathbb{N}$ considered as a subspace of $(\mathbb{R}, d)$ where $d(x, y) = |x - y|$, is

A) closed but not complete
B) complete but not closed
C) both closed and complete
D) neither closed nor complete.

(3) Let Y be a totally bounded subset of a metric space X. Then the closure $\overline{Y}$ of Y

A) is totally bounded
B) may not be totally bounded even if X is complete
C) is totally bounded if and only if X is complete
D) is totally bounded if and only if X is compact.

(4) Let X, Y be metric spaces, $f : X \to Y$ be a continuous function, A be a bounded subset of X and let $B = f(A)$. Then B is

A) bounded
B) bounded if A is also closed
C) bounded if A is compact
D) bounded if A is complete.

(5) Let X be a connected metric space and U be an open subset of X. Then

A) U cannot be closed in X
B) if U is closed in X, then $U = X$
C) if U is closed in X, then $U = \phi$, the empty set
D) if U is closed in X and U is non-empty, then $U = X$.

(6) Let X be a connected metric space and $f : X \to \mathbb{R}$ be a continuous function. Then $f(X)$

A) is whole of $\mathbb{R}$
B) is a bounded subset of $\mathbb{R}$
C) is an interval in $\mathbb{R}$
D) may not be an interval in $\mathbb{R}$.

(7) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined as

$$f(x,\, y) = \begin{cases} \frac{xy^2}{x^2+y^4}, & \text{if } x \neq 0, \\ 0, & \text{if } x = 0. \end{cases}$$

Let $D_u f(0,\, 0)$ denote the directional derivative of f at $(0,\, 0)$ in the direction $u = (u_1,\, u_2) \neq (0,\, 0)$. Then f is

A) continuous at $(0, 0)$ and $D_u f(0,\, 0)$ exist for all u
B) continuous at $(0, 0)$ but $D_u f(0,\, 0)$ does not exist for some $u \neq (0,\, 0)$
C) not continuous at $(0, 0)$ but $D_u f(0,\, 0)$ exist for all u
D) not continuous at $(0, 0)$ and $D_u f(0,\, 0)$ does not exist for some $u \neq (0,\, 0)$.

(8) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined as

$$f(x,\, y) = \frac{x^2 - y^2}{1 + x^2 + y^2}$$

Then

A) $\frac{\partial^2 f}{\partial x \partial y}(0,\, 0)$ and $\frac{\partial^2 f}{\partial y \partial x}(0,\, 0)$ exist but are not equal
B) $\frac{\partial^2 f}{\partial x \partial y}(0,\, 0)$ exist but $\frac{\partial^2 f}{\partial y \partial x}(0,\, 0)$ does not exist
C) $\frac{\partial^2 f}{\partial y \partial x}(0,\, 0)$ exist but $\frac{\partial^2 f}{\partial x \partial y}(0,\, 0)$ does not exist
D) $\frac{\partial^2 f}{\partial x \partial y}(0,\, 0)$ and $\frac{\partial^2 f}{\partial y \partial x}(0,\, 0)$ exist and are equal.

(9) The sequence

$$\left\langle \frac{2^{n+1} + 3^{n+1}}{2^n + 3^n} \right\rangle$$

converges to

A) 1 B) 2 C) 3 D) 5.

(10) The limit of the sequence $\langle \sqrt{(n+1)(n+2)} - n \rangle$ as $n \to \infty$ is

A) $\sqrt{2} - 1$ B) 3 C) 3/2 D) 0.

(11) The radius of convergence of the series

$$\sum_{n=0}^{\infty} \frac{1}{2^n} x^{3n}$$

is

A) 1 B) ∞ C) 1/2 D) $2^{1/3}$.

(12) Which one of the following sequence converges uniformly on the indicated set?

A) $f_n(x) = (1 - |x|)^n; \quad x \in (-1,\, 1)$

B) $f_n(x) = \frac{1}{n}\sin nx; \quad x \in \mathbb{R}$
C) $f_n(x) = x^n; \quad x \in [0,\, 1]$
D) $f_n(x) = \frac{1}{1+x^n}; \quad x \in [0, \infty)$.

(13) Which one of the following integrals is convergent?

A) $\int_1^\infty \frac{1}{x^2}dx$ B) $\int_1^\infty \frac{1}{\sqrt{x}}dx$
C) $\int_0^1 \frac{1}{x^2}dx$ D) $\int_0^\infty \frac{1}{\sqrt{x}}$.

(14) The value of the integral

$$\int_{-\infty}^{\infty} e^{-x^2}dx$$

is

A) 0 B) $\sqrt{2\pi}$ C) $\sqrt{\pi}$ D) $\sqrt{\pi/2}$.

(15) Let $f : I \to \mathbb{R}$ be an increasing function where I is an interval in $\mathbb{R}$. Then

A) f^2 is always increasing
B) f^2 is always decreasing
C) f^2 is constant $\Rightarrow f$ is constant
D) f^2 may be neither decreasing nor increasing.

(16) Consider the function $f(x) = x^2$ on $[0,\, 1]$ and the partition P of $[0,\, 1]$ given by

$$P = \left\{ 0 < \frac{1}{n} < \frac{2}{n} < \cdots < \frac{n-1}{n} < 1 \right\}.$$

Then the upper and the lower Riemann sums of f are

A) $U(f,\, P) = (1+\frac{1}{n})(2-\frac{1}{n})/6$ and $L(f,\, P) = (1+\frac{1}{n})(2+\frac{1}{n})/6$
B) $U(f,\, P) = (1+\frac{1}{n})(2+\frac{1}{n})/6$ and $L(f,\, P) = (1-\frac{1}{n})(2-\frac{1}{n})/6$
C) $U(f,\, P) = (1+\frac{1}{n})(2+\frac{1}{n})/6$ and $L(f,\, P) = (1-\frac{1}{n})(2+\frac{1}{n})/6$
D) $U(f,\, P) = (1-\frac{1}{n})(2+\frac{1}{n})/6$ and $L(f,\, P) = (1+\frac{1}{n})(2-\frac{1}{n})/6$.

(17) Which one of the following is true?

A) If $\sum a_n$ diverges and $a_n > 0$, then $\sum \frac{a_n}{1+a_n}$ diverges
B) If $\sum a_n$ and $\sum b_n$ diverge, then $\sum(a_n + b_n)$ diverges
C) If $\sum a_n$ and $\sum b_n$ diverge, then $\sum(a_n + b_n)$ converges
D) If $\sum a_n$ converges and $\sum b_n$ diverges, then $\sum(a_n + b_n)$ converges.

(18) If $\sum a_n = A$, $\sum |a_n| = B$ and A and B are finite, then

A) $|A| = B$ B) $A \leq B$
C) $|A| \geq B$ D) $A = B$.

(19) If $x_n = 1 + (-1)^n + \frac{1}{2^n}$, then

A) $\limsup x_n = 1$
B) $\liminf x_n = 1$
C) x_n is a convergent sequence
D) $\limsup x_n \neq \liminf x_n$.

(20) Let $\langle x_n \rangle$ be the sequence defined by $x_1 = 2$ and $x_{n+1} = \frac{1}{2}(x_n + \frac{2}{x_n})$. Then

A) $\langle x_n \rangle$ converges to rational number
B) $\langle x_n \rangle$ is an increasing sequence
C) $\langle x_n \rangle$ converges to $2\sqrt{2}$
D) $\langle x_n \rangle$ is a decreasing sequence.

(21) Which one of the following series converges?

A) $\sum \cos \frac{1}{n^2}$
B) $\sum \sin \frac{1}{n^2}$
C) $\sum \frac{1}{n^{1+1/n}}$
D) $\sum n^{\cos 3}$.

(22) The sum of the series

$$\sum_{k=1}^{\infty} \frac{1}{(2k-1)^2}$$

is

A) $\frac{\pi^2}{8}$
B) $\frac{\pi^2}{6}$
C) $\frac{\pi}{2}$
D) 1.

(23) Which one of the following set is not countable?

A) $\mathbb{N}^r$, where $r \geq 1$ and $\mathbb{N}$ is the set of natural numbers
B) $\{0, 1\}^{\mathbb{N}}$, the set of all the sequences which takes values 0 and 1
C) $\mathbb{Z}$, set of integers
D) $\sqrt{2}\mathbb{Q}$, $\mathbb{Q}$ is set of rational numbers.

(24) Let $f : [0, 1] \to \mathbb{R}$ be a continuous function such that $f(x^2) = f(x)$ for all $x \in [0, 1]$. Which one of the following is not true in general?

A) f is constant
B) f is uniformly continuous
C) f is differentiable
D) $f(x) \geq 0 \; \forall \, x \in [0, 1]$.

(25) Let $f : [0, 1] \to [0, 1]$ be a continuous function and $I : [0, 1] \to [0, 1]$ be the identity function. Then f and I

A) agree exactly at one point
B) agree at least at one point
C) may not agree at any point
D) agree at most at one point.

(26) For $x \in \mathbb{R}$, let $[x]$ denote the greatest integer n such that $n \leq x$. The function $h(x) = x[x]$ is

A) continuous everywhere
B) continuous only at $x = \pm 1, \pm 2, \pm 3, \cdots$
C) continuous if $x \neq \pm 1, \pm 2, \pm 3, \cdots$
D) bounded on $\mathbb{R}$.

(27) Let $\langle x_n \rangle$ be an unbounded sequence in $\mathbb{R}$. Then

A) $\langle x_n \rangle$ has a convergent subsequence
B) $\langle x_n \rangle$ has a subsequence $\langle x_{n_k} \rangle$ such that $x_{n_k} \to 0$
C) $\langle x_n \rangle$has a subsequence $\langle x_{n_k} \rangle$ such that $\frac{1}{x_{n_k}} \to 0$
D) Every subsequence of $\langle x_n \rangle$ is unbounded.

(28) Consider the function $g : \mathbb{R} \to \mathbb{R}$ defined by

$$g(x) = \begin{cases} 0, & \text{if } x \geq 0, \\ e^{-1/x^2}, & \text{if } x < 0. \end{cases}$$

Which one of the following is not true?

A) g has derivatives of all orders at every point
B) $g^n(0) = 0$ for all $n \in \mathbb{N}$
C) Taylor Series expansion of g about $x = 0$ converges to g for all x
D) Taylor Series expansion of g about $x = 0$ converges to g for all $x \geq 0$.

(29) The function

$$f(x) = x \sin x + \frac{1}{1 + x^2}; \quad x \in I$$

where $I \subseteq \mathbb{R}$ is

A) uniformly continuous if $I = \mathbb{R}$
B) uniformly continuous if I is compact
C) uniformly continuous if I is closed
D) not uniformly continuous on $[0, 1]$.

(30) Let the function $f : \mathbb{R} \to \mathbb{R}$ be defined as

$$f(x) = \begin{cases} x^2, & \text{if } x \in (0,\, 2) \cap \mathbb{Q}, \\ 2x - 1, & \text{if } x \in (0,\, 2) \cap (\mathbb{R} \setminus \mathbb{Q}). \end{cases}$$

Which one of the following is not true?

A) f is continuous at $x = 1$
B) f is differentiable at $x = 1$
C) f is not differentiable at $x = 1$
D) f is differentiable only at $x = 1$.

(31) Let R be a finite commutative ring with unity and P be an ideal in R satisfying: $ab \in P \implies a \in P$ or $b \in P$, for any $a, b \in R$. Consider the statements:
(i) P is a finite ideal
(ii) P is a prime ideal
(iii) P is a maximal ideal.
Then

A) (i),(ii) and (iii) are all correct
B) None of (i),(ii) or (iii) is correct
C) (i) and (ii) are correct but (iii) is not correct
D) (i) and (ii) are not correct but (iii) is correct.

(32) Let $\phi : R \to R'$ be a non-zero mapping such that $\phi(a + b) = \phi(a) + \phi(b)$ and $\phi(ab) = \phi(a)\phi(b)$ for all $a, b \in R$, where R, R' are rings with unity. Then

A) $\phi(1) = 1$ for all rings with unity R, R'
B) $\phi(1) \neq 1$ for any rings with unity R, R'
C) $\phi(1) \neq 1$ if R' is an integral domain or if ϕ is onto
D) $\phi(1) = 1$ if R' is an integral domain or if ϕ is onto.

(33) Let R be a ring, L be a left ideal of R and let $\lambda(L) = \{x \in R \mid xa = 0\ \forall a \in L\}$. Then

A) $\lambda(L)$ is not a two-sided ideal of R
B) $\lambda(L)$ is a two-sided ideal of R
C) $\lambda(L)$ is a left but not right ideal of R
D) $\lambda(L)$ is a right but not left ideal of R.

(34) Let $S = \{a + ib \mid a, b \in \mathbb{Z}, b \text{ is even}\}$. Then

A) S is both a subring and an ideal of $\mathbb{Z}[i]$
B) S is neither an ideal nor a subring of $\mathbb{Z}[i]$
C) S is an ideal of $\mathbb{Z}[i]$ but not a subring of $\mathbb{Z}[i]$
D) S is a subring of $\mathbb{Z}[i]$ but not an ideal of $\mathbb{Z}[i]$.

(35) The set of all ring homomorphism $\phi : \mathbb{Z} \to \mathbb{Z}$

A) is an infinite set
B) has exactly two elements
C) is a singleton set
D) is an empty set.

(36) Let F be a field of characteristic 2. Then

A) either F has 2^n elements or is an infinite field
B) F is an infinite field
C) F is a finite field with 2^n elements
D) either F is an infinite field or a finite field with $2n$ elements.

(37) Consider the following classes of commutative rings with unity: ED is the class of Euclidean domain, PID is the class of principal ideal domain, UFD is the class of unique factorization domain and ID is the class of integral domain. Then

A) PID $\subset$ ED $\subset$ UFD $\subset$ ID
B) ED $\subset$ UFD $\subset$ PID $\subset$ ID
C) ED $\subset$ PID $\subset$ UFD $\subset$ ID
D) UFD $\subset$ PID $\subset$ ED $\subset$ ID.

(38) Consider the polynomial ring $\mathbb{Z}[x]$ and $\mathbb{Q}[x]$. Then

A) $\mathbb{Z}[x]$ and $\mathbb{Q}[x]$ both are Euclidean domains
B) $\mathbb{Z}[x]$ and $\mathbb{Q}[x]$ both are not Euclidean domains
C) $\mathbb{Z}[x]$ is a Euclidean domain but $\mathbb{Q}[x]$ is not a Euclidean domain
D) $\mathbb{Q}[x]$ is a Euclidean domain but $\mathbb{Z}[x]$ is not a Euclidean domain.

(39) Let R be a commutative ring with unity such that the polynomial ring $R[x]$ is a principal ideal domain. Then

A) R is a field
B) R is a PID but not a field
C) R is a UFD but not a field
D) R is not a field but is an integral domain.

(40) Let T be a linear transformation on $\mathbb{R}^3$ defined by $T(x_1, x_2, x_3) = (3x_1, x_1 - x_2, 2x_1 + x_2 + x_3)$. What is T^{-1}?

A) $T^{-1}(x_1, x_2, x_3) = (\frac{x_1}{3}, \frac{x_1}{3} + x_2, -x_1 + x_2 + x_3)$
B) $T^{-1}(x_1, x_2, x_3) = (\frac{x_1}{3}, \frac{x_1}{3} - x_2, x_1 + x_2 + x_3)$
C) $T^{-1}(x_1, x_2, x_3) = (\frac{x_1}{3}, \frac{x_1}{3} - x_2, -x_1 + x_2 + x_3)$
D) $T^{-1}(x_1, x_2, x_3) = (\frac{x_1}{3}, \frac{x_1}{3} + x_2, x_1 + x_2 + x_3)$.

(41) Let V be the vector space of all $n \times n$ matrices over a field F. Which one of the following is not a subspace of V?

A) All upper triangular matrices of order n
B) All non-singular matrices of order n
C) All symmetric matrices of order n
D) All matrices of order n, the sum of whose diagonal entries is zero.

(42) Let V be the vector space of all $n \times n$ matrices over a field. Let V_1 be the subspace of V consisting of all symmetric matrices of order n and V_2 be the subspace of V consisting of all skew-symmetric matrices of order n. Which one of the following is not a subspace of V?

A) $V_1 + V_2$ B) $V_1 \cup V_2$ C) $V_1 \oplus V_2$ D) $V_1 \cap V_2$.

(43) Let $V = \mathbb{R}^3$ be the real inner product space with the usual inner product. A basis for the subspace $u^\perp$ of V, where $u = (1, 3, -4)$, is

A) $\{(1, 0, 3), (0, 1, 4)\}$, B) $\{(3, -1, 0), (-6, 2, 0)\}$
C) $\{(-3, 1, 0), (4, 0, 1)\}$ D) $\{(3, 1, 0), (-4, 0, 1)\}$.

(44) The matrix A that represents the linear operator T on $\mathbb{R}^2$, where T is the reflection in $\mathbb{R}^2$ about the line $y = -x$ is

A) $A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
B) $A = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$
C) $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
D) $A = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$.

(45) Consider the subspace U of $\mathbb{R}^4$ spanned by the vectors $v_1 = (1, 1, 1, 1), v_2 = (1, 1, 2, 4), v_3 = (1, 2, -4, -3)$. An orthonormal basis of U is

A) $\{\frac{1}{2}(1, 1, 1, 1), \frac{1}{\sqrt{6}}(-1, -1, 0, 2), \frac{1}{\sqrt{2}}(1, 3, -6, 2)\}$
B) $\{\frac{1}{2}(1, 1, 1, 1), \frac{1}{2\sqrt{6}}(-1, -1, 0, 2), \frac{1}{\sqrt{2}}(1, 3, 6, -2)\}$
C) $\{\frac{1}{2}(1, 1, 1, 1), \frac{1}{\sqrt{6}}(-1, -1, 0, 2), \frac{1}{5\sqrt{2}}(1, 3, -6, 2)\}$
D) $\{(1, 1, 1, 1), (-1, -1, 0, 2), (1, 3, -6, 2)\}$.

(46) Let V be a vector space over $\mathbb{Z}_5$ of dimension 3. The number of elements in V is

A) 5 B) 125 C) 243 D) 3.

(47) Let W be the subspace of $\mathbb{R}^4$ spanned by the vectors $u_1 = (1, -2, 5, -3), u_2 = (2, 3, 1, -4), u_3 = (3, 8, -3, -5)$. The dimension of W is

A) 1 B) 2 C) 3 D) 4.

(48) Let λ be a non-zero characteristic root of a non-singular matrix A of order 2×2. Then a characteristic root of the matrix adj.A is

A) $\frac{\lambda}{|A|}$ B) $\frac{|A|}{\lambda}$ C) $\lambda|A|$ D) $\frac{1}{\lambda}$.

(49) Let $A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$ be a 2×2 matrix. Then the expression $A^5 - 2A^4 - 3A^3 + A^2$ is equal to

A) $2A + 3I$ B) $3A + 2I$ C) $2A - 3I$ D) $3A - 2I$.

(50) The number of elements in the group $Aut\ \mathbb{Z}_{200}$ of all automorphisms of $\mathbb{Z}_{200}$ is

A) 78 B) 80 C) 84 D) 82.

(51) Let $A = \begin{pmatrix} 2 & 6 \\ 3 & 5 \end{pmatrix}$ be a matrix over the integers modulo 11. The inverse of A is

A) $A = \begin{pmatrix} 8 & 9 \\ 10 & 9 \end{pmatrix}$

B) $A = \begin{pmatrix} 10 & 8 \\ 9 & 9 \end{pmatrix}$

C) $A = \begin{pmatrix} 9 & 10 \\ 9 & 8 \end{pmatrix}$

D) $A = \begin{pmatrix} 9 & 9 \\ 10 & 8 \end{pmatrix}$.

(52) The order of the group $\left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \middle| ad - bc = 1 \text{ and } a, b, c, d \in \mathbb{Z}_3 \right\}$ relative to matrix multiplication is

A) 18 B) 20 C) 24 D) 22.

(53) The number of subgroups of the group $\mathbb{Z}_{200}$ is

A) 8 B) 14
C) 12 D) 10.

(54) Let $G = U(32)$ and $H = \{1, 31\}$. The quotient group G/H is isomorphic to

A) $\mathbb{Z}_8$
B) $\mathbb{Z}_4 \oplus \mathbb{Z}_2$
C) $\mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_2$
D) The dihedral group D_4.

(55) The number of sylow 5-subgroups of the group $\mathbb{Z}_6 \oplus \mathbb{Z}_5$ is

A) 6 B) 4 C) 12 D) 1.

(56) The singular solution of the first order differential equation $p^3 - 4xyp + 8y^2 = 0$ is

A) $27x - 4y^3 = 0$ B) $27y - 4x^2 = 0$
C) $27y - 4x^3 = 0$ D) $27y + 4x^3 = 0$.

(57) The general solution of the system of first order differential equations

$$\frac{dx}{dt} + \frac{dy}{dt} = x + t,$$
$$\frac{dx}{dt} - \frac{d^2y}{dt^2} = 0$$

is given by

A) $x = \frac{1}{2}t + c_1t^2 + c_2t;\ y = \frac{1}{2}t - c_1t + c_2$
B) $x = \frac{1}{2}t^2 + c_1t + c_2;\ y = \frac{1}{6}t^3 + \frac{1}{2}c_1t^2 + (c_2 - c_1)t + c_3$
C) $x = \frac{1}{2}t^2 - c_1t + c_2t^2;\ y = \frac{1}{6}t^2 + \frac{1}{2}c_1t^2 + (c_2 - c_1)t^2 + c_3$
D) $x = \frac{1}{3}t^2 + c_1t + c_2;\ y = \frac{1}{6}t^3 - \frac{1}{2}c_1t + (c_2 - c_1)t^2 + c_3$.

(58) Consider the following statements regarding the two solutions $y_1(x) = \sin x$ and $y_2(x) = \cos x$ of $y'' + y = 0$:
(i) They are linearly dependent solutions of $y'' + y = 0$.
(ii) Their wronskian is 1.
(iii) They are linearly independent solutions of $y'' + y = 0$.
which of the statements is true?

A) (i) and (ii) B) (ii) and (iii)
C) (iii) D) (i).

(59) The general solution of $\frac{d^4y}{dx^4} - 5\frac{d^3y}{dx^3} + 6\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 8y = 0$ is

A) $y = c_1 + c_2x + c_3x^2 + c_4e^x$
B) $y = c_1 - c_2x + c_3x^3 + c_4e^{-x}$
C) $y = (c_1 + c_2x + c_3x^2)e^{2x} + c_4e^x$

D) $y = (c_1 + c_2x + c_3x^2)e^{2x} + c_4e^{-x}$.

(60) The solution of the initial value problem $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 25y = 0,\ y(0) = -3,\ y'(0) = -1$ is

A) $y = e^{3x}(2\cos 4x + 3\sin 2x)$
B) $y = e^{-3x}(2\sin 2x - 3\cos 2x)$
C) $y = e^{3x}(2\sin 4x - 3\cos 4x)$
D) $y = e^{3x}(2\sin 4x + 3\cos 4x)$.

(61) The sturm-Liouville problem given by $y'' + \lambda y = 0,\ y(0) = 0,\ y(\pi) = 0$ has a trivial solution if

A) $\lambda \leq 0$
B) $\lambda > 0$
C) $0 < \lambda < 1$
D) $\lambda \geq 1$.

(62) The initial value problem $y' = 1 + y^2, y(0) = 1$ has the solution given by

A) $y = \tan(x - \frac{\pi}{4})$
B) $y = \tan(x + \frac{\pi}{4})$
C) $y = \tan(x - \frac{\pi}{2})$
D) $y = \tan(x + \frac{\pi}{2})$.

(63) The series expansion that gives y as a function of x in neighborhood of $x = 0$ when $\frac{dy}{dx} = x^2 + y^2$; with boundary conditions $y(0) = 0$ is given by

A) $y = \frac{1}{3}x^3 + \frac{1}{63}x^7 + \frac{2}{2079}x^{11} + \cdots$
B) $y = \frac{1}{2}x^3 + \frac{1}{8}x^5 + \frac{1}{32}x^7 + \cdots$
C) $y = x^2 + \frac{1}{2!}x^3 + \frac{1}{3!}x^4 + \cdots$
D) $y = \frac{1}{2!}x^2 + \frac{1}{4!}x^4 + \frac{1}{6!}x^6 + \cdots$.

(64) The value of $y(0.2)$ obtained by solving the equation $\frac{dy}{dx} = \log(x + y),\ y(0) = 1$ by modified Euler's method is equal to

A) 1.223
B) 1.0082
C) 2.381
D) 1.639.

(65) Reciprocal square root iteration formula for $N^{-1/2}$ is given by

A) $x_{i+1} = \frac{x_i}{2}(3 - x_i^2N)$
B) $x_{i+1} = \frac{x_i}{9}(4 - x_i^2N)$
C) $x_{i+1} = \frac{1}{16}(8 - x_i^2N)$
D) $x_{i+1} = \frac{x_i}{4}(10 - x_i^2N)$.

(66) If the formula $\int_0^h f(x)\,dx = h[af(0) + bf(\frac{h}{3}) + cf(h)]$ is exact for polynomials of as high order as possible, then $[a, b, c]$ is

A) $[0, 2, 3]$ B) $[1, 5, \frac{9}{4}]$
C) $[\frac{3}{4}, 2, 9]$ D) $[0, \frac{3}{4}, \frac{1}{4}]$.

(67) If f is continuous, $f(x_1)$ and $f(x_2)$ are of opposite sign and $f(\frac{x_1+x_2}{2})$ has same sign as $f(x_1)$, then

A) $(\frac{x_1+x_2}{2}, x_2)$ must contain at least one zero of $f(x)$
B) $(\frac{x_1+x_2}{2}, x_2)$ contain no zero of $f(x)$
C) $(x_1, \frac{x_1+x_2}{2})$ must contain at least one zero of $f(x)$
D) $(\frac{x_1+x_2}{2}, x_2)$ has no zero of $f(x)$.

(68) The first iteration solution of system of equations

$$2x_1 - x_2 = 7$$
$$-x_1 + 2x_2 - x_3 = 1$$
$$-x_2 + 2x_3 = 1$$

by Gauss-Seidel method with initial approximation $x^{(0)} = 0$ is

A) $[3.5, 2.25, 1.625]$
B) $[4.625, 3.625, 2.315]$
C) $[5, 3, 1]$
D) $[5.312, 4.312, 2.656]$.

(69) The partial differential equation for the family of surfaces $z = ce^{\omega t}\cos(\omega x)$, where c and ω are arbitrary constants, is

A) $z_{xx} + z_{tt} = 0$
B) $z_{xx} - z_{tt} = 0$
C) $z_{xt} + z_{tt} = 0$
D) $z_{xt} + z_{xx} = 0$.

(70) The integral surface of the linear partial differential equation $x(y^2 + z)p - y(x^2 + z)q = (x^2 - y^2)z$ which contains the straight line $x - y = 0$, $z = 1$ is

A) $x^2 + y^2 + 2xyz - 2z + 2 = 0$
B) $x^2 + y^2 - 2xyz - 2z + 2 = 0$
C) $x^2 + y^2 - 2xyz + 2z + 2 = 0$
D) $x^2 + y^2 + 2xyz + 2z + 2 = 0$.

(71) The solution of heat equation $\frac{\partial^2 z}{\partial x^2} = \frac{1}{k}\frac{\partial z}{\partial t}$ for which a solution tends to zero as $t \to \infty$ is

A) $z(x,t) = \sum_{n=0}^{\infty} c_n \cos(nx + \epsilon_n)e^{-n^2kt}$
B) $z(x,t) = \sum_{n=0}^{\infty} c_n \cos(nx + \epsilon_n)e^{n^2kt}$
C) $z(x,t) = \sum_{n=0}^{\infty} c_n \sin(nx + \epsilon_n)e^{n^2kt}$
D) $z(x,t) = \sum_{n=-\infty}^{\infty} c_n \sin(nx + \epsilon_n)e^{n^2kt}$.

(72) The complete integral of the equation $p^2y(1 + x^2) = qx^2$ is

A) $z = a(1 + x^2) + \frac{1}{2}a^2y^2 + b$
B) $z = \frac{1}{2}a^2\sqrt{1 + x^2} + a^2y^2 + b$
C) $z = a\sqrt{1 + x^2} + \frac{1}{2}a^2y^2 + b$
D) $z = a(1 + x^2) + \frac{1}{2}ay + b$.

(73) The general integral of the partial differential equation $z(xp - yq) = y^2 - x^2$ is

A) $x^2 + y^2 + z^2 = f(xy)$
B) $x^2 - y^2 + z^2 = f(xy)$
C) $x^2 - y^2 - z^2 = f(xy)$
D) $x^2 + y^2 - z^2 = f(xy)$.

(74) The solution of the partial differential equation $\frac{\partial^4 z}{\partial x^4} + \frac{\partial^4 z}{\partial y^4} = 2\frac{\partial^4 z}{\partial x^2 \partial y^2}$ is

A) $z = x\phi_1(x + y) + \phi_2(x + y) + x\psi_1(x + y) + \psi_2(x + y)$
B) $z = x\phi_1(x - y) + \phi_2(x - y) + x\psi_1(x - y) + \psi_2(x - y)$
C) $z = x\phi_1(x + y) + \phi_2(x - y) + x\psi_1(x + y) + \psi_2(x - y)$
D) $z = x\phi_1(x - y) + \phi_2(x - y) + x\psi_1(x + y) + \psi_2(x + y)$.

(75) The eigen values and eigen functions of the vibrating string problem $u_{tt} - c^2u_{xx} = 0$, $0 \le x \le l$, $t > 0$, $u(x, 0) = f(x)$, $0 \le x \le l$, $u_t(x, 0) = g(x)$, $0 \le x \le l$, $u(0, t) = 0$, $u(l, t) = 0$, $t \ge 0$ are

A) $(\frac{n\pi}{l})^2, \sin\frac{n\pi x}{l}, n = 1, 2, 3, \cdots$
B) $(\frac{n\pi}{l})^2, \cos\frac{n\pi x}{l}, n = 1, 2, 3, \cdots$
C) $\frac{n\pi}{l}, \sin\frac{n\pi x}{l}, \cos\frac{n\pi x}{l}, n = 1, 2, 3, \cdots$
D) All the above.

(1) B
(2) C
(3) A
(4) C
(5) D
(6) C
(7) C
(8) D
(9) C
(10) C
(11) D
(12) B
(13) A
(14) C
(15) A
(16) B
(17) A
(18) B
(19) D
(20) D
(21) B
(22) A
(23) B
(24) D
(25) B
(26) C
(27) C
(28) C
(29) B
(30) C
(31) A
(32) D
(33) B
(34) D
(35) B
(36) A
(37) C
(38) D
(39) A
(40) C
(41) B
(42) B

(43) C
(44) D
(45) C
(46) B
(47) B
(48) B
(49) A
(50) B
(51) D
(52) C
(53) C
(54) A
(55) D
(56) C
(57) B
(58) C
(59) D
(60) C
(61) A
(62) B
(63) A
(64) B
(65) A
(66) D
(67) A
(68) A
(69) A
(70) B
(71) A
(72) C
(73) A
(74) D
(75) A

Application Form Number:	
Name:	
Signature:	

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M.A./M.Sc ADMISSION ENTRANCE TEST — 2014

INSTRUCTIONS TO THE CANDIDATES

(1) Duration: 3 Hours

(2) Enter your name , application number and also sign in the space provided above. Also write your application form number on the coding sheet.

(3) Do not open the question paper booklet until the Invigilator gives the signal for the commencement of the examination.

(4) The question paper shall be of 250 marks.

(5) The paper will consist of two parts: Part I (150 marks / 50 questions of 3 marks each) and Part II (100 marks / 20 questions of 5 marks each).

(6) **Part-I contains 50 Multiple Choice Questions having exactly one correct answer. Shade exactly one option. For each correct answer, three marks will be given and for an incorrect answer one mark will be deducted.**

(7) **Part-II consists of 20 Multiple Choice Questions. Questions may have multiple correct answers and carry five marks. Shade all correct option(s). Five marks will be given only if all correct choice(s) are shaded. There will be no negative marking in this part.**

(8) The answers should be given only in the coding sheet. Do not write or mark anything in the question paper booklet.

(9) Four rough work sheets are provided.

(10) You are advised to complete answering 10 minutes before the end of the examination and verify all your entries.

(11) **Every question paper and OMR sheet has one of the four codes: A, B, C, D. Check whether the answer booklet code matches with question paper code.**

(12) At the end of the examination when the Invigilator announces **Stop writing**, you must stop immediately and place the coding sheet, question paper booklet, rough work sheets, and acknowledgement letter on the table. You should not leave the hall until all the above sheets are collected.

Part I: For each correct answer 3 marks will be given and for an incorrect answer one mark will be deducted. The symbols $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$, and $\mathbb{C}$ respectively denote the sets of integers, rational numbers, real numbers and complex numbers.

(1) How many elements are there in $\mathbb{Z}[i]/\langle 3+i\rangle$?

A) infinite
B) 3
C) 10
D) finite but not 3 or 10.

(2) Let P be the set of all $n \times n$ complex Hermitian matrices. Then P is a vector space over the filed of

A) $\mathbb{C}$
B) $\mathbb{R}$ but not $\mathbb{C}$
C) both $\mathbb{R}$ and $\mathbb{C}$
D) $\mathbb{C}$ but not $\mathbb{R}$.

(3) Which one of the following is true?

A) There are infinitely many one-one linear transformations from $\mathbb{R}^4$ to $\mathbb{R}^3$
B) The dimension of the vector space of all 3×3 skew-symmetric matrices over the field of real numbers is 6
C) Let F be a field and A a fixed $n \times n$ matrix over F. If $T : M_n(F) \to M_n(F)$ is a linear transformation such that $T(B) = AB$ for every $B \in M_n(F)$, then the characteristic polynomial for A is the same as the characteristic polynomial for T
D) A two-dimensional vector space over a field with 2 elements has exactly 3 different basis.

(4) Let V and W be vector spaces over a filed F. Let $S : V \to W$ and $T : W \to V$ be linear transformations. Then which one of the following is true?

A) If ST is one-to-one, then S is one-to-one
B) If $V = W$ and V is finite-dimensional such that $TS = I$, then T is invertible
C) If $\dim V = 2$ and $\dim W = 3$, then ST is invertible
D) If TS is onto, then S is onto.

(5) The order of the automorphism group of Klein's group is

A) 3
B) 4
C) 6
D) 24.

(6) Which one of the following group is cyclic?

A) The group of positive rational numbers under multiplication

B) The dihedral group of order 30
C) $\mathbb{Z}_3 \oplus \mathbb{Z}_{15}$
D) Automorphism group of $\mathbb{Z}_{10}$.

(7) Which one of the following is a field?

A) An infinite integral domain
B) $\mathbb{R}[x]/\langle x^2 - 2\rangle$
C) $\mathbb{Z}_3 \oplus \mathbb{Z}_{15}$
D) $\mathbb{Q}[x]/\langle x^2 - 2\rangle$.

(8) Which one of the following is true for the transformation $T : P_2(\mathbb{R}) \to P_2(\mathbb{R})$ defined by $T(f) = f + f' + f''$?

A) T is one-to-one but not onto
B) T is onto but not one-to-one
C) T is invertible
D) the matrix of T with respect to the basis $\{1, x, x^2\}$ is upper triangular.

(9) In $\mathbb{Z}[x]$, the ideal of $\langle x\rangle$ is

A) maximal but not prime
B) prime but not maximal
C) both prime and maximal
D) neither prime not maximal.

(10) Which one of the following is true for the transformation $T : M_n \to \mathbb{C}$ defined by $T(A) = \operatorname{tr} A = \sum_{i=1}^{n} A_{ii}$?

A) Nullity of T is $n^2 - 1$
B) Rank of T is n
C) T is one-to-one
D) $T(AB) = T(A)T(B)$ for all $A, B \in M_{n\times n}$.

(11) Let $W_1 = \{A \in M_n(\mathbb{C}) : A_{ij} = 0 \;\forall\; i \le j\}$ and W_2 is the set of symmetric matrices of order n. Then the dimension of $W_1 + W_2$ is

A) n B) $2n$ C) n^2 D) $n^2 - n$.

(12) The logarithmic map from the multiplicative group of positive real numbers to the additive group of real number is

A) a one-to-one but not an onto homomorphism
B) an onto but not a one-to-one homomorphism
C) not a homomorphism
D) an isomorphism.

(13) If f is a group homomorphism from $(\mathbb{Z}, +)$ to $(\mathbb{Q} - \{0\}, \cdot)$ such that $f(2) = 1/3$, then the value $f(-8)$ is

A) 81 B) 1/81 C) 1/27 D) 27.

(14) The quotient group $\mathbb{Q}_8/\{1, -1\}$ is isomorphic to

A) $(\mathbb{Q}_8, \cdot)$
B) $(\{1, -1\}, \cdot)$
C) $(V_4, +)$
D) $(\mathbb{Z}_4, +)$.

(15) The converse of Lagrange's theorem does not hold in

A) A_4, the alternating group of degree 4
B) $A_4 \times \mathbb{Z}_2$
C) the additive group of integers modulo 4
D) Klein's four group.

(16) The ring $(R, +, \cdot)$ is an integral domain when R is

A) $M_2(\mathbb{Z})$
B) $\mathbb{Z}_7$
C) $\mathbb{Z}_6$
D) $C[0, 1]$ of all continuous functions from $[0, 1]$ to $\mathbb{R}$.

(17) The polynomial ring $\mathbb{Z}[x]$ is

A) a field
B) a principal ideal domain
C) unique factorization domain
D) Euclidian domain.

(18) An algebraic number is a root of a polynomial whose coefficients are rational. The set of algebraic numbers is

A) finite
B) countably infinite
C) uncountable
D) none of these.

(19) Let $f : A \to A$ and $B \subset A$. Then which one of the following is always true?

A) $B \subset f^{-1}(f(B))$
B) $B = f^{-1}(f(B))$
C) $B \subset f(f^{-1}(B))$
D) $B = f(f^{-1}(B))$.

(20) Which one of the following does not imply $a = 0$?

A) For all $\epsilon > 0$, $0 \leq a < \epsilon$
B) For all $\epsilon > 0$, $-\epsilon < a < \epsilon$
C) For all $\epsilon > 0$, $a < \epsilon$
D) For all $\epsilon > 0$, $0 \leq a \leq \epsilon$.

(21) Let X and Y be metric spaces and $f : X \to Y$ be continuous. Then f maps

A) open sets to open sets and closed sets to closed sets

B) compact sets to bounded sets
C) connected sets to compact sets
D) bounded sets to compact sets.

(22) Suppose $f : [0, 1] \to \mathbb{R}$ is bounded. Then

A) f is Riemann integrable on $[0, 1]$
B) f is continuous on $[0, 1]$ except for finitely many points implies f is Riemann integrable on $[0, 1]$
C) f is Riemann integrable on $[0, 1]$ implies f is continuous on $[0, 1]$
D) f is Riemann integrable on $[0, 1]$ implies f is monotone function.

(23) Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = \sin x^3$, then f is

A) uniformly continuous
B) not differentiable
C) continuous but not uniformly continuous
D) not continuous.

(24) Consider the sequence $\langle f_n \rangle$ defined by $f_n(x) = 1/(1 + x^n)$ for $x \in [0, 1]$. Let $f(x) = \lim_{n\to\infty} f_n(x)$. Then

A) For $0 < a < 1$, $\langle f_n \rangle$ converges uniformly to f on $[0, a]$
B) the sequence $\langle f_n \rangle$ converges uniformly to f on $[0, 1]$
C) the sequence $\langle f_n \rangle$ converges uniformly to f on $[1/2, 1]$
D) the sequence $\langle f_n \rangle$ converges uniformly to f on $[0, 1]$.

(25) The open unit ball $B((0, 0), 1)$ in the metric space $(\mathbb{R}^2, d)$ where the metric d is defined by $d((x_1, y_1), (x_2, y_2)) = |x_1 - x_2| + |y_1 - y_2|$ is the inside portion of

A) the circle centered at the origin and radius 1
B) the rectangle with vertices at $(0, 1)$, $(1, 0)$, $(-1, 0)$, $(0, -1)$
C) the rectangle with vertices at $(1, 1)$, $(1, -1)$, $(-1, 1)$, $(-1, -1)$
D) the triangle with vertices $(0, 1)$, $(-1, -1)$, $(1, -1)$.

(26) Let $\langle a_n \rangle$ and $\langle b_n \rangle$ be two sequences of real numbers such that $a_n = b_n - b_{n+1}$ for $n \in \mathbb{N}$. If $\sum b_n$ is convergent, then which of the following is true?

A) $\sum a_n$ may not converge
B) $\sum a_n$ is convergent and $\sum a_n = b_1$
C) $\sum a_n$ is convergent and $\sum a_n = 0$
D) $\sum a_n$ is convergent and $\sum a_n = a_1 - b_1$.

(27) Let f be a real-valued function on $[0, 1]$ such that $f(0) = -1$ and $f(1) = 1/2$, then there always exists a $t \in (0, 1)$ such that

A) $f'(t) = -2$ B) $f'(t) = 1$
C) $f'(t) = 3/2$ D) $f'(t) = -1/2$.

(28) Let S and T be subsets of $\mathbb{R}$. Select the incorrect statement:

A) $(\text{int}\, S) \cap (\text{int}\, T) = \text{int}(S \cap T)$
B) $(\text{int}\, S) \cup (\text{int}\, T) \subset \text{int}(S \cup T)$
C) $\overline{S}$ is closed in $\mathbb{R}$
D) $\overline{T}$ is the largest closed set containing T.

(29) The number of solutions of the equation $3^x + 4^x = 5^x$ in the set of positive real numbers is exactly

A) 1 B) 2 C) 3 D) 5.

(30) Let $f : \mathbb{R} \to \mathbb{R}$ be differentiable and f' be bounded. Then

A) f has a local maximum at exactly one point of $\mathbb{R}$
B) f has a local maximum at exactly two point of $\mathbb{R}$
C) f is uniformly continuous on $\mathbb{R}$
D) $f + f'$ is uniformly continuous on $\mathbb{R}$.

(31) Let $a_n = 2^n + n^2$ for $n \leq 100$ and $a_n = 3 + (-1)^n \frac{n^2}{2^n+1}$ for $n > 100$. Then

A) $\langle a_n \rangle$ is a Cauchy sequence
B) $\langle a_n \rangle$ is an unbounded sequence
C) $\langle a_n \rangle$ has exactly three limit points
D) $\langle a_n \rangle$ has two convergent subsequences converging to two different points.

(32) Let the function $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(0,0) = 0$ and $f(x,y) = \frac{x^3 - y^3}{x^2 + y^2}$ for $(x,y) \neq (0,0)$. Then

A) f is continuous on $\mathbb{R}^2$
B) f is continuous at all points of $\mathbb{R}^2$ except at $(0,0)$
C) $f_x(0,0) = f_y(0,0)$
D) f is bounded.

(33) Let $f : [0,1] \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} 1/2 & \text{if } x = 1/4 \\ 1/4 & \text{if } x = 1/2 \\ 0 & \text{if } x \in [0,1] \setminus \{1/4, 1/2\}. \end{cases}$$

Then

A) f is Riemann integrable and $\int_0^1 f(x)dx = 3/4$

B) f is Riemann integrable and $\int_0^1 f(x)dx = 1/4$
C) f is Riemann integrable and $\int_0^1 f(x)dx = 0$
D) f is not Riemann integrable.

(34) Let $f : [0, \pi/2] \to \mathbb{R}$ be continuous and satisfy $\int_0^{\sin x} f(t)dt = \sqrt{3}x/2$ for $0 \le x \le \pi/2$. Then $f(1/2)$ equals

A) $1/2$ B) $1/\sqrt{2}$ C) $1/\sqrt{3}$ D) 1.

(35) For $n \in \mathbb{N}$, let $f_n(x) = \frac{\sin x}{x} + \frac{\cos x}{\sqrt{n}}$ for $x \in (0, \pi/2]$. Then

A) $\langle f_n \rangle$ converges uniformly on $(0, \pi/2)$ but not on $(0, \pi/2]$
B) $\langle f_n \rangle$ converges uniformly on $(0, \pi/2]$
C) $\langle f_n \rangle$ converges uniformly on $(0, \pi/4)$ but not on $(0, \pi/4]$
D) none of these.

(36) Define a metric d on $\mathbb{R}$ by $d(x, x) = 0$ for any x and $d(x, y) = 1$ for any x, y with $x \ne y$. Let $\langle a_n \rangle$ be a Cauchy sequence in $\langle \mathbb{R}, d \rangle$. Then

A) $\langle a_n \rangle$ is a constant sequence
B) $\langle a_n \rangle$ contains infinitely many points
C) $\langle a_n \rangle$ contains at most finite number of distinct points
D) none of these.

(37) The singular solution of $y = px + p^3$, $p = dy/dx$ is

A) $4y^3 + 27x^2 = 0$ B) $4x^2 + 27y^3 = 0$
C) $4y^2 - 27x^3 = 0$ D) $4x^3 + 27y^2 = 0$.

(38) Consider the following initial value problem: $(x + 1)^2 y'' - 2(x + 1)y' + 2y = 1$ subject to the condition $y(0) = 0$ and $y'(0) = 1$. Given that $x + 1$ and $(x + 1)^2$ are linearly independent solutions of the corresponding homogeneous equation, the value of $y(1/2)$ is equal to

A) $5/16$ B) $7/8$ C) 0 D) $1/24$.

(39) Assume that all the roots of the polynomial equation $a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0 = 0$ have negative real parts. If $u(t)$ is any solution to the differential equation: $a_n u^{(n)} + a_{n-1}u^{(n-1)} + \cdots + a_1 u' + a_1 u = 0$, the value of the limit $\lim_{t\to\infty} u(t)$ is

A) 0 B) n C) ∞ D) 1.

(40) The initial value problem $y' = y^{2/3}$ with $0 \le x \le a$ for any positive real number a and $y(0) = 0$ has

A) infinitely many solutions
B) more than one but finitely many solutions
C) unique solution
D) no solution.

(41) One of the particular integrals of the partial differential equation $r - 2s + t = \cos(2x + 3y)$ is

A) $-\cos(2x + 3y)$ B) $\cos(2x + 3y)$
C) $\sin(2x + 3y)$ D) none of these.

(42) The region in which the equation $xu_{xx} + u_{yy} = x^2$ is hyperbolic is

A) the whole plane $\mathbb{R}^2$ B) the half plane $x > 0$
C) the half plane $y > 0$ D) the half plane $x < 0$.

(43) The solution of Cauchy problem $u_t + uu_s = x$, $u(x, 0) = 1$ is $u(x, t) =$

A) $x \tanh t + \operatorname{sech} t$ B) $\tanh t + \operatorname{sech} t$
C) $(x^2 + t^2) \sin t$ D) none of these.

(44) The integral surface that satisfies the first order partial differential equation:

$$(x^2 - y^2 - z^2)\frac{\partial z}{\partial x} + 2xy\frac{\partial z}{\partial y} = 2xz$$

is given by

A) $\phi(xy/z, y^2/(x^2 + z^2)) = 0$
B) $\phi(y/z, (x^2 + y^2 + z^2)/x) = 0$
C) $\phi(y/z, (x^2 + y^2 + z^2)/z) = 0$
D) $\phi(y/(zx), x^2/(y^2 + z^2)) = 0$.

(45) Consider the diffusion equation $u_{xx} = u_t$ with $0 < x < \pi$ and $t > 0$, subject to the initial and boundary conditions: $u(x, 0) = 4 \sin 2x$ for $0 < x < \pi$ and $u(0, t) = 0 = u(\pi, t)$ for $t > 0$. Then, $u(\pi/8, 1)$ is equal to:

A) $4e^{-4}/\sqrt{2}$ B) $4e^{-9}/\sqrt{2}$ C) $4/e^2$ D) $4/\sqrt{e}$.

(46) The general solution to the second order partial differential equation $u_{xx} + u_{xy} - 2u_{yy} = (y + 1)e^x$ is given by

A) $\phi_1(y - x) + \phi_2(y + 2x) + xe^y$
B) $\phi_1(y + x) + \phi_2(y - 2x) + ye^x$
C) $\phi_1(y + x) + \phi_2(y - 2x) + xe^{-y}$
D) $\phi_1(y - x) + \phi_2(y + 2x) + ye^{-x}$.

(47) The trajectories of the system of differential equations $dx/dt = y$ and $dy/dt = -x$ are

A) ellipses B) hyperbolas C) circles D) spirals.

(48) The backward Euler method for solving the differential equation $y' = f(x, y)$ is

A) $y_{n+1} = y_n + hf(x_n, y_n)$
B) $y_{n+1} = y_n + hf(x_{n+1}, y_{n+1})$
C) $y_{n+1} = y_{n-1} + 2hf(x_n, y_n)$
D) $y_{n+1} = (1 + h)f(x_{n+1}, y_{n+1})$.

(49) The Newton-Raphson formula for finding approximate root of the equation $f(x) = 0$ is

A) $x_{n+1} = f(x_n)/f'(x_n),\ f'(x_n) \neq 0$
B) $x_{n+1} = x_n + f(x_n)/f'(x_n),\ f'(x_n) \neq 0$
C) $x_{n+1} = x_{n-1} - f(x_n)/f'(x_n),\ f'(x_n) \neq 0$
D) $x_{n+1} = x_n - f(x_n)/f'(x_n),\ f'(x_n) \neq 0$.

(50) If Euler's method is used to solve the initial value problem $y' = -2ty^2$, $y(0) = 1$ numerically with step size $h = -0.2$, the approximate value of $y(0.6)$ is

A) 0.7845 B) 0.8745 C) 0.8754 D) 0.7875.

Part II: Questions may have multiple correct answers and carry five marks. Five marks will be given only if all correct choices are marked. There will be no negative marks.

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(51) There exists a finite field of order

A) 6 B) 12 C) 81 D) 121.

(52) If S_3 and A_3 respectively denote the permutation group and alternating group, then

A) A_3 is the Sylow 3-subgroup of S_3
B) Sylow 2-subgroup of S_3 is unique
C) $\{I, (12)\}$, $\{I, (12)\}$, $\{I, (23)\}$ are Sylow 2-subgroup of S_3
D) A_3 is not a normal subgroup of S_3.

(53) Let G be a group of order 105 and H be its subgroup of order 35. Then

A) H is a normal subgroup of G

B) H is cyclic
C) G is simple
D) H has a normal subgroup K of order 5 and K is normal in G.

(54) The quotient group $\mathbb{R}/\mathbb{Z}$ is

A) an infinite Abelian group
B) cyclic
C) the same as $\{r + \mathbb{Z} : 0 \leq r < 1\}$
D) isomorphic to the multiplicative group of all complex numbers of unit modulus.

(55) Which of the following pairs of groups are isomorphic to each other?

A) $\langle \mathbb{Z}, + \rangle$, $\langle \mathbb{Q}, + \rangle$
B) $\langle \mathbb{Q}, + \rangle$, $\langle \mathbb{R}^+, \cdot \rangle$
C) $\langle \mathbb{R}, + \rangle$, $\langle \mathbb{R}^+, \cdot \rangle$
D) $\mathrm{Aut}(\mathbb{Z}_3)$, $\mathrm{Aut}(\mathbb{Z}_4)$.

(56) Let V and W be finite-dimensional vector spaces and $T : V \to W$ be a linear transformation. Then

A) $\dim V < \dim W \Rightarrow T$ cannot be onto
B) $\dim V > \dim W \Rightarrow T$ cannot be one-to-one
C) $\dim V + \mathrm{null}\, T = \mathrm{rank}\, T$
D) $\dim V = \dim W \Rightarrow T$ is invertible .

(57) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation given by the formula $T(x, y, z) = A(x\ y\ z)^t$ where A is a 3×3 real orthogonal matrix of determinant 1. Then

A) T is an isometry of $\mathbb{R}^3$
B) the matrix of T with respect to the usual basis of $\mathbb{R}^3$ is A^t
C) the eigenvalues of T are either 1 or -1
D) T is surjective.

(58) Choose the correct statements

A) $\cup_{n=1}^{\infty}[1/n, 2] = [0, 2]$
B) $\cup_{n=1}^{\infty}(1/n, 2] = (0, 2]$
C) $\cap_{n=1}^{\infty}(1 - 1/n, 2] = (1, 2]$
D) $\cap_{n=1}^{\infty}[1 - 1/n, 2] = [1, 2]$.

(59) If $\mathbb{Q} \subset A \subset \mathbb{R}$, which of the following must be true?

A) If A is open, then $A = \mathbb{R}$
B) If A is closed, then $A = \mathbb{R}$
C) If A is uncountable, then A is closed
D) If A is countable, then A is closed.

(60) The function $f : [0, 1] \to [0, 1]$ defined by $f(0) = 0$ and $f(x) = x^2 \sin(1/x)$ for $x \neq 0$, is

A) differentiable on $(0, 1)$
B) is continuous on $[0, 1]$
C) is continuous on $[0, 1]$ but not differentiable at 0
D) is uniformly continuous.

(61) Let $f : [a, b] \to [a, b]$ be a continuous function. Then

A) $\lim_{n\to\infty} \int_a^b f(x) \sin nx dx = \pi$
B) $\lim_{n\to\infty} \int_a^b f(x) \cos nx dx = \pi$
C) $\lim_{n\to\infty} \int_a^b f(x) \sin nx dx = 0$
D) $\lim_{n\to\infty} \int_a^b f(x) \cos nx dx = 0$.

(62) Which of the following statements about a sequence of real numbers are true?

A) Every bounded sequence has a convergent subsequence
B) Every sequence has a monotonic subsequence
C) Every sequence has a limit point
D) Every sequence has a countable number of terms.

(63) Let $\langle a_n \rangle = \langle 1, 1, 1/2, 1, 1/2, 1/3, 1, 1/2, 1/3, 1/4, \dots \rangle$ be a sequence of real numbers. Then

A) $\langle a_n \rangle$ has infinite number of limit points
B) $\limsup_{n\to\infty} a_n = 1$
C) $\liminf_{n\to\infty} a_n = 0$
D) $\langle a_n \rangle$ has infinite number of convergent subsequences.

(64) Let $f : \mathbb{R} \to \mathbb{R}$ be given by $f(x) = \min\{x, x + 1, |x - 2|\}$. Then

A) f is continuous on $\mathbb{R}$
B) f is not differentiable at exactly two points
C) f increases on the interval $(-\infty, 1]$
D) f decreases on the interval $[1, 2]$.

(65) Let $F_n = [-1/n, 1/n]$ for each $n \in \mathbb{N}$ and let $F = \cap_{n=1}^{\infty} F_n$. Then

A) F contains finite number of points
B) $\sup\{|x - y| : x, y \in F\} = 0$
C) $\inf\{|x - y| : x, y \in F\} = 0$
D) F is a closed set.

(66) Let $d_1, d_2, d_3, d_4 : \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$d_1(x,y) = \sqrt{|x-y|},$$
$$d_2(x,y) = |x^2 - y^2|,$$
$$d_3(x,y) = |\sin x - \sin y|,$$
$$d_4(x,y) = |\tan^{-1} x - \tan^{-1} y|.$$

Then which of the following is/are metric on $\mathbb{R}$?

A) d_1 B) d_2 C) d_3 D) d_4.

(67) Which of the following is/are true for the initial value problem: $xy' = 2y$, $y(a) = b$:

A) there is unique solution near (a, b) if $b \neq 0$
B) there is no solution if $a = 0$ but $b \neq 0$
C) there are infinitely many solutions if $a = b = 0$
D) the function $y = x^2$ if $y \leq 0$ and $y = cx^2$ if $x \geq 0$ is one of the solutions.

(68) The solution of the partial differential equation $z = pq$ where $p = \partial z/\partial x$ and $q = \partial z/\partial y$ is

A) $z = (x + a)(x + b)$ B) $4z = (ax + y/a + b)^2$
C) $z = ax + a^2 + by$ D) none of these.

(69) Consider the second order Sturm - Liouville problem: $x^2y'' + xy' + \lambda y = 0$ where $\lambda \geq 0$, subject to the conditions: $y'(1) = y'(e^{2\pi}) = 0$. Pick out the true statements

A) For $\lambda = 1$, the given problem has infinitely many solutions
B) For $\lambda = 0$, only solution to the given problem is the trivial solution
C) The characteristic values λ_n of the given problem can be arranged in a monotonically increasing sequence
D) For $\lambda = 1/16$, a non-trivial solution exists.

(70) For any integer $n \geq 2$, let $S_n = \{(x, y) \in \mathbb{R}^2 : (x - \frac{1}{2})^2 + y^2 = \frac{1}{n^2}\}$ and $S = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1\}$. The second order partial differential equation: $(x^2 - 1)u_{xx} + 2yu_{xy} - u_{yy} = 0$ is

A) elliptic on $\cup_{n=2}^{\infty} S_n$
B) elliptic on $\cup_{n=3}^{\infty} S_n$ and parabolic on S_2
C) hyperbolic in $\mathbb{R}^2 - S$
D) parabolic on $S \cap (\cup_{n=2}^{\infty} S_n)$.

Rough Work

M.A./M.Sc ADMISSION ENTRANCE TEST — 2014

(1) C
(2) B
(3) D
(4) B
(5) C
(6) D
(7) D
(8) D
(9) B
(10) A
(11) C
(12) D
(13) A
(14) C
(15) A
(16) B
(17) C
(18) B
(19) A
(20) C
(21) B
(22) B
(23) C
(24) A
(25) B
(26) B
(27) C
(28) D
(29) A
(30) C
(31) A
(32) A
(33) C
(34) D
(35) B
(36) C
(37) D
(38) B
(39) A

(40) A
(41) A
(42) D
(43) A
(44) C
(45) B
(46) B
(47) C
(48) B
(49) D
(50) A
(51) C,D
(52) A,C
(53) A,B,D
(54) A,C,D
(55) C,D
(56) A,B
(57) A,C,D
(58) B,D
(59) B
(60) A,B,D
(61) C,D
(62) A,B,D
(63) A,B,C,D
(64) A,B,C,D
(65) A,B,C,D
(66) A,D
(67) B,C,D
(68) A,B
(69) A,C
(70) C,D

Some Useful Links:

1. **Free Maths Study Materials (**https://pkalika.in/2020/04/06/free-maths-study-materials/**)**

2. **BSc/MSc Free Study Materials (**https://pkalika.in/2019/10/14/study-material/**)**

3. **MSc Entrance Exam Que. Paper: (**https://pkalika.in/2020/04/03/msc-entrance-exam-paper/**)**
[JAM(MA), JAM(MS), BHU, CUCET, ...etc]

4. **PhD Entrance Exam Que. Paper: (**https://pkalika.in/que-papers-collection/**)**
[CSIR-NET, GATE(MA), BHU, CUCET,IIT, NBHM, ...etc]

5. **CSIR-NET Maths Que. Paper: (**https://pkalika.in/2020/03/30/csir-net-previous-yr-papers/**)**
[Upto 2019 Dec]

6. **Practice Que. Paper: (**https://pkalika.in/2019/02/10/practice-set-for-net-gate-set-jam/**)**
[Topic-wise/Subject-wise]

7. **List of Maths Suggested Books** (https://pkalika.in/suggested-books-for-mathematics/)

8. **CSIR-NET Mathematics Details Syllabus (**https://wp.me/p6gYUB-Fc)