

CUCET-2020 PhD Maths Que Paper

Credit:

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P Kalika Maths

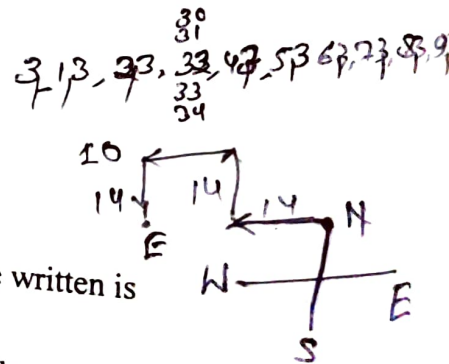
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PART-A

Instructions: Part-A consists of 50 questions. Questions No. 1 -10 (English), Questions No. 11-25 (General Knowledge and Numerical Ability) and Questions No. 26-50 (Research Methodology)

1. Find the suitable antonym of 'Accentuate'
(A) Extenuate (B) Attenuate (C) Supplicate (D) Slanting
2. Correct the given sentence:
Rural-area people can cope well with physical strain as they are used to working hardly.
(A) Work hard (B) The working hard
(C) Hardly working (D) Hard working
3. Change the speech of the given sentence: My mother confessed with regret that she had squandered all her money.
(A) My mother said, "Alas! I have squandered all my money."
(B) My mother said, "I have been very foolish to squander all my money."
(C) My mother said, "How stupid of me that I have squandered all my money."
(D) My mother said, "I am a fool to squander all my money."
4. Rule by the Officers
(A) Officialdom (B) Autocracy
(C) Bureaucracy (D) Celibacy
5. Give the synonym of 'ADVERT'
(A) Insinuate (B) Eulogize
(C) Scold (D) Lucky
6. What does phrase "to run across" mean
(A) to have an appointed meeting (B) to meet by chance
(C) to run in the playground (D) to run very fast
7. Complete the given sentence:
Bread is usually made _____ wheat.
(A) of (B) from (C) with (D) by
8. Select the pair which has the same relationship. CORPOREAL: SPIRITUAL
(A) MESA: PLATEAU (B) PEDAGOGUE: TEACHER
(C) FOREIGNER: IMMIGRANT (D) MORON: SAVANT
9. Choose the correct spelling
(A) Mauseleum (B) Moausoleum
(C) Mausoleum (D) Maousoleum
10. Select the correct plural form of 'Codex'
(A) Codex (B) Codices
(C) Codium (D) Codexes
11. A person writes all the numbers from 0 to 99. The number of times digit 3 will be written is
(A) 18 (B) 19 (C) 20 (D) 21
12. Starting from point A, Ajit walks 14 metres towards west, he then turns to his right and walks 14 m and then turns to his left and walks 10 m. He again turns to his left and walks 14 m and reaches to the point E. The shortest distance between A and E is
(A) 38 m (B) 42 m
(C) 52 m (D) 24 m



13. A, B, C, D, E and F are sitting around a round table. A is between E and F. E is opposite to D and C is not in either of the neighbouring seats of E. The person opposite to B is
 (A) B (B) C (C) D (D) F $7x + 84 = 38$
 $5x = 207$
 $10x = 99$
14. In certain code TEACHER is written as VGCEJGT. The code of CHILDREN will be
 (A) EKNJFTGP (B) EJKNFTGP (C) KNJFGTP (D) KNJFGTP
15. A person has to buy both apples and mangoes. The cost of one apple is Rs 7/- whereas that of mango is Rs 5/- If the person has Rs. 38, the number of apples he can buy is
 (A) 1 (B) 2 (C) 3 (D) 4 45
 65
 110
 23
 20
16. The mean mark obtained by a class of 40 students is 65; the mean mark of half of the students is found to be 45. The mean marks of the remaining students is
 (A) 85 (B) 60 (C) 70 (D) 65
17. Anil is twice as old as Sunita. Three years ago, he was three times as old as Sunita. The present age of Anil is
 (A) 6 years (B) 8 years (C) 12 years (D) 16 years
18. Complete the series BB, FE, II, ML, PP: by choosing one of the following option given below:
 (A) TS (B) ST (C) RS (D) SR
19. Two numbers are in the ratio 3:5. If 9 is subtracted from the numbers, the ratio becomes 12:23. The numbers are
 (A) 30, 50 (B) 36, 60 (C) 33, 55 (D) 42, 70 $3x, 5x$
 $\frac{3x-9}{5x-9} = \frac{12}{23}$
20. The next term in the series: 2, 7, 28, 63, 126, ___ is
 (A) 215 (B) 245 (C) 276 (D) 296 $69x - 207 = 60x$
 $9x = 99$
 $x = 11$
21. The primary source of organic pollution in fresh water bodies is
 (A) Run off from urban areas (B) Run off from agricultural forms
 (C) Sewage effluents (D) Industrial effluents
22. 'Lahar' is a natural disaster involving
 (A) Eruption of large amount of mud (B) Eruption of methane
 (C) Strong water waves (D) Strong wind and water waves
23. The National Disaster Management Authority functions under the Union Ministry of
 (A) Environment (B) Water Resources
 (C) Home Affairs (D) Defence
24. Which one of the following greenhouse gases has the shortest residence time in the atmosphere?
 (A) Chlorofluorocarbon (B) Carbon dioxide
 (C) Methane (D) Nitrous oxide
25. One of the anthropogenic sources of gaseous pollutants chlorofluorocarbons (CFCs) in air is
 (A) Cement Industry (B) Fertiliser industry
 (C) Foam industry (D) Pesticide industry
26. A thesis statement is
 (A) An observation (B) A fact (C) An assertion (D) A discussion
27. Which one of the following is a non-probability sampling?
 (A) Simple Random (B) Purposive (C) Systematic (D) Stratified
28. The research stream of immediate application is
 (A) Action Research (B) Empirical Research
 (C) Conceptual Research (D) Fundamental Research

29. What is a Research Design?

- (A) A way of conducting research that is not grounded in theory
- (B) The choice between using qualitative or quantitative methods
- (C) The style in which you present your research findings e.g. a graph
- (D) A framework for every stage of the collection and analysis of data

30. "Sampling Cases" means

- (A) Sampling using a sampling frame
- (B) Identifying people who are suitable for research
- (C) Literally the researcher's brief case
- (D) Sampling of people, newspapers, television programmes etc.

31. The frequency distribution of a research data which is symmetrical in shape similar to a normal distribution but center peak is much higher, is

- (A) Skewed
- (B) Mesokurtic
- (C) Leptokurtic
- (D) Platykurtic

32. When planning to do a social research, it is better to

- (A) Approach the topic with an open mind
- (B) Do a pilot study before getting stuck into it
- (C) Be familiar with literature on the topic
- (D) Forget about theory because this is a very practical

33. The conclusions/findings of which type of research cannot be generalized to other situations?

- (A) Causal Comparative Research
- (B) Historical Research
- (C) Descriptive Research
- (D) Experimental Research

34. Jean Piaget gave a theory of cognitive development of humans on the basis of his:

- (A) Evaluation Research
- (B) Fundamental Research
- (C) Applied Research
- (D) Action Research

35. "Male and female students perform equally well in a numerical aptitude test." This statement indicates a:

- (A) Statistical hypothesis
- (B) Research hypothesis
- (C) Null hypothesis
- (D) Directional hypothesis

36. Which of the following statements is not true in the context of participatory research?

- (A) It recognizes knowledge as power
- (B) It emphasises on people as experts
- (C) It is a collective process of enquiry
- (D) Its sole purpose is production of knowledge

37. Which of the following statements is true in the context of the testing of a hypothesis?

- (A) It is only the alternative hypothesis that can be tested
- (B) It is only the null hypothesis that can be tested
- (C) Both, the alternative and the null hypotheses can be tested
- (D) Both, the alternative and the null hypotheses cannot be tested

38. Ethical norms in research do not involve guidelines for:

- (A) Thesis format
- (B) Copyright
- (C) Patenting policy
- (D) Data sharing policies

39. In qualitative research paradigm, which of the following features may be considered critical?

- (A) Data collection with standardized research tools
- (B) Sampling design with probability sample techniques
- (C) Data collection with bottom-up empirical evidences
- (D) Data gathering to take with top-down systematic evidences

40. From the following list of statement identify the set, which has positive implications for 'research ethics':
 (A) A researcher critically looks at the findings of another research
 (B) Related studies are cited without proper references
 (C) Conduct of practitioner is screened in terms of reported research evidences
 (D) Both policy making and policy implementing processes are regulated in terms of preliminary studies
41. A research intends to explore the effect of possible factors for the organization of effective mid-day meal interventions. Which research method will be most appropriate for this study?
 (A) Historical method (B) Descriptive survey method
 (C) Experimental method (D) Ex-post facto method
42. Which of the following is an initial mandatory requirement for pursuing research?
 (A) Developing a research design (B) Formulating a research question
 (C) Deciding about the data analysis procedure (D) Formulating a research hypothesis
43. The format of thesis writing is the same as in
 (A) Preparation of a research paper/article (B) Writing of seminar presentation
 (C) A research dissertation (D) Presenting a workshop/conference paper
44. Which of the following is not the critical feature of qualitative research?
 (A) Actual settings are the direct source of data.
 (B) Data take the forms of words or pictures.
 (C) Seeking to establish relationships among measured social facts.
 (D) Researcher becomes immersed in the situation, present or past related to the phenomena.
45. Research ethics has a direct connection more often with which stages of research?
 (A) Defining and delimiting the scope of research.
 (B) Problem formulation and reporting of research findings.
 (C) Defining the population and deciding the sampling technique for research.
 (D) Deciding about statistical techniques and data analysis.
46. If a researcher conducts a research on finding out which administrative style contributes more to institutional effectiveness? This will be an example of
 (A) Evaluation Research (B) Fundamental Research
 (C) Applied Research (D) Action Research
47. A null hypothesis is
 (A) When there is no difference between the variables
 (B) The same as research hypothesis
 (C) Subjective in nature
 (D) When there is difference between the variables
48. The research which is exploring new facts through the study of the past is called
 (A) Philosophical research (B) Historical research
 (C) Mythological research (D) Content analysis
49. Action research is
 (A) An applied research
 (B) A research carried out to solve immediate problems
 (C) A longitudinal research
 (D) Simulative research
50. The process not needed in Experimental Researches is
 (A) Observation (B) Manipulation
 (C) Controlling (D) Content Analysis

PART-B

51. Which of the following is a convergent series?

(A) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1} - \sqrt{n}}$

(B) $\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$

(C) $\sum_{n=1}^{\infty} (-1)^n \log n$

(D) $\sum_{n=1}^{\infty} \frac{\log n}{n}$

52. Let f be a twice differentiable function on \mathbb{R} . Given that $f''(x) > 0$ for all $x \in \mathbb{R}$

(A) $f(x) = 0$ has exactly two solutions on \mathbb{R}

(B) $f(x) = 0$ has a positive solution if $f(0) = 0$ and $f'(0) = 0$

(C) $f(x) = 0$ has no positive solution if $f(0) = 0$ and $f'(0) > 0$

(D) $f(x) = 0$ has no positive solution if $f(0) = 0$ and $f'(0) < 0$

53. Which of the following real-valued functions is uniformly continuous on $(0, 1)$?

(A) $f(x) = 1/x$.

(B) $f(x) = \sin x/x$.

(C) $f(x) = \sin(1/x)$.

(D) $f(x) = \cos x/x$.

54. In which of the following cases, there is no continuous function f from the set S onto the set T ?

(A) $S = [0, 1], T = \mathbb{R}$.

(B) $S = (0, 1), T = \mathbb{R}$.

(C) $S = (0, 1), T = (0, 1)$.

(D) $S = \mathbb{R}, T = (0, 1)$.

55. Let X be a metric space and $A \subseteq X$ be a connected set with at least two distinct points. Then the number of distinct points in A is

(A) 2

(B) more than 2, but finite

(C) countably infinite.

(D) uncountable

56. The system of equations

$$x + y + z = 1$$

$$2x + 3y - z = 5$$

$$x + 2y - kz = 4$$

where $k \in \mathbb{R}$, has an infinite number of solutions for

(A) $k = 0$.

(B) $k = 1$.

(C) $k = 2$.

(D) $k = 3$.

57. The number of limit point(s) of the set $\left\{1 + \frac{1}{n} : n \in \mathbb{N}\right\}$ is

(A) 2

(B) 1

(C) finite many

(D) infinite many

58. Consider \mathbb{R}^3 with the standard inner product. Let W be the subspace of \mathbb{R}^3 spanned by $(1, 0, -1)$. Which of the following is a basis for the orthogonal complement of W ?

(A) $\{(1, 0, 1), (0, 1, 0)\}$.

(B) $\{(1, 2, 1), (0, 1, 1)\}$.

(C) $\{(2, 1, 2), (4, 2, 4)\}$.

(D) $\{(2, -1, 2), (1, 3, 1), (-1, -1, -1)\}$.

59. The row space of a 20×50 matrix A has dimension 13. What is the dimension of the space of $Ax = 0$?

(A) 7

(B) 13

(C) 33

(D) 37

60. Let A, B be $n \times n$ real matrices. Which of the following statement is correct?

(A) $\text{rank}(A+B) = \text{rank}(A) + \text{rank}(B)$
 (C) $\text{rank}(A+B) = \min \{\text{rank}(A), \text{rank}(B)\}$

(B) $\text{rank}(A+B) \leq \text{rank}(A) + \text{rank}(B)$
 (D) $\text{rank}(A+B) = \max \{\text{rank}(A), \text{rank}(B)\}$

61. Let $M = \{(a_1, a_2, a_3) : a_i \in \{1, 2, 3, 4\}, a_1 + a_2 + a_3 = 6\}$. Then the number of elements in M is

(A) 8
 (C) 10

(B) 9
 (D) 12

62. Let $f(x)$ be the minimal polynomial of the 4×4 matrix

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Then the $f(A)$ is equal to

(A) 0
 (C) 2

(B) 1
 (D) 4

63. A linear transformation T rotates each vector in R^2 clockwise through 90° . The matrix T relative to the standard ordered basis $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$ is

(A) $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

(B) $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(D) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

64. Let $p(z), q(z)$ be two non-zero complex polynomials. Then $p(z), \overline{q(z)}$ is analytic if and only if

(A) $p(z)$ is constant.
 (C) $q(z)$ is a constant.

(B) $p(z)q(z)$ is constant
 (D) $\overline{p(z)}, q(z)$ is a constant

65. $\int_{|z+1|} \frac{z^2}{4-z^2} dz =$

(A) 0
 (C) $2\pi i$

(B) $-2\pi i$
 (D) 1

66. The residue of the function $f(z) = e^{-e^{1/z}}$ at $z = 0$ is

(A) $-e^{-1}$
 (C) $1+e^{-1}$

(B) e^{-1}
 (D) $1-e^{-1}$

67. Let C be the circle $|z| = 3/2$ in the complex plane that is oriented in the counter clockwise direction. The value of a for which

$$\oint_C \left(\frac{z+1}{z^2-3z+2} + \frac{a}{z-1} \right) dz = 0$$

(A) 1
 (C) 2

(B) -1
 (D) -2

68. Let f be a real valued harmonic function on C , that is, f satisfies the equation $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$.

Define the functions

$$g = \left(\frac{\partial f}{\partial x} \right) - i \left(\frac{\partial f}{\partial y} \right) \text{ and } h = \frac{\partial f}{\partial x} + i \frac{\partial f}{\partial y}. \text{ Then}$$

$$\begin{aligned} \frac{\partial f}{\partial x} &= f_{xx} \\ \frac{\partial f}{\partial y} &= -if_{yy} \\ f_{xx} + f_{yy} &= 0 \end{aligned}$$

$$\begin{aligned} \frac{\partial f}{\partial x} &= 0 \\ \frac{\partial f}{\partial y} &= 0 \end{aligned}$$

$$f_{xx}$$

- (A) g and h are both holomorphic functions
 (B) g is holomorphic, but h need not be holomorphic
 (C) h is holomorphic, but g need not be holomorphic
 (D) both g and h are identically equal to the zero function
69. Let G be a simple group of order 168. What is the number of subgroups of G of order 7?
- (A) 1
 (B) 7
 (C) 8
 (D) 28
70. A polynomial of odd degree with real coefficients must have
- (A) at least one real root.
 (B) no real root
 (C) only real roots.
 (D) at least one root which is not real
71. A group G is generated by the elements x, y with the relations $x^3 = y^3 = (xy)^2 = 1$. The order of G is
- (A) 4
 (B) 6
 (C) 8
 (D) 12

$$\begin{array}{r|l} 2 & 168 \\ \hline 2 & 84 \\ 2 & 42 \\ 3 & 14 \\ \hline & 7 \end{array}$$

$$3 \times 3 \times 7$$

$$7k+1$$

72. Let R be a Euclidean domain such that R is not a field. Then the polynomial ring $R[X]$ is always
- (A) a Euclidean domain
 (B) a principal ideal domain, but not a Euclidean domain
 (C) a unique factorization domain, but not a principal ideal domain
 (D) not a unique factorization domain

$$\begin{aligned} 2^2 \times 3^3 \\ (2+1)(3+1) \\ 3 \times 4 = 12 \end{aligned}$$

$$\begin{array}{r|l} 2 & 108 \\ \hline 2 & 54 \\ 3 & 27 \\ 3 & 9 \\ \hline & 3 \end{array}$$

73. Up to isomorphism, the number of abelian group of order 108 is:
- (A) 12
 (B) 9
 (C) 6
 (D) 5

74. Let A be a connected open subset of \mathbb{R}^2 . The number of continuous surjective functions from \bar{A} (the closure of A in \mathbb{R}^2) to \mathbb{Q} is:

- (A) 1
 (B) 0
 (C) 2
 (D) not finite

75. Which of the following subsets of \mathbb{R}^n is compact (with respect to the usual topology of \mathbb{R}^n)?

- (A) $\{(x_1, x_2, \dots, x_n) : |x_i| < 1, 1 \leq i \leq n\}$.
 (B) $\{(x_1, x_2, \dots, x_n) : x_1 + x_2 + \dots + x_n = 0\}$.
 (C) $\{(x_1, x_2, \dots, x_n) : x_i \geq 0, 1 \leq i \leq n\}$.
 (D) $\{(x_1, x_2, \dots, x_n) : 1 \leq x_i \leq 2^i, 1 \leq i \leq n\}$.

$$\begin{aligned} \frac{dy}{y+17} &= dx \\ \log|y+17| &= x+C \\ y+17 &= e^{x+A} \\ y+17 &= 17e^x \end{aligned}$$

76. Let $y_1(x)$ and $y_2(x)$ be the solutions of the differential equation $\frac{dy}{dx} = y + 17$ with initial conditions

$$y_1(0) = 0, y_2(0) = 1. \text{ Then}$$

- (A) y_1 and y_2 will never intersect.
 (B) y_1 and y_2 will intersect at $x = 17$
 (C) y_1 and y_2 will intersect at $x = e$
 (D) y_1 and y_2 will intersect at $x = 1$

77. Consider the initial value problem (IVP)

$$\frac{dy}{dx} = y^2, y(0) = 1, (x, y) \in \mathbb{R} \times \mathbb{R}.$$

Then there exists a unique solution of IVP on

- (A) $(-\infty, \infty)$. (B) $(-\infty, 1)$.
(C) $(-2, 2)$. (D) $(-1, \infty)$.
78. The boundary value problem $x^2 y'' - 2xy' + 2y = 0$, subject to the boundary conditions $y(1) + ay'(1) = 1, y(2) + by'(2) = 2$ has a unique solution if
- (A) $a = -1, b = 2$. (B) $a = -1, b = -2$.
(C) $a = -2, b = 2$. (D) $a = -3, b = 2/3$.

79. Let $y(x)$ be a continuous solution of the initial value problem

$$y' + 2y = f(x), \quad y(0) = 0, \quad \text{where } f(x) = \begin{cases} 1, & 0 \leq x \leq 1 \\ 0, & x > 1 \end{cases}$$

Then $y(3/2)$ is equal to

80. Wronskian of the following differential equation is

$$y''(t) - 6y'(t) + 9y(t) = t,$$

- (A) $W(t) = e^{6t}$.
(B) $W(t) = e^{-6t}$.
(C) $W(t) = 1$.
(D) $W(t) = -1$.

81. The second order PDE $u_{yy} - yu_{xx} + x^3 = 0$ is

- (A) elliptic for all $x \in \mathbb{R}, y \in \mathbb{R}$.
(B) parabolic for all $x \in \mathbb{R}, y \in \mathbb{R}$.
(C) elliptic for all $x \in \mathbb{R}, y < 0$.
(D) hyperbolic for all $x \in \mathbb{R}, y < 0$.

82. Let $u(x, t)$ satisfy the initial boundary value problem

$$u_t = u_{xx}; \quad x \in (0, 1), t > 0$$

$$u(x, 0) = \sin(\pi x); \quad x \in [0, 1]$$

$$u(0, t) = u(1, t) = 0, t > 0$$

Then for $x \in (0, 1), u(x, 1/\pi^2)$ is equal to

- (A) $e \sin(\pi x)$
(B) $e^{-1} \sin(\pi x)$
(C) $\sin(\pi x)$
(D) $\sin(x/\pi)$

$$\frac{y^{-2+1}}{-2+1} = x + C$$

$$\frac{1}{y} = x + C$$

$$1 = C, (C = -)$$

$$\frac{1}{y} = x + 1$$

$$y = \frac{1}{1+x}$$

$$x = \log x$$

$$x = 3e^{3x} + e^{3x}$$

$$y' + 2y = 1$$

$$\frac{dy}{dx} = 1 - 2y$$

$$\log(1-2y) = x + C$$

$$\frac{1}{-2} \log(1-2y)$$

$$y'(t) = 8e^{6t}$$

$$y''(t) = 36e^{6t}$$

$$36e^{6t} - 36e^{6t} + 9e^{6t} = t$$

$$m^2 - 6m + 9 = 0$$

$$m = \frac{6 \pm \sqrt{36 - 36}}{2}$$

$$m = 3$$

$$y = e^{3x}$$

$$y_2 = xe^{3x}$$

$$\begin{vmatrix} e^{3x} & xe^{3x} \\ 3e^{3x} & e^{3x}(3x+1) \end{vmatrix}$$

$$3x e^{6x} + e^{3x} + 3x e^{6x}$$

$$= e^{3x}$$

$$D(D+1) - 2D + 2 = 0$$

$$D^2 - D - 2D + 2 = 0$$

$$D(D+1) - 2(D-1) = 0$$

$$(D-2)(D+1) = 0$$

$$y(t) = C_1 e^{2t} + C_2 e^{-t}$$

$$t = \log x$$

$$x = e^t$$

$$-C_1 + 2C_2 = 1$$

$$-4C_1 = 2$$

$$C_1 + C_2 - 2C_1 + C_2 = 1$$

$$4C_1 + 2C_2 - 8C_1 - 2C_2 = 2$$

[8]

$$y(x) = C_1 x^2 + C_2 x$$

$$y'(x) = 2C_1 x + C_2$$

83. Consider the initial value problem

$$\frac{\partial u}{\partial x} + 2 \frac{\partial u}{\partial y} = 0, \quad u(0, y) = 4e^{-2y}.$$

Then the value of $u(1, 1)$ is

(A) $4e^{-2}$.

(C) $2e^{-4}$.

(B) $4e^2$.

(D) $4e^4$.

84. Solution of following partial differential equation is

$$\frac{\partial z}{\partial x} \sqrt{x} + \frac{\partial z}{\partial y} \sqrt{y} = \sqrt{z}$$

(A) $f(\sqrt{x} - \sqrt{y}, \sqrt{y} - \sqrt{z}) = 0$.

(B) $f(\sqrt{x} + \sqrt{y}, \sqrt{y} + \sqrt{z}) = 0$.

(C) $f(\sqrt{x} - \sqrt{y}, \sqrt{y} + \sqrt{z}) = 0$.

(D) $f(\sqrt{x} + \sqrt{y}, \sqrt{y} - \sqrt{z}) = 0$.

85. Solution of following partial differential equation is

$$\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = 1$$

(A) $z = ax + \sqrt{1 + a^2}y + c$.

(B) $z = ax + a/\sqrt{1 + a^2}y + c$.

(C) $z = ax + a/\sqrt{1 - a^2}y + c$.

(D) $z = ax + \sqrt{1 - a^2}y + c$.

86. Let $f(x) = ax + 100$ for $a \in \mathbb{R}$. Then the iteration

$$x_{n+1} = f(x_n) \text{ for } n \geq 0 \text{ and } x_0 = 0 \text{ converges for}$$

(A) $a = 5$.

(C) $a = 1$.

(D) $a = 0.1$.

(B) $a = 10$.

87. The order of convergence for Secant method is

(A) 1.618

(B) 1.718

(C) 1

(D) 2

88. Which of the following is not a single step method to solve ordinary differential equation numerically

(A) Euler method

(B) Heun method

(C) Picard's method

(D) Milne method

89. In Newton-Cotes quadrature formulas, two intervals ($n = 2$) considered at a time represent for

(A) Simpson's $3/8$ rule

(B) Simpson's $1/3$ rule

(C) Trapezoidal's rule

(D) Boole's rule

90. The curve of fixed length l , that joins the points $(0,0)$ and $(1,0)$ lies above the x -axis, and encloses the maximum area between itself and the x -axis, is a segment of

(A) a straight line
(B) a parabola
(C) an ellipse
(D) a circle

$$\int_0^1 \sqrt{1-y^2}$$

91. For the homogeneous Fredholm integral equation

$\varphi(x) = \lambda \int_0^1 e^{x+t} \varphi(t) dt$, a non-trivial solution exists, when λ has the value

(A) $\lambda = \frac{2}{e-1}$

(B) $\lambda = \frac{2}{e^2+1}$

(C) $\lambda = \frac{2}{e+1}$

(D) $\lambda = \frac{2}{e^2-1}$

92. If A is 5×5 real matrix with trace 15 and if 2 and 3 are eigenvalues of A , each with algebraic multiplicity 2, then the determinant of A is equal to

(A) 0
(B) 180
(C) 120
(D) 24

$$\begin{aligned} a_1 + a_2 + a_3 + a_4 + a_5 &= 15 \\ 2 + 2 + 3 + 3 + a_5 &= 15 \\ a_5 &= 5 \\ 4 \times 3 \times 5 &= 60 \end{aligned}$$

93. The integral equation

$$y(x) = \lambda \int_0^1 (3x-2)ty(t)dt, \quad \text{with } \lambda \text{ as a parameter, has}$$

(A) only one characteristic number
(B) two characteristic numbers
(C) more than two characteristic numbers
(D) no characteristic number

94. The resolvent kernel $R(x, t, \lambda)$ for the Volterra integral equation

$$\varphi(x) = x + \lambda \int_a^x \varphi(s)ds, \text{ is}$$

(A) $e^{\lambda(x+t)}$

(B) $e^{\lambda(x-t)}$

(C) $\lambda e^{\lambda(x+t)}$

(D) $e^{\lambda xt}$

$$-x$$

$$\int -x$$

$$\begin{aligned} F &= y(3x-4) = 3xy-4y \\ \frac{\partial F}{\partial y} - \frac{d}{dx} \left(\frac{\partial F}{\partial y'} \right) &= 0 \\ (3x-4) - \frac{d}{dx}(0) &= 0 \\ 3x-4 &= 0 \end{aligned}$$

95. The variational problem of extremizing the functional

$$I(y(x)) = \int_1^3 y(3x-y)dx; \quad y(3) = 9/2, \quad y(1) = 1 \text{ has}$$

(A) a unique solution
(B) exactly two solutions
(C) an infinite number of solutions
(D) no solution

96. From the six letters A, B, C, D, E and F , three letters are chosen at random with replacement. What is the probability that either the word BAD or the word CAD can be formed from the chosen letters

(A) $1/216$

(B) $3/216$

(C) $12/216$

(D) $36/216$

$${}^6C_3 \times \frac{3!}{6!} + \frac{3!}{6!}$$

$$\frac{6! \times 3!}{24 \times 6!}$$

97. Suppose observations on the pair (X, Y) are:

X	1	7	5	9	11	3
Y	20	68	58	70	181	37

Let r_p and r_s respectively denote the Pearson's and Spearman's rank correlation coefficient between X and Y based on the above data. Then which of the following is true?

- (A) $r_p = 1, r_s = 1$.
 (B) $0 < r_p < 1, r_s = 1$.
 (C) $r_p = 1, 0 < r_s < 1$.
 (D) $0 < r_p < 1, 0 < r_s < 1$.

98. What is the smallest positive integer in the set $\{24x + 60y + 2000z \mid x, y, z \in \mathbb{Z}\}$?

- (A) 2
 (B) 4
 (C) 6
 (D) 24

99. Assume that $X \sim \text{Binomial}(n, p)$ for some $n \geq 1$ and $0 < p < 1$ and $Y \sim \text{Poisson}(\lambda)$ for some $\lambda > 0$. Suppose $E[X] = E[Y]$. Then

- (A) $\text{Var}(X) = \text{Var}(Y)$
 (B) $\text{Var}(X) < \text{Var}(Y)$
 (C) $\text{Var}(X) > \text{Var}(Y)$
 (D) $\text{Var}(X)$ maybe larger or smaller than $\text{Var}(Y)$ depending on the values of n, p and λ

100. Consider the following linear programming problem:

Maximize $Z = 3x + 2y$

Subject to

$$x + y \geq 1$$

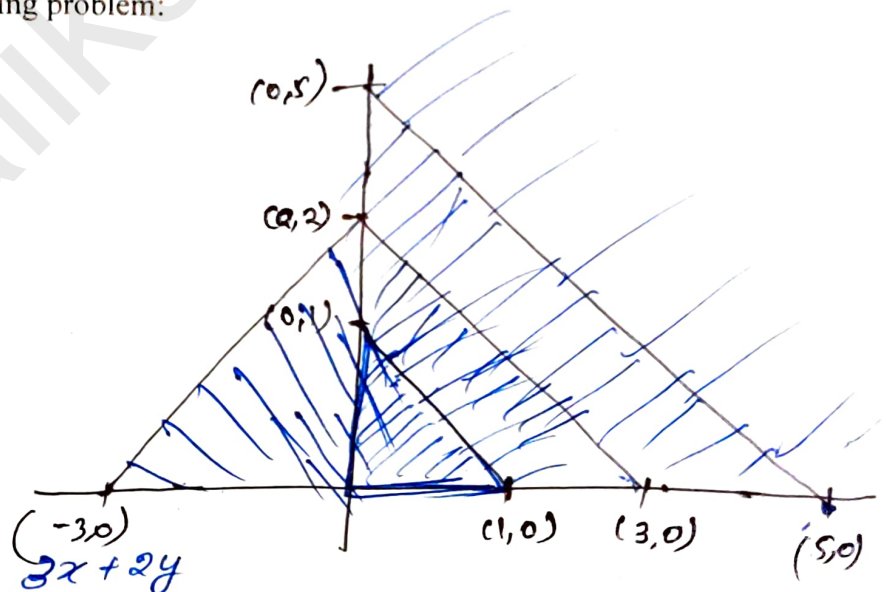
$$x + y \leq 5$$

$$2x + 3y \leq 6$$

$$-2x + 3y \leq 6$$

The problem is

- (A) exactly one optimal solution
 (B) an unbounded solution
 (C) more than one optimal solution
 (D) no feasible solution



$(0, 5)$	$\rightarrow 10$
$(5, 0)$	$\rightarrow 15$
$(3, 0)$	$\rightarrow 9$
$(-3, 0)$	$\rightarrow -9$
$(1, 0)$	$\rightarrow 3$
$(0, 2)$	$\rightarrow 4$
$(0, 1)$	$\rightarrow 2$