CSIR November 2020

Application No.	
Candidate Name	
Roll No.	
Test Date	30/11/2020
Test Time	3:00 PM - 6:00 PM
Subject	Mathematical Sciences

Section : **Part A General Aptitude**

Q.1

The first day of the year 2020 was a Wednesday. The first day of 2021 would be a

1. Wednesday
2. Thursday
3. Friday
4. Monday

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **7720331003**
Option 1 ID : **7720334009**
Option 2 ID : **7720334010**
Option 3 ID : **7720334011**
Option 4 ID : **7720334012**
Status : **Answered**
Chosen Option : **3**

Q.2

In an examination, each of the two brilliant students got 100 out of 100 and each of the remaining six students scored less than 12. There is no provision of getting negative marks. If N= Median score of the students and M= Mean score of the students, which of the following is true?

1. $M > 2N$
2. $3N/2 < M \le 2N$
3. $N \le M \le 3N/2$
4. From the above information nothing can be said about the relationship between the Median score of the students and the Mean score of the students.

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**772033998**
Option 1 ID :	**7720333989**
Option 2 ID :	**7720333990**
Option 3 ID :	**7720333991**
Option 4 ID :	**7720333992**
Status :	**Answered**
Chosen Option :	**2**

Q.3

Milk rises rapidly at boiling point because

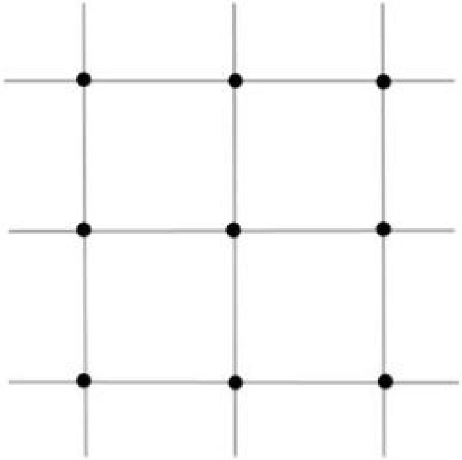
1. it becomes hotter than the vessel at boiling point
2. its temperature rapidly increases at boiling point
3. its bulk density rapidly decreases and it becomes more buoyant at boiling point
4. its vapour pressure rapidly decreases at boiling point

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**772033997**
Option 1 ID :	**7720333985**
Option 2 ID :	**7720333986**
Option 3 ID :	**7720333987**
Option 4 ID :	**7720333988**
Status :	**Answered**
Chosen Option :	**2**

Q.4

Dots are placed at the intersections of a grid.



How many triangles one can draw by joining these dots?

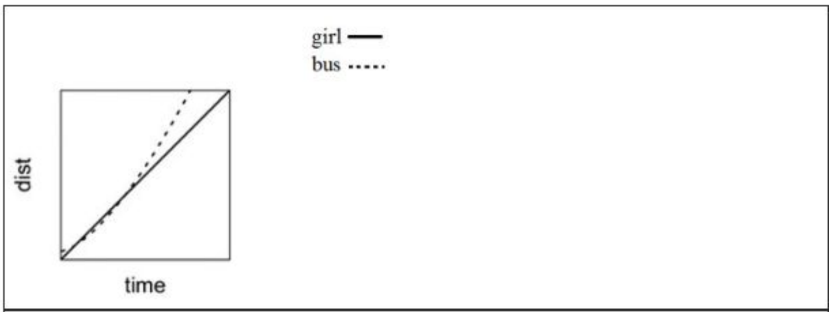
1. 74
2. 76
3. 78
4. 84

Options 1. 1
2. 2
3. 3
4. 4

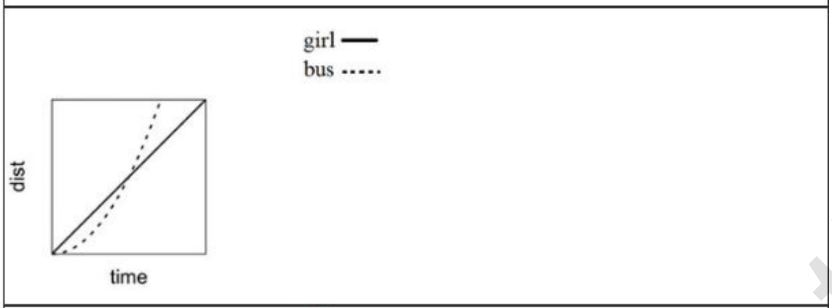
Question Type :	**MCQ**
Question ID :	**7720331010**
Option 1 ID :	**7720334037**
Option 2 ID :	**7720334038**
Option 3 ID :	**7720334039**
Option 4 ID :	**7720334040**
Status :	**Answered**
Chosen Option :	**3**

Q.5

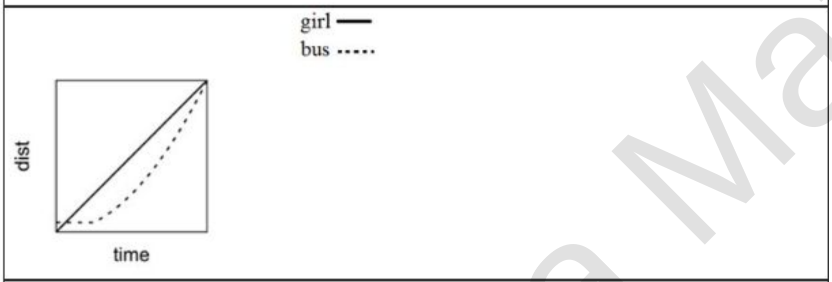
A girl is running at a constant speed along a straight path to catch a bus that is some distance away from her and stationary. Before she reaches the bus, it accelerates away from her. Which of the following graphs is a possible depiction of their motion?



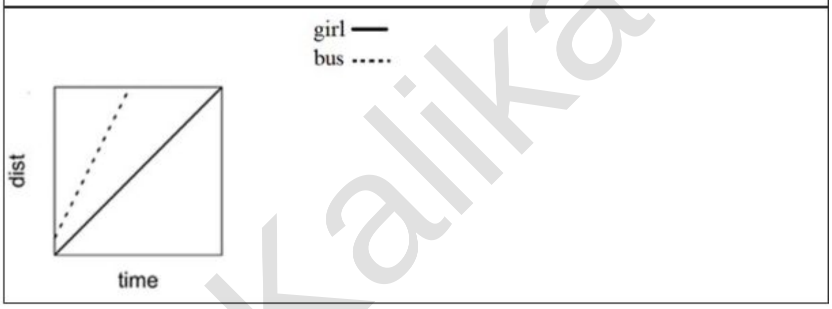
1.



2.



3.



4.

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **7720331013**
Option 1 ID : **7720334049**
Option 2 ID : **7720334050**
Option 3 ID : **7720334051**
Option 4 ID : **7720334052**
Status : **Answered**
Chosen Option : **4**

Q.6

Probability of selection of Alex for a post is 8/11 and for Gafoor it is 5/14. The selection of one is independent of the other. What is the probability of selection of only one of them for the post?

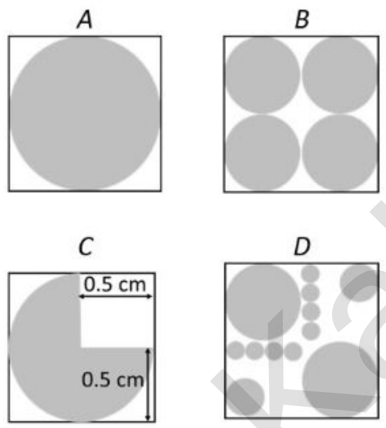
1. 25/72
2. 67/154
3. 87/154
4. 15/72

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **7720331008**
Option 1 ID : **7720334029**
Option 2 ID : **7720334030**
Option 3 ID : **7720334031**
Option 4 ID : **7720334032**
Status : **Answered**
Chosen Option : **2**

Q.7

The shaded circles having diameter of 1, 0.5, 0.25 and 0.125 cm are inside squares of side 1 cm. The ratio of shaded area in *A* and *B* is one. The ratio of shaded area in *C* and *D* would be



1. 0.75
2. 1
3. 1.25
4. 1.5

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **7720331004**
Option 1 ID : **7720334013**
Option 2 ID : **7720334014**
Option 3 ID : **7720334015**
Option 4 ID : **7720334016**
Status : **Not Answered**
Chosen Option : **--**

Q.8

The number of unit squares that can be fitted inside a circle of some finite diameter d is at the most

1. $(\pi d^2/4)-1$
2. πd^2
3. $(\pi d^2)/4$
4. $(\pi (d-1)^2)/4$

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**7720331015**
Option 1 ID :	**7720334057**
Option 2 ID :	**7720334058**
Option 3 ID :	**7720334059**
Option 4 ID :	**7720334060**
Status :	**Answered**
Chosen Option :	**4**

Q.9

The coding genes form a tiny fraction of about 2% of the total human genome. If the total human genome is written on a pack of 50 cards, the number of cards corresponding to coding genes would be

1. One card
2. Two cards
3. Three cards
4. Four cards

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**7720331006**
Option 1 ID :	**7720334021**
Option 2 ID :	**7720334022**
Option 3 ID :	**7720334023**
Option 4 ID :	**7720334024**
Status :	**Not Answered**
Chosen Option :	**--**

Q.10

Four locations A, B, C and D are along a straight road. Distance between A and C is 30 km. From A, a bus starts at 8 am to reach B, and another bus from C starts at 8:30 am to reach D. The buses move away from A, in the same direction with speed of the first being double of the second. They meet at 9 am, at a place covering 80% of the respective distances they are to travel. Then what is the distance between B and D?

1. 37.5 km
2. 25.0 km
3. 12.5 km
4. 7.5 km

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **7720331007**
Option 1 ID : **7720334025**
Option 2 ID : **7720334026**
Option 3 ID : **7720334027**
Option 4 ID : **7720334028**
Status : **Answered**
Chosen Option : **4**

Q.11

A box contains 3 white and 5 black balls. What is the probability of choosing 1 white and 1 black ball, if two balls are drawn at random, one by one, with replacement?

1. 15/64
2. 15/32
3. 3/32
4. 3/28

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **7720331002**
Option 1 ID : **7720334005**
Option 2 ID : **7720334006**
Option 3 ID : **7720334007**
Option 4 ID : **7720334008**
Status : **Not Answered**
Chosen Option : **--**

Q.12

Along the common radial direction from the center of two concentric circles of radii 100 m and 150 m, point A is on the circumference of the inner circle and point B is on the circumference of the outer circle. The points A and B start moving on the respective circles with a speed of 8 m/s, at the same instant, but in opposite directions. After how many seconds, approximately, would they again cross each other along a common radial direction?

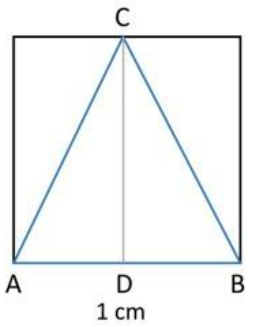
1. 31
2. 37
3. 47
4. 53

Options
1. 1
2. 2
3. 3
4. 4

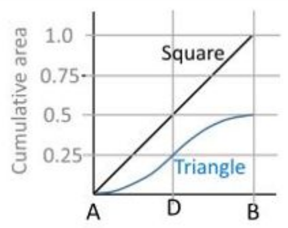
Question Type :	**MCQ**
Question ID :	**772033999**
Option 1 ID :	**7720333993**
Option 2 ID :	**7720333994**
Option 3 ID :	**7720333995**
Option 4 ID :	**7720333996**
Status :	**Not Answered**
Chosen Option :	**--**

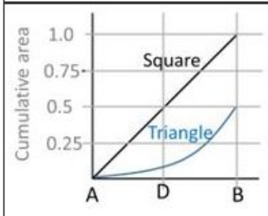
Q.13

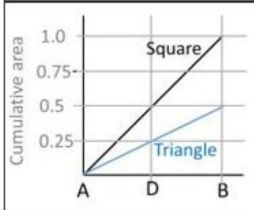
An isosceles triangle, ABC, is inside a square of side of 1 cm where AD=DB.

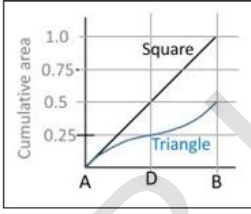


Which one of the following graphs represents cumulative area (cm^2) of the square and the triangle when moving from A to B?

1.


2.


3.


4.


Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **7720331005**
Option 1 ID : **7720334017**
Option 2 ID : **7720334018**
Option 3 ID : **7720334019**
Option 4 ID : **7720334020**
Status : **Not Answered**
Chosen Option : **--**

Q.14

Three liquids, A, B and C having densities of 3, 2 and 1 g/cc respectively, are mixed in the volume proportion of 1:2:3 to form a 6 ml solution. What will be the density (in g/cc) of the solution?

1. 1.7
2. 2.0
3. 2.5
4. 3.0

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**7720331011**
Option 1 ID :	**7720334041**
Option 2 ID :	**7720334042**
Option 3 ID :	**7720334043**
Option 4 ID :	**7720334044**
Status :	**Not Answered**
Chosen Option :	**--**

Q.15

Suppose the standard deviation of the body temperatures of 17 persons measured in degrees Fahrenheit is 2.7. Which of the following is the standard deviation of the above 17 temperatures measured in degrees Celsius?

1. 3.5
2. 2.7
3. 2.5
4. 1.5

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**7720331009**
Option 1 ID :	**7720334033**
Option 2 ID :	**7720334034**
Option 3 ID :	**7720334035**
Option 4 ID :	**7720334036**
Status :	**Answered**
Chosen Option :	**4**

Q.16

Fifteen females participate in a singles badminton tournament. If a player is eliminated as soon as she loses a match, how many matches are required to determine the winner?

1. 30
2. 29
3. 15
4. 14

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**7720331001**
Option 1 ID :	**7720334001**
Option 2 ID :	**7720334002**
Option 3 ID :	**7720334003**
Option 4 ID :	**7720334004**
Status :	**Answered**
Chosen Option :	**4**

Q.17

A tap takes 6 hours to fill a water tank, a second tap takes 5 hours, a third 4 hours and a fourth 3 hours. How long, approximately, would it take to fill the tank if all the taps are used together?

1. 1 hour
2. 1.5 hours
3. 2 hours
4. 2.5 hours

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**7720331014**
Option 1 ID :	**7720334053**
Option 2 ID :	**7720334054**
Option 3 ID :	**7720334055**
Option 4 ID :	**7720334056**
Status :	**Answered**
Chosen Option :	**2**

Q.18

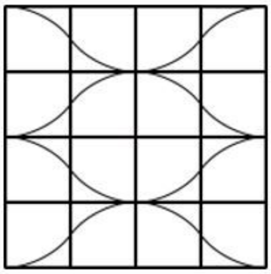
A 3.1m x 2.2m x 2.1m block of granite is cut to have maximum number of 3m x 2m sized slabs having thickness of 4 cm. These slabs are used to make a 1.5m wide pavement. What is the maximum length (in meters) of pavement that can be made using these slabs?

1. 200
2. 220
3. 400
4. 440

Options
1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**772033996**
Option 1 ID :	**7720333981**
Option 2 ID :	**7720333982**
Option 3 ID :	**7720333983**
Option 4 ID :	**7720333984**
Status :	**Not Answered**
Chosen Option :	**--**

Q.19



Which combination of square tiles shown below can produce the given pattern on the floor?

1.  and 
2.  and 
3.  and 
4.  and 

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **7720331000**
Option 1 ID : **7720333997**
Option 2 ID : **7720333998**
Option 3 ID : **7720333999**
Option 4 ID : **7720334000**
Status : **Answered**
Chosen Option : **2**

Q.20
A fair coin is tossed 8 times independently. What is the probability that the third toss results in a head?

1. 1/8
2. 1/4
3. 1/2
4. $3/2^8$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **7720331012**
Option 1 ID : **7720334045**
Option 2 ID : **7720334046**
Option 3 ID : **7720334047**
Option 4 ID : **7720334048**
Status : **Answered**
Chosen Option : **3**

Section : **Part B Mathematical Sciences**

Q.1 Let $S = \{u_1, \dots, u_k\}$ be a subset of non-zero vectors from $\mathbb{R}^n$. Now, consider the two statements given below:

I: If S is linearly dependent set in $\mathbb{R}^n$ then u_k is a linear combination of $u_1, \dots, u_{k-1}$.

II: If S is linearly independent set in $\mathbb{R}^n$ then $k < n$.

Which of the following statements is true?

1. Statement I is FALSE and Statement II is TRUE
2. Statement I is TRUE and Statement II is FALSE
3. Both Statement I and Statement II are FALSE
4. Both Statement I and Statement II are TRUE

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **7720331022**
Option 1 ID : **7720334085**
Option 2 ID : **7720334086**
Option 3 ID : **7720334087**
Option 4 ID : **7720334088**
Status : **Answered**
Chosen Option : **4**

Q.2 Let a, b and c be distinct integers. Let A be the matrix

$$A = \begin{pmatrix} a^2 & b^2 & c^2 \\ a^5 & b^5 & c^5 \\ a^{11} & b^{11} & c^{11} \end{pmatrix}.$$

Which among the following is the set of all possible ranks of A?

1. $\{3\}$
2. $\{2,3\}$
3. $\{1,2,3\}$
4. $\{0,1,2,3\}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **7720331025**
Option 1 ID : **7720334097**
Option 2 ID : **7720334098**
Option 3 ID : **7720334099**
Option 4 ID : **7720334100**
Status : **Answered**
Chosen Option : **3**

Q.3 Which of the following is an inner product on the vector space of all real valued continuous functions on $[0,1]$?

1. $\langle f, g\rangle = \left|\int_0^1 f(t)g(t)dt\right|$
2. $\langle f, g\rangle = \int_0^1 |f(t)g(t)|\, dt$
3. $\langle f, g\rangle = f(0)g(0) + f(1)g(1)$
4. $\langle f, g\rangle = \int_0^1 f(t)g(t)dt$

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**7720331026**
Option 1 ID :	**7720334101**
Option 2 ID :	**7720334102**
Option 3 ID :	**7720334103**
Option 4 ID :	**7720334104**
Status :	**Answered**
Chosen Option :	**3**

Q.4

$$\lim_{n\to\infty} \frac{((n+1)(n+2)\cdots(n+n))^{1/n}}{n}$$

1. is equal to $\frac{e}{4}$
2. is equal to $\frac{4}{e}$
3. is equal to e
4. does not exist

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**7720331017**
Option 1 ID :	**7720334065**
Option 2 ID :	**7720334066**
Option 3 ID :	**7720334067**
Option 4 ID :	**7720334068**
Status :	**Answered**
Chosen Option :	**1**

Q.5

Suppose that A, B are two non-empty subsets of $\mathbb{R}$ and $C = A \cap B$.

Which of the following conditions imply that C is empty?

1. A and B are open and C is compact
2. A and B are open and C is closed
3. A and B are both dense in $\mathbb{R}$
4. A is open and B is compact

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**7720331016**
Option 1 ID :	**7720334061**
Option 2 ID :	**7720334062**
Option 3 ID :	**7720334063**
Option 4 ID :	**7720334064**
Status :	**Answered**
Chosen Option :	**3**

Q.6 Let A be an $n \times n$ matrix of rank 1. Let $\alpha = \det(I + A)$, where I is the identity matrix and let $\beta = \text{trace } A$. Which of the following is true?

1. $\beta - \alpha = 1$
2. $\alpha - \beta = 1$
3. $\alpha < \beta + 1$
4. $\alpha > \beta + 1$

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**7720331024**
Option 1 ID :	**7720334093**
Option 2 ID :	**7720334094**
Option 3 ID :	**7720334095**
Option 4 ID :	**7720334096**
Status :	**Answered**
Chosen Option :	**4**

Q.7

An infinite binary word a is a string $(a_1a_2a_3\cdots)$, where each $a_n \in \{0,1\}$. Fix a word $s = (s_1s_2s_3\cdots)$, where $s_n = 1$ if and only if n is prime. Let $S = \{a = (a_1a_2a_3\cdots) \mid \exists m \in \mathbb{N} \text{ such that } a_n = s_n, \forall n \geq m\}$. What is the cardinality of S?

1. 1
2. Finite but more than 1
3. Countably infinite
4. Uncountable

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **7720331019**
Option 1 ID : **7720334073**
Option 2 ID : **7720334074**
Option 3 ID : **7720334075**
Option 4 ID : **7720334076**
Status : **Answered**
Chosen Option : **3**

Q.8

Let $Y = \{1,2,3,\ldots,100\}$ and let $h: Y \to Y$ be a strictly increasing function. The total number of functions $g: Y \to Y$ satisfying

$g(h(j)) = h(g(j)), \ \forall j \in Y$ is

1. 0
2. 100!
3. 100^{100}
4. 100^{98}

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **7720331018**
Option 1 ID : **7720334069**
Option 2 ID : **7720334070**
Option 3 ID : **7720334071**
Option 4 ID : **7720334072**
Status : **Answered**
Chosen Option : **2**

Q.9 Let M be a 5×5 matrix with real entries such that $\text{Rank}(M) = 3$. Consider the linear system $M\text{x} = \text{b}$. Let the row-reduced echelon form of the augmented matrix $[M \;\; \text{b}]$ be R and let $R[i,:]$ denote the $i-th$ row of R. Suppose that the linear system admits a solution. Which of the following statements is necessarily true?

1. $R[3,:] = [0 \quad 1 \quad 0 \quad * \quad * \quad *]$
2. $R[5,:] = [0 \quad 0 \quad 1 \quad 0 \quad * \quad *]$
3. $R[4,:] = [0 \quad 0 \quad 0 \quad 1 \quad * \quad *]$
4. $R[4,:] = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **7720331023**
Option 1 ID : **7720334089**
Option 2 ID : **7720334090**
Option 3 ID : **7720334091**
Option 4 ID : **7720334092**
Status : **Answered**
Chosen Option : **4**

Q.10 The sum of the infinite series

$$S = \frac{1}{2} - \frac{1}{3 \times 1!} + \frac{1}{4 \times 2!} - \frac{1}{5 \times 3!} + \cdots$$

is equal to

1. $2 - \frac{1}{e}$
2. $1 - \frac{2}{e}$
3. $\frac{2}{e} - 1$
4. $\frac{1}{e} - 2$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **7720331021**
Option 1 ID : **7720334081**
Option 2 ID : **7720334082**
Option 3 ID : **7720334083**
Option 4 ID : **7720334084**
Status : **Answered**
Chosen Option : **2**

Q.11

Let us define a matrix $A \in M_n(\mathbb{R})$ to be *positive* if for every column vector $v \in \mathbb{R}^n$ we have $\langle Av, v\rangle \geq 0$, where $\langle \cdot,\cdot \rangle$ is the standard inner product on $\mathbb{R}^n$.

Let $A_{\alpha,\beta} = \begin{pmatrix} \alpha & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & \beta \end{pmatrix}$. Let $S = \{(\alpha,\beta) \in \mathbb{R}^2 : A_{\alpha,\beta} \text{ is positive}\}$. Which of the following statements is true?

1. S is empty
2. $(\alpha,\beta) \in S$ if and only if $\alpha\beta > 0$
3. $(\alpha,\beta) \in S$ if and only if $\alpha + \beta + 4 > 0$
4. $S = \mathbb{R}^2$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **7720331027**
Option 1 ID : **7720334105**
Option 2 ID : **7720334106**
Option 3 ID : **7720334107**
Option 4 ID : **7720334108**
Status : **Answered**
Chosen Option : **2**

Q.12 Let f be a nonconstant polynomial of degree k and let $g: \mathbb{R} \to \mathbb{R}$ be a bounded continuous function. Which of the following statements is necessarily true?

1. There always exists $x_0 \in \mathbb{R}$ such that $f(x_0) = g(x_0)$
2. There is no $x_0 \in \mathbb{R}$ such that $f(x_0) = g(x_0)$
3. There exists $x_0 \in \mathbb{R}$ such that $f(x_0) = g(x_0)$ if k is even
4. There exists $x_0 \in \mathbb{R}$ such that $f(x_0) = g(x_0)$ if k is odd

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **7720331020**
Option 1 ID : **7720334077**
Option 2 ID : **7720334078**
Option 3 ID : **7720334079**
Option 4 ID : **7720334080**
Status : **Answered**
Chosen Option : **1**

Q.13 Which of the following statements is NOT true?

1. The polynomial ring $\mathbb{Z}[x]$ is a Principal Ideal Domain (PID)
2. The polynomial ring $\mathbb{Q}[x]$ is a Principal Ideal Domain (PID)
3. The polynomial ring $\mathbb{Z}[x]$ is a Unique Factorization Domain (UFD)
4. The polynomial ring $\mathbb{Q}[x]$ is a Unique Factorization Domain (UFD)

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **7720331034**
Option 1 ID : **7720334133**
Option 2 ID : **7720334134**
Option 3 ID : **7720334135**
Option 4 ID : **7720334136**
Status : **Answered**
Chosen Option : **4**

Q.14 Let G be a group of order 2020. Which of the following statements is necessarily true?

1. G is not a simple group
2. G has exactly four proper subgroups
3. G is a cyclic group
4. G is abelian

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **7720331033**
Option 1 ID : **7720334129**
Option 2 ID : **7720334130**
Option 3 ID : **7720334131**
Option 4 ID : **7720334132**
Status : **Answered**
Chosen Option : **3**

Q.15

Let f be a holomorphic function on the disc $\{z \in \mathbb{C}: |z| < 2\}$. Assume that the only zero of f in the closed unit disc $\{z \in \mathbb{C}: |z| \le 1\}$ is a simple zero at the origin. Let γ be the positively oriented circle $\{z \in \mathbb{C}: |z| = 1\}$. The integral

$$\int_\gamma \frac{dz}{f(z)}$$ equals

1. $2\pi i f'(0)$
2. $2\pi i f''(0)$
3. $2\pi i / f'(0)$
4. $2\pi i / f''(0)$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **7720331031**
Option 1 ID : **7720334121**
Option 2 ID : **7720334122**
Option 3 ID : **7720334123**
Option 4 ID : **7720334124**
Status : **Answered**
Chosen Option : **1**

Q.16

Let f, g be entire functions such that $\lim_{z\to\infty} \frac{f(z)}{z^n} = \lim_{z\to\infty} \frac{g(z)}{z^n} = 1$ for some fixed positive integer n. Which of the following statements is true?

1. $f = g$
2. $f - g$ is necessarily a polynomial of degree at most $n - 1$
3. there exist f, g with these properties such that $f - g$ is a polynomial of degree n
4. there exist f, g with these properties such that $f - g$ is not a polynomial

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **7720331030**
Option 1 ID : **7720334117**
Option 2 ID : **7720334118**
Option 3 ID : **7720334119**
Option 4 ID : **7720334120**
Status : **Answered**
Chosen Option : **2**

Q.17 Define $f: \mathbb{C} \to \mathbb{C}$ by $f(z) = ||z|^2 - 1|^2$. Which of the following statements is true?

1. f is complex differentiable at all complex numbers except for $z \in \{0,1\}$
2. f is complex differentiable only at $z = 0$
3. f is complex differentiable only at $z = 1$
4. f is complex differentiable only for $z \in \{0,1\}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **7720331029**
Option 1 ID : **7720334113**
Option 2 ID : **7720334114**
Option 3 ID : **7720334115**
Option 4 ID : **7720334116**
Status : **Not Answered**
Chosen Option : **--**

Q.18 A function $f: \mathbb{C} \mapsto \mathbb{C}$ is said to be analytic at ∞, if the function g defined by $g(w) = f(\frac{1}{w})$ is analytic at 0 with an appropriate value given for $g(0)$. Which of the following statements is true?

1. Any non-constant polynomial is analytic at ∞
2. If f is analytic at ∞ then f is bounded
3. For any z_0 in $\mathbb{C}$, the function $f(z) = e^{\frac{1}{z-z_0}}$ is analytic at ∞
4. Any entire function can be extended to an analytic function at ∞

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **7720331028**
Option 1 ID : **7720334109**
Option 2 ID : **7720334110**
Option 3 ID : **7720334111**
Option 4 ID : **7720334112**
Status : **Not Answered**
Chosen Option : **--**

Q.19 The last two digits of 3^{2019} are

1. 27
2. 37
3. 57
4. 67

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **7720331032**
Option 1 ID : **7720334125**
Option 2 ID : **7720334126**
Option 3 ID : **7720334127**
Option 4 ID : **7720334128**
Status : **Not Answered**
Chosen Option : **--**

Q.20 Suppose $f:[0,1]\times[0,1]\to(0,1)\times(0,1)$ is a continuous non-constant function. Which of the following statements is NOT true?

1. Image of f is uncountable
2. Image of f is a path connected set
3. Image of f is a compact set
4. Image of f has non-empty interior

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **7720331035**
Option 1 ID : **7720334137**
Option 2 ID : **7720334138**
Option 3 ID : **7720334139**
Option 4 ID : **7720334140**
Status : **Not Answered**
Chosen Option : **--**

Q.21

Consider the ODE $t\dot{y} - 3y = t^2 y^{\frac{1}{2}}, y(1) = 1$. Find the value of $y(2)$.

1. 14
2. 16
3. 0
4. 8

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**7720331036**
Option 1 ID :	**7720334141**
Option 2 ID :	**7720334142**
Option 3 ID :	**7720334143**
Option 4 ID :	**7720334144**
Status :	**Not Answered**
Chosen Option :	**--**

Q.22

For $\lambda \in \mathbb{R}$, consider the system of differential equations

$$\begin{aligned} x_1' &= x_1 + 2x_2 + 2x_3, \\ x_2' &= 2x_2 + x_3, \\ x_3' &= -x_3 + 2x_2 + \lambda x_3. \end{aligned}$$

If $\vec{x}(t) = \vec{\alpha}\, te^{2t}$ (for some $\vec{\alpha}$) is a solution of the system then the value of λ is equal to

1. 2
2. 4
3. 6
4. 1

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**7720331037**
Option 1 ID :	**7720334145**
Option 2 ID :	**7720334146**
Option 3 ID :	**7720334147**
Option 4 ID :	**7720334148**
Status :	**Not Answered**
Chosen Option :	**--**

Q.23 Consider the Newton-Raphson method applied to approximate the square root of a positive number α. A recursion relation for the error $e_n = x_n - \sqrt{\alpha}$ is given by

1. $e_{n+1} = \frac{1}{2}(e_n + \frac{\alpha}{e_n})$
2. $e_{n+1} = \frac{1}{2}(e_n - \frac{\alpha}{e_n})$
3. $e_{n+1} = \frac{1}{2}\frac{e_n^2}{e_n+\sqrt{\alpha}}$
4. $e_{n+1} = \frac{e_n^2}{e_n+2\sqrt{\alpha}}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **7720331040**
Option 1 ID : **7720334157**
Option 2 ID : **7720334158**
Option 3 ID : **7720334159**
Option 4 ID : **7720334160**
Status : **Not Answered**
Chosen Option : **--**

Q.24 A body moves freely in a uniform gravitational field. The trajectory lies on which of the following curves in phase space?

1. Straight line in phase space
2. Parabola in phase space
3. Hyperbola in phase space
4. Ellipse in phase space

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **7720331043**
Option 1 ID : **7720334169**
Option 2 ID : **7720334170**
Option 3 ID : **7720334171**
Option 4 ID : **7720334172**
Status : **Not Answered**
Chosen Option : **--**

Q.25 Which of the following is a solution to $u_x + x^2 u_y = 0$ with $u(x,0) = e^x$?

1. e^x
2. $e^{(x^3+y)^{1/3}}$
3. $e^{(x^3-3y)^{1/3}}$
4. $(x^2y+1)e^x$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **7720331038**
Option 1 ID : **7720334149**
Option 2 ID : **7720334150**
Option 3 ID : **7720334151**
Option 4 ID : **7720334152**
Status : **Not Answered**
Chosen Option : **--**

Q.26 Consider the differential equation

$$x^2y'' - 2x(x+1)y' + 2(x+1)y = 0.$$

If a polynomial is a solution then the degree of the polynomial is equal to

1. 1
2. 2
3. 3
4. 4

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **7720331039**
Option 1 ID : **7720334153**
Option 2 ID : **7720334154**
Option 3 ID : **7720334155**
Option 4 ID : **7720334156**
Status : **Not Answered**
Chosen Option : **--**

Q.27 Consider continuous solutions f of the following integral equation in $[0,1]$.

$$f^2(t) = 1 + 2\int_0^t f(s)\, ds, \quad \forall t \in [0,1].$$

Which of the following statements is true?

1. There is no solution
2. There is exactly one solution
3. There are exactly two solutions
4. There are more than two solutions

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **7720331042**
Option 1 ID : **7720334165**
Option 2 ID : **7720334166**
Option 3 ID : **7720334167**
Option 4 ID : **7720334168**
Status : **Not Answered**
Chosen Option : **--**

Q.28 The Euler equations satisfied by the extremals of the functional

$$J(y) = \int_0^5 [y^2 + x^3 y']dx$$

define a solution curve in the (x, y) - plane which is

1. linear
2. quadratic
3. cubic
4. trigonometric

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **7720331041**
Option 1 ID : **7720334161**
Option 2 ID : **7720334162**
Option 3 ID : **7720334163**
Option 4 ID : **7720334164**
Status : **Not Answered**
Chosen Option : **--**

Q.29 Let $X_0 = 0$ and for $k \geq 1$ let X_k be a random variable with Binomial$(k, \frac{1}{2})$ distribution. Let N be a Poisson random variable with mean 1. Assume that for every $k \geq 1$, X_k and N are independent and set $Y = X_N$. Given that $Y = 3$, what is the probability that $N = 3$?

1. $\frac{1}{6}e^{-1}$
2. $e^{-\frac{1}{2}}$
3. $\frac{1}{6}e^{-\frac{1}{2}}$
4. $\frac{1}{48}e^{-\frac{1}{2}}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **7720331045**
Option 1 ID : **7720334177**
Option 2 ID : **7720334178**
Option 3 ID : **7720334179**
Option 4 ID : **7720334180**
Status : **Not Answered**
Chosen Option : **--**

Q.30 Let $X_1, X_2, \ldots$ be i.i.d. Normal random variables with mean 2 and variance 3. Let N be a Poisson random variable with mean 4 that is independent of $\{X_1, X_2, \ldots\}$. Let $Y = X_1 + \cdots + X_N$ if $N \geq 1$ and $Y = 0$ if $N = 0$. What is the variance of Y?

1. 12
2. 16
3. 20
4. 28

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **7720331047**
Option 1 ID : **7720334185**
Option 2 ID : **7720334186**
Option 3 ID : **7720334187**
Option 4 ID : **7720334188**
Status : **Not Answered**
Chosen Option : **--**

Q.31

A sample of size n is to be drawn from a population of N households to estimate the proportion of the households which have more than one earning member. Let

$0 < n < N$.

Sample I is drawn using Simple Random Sampling without replacement.

Sample II is drawn using Simple Random Sampling with replacement.

Let p_1 and p_2 denote the sample proportions in samples I and II respectively. Let

$\sigma_i^2 = \text{Var}(p_i)$ for $i = 1,2$. Which of the following statements is true?

1. p_1 is an unbiased estimate of the population proportion but p_2 is not
2. p_2 is an unbiased estimate of the population proportion but p_1 is not
3. $\sigma_1^2 < \sigma_2^2$
4. $\sigma_2^2 < \sigma_1^2$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **7720331054**
Option 1 ID : **7720334213**
Option 2 ID : **7720334214**
Option 3 ID : **7720334215**
Option 4 ID : **7720334216**
Status : **Not Answered**
Chosen Option : **--**

Q.32 Suppose that X follows a distribution with probability density function

$f_\theta(x) \propto x^{\theta-1}(1-x)^{\theta-1};\ 0 < x < 1,\ \theta > 0$. The uniformly most powerful critical region for testing $H_0: \theta = 1$ against $H_1: \theta > 1$ based on a single observation is of the form

1. $(1-\alpha, 1]$
2. $[0, \alpha)$
3. $(\frac{1-\alpha}{2}, \frac{1+\alpha}{2})$
4. $[0, \frac{\alpha}{2}) \cup (1-\frac{\alpha}{2}, 1]$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **7720331049**
Option 1 ID : **7720334193**
Option 2 ID : **7720334194**
Option 3 ID : **7720334195**
Option 4 ID : **7720334196**
Status : **Not Answered**
Chosen Option : **--**

Q.33 Consider a linear regression model of the form $y_i = \alpha + \beta x_i + \epsilon_i$ based on the data $\{(x_i, y_i): i = 1,2,\ldots,50\}$ (here ϵ_i are the error terms). Assume that not all x_i are the same. Which of the following is an admissible value of the *leverage* of the 11^{th} observation?

1. 0
2. 0.01
3. 0.1
4. 1.1

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**7720331052**
Option 1 ID :	**7720334205**
Option 2 ID :	**7720334206**
Option 3 ID :	**7720334207**
Option 4 ID :	**7720334208**
Status :	**Not Answered**
Chosen Option :	**--**

Q.34 Let X be a uniform $(0,1)$ random variable. Suppose that given X, the random variable Y is uniform on $(0, X)$. Given X and Y, the random variable Z is uniform on $(Y, 1)$. What is the value of $E(Z)$?

1. 1/8
2. 3/8
3. 5/8
4. 1/2

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MCQ**
Question ID :	**7720331044**
Option 1 ID :	**7720334173**
Option 2 ID :	**7720334174**
Option 3 ID :	**7720334175**
Option 4 ID :	**7720334176**
Status :	**Not Answered**
Chosen Option :	**--**

Q.35

Let (N_t^1) and (N_t^2) be two independent Poisson processes with intensities a, b respectively, with $a > b$. Let $M_t = \max\{N_t^1, N_t^2\}$ and $X_t = \max(N_t^1 - N_t^2, 0)$. Then which of the following is true?

1. X is a Poisson process with intensity $a - b$
2. X is a birth-and-death process with birth-rate a and death- rate b
3. M is a Poisson process with intensity $\max(a, b)$
4. M is a pure birth process with birth-rate $a + b$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **7720331046**
Option 1 ID : **7720334181**
Option 2 ID : **7720334182**
Option 3 ID : **7720334183**
Option 4 ID : **7720334184**
Status : **Not Answered**
Chosen Option : **--**

Q.36 Consider the following Linear Programming Problem.

Maximise $4x + 5y$
subject to

$$2x + 3y \leq 14$$

$$x + 2y \leq 9$$

$$x + y \leq 6$$

$$x \geq 0,\ y \geq 0$$

What is the optimal value of the objective function?

1. 24
2. 26
3. 27
4. 25

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **7720331055**
Option 1 ID : **7720334217**
Option 2 ID : **7720334218**
Option 3 ID : **7720334219**
Option 4 ID : **7720334220**
Status : **Answered**
Chosen Option : **1**

Q.37 Let $X_1, X_2, \ldots, X_n$ be i.i.d. $N(\theta, 1)$ random variables. Suppose the prior distribution of θ is $N(0, \sigma^2)$. Suppose we have squared error loss function. Let $\bar{X} = \frac{\sum_{i=1}^{n} X_i}{n}$. Then, the Bayes estimator of θ is

1. $\frac{\bar{X}}{1+\sigma^2}$
2. $\bar{X}$
3. $\frac{n\bar{X}\sigma^2}{1+n\sigma^2}$
4. $\frac{n(\bar{X}+\sigma^2)}{1+n\sigma^2}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **7720331050**
Option 1 ID : **7720334197**
Option 2 ID : **7720334198**
Option 3 ID : **7720334199**
Option 4 ID : **7720334200**
Status : **Not Answered**
Chosen Option : **--**

Q.38 Suppose that $(X_1, Y_1), (X_2, Y_2), \ldots, (X_n, Y_n)$ are independent and they have a common distribution, which is uniform on the triangle with vertices $(0,0)$, $(\theta, 0)$ and $(0, \theta)$, where $\theta > 0$. A sufficient statistic for θ is

1. $\max_{1 \le i \le n} X_i + \max_{1 \le i \le n} Y_i$
2. $\max_{1 \le i \le n} (X_i + Y_i)$
3. $\max_{1 \le i \le n} |X_i - Y_i|$
4. $\max\{\max_{1 \le i \le n} X_i, \max_{1 \le i \le n} Y_i\}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **7720331053**
Option 1 ID : **7720334209**
Option 2 ID : **7720334210**
Option 3 ID : **7720334211**
Option 4 ID : **7720334212**
Status : **Not Answered**
Chosen Option : **--**

Q.39 Let X be a $p \times p$ matrix valued random variable having the Wishart distribution with parameters Σ (variance matrix) and m (degrees of freedom). Which of the following is necessarily true?

($A_{i,j}$ denotes the entry in i^{th} row and j^{th} column of a matrix A.)

1. $cX_{1,2}$ has Chi-square distribution with $m - p + 1$ degrees of freedom for a suitable constant c
2. $cX_{1,1}$ has Chi-square distribution with $m - p + 1$ degrees of freedom for a suitable constant c
3. $Y = AX$ has the Wishart distribution with variance matrix $A\Sigma A^T$ and degrees of freedom m for any non-singular $p \times p$ matrix A
4. $k \times k$ submatrix of X also has a Wishart distribution for $k < p$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **7720331051**
Option 1 ID : **7720334201**
Option 2 ID : **7720334202**
Option 3 ID : **7720334203**
Option 4 ID : **7720334204**
Status : **Not Answered**
Chosen Option : **--**

Q.40 Let $X_1, X_2, \ldots, X_n$ be i.i.d $N(\theta, \sigma^2)$ random variables where σ^2 is known. Let $\bar{X} = \frac{\sum_{i=1}^{n} X_i}{n}$. Then Minimum Variance Unbiased Estimator of $e^{2\theta}$ is given by

1. $\exp(2\bar{X} - \frac{\sigma^2}{n})$
2. $\exp(2\bar{X} - \frac{4\sigma^2}{n})$
3. $\exp(2\bar{X} - \frac{2\sigma^2}{n})$
4. $\exp(2n\bar{X} - \frac{2\sigma^2}{n})$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MCQ**
Question ID : **7720331048**
Option 1 ID : **7720334189**
Option 2 ID : **7720334190**
Option 3 ID : **7720334191**
Option 4 ID : **7720334192**
Status : **Not Answered**
Chosen Option : **--**

Section : **Part C Mathematical Sciences**

Q.1 Which of the following functions are uniformly continuous on $(0,1)$?

1. $\frac{1}{x}$
2. $\sin\frac{1}{x}$
3. $x\sin\frac{1}{x}$
4. $\frac{\sin x}{x}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**7720331059**
Option 1 ID :	**7720334233**
Option 2 ID :	**7720334234**
Option 3 ID :	**7720334235**
Option 4 ID :	**7720334236**
Status :	**Answered**
Chosen Option :	**1,2,3,4**

Q.2 A 100×100 matrix $A = (a_{i,j})$ is such that $a_{i,j} = i$ if $i + j = 101$ and $a_{i,j} = 0$ otherwise. Which of the following statements are true about A?

1. A is similar to a diagonal matrix over $\mathbb{R}$
2. A is not similar to a diagonal matrix over $\mathbb{C}$
3. One of the eigenvalues of A is 10
4. None of the real eigenvalues of A exceeds 51

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**7720331070**
Option 1 ID :	**7720334277**
Option 2 ID :	**7720334278**
Option 3 ID :	**7720334279**
Option 4 ID :	**7720334280**
Status :	**Not Answered**
Chosen Option :	**--**

Q.3

Let A be a 3×3 nilpotent matrix. Which of the following statements are necessarily true?

1. $(I + A)^n = I$ for some $n > 0$ where I is the identity matrix
2. The column space of A is $\{0\}$
3. The eigenvalues of A are roots of 1
4. A^3 is diagonalizable

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **7720331069**
Option 1 ID : **7720334273**
Option 2 ID : **7720334274**
Option 3 ID : **7720334275**
Option 4 ID : **7720334276**
Status : **Answered**
Chosen Option : **1,3**

Q.4

Let $C_0(\mathbb{R})$ be the space of all continuous functions on $\mathbb{R}$ such that $\lim_{x\to\pm\infty} f(x) = 0$. Let $C_0(\mathbb{R})$ be equipped with $\|\cdot\|$, the norm of uniform convergence. Let (f_n) be a sequence in $C_0(\mathbb{R})$ and $f \in C_0(\mathbb{R})$. Which of the following statements are correct?

1. If $f_n \to f$ uniformly on compact sets then $\| f_n - f \| \to 0$
2. If $\| f_n - f \| \to 0$ then $f_n \to f$ uniformly on compact sets
3. $f_n \to f$ uniformly on compact sets if and only if $\| f_n - f \| \to 0$
4. Neither of the two statements "$f_n \to f$ uniformly on compact sets" and "$\| f_n - f \| \to 0$" imply the other

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **7720331065**
Option 1 ID : **7720334257**
Option 2 ID : **7720334258**
Option 3 ID : **7720334259**
Option 4 ID : **7720334260**
Status : **Answered**
Chosen Option : **1,2,3**

Q.5 Let $\{v_1, v_2, v_3\}$ be an orthonormal basis of $\mathbb{R}^3$. Let V be the 3×3 matrix whose columns are v_1, v_2, v_3. Which of the following statements are necessarily true?

1. $VV^T = I$
2. $V^TV = I$
3. $V = V^T$
4. Determinant of V is not zero

Options
1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **7720331071**
Option 1 ID : **7720334281**
Option 2 ID : **7720334282**
Option 3 ID : **7720334283**
Option 4 ID : **7720334284**
Status : **Answered**
Chosen Option : **3,4**

Q.6 Let $\mathbb{N} = \{1,2,3, \dots\}$ be the set of natural numbers. Which of the following functions from $\mathbb{N} \times \mathbb{N}$ to $\mathbb{N}$ are injective?

1. $f_1(m,n) = 2^m 3^n$
2. $f_2(m,n) = mn + m + n$
3. $f_3(m,n) = m^2 + n^3$
4. $f_4(m,n) = m^2 n^3$

Options
1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **7720331056**
Option 1 ID : **7720334221**
Option 2 ID : **7720334222**
Option 3 ID : **7720334223**
Option 4 ID : **7720334224**
Status : **Answered**
Chosen Option : **1,2,3,4**

Q.7 Let $W_1 = \left\{ \begin{bmatrix} a & b \\ -b & 2a \end{bmatrix} : a, b \in \mathbb{R} \right\}$ and $W_2 = \left\{ \begin{bmatrix} a & b \\ -b & -a \end{bmatrix} : a, b \in \mathbb{R} \right\}$ be two subspaces of $\mathbb{M}_2(\mathbb{R})$. Which of the following statements are true?

1. $W_1 \cap W_2 = \left\{ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \right\}$
2. $\left\{ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \right\}$ is a proper subset of $W_1 \cap W_2$
3. $W_1 + W_2 = \mathbb{M}_2(\mathbb{R})$
4. $W_1 + W_2$ is a proper subset of $\mathbb{M}_2(\mathbb{R})$

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**7720331068**
Option 1 ID :	**7720334269**
Option 2 ID :	**7720334270**
Option 3 ID :	**7720334271**
Option 4 ID :	**7720334272**
Status :	**Answered**
Chosen Option :	**1,3,4**

Q.8 Let $f(x,y) = (u(x,y), v(x,y)) : \mathbb{R}^2 \to \mathbb{R}^2$ be a differentiable function. Let A denote the matrix of the derivative of f at the origin $(0,0)$ with respect to the standard basis of $\mathbb{R}^2$. Assume $f(y,-x) = (v(x,y), -u(x,y))$ for all $(x,y) \in \mathbb{R}^2$. Which of the following statements are possibly true?

1. $A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
2. $A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$
3. $A = \begin{pmatrix} 1 & 2 \\ -1 & -2 \end{pmatrix}$
4. $A = \begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**7720331063**
Option 1 ID :	**7720334249**
Option 2 ID :	**7720334250**
Option 3 ID :	**7720334251**
Option 4 ID :	**7720334252**
Status :	**Not Answered**
Chosen Option :	**--**

Q.9 Let $f(x) = e^x$ for $x \in \mathbb{R}$. Which of the following statements are correct?

1. There is a real $C > 0$ such that $|f(x) - 1 - x| \leq Cx^2$ for all $x \in \mathbb{R}$
2. There is a real $C > 0$ such that $|f(x) - 1 - x - \frac{x^2}{2}| \leq C|x|^3$ for all $x \in [-1,1]$
3. There is a real $C > 0$ such that $|f(x) - 1 - x - \frac{x^2}{2} - \frac{x^3}{3!}| \leq Cx^4$ for all $x \in \mathbb{R}$
4. There is a real $C > 0$ such that $|f(x) - 1 - x - \frac{x^2}{2} - \frac{x^3}{3!}| \leq Cx^4$ for all $x \in [-1,1]$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **7720331058**
Option 1 ID : **7720334229**
Option 2 ID : **7720334230**
Option 3 ID : **7720334231**
Option 4 ID : **7720334232**
Status : **Not Answered**
Chosen Option : **--**

Q.10 Let $f : [0,1] \to (0,1)$ be a function. Which of the following statements are FALSE?

1. If f is onto, then f is continuous
2. If f is continuous, then f is not onto
3. If f is one-to-one, then f is continuous
4. If f is continuous, then f is not one-to-one

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **7720331072**
Option 1 ID : **7720334285**
Option 2 ID : **7720334286**
Option 3 ID : **7720334287**
Option 4 ID : **7720334288**
Status : **Answered**
Chosen Option : **2,4**

Q.11 For any two non-negative integers n, k define $f_{n,k}(x)$ on $[0,1]$ by

$$f_{n,k}(x) = \begin{cases} x^n \sin(\frac{\pi}{2x}) - x^k & x \neq 0 \\ 0 & x = 0. \end{cases}.$$

In which of the following cases is the function $f_{n,k}$, a function of bounded variation?

1. for all $n \geq 1$ and for all $k \geq 0$
2. for all $n \geq 1$ and $k = 0$
3. for all $n \geq 0$ and for all $k \geq 2$
4. for all $n \geq 2$ and for all $k \geq 0$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **7720331062**
Option 1 ID : **7720334245**
Option 2 ID : **7720334246**
Option 3 ID : **7720334247**
Option 4 ID : **7720334248**
Status : **Answered**
Chosen Option : **1**

Q.12 Which of the following functions f admit an inverse in an open neighbourhood of the point $f(p)$?

1. For $p = (1,0)$ and $f(x,y) = (x^3 \exp y + y - 2x, 2xy + 2x)$
2. For $p = (1,\pi)$ and $f(r,\theta) = (r\cos\theta, r\sin\theta)$
3. For $p = 0$ and $f(x) = \begin{cases} x + 2x^2 \sin\frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$
4. For $p = 0$ and $f(x) = \begin{cases} x^2 \sin\frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **7720331064**
Option 1 ID : **7720334253**
Option 2 ID : **7720334254**
Option 3 ID : **7720334255**
Option 4 ID : **7720334256**
Status : **Answered**
Chosen Option : **1,2**

Q.13 Let $M \in \mathbb{M}_n(\mathbb{R})$ with $M \neq \mathbf{0}, I_n$ but $M^2 = M$. Which of the following statements are true?

1. $\text{Null}(M)$ is the eigenspace of M corresponding to the eigenvalue 0
2. Let $\mathbf{x} \in \text{Col}(M)$ with $\mathbf{x} \neq \mathbf{0}$. Then $\mathbf{x}$ is an eigenvector of M corresponding to the eigenvalue 1
3. Let $\mathbf{x} \notin \text{Null}(M)$. Then $\mathbf{x}$ is an eigenvector of M corresponding to the eigenvalue 1
4. $\mathbb{R}^n = \text{Col}(M) + \text{Null}(M)$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **7720331067**
Option 1 ID : **7720334265**
Option 2 ID : **7720334266**
Option 3 ID : **7720334267**
Option 4 ID : **7720334268**
Status : **Answered**
Chosen Option : **1,3,4**

Q.14 Consider the identity function $f(x) = x$ on $I := [0,1]$. Let P_n be the partition that divides I into n equal parts. If $U(f, P_n)$ and $L(f, P_n)$ are the upper and lower Riemann sums, respectively, and $A_n = U(f, P_n) - L(f, P_n)$ then

1. $lim_{n\to\infty} nA_n = 0$
2. $\sum_{n=1}^{\infty} A_n$ is convergent
3. A_n is strictly monotonically decreasing
4. $\sum_{n=1}^{\infty} A_n A_{n+1} = 1$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **7720331061**
Option 1 ID : **7720334241**
Option 2 ID : **7720334242**
Option 3 ID : **7720334243**
Option 4 ID : **7720334244**
Status : **Answered**
Chosen Option : **1,2,4**

Q.15 Let $\{y_1, y_2, y_3, y_4\}$ be an orthonormal basis of $\mathbb{R}^4$. Which of the following are orthonormal bases?

1. $\{y_1 + y_2, y_1 - y_2, y_3, y_4\}$
2. $\{y_3, y_4, -y_1, y_2\}$
3. $\left\{\frac{y_1+y_2}{2}, \frac{y_1-y_2}{2}, y_3, y_4\right\}$
4. $\left\{\frac{3y_1+4y_2}{5}, \frac{4y_1-3y_2}{5}, y_3, y_4\right\}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**7720331073**
Option 1 ID :	**7720334289**
Option 2 ID :	**7720334290**
Option 3 ID :	**7720334291**
Option 4 ID :	**7720334292**
Status :	**Answered**
Chosen Option :	**1,3,4**

Q.16

Let A be an $m \times n$ matrix. let $A(1,:), A(:,1)$ and $A(1,1)$ be the matrices obtained from A by deleting row 1 deleting column 1 and deleting both row 1 and column 1 respectively. Which of the following hold?

1. $(\text{rank } A) - 2 \leq \text{rank } A(1,1) \leq \text{rank } A$
2. $\text{rank } A(1,:) = \text{rank } A(:,1)$
3. $\text{rank } A = \text{rank } A(1,:) = \text{rank } A(:,1)$, then $\text{rank } A = \text{rank} A(1,1)$
4. $\text{rank } A(1,:) + \text{rank } A(:,1) + 2 \geq 2 \text{ rank } A$

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**7720331066**
Option 1 ID :	**7720334261**
Option 2 ID :	**7720334262**
Option 3 ID :	**7720334263**
Option 4 ID :	**7720334264**
Status :	**Answered**
Chosen Option :	**1,4**

Q.17 Let $\{x_n\}$ be a sequence of positive real numbers. Which of the following statements are true?

1. If the two subsequences $\{x_{2n}\}$ and $\{x_{2n+1}\}$ converge, then the sequence $\{x_n\}$ converges
2. If $\{(-1)^n x_n\}$ converges, then the sequence $\{x_n\}$ converges
3. If $\lim_{n\to\infty} \frac{x_{n+1}}{x_n}$ exists then $\{(x_n)^{1/n}\}$ is bounded
4. If the sequence $\{x_n\}$ is unbounded then every subsequence is unbounded

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**7720331057**
Option 1 ID :	**7720334225**
Option 2 ID :	**7720334226**
Option 3 ID :	**7720334227**
Option 4 ID :	**7720334228**
Status :	**Answered**
Chosen Option :	**1,2,3**

Q.18 Let x, y be real numbers such that $0 < y \leq x$ and let n be a positive integer. Which of the following statements are true?

1. $ny^{n-1}(x-y) \leq x^n - y^n$
2. $nx^{n-1}(x-y) \leq x^n - y^n$
3. $ny^{n-1}(x-y) \geq x^n - y^n$
4. $nx^{n-1}(x-y) \geq x^n - y^n$

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**7720331060**
Option 1 ID :	**7720334237**
Option 2 ID :	**7720334238**
Option 3 ID :	**7720334239**
Option 4 ID :	**7720334240**
Status :	**Answered**
Chosen Option :	**3,4**

Q.19

Let $\mathbb{N} = \{1,2 \ldots\ldots\}$ denote the set of positive integers. For $n \in \mathbb{N}$, let

$$A_n = \{(x, y, z) \in \mathbb{N}^3 : x^n + y^n = z^n \text{ and } \quad z < n\}.$$

Let $F(n)$ be the cardinality of the set A_n. Which of the following statements are true?

1. $F(n)$ is always finite for $n \geq 3$
2. $F(2) = \infty$
3. $F(n) = 0$ for all n
4. $F(n)$ is non zero for some $n > 2$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **7720331078**
Option 1 ID : **7720334309**
Option 2 ID : **7720334310**
Option 3 ID : **7720334311**
Option 4 ID : **7720334312**
Status : **Answered**
Chosen Option : **1,4**

Q.20

Which of the following statements are true for $\alpha \in \mathbb{R}$?

1. If α^3 is algebraic over $\mathbb{Q}$, then α is algebraic over $\mathbb{Q}$
2. α could be algebraic over $\mathbb{Q}[\sqrt{2}]$ but may not be algebraic over $\mathbb{Q}$
3. α need not be algebraic over any subfield of $\mathbb{R}$
4. There is an α which is not algebraic over $\mathbb{Q}[\sqrt{-1}]$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **7720331082**
Option 1 ID : **7720334325**
Option 2 ID : **7720334326**
Option 3 ID : **7720334327**
Option 4 ID : **7720334328**
Status : **Answered**
Chosen Option : **2,3,4**

Q.21

For a positive integer n, let $\varphi(n)$ be the Euler's φ-function. Which of the following statements are true for $n > 3$?

1. $\varphi(n)$ can never divide n
2. $\varphi(n)|n \implies$ there exist integers x, y such that $nx + 6y = 1$
3. $\varphi(n)|n \implies n$ can have at most two distinct prime divisors
4. $\varphi(n) \mid \varphi(nk)$ for every positive integer k

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **7720331079**
Option 1 ID : **7720334313**
Option 2 ID : **7720334314**
Option 3 ID : **7720334315**
Option 4 ID : **7720334316**
Status : **Answered**
Chosen Option : **3,4**

Q.22 Let $\mathbb{D} \subset \mathbb{C}$ be the open unit disc. Consider the family $\mathcal{F}$ of all holomorphic maps $f: \mathbb{D} \to \mathbb{D}$ such that $f(0) = 1/2$. For $f \in \mathcal{F}$, the possible values of $|f'(0)|$ are

1. $7/8$
2. $5/6$
3. $3/4$
4. 1

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **7720331077**
Option 1 ID : **7720334305**
Option 2 ID : **7720334306**
Option 3 ID : **7720334307**
Option 4 ID : **7720334308**
Status : **Not Answered**
Chosen Option : **--**

Q.23

Let $p > 2019$ be a prime number. Consider the polynomial

$$f(x) = (x^2 - 3)(x^2 - 673)(x^2 - 2019)$$

How many roots can f possibly have in the finite field $\mathbb{F}_p$?

1. 0
2. 2
3. 3
4. 6

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **7720331083**
Option 1 ID : **7720334329**
Option 2 ID : **7720334330**
Option 3 ID : **7720334331**
Option 4 ID : **7720334332**
Status : **Not Answered**
Chosen Option : **--**

Q.24 Let G be the cyclic group of order 8 and $H = S_5$ be the permutation group of 5 elements. Which of the following statements are necessarily true?

1. There exists no nontrivial group homomorphism from G to H
2. There exists no injective group homomorphism from G to H
3. There exists no surjective group homomorphism from G to H
4. There are more than 20 different group homomorphisms from G to H

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **7720331081**
Option 1 ID : **7720334321**
Option 2 ID : **7720334322**
Option 3 ID : **7720334323**
Option 4 ID : **7720334324**
Status : **Not Answered**
Chosen Option : **--**

Q.25

Let $X = \mathbb{N} \cup \{\infty, -\infty\}$. Let τ be the topology on X consisting of subsets U of X such that either $U \subset \mathbb{N}$ or $X \setminus U$ is finite. Let $A = \mathbb{N} \cup \{\infty\}$ and $B = \mathbb{N} \cup \{-\infty\}$.

Which of the following subsets are compact?

1. A
2. $X \setminus A$
3. $A \cup B$
4. $A \cap B$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **7720331085**
Option 1 ID : **7720334337**
Option 2 ID : **7720334338**
Option 3 ID : **7720334339**
Option 4 ID : **7720334340**
Status : **Not Answered**
Chosen Option : **--**

Q.26

For $z \in \mathbb{C}$, let $\Re z$ denotes its real part. Let f be an entire function satisfying

$|f(z)| \leq |z||\Re z|$ on $\mathbb{C}$. Which of the following statements are true?

1. $f(0) = 0$
2. $f'(0) = 0$
3. The only entire function satisfying the given property is $f(z) \equiv 0$
4. There exists a non-constant entire function satisfying the given property

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **7720331076**
Option 1 ID : **7720334301**
Option 2 ID : **7720334302**
Option 3 ID : **7720334303**
Option 4 ID : **7720334304**
Status : **Not Answered**
Chosen Option : **--**

Q.27

Fix a positive real number c. Consider the locus of all points $z \in \mathbb{C}$ such that

$\left|\frac{z-i}{z+i}\right| = c.$ Which of the following statements are true?

1. If $c > 1$, the locus is a circle centered on the imaginary axis
2. If $c < 1$, the locus is a circle centered on the real axis
3. If $c = 1$, the locus is a straight line parallel to the imaginary axis
4. If $c = 1$, the locus is a straight line not passing through the origin

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**7720331075**
Option 1 ID :	**7720334297**
Option 2 ID :	**7720334298**
Option 3 ID :	**7720334299**
Option 4 ID :	**7720334300**
Status :	**Not Answered**
Chosen Option :	**--**

Q.28 Which of the following statements are true?

1. Any compact topological space is metrizable
2. Any continuous image of a Hausdorff topological space is Hausdorff
3. If $f: X \to Y$ is continuous and one-to-one, and Y is Hausdorff, then X is Hausdorff
4. Intersection of two connected subsets of $\mathbb{R}$ with the usual topology is either empty or connected

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**7720331084**
Option 1 ID :	**7720334333**
Option 2 ID :	**7720334334**
Option 3 ID :	**7720334335**
Option 4 ID :	**7720334336**
Status :	**Answered**
Chosen Option :	**2,3,4**

Q.29

For positive integers m and n, let gcd(m, n) denote their greatest common divisor.
Let $m > n$ be such that gcd$(m, n) = 1$. Which of the following statements are true?

1. gcd$(m-n, m+n) = 1$
2. gcd$(m-n, m+n)$ can have a prime divisor
3. There exists integers x, y such that $nx - my = 3$
4. gcd$(m-n, m+n)$ can be an odd prime

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**7720331080**
Option 1 ID :	**7720334317**
Option 2 ID :	**7720334318**
Option 3 ID :	**7720334319**
Option 4 ID :	**7720334320**
Status :	**Answered**
Chosen Option :	**2,3**

Q.30

Consider the set $S := \{\exp(2\pi i\theta) : \theta \text{ is a rational number}\}$. For each $z \in S$ the set $\{z^{n!} : n \text{ is a positive integer}\}$ is

1. countable
2. countably infinite
3. uncountable
4. finite

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**7720331074**
Option 1 ID :	**7720334293**
Option 2 ID :	**7720334294**
Option 3 ID :	**7720334295**
Option 4 ID :	**7720334296**
Status :	**Not Answered**
Chosen Option :	**--**

Q.31

Which of the following are solutions of the Laplace equation $u_{xx} + u_{yy} = 0$ in the unit disk $D = \{(x, y): x^2 + y^2 < 1\}$?

1. $x^5 + 2x^2y^3 - y^5$
2. $x^2 + 2xy - y^2$
3. $\cos(y)e^x + \sin(x)e^y$
4. $\frac{1+x}{1+2x+x^2+y^2}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **7720331089**
Option 1 ID : **7720334353**
Option 2 ID : **7720334354**
Option 3 ID : **7720334355**
Option 4 ID : **7720334356**
Status : **Not Answered**
Chosen Option : **--**

Q.32

Let x and y be continuously differentiable functions on $[0, \infty)$ that satisfy the respective initial value problems (ODEs)

$$\frac{dx}{dt} + (\sin t - 1)x = \log(1 + t), \quad x(0) = 1;$$
$$\frac{dy}{dt} + (\sin t - 1)y = t, \quad y(0) = 2.$$

Define $z(t) = y(t) - x(t)$ for $t \geq 0$. Which of the following statements are true?

1. $z(1) \leq 1$
2. $z(2) > z(1)$
3. $z(1) > 1$
4. $z(2) \leq z(1)$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **7720331087**
Option 1 ID : **7720334345**
Option 2 ID : **7720334346**
Option 3 ID : **7720334347**
Option 4 ID : **7720334348**
Status : **Not Answered**
Chosen Option : **--**

Q.33 Let $K(x,y)$ be a kernel in $[0,1]\times[0,1]$, defined as $K(x,y)=\sin(xy)$. The integral equation $u(x)=\sin x+\int_0^1 K(x,y)u(y)\,dy$

1. is uniquely solvable and the solution is differentiable
2. has more than one differentiable solution
3. has no solution
4. is solvable and the solution is not differentiable

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **7720331096**
Option 1 ID : **7720334381**
Option 2 ID : **7720334382**
Option 3 ID : **7720334383**
Option 4 ID : **7720334384**
Status : **Not Answered**
Chosen Option : **--**

Q.34 Consider the numerical integration formula

$$\int_{-1}^{1} g(x)\,dx \approx g(\alpha)+g(-\alpha),$$

where $\alpha=(0.2)^{1/4}$. Which of the following statements are true?

1. The integration formula is exact for polynomials of the form $a+bx$, for all $a,b\in\mathbb{R}$
2. The integration formula is exact for polynomials of the form $a+bx+cx^2$, for all $a,b,c\in\mathbb{R}$
3. The integration formula is exact for polynomials of the form $a+bx+cx^2+dx^3$, for all $a,b,c,d\in\mathbb{R}$
4. The integration formula is exact for polynomials of the form $a+bx+cx^3+dx^4$, for all $a,b,c,d\in\mathbb{R}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **7720331091**
Option 1 ID : **7720334361**
Option 2 ID : **7720334362**
Option 3 ID : **7720334363**
Option 4 ID : **7720334364**
Status : **Not Answered**
Chosen Option : **--**

Q.35 Consider the partial differential equation (PDE)

$$x(\frac{\partial u}{\partial x})^2 + y(\frac{\partial u}{\partial y})^2 + (x+y)\frac{\partial u}{\partial x}\frac{\partial u}{\partial y} - u(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}) + 1 = 0.$$

Which of the following statements are true?

1. The general solution of the PDE can be expressed in the form $u(x,y) = ax + by + \frac{1}{a+b}$, where a and b are arbitrary constants.
2. The general solution of the PDE can be expressed in the form $u(x,y) = f(ax+by) + \frac{1}{a+b}$, where a and b are arbitrary constants and f is an arbitrary function.
3. The Charpit's equations are
$$\frac{dx}{p^2+pq} = \frac{dy}{q^2+pq} = \frac{du}{p(p^2+pq)+q(p^2+pq)} = \frac{dp}{0} = \frac{dq}{0}$$
4. The Charpit's equations are
$$\frac{dx}{2px+(x+y)q-u} = \frac{dy}{2qy+(x+y)p-u}$$
$$= \frac{du}{p(2px+(x+y)q-u)+q(2qy+(x+y)p-u)} = \frac{dp}{0} = \frac{dq}{0}$$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **7720331090**
Option 1 ID : **7720334357**
Option 2 ID : **7720334358**
Option 3 ID : **7720334359**
Option 4 ID : **7720334360**
Status : **Not Answered**
Chosen Option : **--**

Q.36 Consider a body of unit mass moving under an attractive central force. At a certain radius R, the body moves in a circular orbit. Which of the following are true?

1. The force must be equal to $-\frac{L^2}{R^3}$ where L is the angular momentum of the body
2. The force can be any strictly negative valued function of R
3. The force can only be of the inverse square law form
4. The force cannot be of the form $-kR$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **7720331097**
Option 1 ID : **7720334385**
Option 2 ID : **7720334386**
Option 3 ID : **7720334387**
Option 4 ID : **7720334388**
Status : **Not Answered**
Chosen Option : **--**

Q.37 Let $f: \mathbb{R}^2 \to \mathbb{R}$ be a locally Lipschitz function. Consider the system of ODEs given by $\dot{x}_1 = \sin(e^{x_2}), \dot{x}_2 = f(x_1, x_2)$ with initial condition $(x_1(0), x_2(0)) = (1,1)$. Which of the following statements is true?

1. There is at most one local solution at time 0
2. There always exists a global solution defined in $[0, \infty)$
3. There might not be any solution around the time 0
4. There is at least one solution around time 0

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **7720331088**
Option 1 ID : **7720334349**
Option 2 ID : **7720334350**
Option 3 ID : **7720334351**
Option 4 ID : **7720334352**
Status : **Not Answered**
Chosen Option : **--**

Q.38

Let $f(x,y) = (y + x(1 - x^2 - y^2), -x + y(1 - x^2 - y^2))$

and consider the ODE $(\dot{x}, \dot{y}) = f(x,y)$ with initial condition $(x(0), y(0)) = (0, \frac{1}{2})$.

Which of the following statements are true?

1. $x^2(t) + y^2(t) \to \infty$ as $t \to \infty$
2. $x^2(t) + y^2(t) \to 0$ as $t \to \infty$
3. $x^2(t) + y^2(t)$ remains bounded as $t \to \infty$
4. $x^2(t) + y^2(t) \to 1$ as $t \to \infty$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **7720331086**
Option 1 ID : **7720334341**
Option 2 ID : **7720334342**
Option 3 ID : **7720334343**
Option 4 ID : **7720334344**
Status : **Not Answered**
Chosen Option : **--**

Q.39

Let S be the set of all continuous functions $f: [0,1] \to [0, \infty)$ that satisfy

$$\int_0^1 x^2 f(x)\, dx = \frac{1}{2}\int_0^1 x f^2(x)\, dx + \frac{1}{8}.$$

Which of the following statements are true?

1. S is an empty set
2. S has at most one element
3. S has at least one element
4. S has more than two elements

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **7720331092**
Option 1 ID : **7720334365**
Option 2 ID : **7720334366**
Option 3 ID : **7720334367**
Option 4 ID : **7720334368**
Status : **Not Answered**
Chosen Option : **--**

Q.40

Let B be the unit ball in $\mathbb{R}^3$ centered at origin. The Euler-Lagrange equation

corresponding to the functional $I(u) = \frac{1}{2}\int_B |\nabla u|^2 dx - \frac{1}{5}\int_B |u|^5 dx$ is

1. $\Delta u = u^4$
2. $\Delta u = -u^4$
3. $\mathbf{det}(D^2 u) = u^4$
4. $\Delta u = -|u|^3 u$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **7720331094**
Option 1 ID : **7720334373**
Option 2 ID : **7720334374**
Option 3 ID : **7720334375**
Option 4 ID : **7720334376**
Status : **Not Answered**
Chosen Option : **--**

Q.41

Let $K(x,y)$ be a kernel in $[0,1] \times [0,1]$, defined as

$$K(x,y) = \begin{cases} x(1-y) & 0 \le x \le y \le 1 \\ y(1-x) & 0 \le y \le x \le 1; \end{cases}$$

and $\mathcal{K}$ be the associated integral operator.

Then $\mathcal{K}: L^2([0,1]) \to L^2([0,1])$

1. is positive definite
2. is self adjoint
3. has a non-zero null space
4. is onto

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **7720331095**
Option 1 ID : **7720334377**
Option 2 ID : **7720334378**
Option 3 ID : **7720334379**
Option 4 ID : **7720334380**
Status : **Not Answered**
Chosen Option : **--**

Q.42

Consider the problem of extremising the functional

$J(y) = \int_1^3 y\,(3x - y)\,dx$ with boundary conditions $y(1) = 1$ and $y(3) = 2$. Which of the following statements are true?

1. There is a unique extremal
2. $y(x) = \frac{3x}{2}$ is an extremal
3. $y(x) = \frac{x}{2} + \frac{1}{2}$ is an extremal
4. There are no extremals

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **7720331093**
Option 1 ID : **7720334369**
Option 2 ID : **7720334370**
Option 3 ID : **7720334371**
Option 4 ID : **7720334372**
Status : **Not Answered**
Chosen Option : **--**

Q.43

$(X_n, n \geq 0)$ is a Markov chain with state space $\{1,2,3,4,5\}$ and transition matrix

$(p_{ij})_{1 \leq i,j \leq 5}$ given by

$$\begin{pmatrix} 1/2 & 0 & 0 & 0 & 1/2 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1/5 & 1/5 & 1/5 & 1/5 & 1/5 \\ 1/3 & 0 & 0 & 0 & 2/3 \end{pmatrix}.$$

Which of the following are true?

1. states 2 and 3 are absorbing
2. state 3 is recurrent
3. $\{1,5\}$ form a recurrent class
4. state 4 is transient

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **7720331100**
Option 1 ID : **7720334397**
Option 2 ID : **7720334398**
Option 3 ID : **7720334399**
Option 4 ID : **7720334400**
Status : **Not Answered**
Chosen Option : **--**

Q.44

Let $X_1, X_2, \dots\dots, X_n$ be a random sample from normal distribution with unknown mean μ and variance 1. Let $\bar{X}$ be the sample mean and $X_{(1)}$ = $\min\{X_1, X_2, \dots, X_n\}$.

Which of the following are true?

1. $\bar{X}$ is complete sufficient for μ
2. $\bar{X}$ is minimal sufficient for μ
3. $\bar{X} - X_{(1)}$ is an ancillary statistic
4. Cov($\bar{X}, X_{(1)}$) = 0

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **7720331111**
Option 1 ID : **7720334441**
Option 2 ID : **7720334442**
Option 3 ID : **7720334443**
Option 4 ID : **7720334444**
Status : **Not Answered**
Chosen Option : **--**

Q.45 Consider the Gauss-Markov Model $\mathbf{Y}_{4\times1} = \mathbf{X}_{4\times4}\boldsymbol{\beta}_{4\times1} + \varepsilon_{4\times1}$, where $\boldsymbol{\beta} = (\beta_1\ \beta_2\ \beta_3\ \beta_4)$ and $\varepsilon \sim N(\mathbf{0}, \sigma^2\mathbf{I})$. Two choices for $\mathbf{X}$ are given below.

Case-1: $\mathbf{X} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

Case-2: $\mathbf{X} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{pmatrix}$

Which of the following statements are true?

1. The contrast $\beta_1 - \beta_2$ is estimable in Case 1, but not in Case 2
2. The contrast $\beta_1 - \beta_2$ is estimable in both cases and $\mathrm{Var}(\hat{\beta}_1 - \hat{\beta}_2)$ is larger in Case 1 than in Case 2
3. The contrast $\beta_1 - \beta_3$ is estimable in Case 1, but not in Case 2
4. The contrast $\beta_1 - \beta_3$ is estimable in both cases and $\mathrm{Var}(\hat{\beta}_1 - \hat{\beta}_3)$ is larger in Case 1 than in Case 2

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **7720331108**
Option 1 ID : **7720334429**
Option 2 ID : **7720334430**
Option 3 ID : **7720334431**
Option 4 ID : **7720334432**
Status : **Not Answered**
Chosen Option : **--**

Q.46

Consider a M/M/1 queue at the steady state with traffic intensity $\rho \in (0,1)$.

Let $f(\rho)$ be the expected waiting time of a customer in the queue. Then

1. f is increasing in ρ
2. f is decreasing in ρ
3. f attains it maximum at $\rho = \frac{1}{2}$
4. f is a smooth function in $(0,1)$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **7720331115**
Option 1 ID : **7720334457**
Option 2 ID : **7720334458**
Option 3 ID : **7720334459**
Option 4 ID : **7720334460**
Status : **Not Answered**
Chosen Option : **--**

Q.47

Suppose X denotes the lifetime of a system and follows the distribution with cumulative distribution function

$$F(t) = 1 - e^{-t^{\gamma}},\ t > 0, \gamma > 0.$$

Let $r(t)$ be the hazard (failure) rate of X. Which of the following are true?

1. $r(t)$ is increasing in t if $\gamma > 1$
2. $r(t)$ is decreasing in t if $\gamma > 1$
3. $r(t)$ is constant in t if $\gamma = 1$
4. $r(t)$ is decreasing in t if $0 < \gamma < 1$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **7720331113**
Option 1 ID : **7720334449**
Option 2 ID : **7720334450**
Option 3 ID : **7720334451**
Option 4 ID : **7720334452**
Status : **Not Answered**
Chosen Option : **--**

Q.48 Let X_1, X_2 be i.i.d. random variables which are uniformly distributed on the interval $(0, \theta)$. Consider the problem of testing $H_0 : \theta = 1$ versus $H_1 : \theta = 2$. Which of the following tests have size ≤ 0.5 and power ≥ 0.5?

1. Reject H_0 if and only if $\max\{X_1, X_2\} \geq 1.6$
2. Reject H_0 if and only if $\min\{X_1, X_2\} \geq 1$
3. Reject H_0 if and only if $X_1 \geq 0.4$
4. Reject H_0 if and only if $X_1 - X_2 \geq 0$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **7720331104**
Option 1 ID : **7720334413**
Option 2 ID : **7720334414**
Option 3 ID : **7720334415**
Option 4 ID : **7720334416**
Status : **Not Answered**
Chosen Option : **--**

Q.49 Let X, Y be random variables with cumulative distribution functions F and G respectively. Assume that F is continuous. Which of the following functions of x are necessarily cumulative distribution functions (CDF)?

1. $\frac{1}{2}(1 - G(-x) + F(x))$
2. $\frac{1}{2}(1 - F(-x) + G(x))$
3. $F(x)G(x)$
4. $(1 - F(x))(1 - G(x))$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **7720331102**
Option 1 ID : **7720334405**
Option 2 ID : **7720334406**
Option 3 ID : **7720334407**
Option 4 ID : **7720334408**
Status : **Not Answered**
Chosen Option : **--**

Q.50

Let $(X_n, n \geq 1)$ be i.i.d. random variables with mean zero and variance one. Which of the following are true as $n \to \infty$?

1. $\frac{X_1+X_2+\cdots+X_n}{\sqrt{n+1}}$ converges in distribution to standard normal
2. $\frac{X_1-X_2+\cdots+(-1)^{n+1}X_n}{\sqrt{n}}$ converges in distribution to standard normal
3. $\frac{X_1+X_2+\cdots+X_n}{n+1}$ converges to zero in probability
4. $\frac{X_1+X_2+\cdots+X_n}{\sqrt{n}}$ converges to zero in probability

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **7720331114**
Option 1 ID : **7720334453**
Option 2 ID : **7720334454**
Option 3 ID : **7720334455**
Option 4 ID : **7720334456**
Status : **Not Answered**
Chosen Option : **--**

Q.51

In a BIBD (Balanced Incomplete Block Design) with v treatments, b blocks, let t_j denote the effect of j^{th} treatment, b_t denote the effect of t^{th} block
$1 \leq j \leq v, 1 \leq t \leq b$

Which of the following statements are necessarily true?

1. t_j is estimable for all $1 \leq j \leq v$
2. $t_j - b_t$ is estimable for all $1 \leq j \leq v, 1 \leq t \leq b$
3. $t_j - t_i$ is estimable for all $1 \leq i < j \leq v$
4. $t_j + t_i$ is estimable for all $1 \leq i < j \leq v$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **7720331112**
Option 1 ID : **7720334445**
Option 2 ID : **7720334446**
Option 3 ID : **7720334447**
Option 4 ID : **7720334448**
Status : **Not Answered**
Chosen Option : **--**

Q.52

Suppose that $X_1, X_2, \dots\dots, X_n$ are independent and identically distributed Poisson (λ) random variables, where $\lambda > 0$ is unknown. Which of the following statements are true?

1. $\frac{1}{n}\sum_{i=1}^{n}(-1)^{X_i}$ is an unbiased estimator of $e^{-2\lambda}$
2. $\frac{1}{n}\sum_{i=1}^{n}(-1)^{X_i}$ is a consistent estimator for $e^{-2\lambda}$
3. $\left(\frac{n-2}{n}\right)^{\sum_{i=1}^{n} X_i}$ is the minimum variance unbiased estimator of $e^{-2\lambda}$
4. $\left(\frac{n-2}{n}\right)^{\sum_{i=1}^{n} X_i}$ is a consistent estimator of $e^{-2\lambda}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **7720331103**
Option 1 ID : **7720334409**
Option 2 ID : **7720334410**
Option 3 ID : **7720334411**
Option 4 ID : **7720334412**
Status : **Not Answered**
Chosen Option : **--**

Q.53

Let $X_1, X_2, \dots\dots, X_n$ be i.i.d. $N(0, \sigma^2)$ random variables. Which of the following are sufficient for σ^2 ?

1. $T_1(X_1, X_2, \dots, X_n) = (X_1, X_2, \dots, X_n)$
2. $T_2(X_1, X_2, \dots, X_n) = (X_1^2, X_2^2, \dots, X_n^2)$
3. $T_3(X_1, X_2, \dots, X_n) = (X_1^2 + X_2^2 + \cdots + X_n^2)$
4. $T_4(X_1, X_2, \dots, X_n) = (X_1^2 + X_2^2,\ X_3^2 + X_4^2 + \cdots + X_n^2)$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **7720331105**
Option 1 ID : **7720334417**
Option 2 ID : **7720334418**
Option 3 ID : **7720334419**
Option 4 ID : **7720334420**
Status : **Not Answered**
Chosen Option : **--**

Q.54

Consider a Markov chain on a finite state space S. Suppose that the transition probability matrix P is such that the transpose of P is also a stochastic matrix. Then which of the following is necessarily true?

1. All states have the same period
2. The chain admits at most one stationary distribution
3. All states are recurrent
4. At least one state has period 1

Options
1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **7720331101**
Option 1 ID : **7720334401**
Option 2 ID : **7720334402**
Option 3 ID : **7720334403**
Option 4 ID : **7720334404**
Status : **Not Answered**
Chosen Option : **--**

Q.55

Consider a population which is distributed according to $f_\theta(x)$. Here f_θ depends on an unknown parameter θ and denotes either a probability mass function or a probability density function.

Consider a random sample $X_1, X_2, \dots \dots, X_n$ from this population. Let $\overline{X}$ denote the corresponding sample mean. Among the following, identify those cases for which $\overline{X}$ is the Minimum Variance Unbiased Estimator of θ.

1. $f_\theta(x) \propto \exp\{-\theta x\}$ for $x \geq 0; \quad \theta > 0$
2. $f_\theta(x) \propto \exp\{-(\theta - x)^2\}$ for $-\infty < x < \infty; \quad -\infty < \theta < \infty$
3. $f_\theta(x) \propto \theta^x / x!$ for $x = 0,1,2,\dots; \quad \theta > 0$
4. $f_\theta(x) \propto \theta^x$ for $x = 0,1,2,\dots; \quad 0 < \theta < 1$

Options
1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **7720331109**
Option 1 ID : **7720334433**
Option 2 ID : **7720334434**
Option 3 ID : **7720334435**
Option 4 ID : **7720334436**
Status : **Not Answered**
Chosen Option : **--**

Q.56

Let $(X_1,Y_1),(X_2,Y_2)\dots,(X_n,Y_n), n\geq 4$ be a random sample from the bivariate normal distribution with $E(X_1)=\mu_1,\ E(Y_1)=\mu_2,\ Var(X_1)=Var(Y_1)=\sigma^2$ and the correlation coefficient between X_1 and Y_1 equal to ρ. All four parameters μ_1,μ_2,σ^2 and ρ are unknown. Also, let

$$S_{xx}=\sum_{i=1}^{n}(X_i-\bar{X})^2,\ S_{yy}=\sum_{i=1}^{n}(Y_i-\bar{Y})^2,\ S_{xy}=\sum_{i=1}^{n}(X_i-\bar{X})(Y_i-\bar{Y}).$$

Which of the following are maximum likelihood estimators of ρ?

1. $\frac{S_{xy}}{\sqrt{S_{xx}\,S_{yy}}}$
2. $\frac{\sum_{i=1}^{n}X_iY_i}{\sqrt{(\sum_{i=1}^{n}X_i^2)(\sum_{i=1}^{n}Y_i^2)}}$
3. $\frac{2S_{xy}}{S_{xx}+S_{yy}}$
4. $\frac{2S_{xy}}{S_{xx}+S_{yy}-2S_{xy}}$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **7720331110**
Option 1 ID : **7720334437**
Option 2 ID : **7720334438**
Option 3 ID : **7720334439**
Option 4 ID : **7720334440**
Status : **Not Answered**
Chosen Option : **--**

Q.57

A sample of size 10 is selected at random from a lot containing 20 items of a manufactured product. Suppose the total number of defectives D in the lot is unknown. Let X be the number of defectives in the sample. We wish to test $H_0: D=7$ against the alternative $H_1: D>7$. Consider the test : Reject H_0 if $X\geq k$ where k is determined such that

$$P_{H_0}(X\geq k)\leq 0.05 < P_{H_0}(X\geq k-1).$$

Which of the following are true?

1. The test is Uniformly Most Powerful test of its size
2. The power function of the test is increasing in D
3. The power function of the test is decreasing in D
4. The test is unbiased

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **7720331107**
Option 1 ID : **7720334425**
Option 2 ID : **7720334426**
Option 3 ID : **7720334427**
Option 4 ID : **7720334428**
Status : **Not Answered**
Chosen Option : **--**

Q.58 Let $X_1, X_2, \ldots, X_n$ and $Y_1, Y_2, \ldots, Y_m$ be two independent random samples from a common continuous distribution F. Let W be the sum of ranks of X observations in the combined ranking of all the $N = n + m$ observations. Which of the following are true?

1. $E(W) = \frac{n(N+1)}{2}$
2. $E(W) = \frac{N(N+1)}{4}$
3. The distribution of W is symmetric about $\frac{n(N+1)}{2}$
4. The distribution of W does not depend on F

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **7720331106**
Option 1 ID : **7720334421**
Option 2 ID : **7720334422**
Option 3 ID : **7720334423**
Option 4 ID : **7720334424**
Status : **Not Answered**
Chosen Option : **--**

Q.59 Which of the following are true?

1. If X_1 and X_2 are independent uniform $(0,1)$ then $X_1 + X_2$ is uniform $(0,2)$
2. If X is uniform $(0, n)$ then X/n is uniform $(0,1)$
3. If X_1 and X_2 are standard normal, then $(X_1 + X_2)/\sqrt{2}$ is standard normal
4. If X is exponential with mean 1, then e^{-X} is uniform $(0,1)$

Options 1. 1
2. 2
3. 3
4. 4

Question Type : **MSQ**
Question ID : **7720331099**
Option 1 ID : **7720334393**
Option 2 ID : **7720334394**
Option 3 ID : **7720334395**
Option 4 ID : **7720334396**
Status : **Not Answered**
Chosen Option : **--**

Q.60

Let A and B be two events. Which of the following are necessarily correct?

1. $P(A \cup B) \leq P(A) + P(B)$
2. $P(A \cup B) \geq P(A) + P(B)$
3. $P(A \cap B) = P(A)P(B)$
4. $P(A \cap B) \geq P(A) + P(B) - 1$

Options 1. 1
2. 2
3. 3
4. 4

Question Type :	**MSQ**
Question ID :	**7720331098**
Option 1 ID :	**7720334389**
Option 2 ID :	**7720334390**
Option 3 ID :	**7720334391**
Option 4 ID :	**7720334392**
Status :	**Not Answered**
Chosen Option :	**--**

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UGC CSIR NET NOVEMBER 2020

Exam Date : 30.11.2020 **Final Answer Key on which result compiled on 28.12.2020** Shift : Second

Subject : (704) Mathematical Sciences

Question ID	Correct Option ID	Question ID	Correct Option ID	Question ID	Correct Option ID
772033996	7720333982	7720331051	7720334203		7720334323, 7720334324
772033997	7720333987	7720331052	7720334207	7720331082	7720334325, 7720334326, 7720334328
772033998	7720333989	7720331053	7720334210	7720331083	7720334330, 7720334332
772033999	7720333995	7720331054	7720334215	7720331084	7720334335, 7720334336
7720331000	7720333997	7720331055	7720334218	7720331085	7720334337, 7720334338, 7720334339
7720331001	7720334004	7720331056	7720334221	7720331086	7720334342, 7720334343
7720331002	7720334006	7720331057	7720334226, 7720334227	7720331087	7720334346, 7720334347
7720331003	7720334011	7720331058	7720334230, 7720334232	7720331088	7720334349, 7720334352
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7720331006	7720334021	7720331061	7720334243, 7720334244	7720331091	7720334361, 7720334364
7720331007	7720334028	7720331062	7720334248	7720331092	7720334365, 7720334366
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7720331009	7720334036	7720331064	7720334253, 7720334254	7720331094	7720334376
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7720331011	7720334041	7720331066	7720334261, 7720334263, 7720334264	7720331096	7720334381
7720331012	7720334047	7720331067	7720334265, 7720334266, 7720334268	7720331097	7720334385, 7720334386
7720331013	7720334051	7720331068	7720334270, 7720334272	7720331098	7720334389, 7720334392
7720331014	7720334053	7720331069	7720334276	7720331099	7720334394, 7720334396
7720331015	7720334057	7720331070	7720334277, 7720334279, 7720334280	7720331100	7720334399, 7720334400
7720331016	7720334061	7720331071	7720334281, 7720334282, 7720334284	7720331101	7720334403
7720331017	7720334066	7720331072	7720334285, 7720334287, 7720334288	7720331102	7720334406, 7720334407
7720331018	7720334071	7720331073	7720334290, 7720334292	7720331103	7720334409, 7720334410, 7720334411, 7720334412
7720331019	7720334075	7720331074	7720334293, 7720334296	7720331104	7720334416
7720331020	7720334080	7720331075	7720334297	7720331105	7720334417, 7720334418, 7720334419, 7720334420
7720331021	7720334082	7720331076	7720334301, 7720334302, 7720334303	7720331106	7720334421, 7720334423, 7720334424
7720331022	7720334087	7720331077	7720334307	7720331107	7720334425, 7720334426,
7720331023	7720334092	7720331078	7720334309, 7720334311		
7720331024	7720334094	7720331079	7720334315, 7720334316		
7720331025	7720334098	7720331080	7720334318, 7720334319		
7720331026	7720334104	7720331081	7720334322,		
7720331027	7720334105				
7720331028	7720334111				
7720331029	7720334116				
7720331030	7720334118				
7720331031	7720334123				
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7720331036	7720334142				
7720331037	7720334145				
7720331038	7720334151				
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7720331046	7720334182				
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7720331048	7720334191				
7720331049	7720334195				
7720331050	7720334199				

NATIONAL TESTING AGENCY

UGC CSIR NET NOVEMBER 2020

Exam Date : 30.11.2020 **Final Answer Key on which result compiled on 28.12.2020** Shift : Second

Subject : (704) Mathematical Sciences

Question ID	Correct Option ID
	7720334428
7720331108	7720334430, 7720334432
7720331109	7720334434, 7720334435
7720331110	7720334439
7720331111	7720334441, 7720334442, 7720334443
7720331112	7720334447
7720331113	7720334449, 7720334451, 7720334452
7720331114	7720334453, 7720334454, 7720334455
7720331115	7720334457, 7720334460

NB:-Correction option ID -------- means the quesion remains cancelled and marks will be given to only those candidates who attempted that question

Some Useful Links:

1. **Free Maths Study Materials (**https://pkalika.in/2020/04/06/free-maths-study-materials/**)**

2. **BSc/MSc Free Study Materials (**https://pkalika.in/2019/10/14/study-material/**)**

3. **MSc Entrance Exam Que. Paper: (**https://pkalika.in/2020/04/03/msc-entrance-exam-paper/**)**
[JAM(MA), JAM(MS), BHU, CUCET, ...etc]

4. **PhD Entrance Exam Que. Paper: (**https://pkalika.in/que-papers-collection/**)**
[CSIR-NET, GATE(MA), BHU, CUCET,IIT, NBHM, ...etc]

5. **CSIR-NET Maths Que. Paper: (**https://pkalika.in/2020/03/30/csir-net-previous-yr-papers/**)**
[Upto 2019 Dec]

6. **Practice Que. Paper: (**https://pkalika.in/2019/02/10/practice-set-for-net-gate-set-jam/**)**
[Topic-wise/Subject-wise]

7. **List of Maths Suggested Books** (https://pkalika.in/suggested-books-for-mathematics/)

8. **CSIR-NET Mathematics Details Syllabus (**https://wp.me/p6gYUB-Fc)

9. **Free Video Lectures for CSIR-NET, GATE, SET, Asst. Prof. ..etc**
https://www.youtube.com/c/pkalika