GATE Mathematics Questions & Answer

[GATE Que. Paper + Answer 2021-2010]

No. of Pages: 221

P Kalika Maths

(A Hub of Study Materials/Solutions for CSIR-NET(JRF), GATE, SET, JAM, CUCET, NBHM, PhD/MSc Entrances, ...etc)

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Your Notes/Remark

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- **GATE Mathematics(MA) Video Solution**
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MA	Mathematics

Calculus: Functions of two or more variables, continuity, directional derivatives, partial derivatives, total derivative, maxima and minima, saddle point, method of Lagrange's multipliers; Double and Triple integrals and their applications to area, volume and surface area; Vector Calculus: gradient, divergence and curl, Line integrals and Surface integrals, Green's theorem, Stokes' theorem, and Gauss divergence theorem.

Linear Algebra: Finite dimensional vector spaces over real or complex fields; Linear transformations and their matrix representations, rank and nullity; systems of linear equations, characteristic polynomial, eigenvalues and eigenvectors, diagonalization, minimal polynomial, Cayley-Hamilton Theorem, Finite dimensional inner product spaces, Gram-Schmidt orthonormalization process, symmetric, skew-symmetric, Hermitian, skew-Hermitian, normal, orthogonal and unitary matrices; diagonalization by a unitary matrix, Jordan canonical form; bilinear and quadratic forms.

Real Analysis: Metric spaces, connectedness, compactness, completeness; Sequences and series of functions, uniform convergence, Ascoli-Arzela theorem; Weierstrass approximation theorem; contraction mapping principle, Power series; Differentiation of functions of several variables, Inverse and Implicit function theorems; Lebesgue measure on the real line, measurable functions; Lebesgue integral, Fatou's lemma, monotone convergence theorem, dominated convergence theorem.

Complex Analysis: Functions of a complex variable: continuity, differentiability, analytic functions, harmonic functions; Complex integration: Cauchy's integral theorem and formula; Liouville's theorem, maximum modulus principle, Morera's theorem; zeros and singularities; Power series, radius of convergence, Taylor's series and Laurent's series; Residue theorem and applications for evaluating real integrals; Rouche's theorem, Argument principle, Schwarz lemma; Conformal mappings, Mobius transformations.

Ordinary Differential equations: First order ordinary differential equations, existence and uniqueness theorems for initial value problems, linear ordinary differential equations of higher order with constant coefficients; Second order linear ordinary differential equations with variable coefficients; Cauchy-Euler equation, method of Laplace transforms for solving ordinary differential equations, series solutions (power series, Frobenius method); Legendre and Bessel functions and their orthogonal properties; Systems of linear first order ordinary differential equations, Sturm's oscillation and separation theorems, Sturm-Liouville eigenvalue problems, Planar autonomous systems of ordinary differential equations: Stability of stationary points for linear systems with constant coefficients, Linearized stability, Lyapunov functions.

Algebra: Groups, subgroups, normal subgroups, quotient groups, homomorphisms, automorphisms; cyclic groups, permutation groups, Group action, Sylow's theorems and their applications; Rings, ideals, prime and maximal ideals, quotient rings, unique factorization domains, Principle ideal domains, Euclidean domains, polynomial rings, Eisenstein's irreducibility criterion; Fields, finite fields, field extensions, algebraic extensions, algebraically closed fields

Functional Analysis: Normed linear spaces, Banach spaces, Hahn-Banach theorem, open mapping and closed graph theorems, principle of uniform boundedness; Inner-product spaces, Hilbert spaces, orthonormal bases, projection theorem, Riesz representation theorem, spectral theorem for compact self-adjoint operators.

Numerical Analysis: Systems of linear equations: Direct methods (Gaussian elimination, LU decomposition, Cholesky factorization), Iterative methods (Gauss-Seidel and Jacobi) and their convergence for diagonally dominant coefficient matrices; Numerical solutions of nonlinear equations: bisection method, secant method, Newton-Raphson method, fixed point iteration; Interpolation: Lagrange and Newton forms of interpolating polynomial, Error in polynomial interpolation of a function; Numerical differentiation and error, Numerical integration: Trapezoidal and Simpson rules, Newton-Cotes integration formulas, composite rules, mathematical errors involved in numerical integration formulae; Numerical solution of initial value problems for ordinary differential equations: Methods of Euler, Runge-Kutta method of order 2.

Partial Differential Equations: Method of characteristics for first order linear and quasilinear partial differential equations; Second order partial differential equations in two independent variables: classification and canonical forms, method of separation of variables for Laplace equation in Cartesian and polar coordinates, heat and wave equations in one space variable; Wave equation: Cauchy problem and d'Alembert formula, domains of dependence and influence, non-homogeneous wave equation; Heat equation: Cauchy problem; Laplace and Fourier transform methods.

Topology: Basic concepts of topology, bases, subbases, subspace topology, order topology, product topology, quotient topology, metric topology, connectedness, compactness, countability and separation axioms, Urysohn's Lemma.

Linear Programming: Linear programming models, convex sets, extreme points; Basic feasible solution, graphical method, simplex method, two phase methods, revised simplex method ; Infeasible and unbounded linear programming models, alternate optima; Duality theory, weak duality and strong duality; Balanced and unbalanced transportation problems, Initial basic feasible solution of balanced transportation problems (least cost method, north-west corner rule, Vogel's approximation method); Optimal solution, modified distribution method; Solving assignment problems, Hungarian method.

GATE-2021 Syllabus: https://wp.me/p6gYUB-WR

GATE 2021 — Graduate Aptitude Test in Engineering 2021
Organising Institute - IIT Bombay

Mathematics (MA)

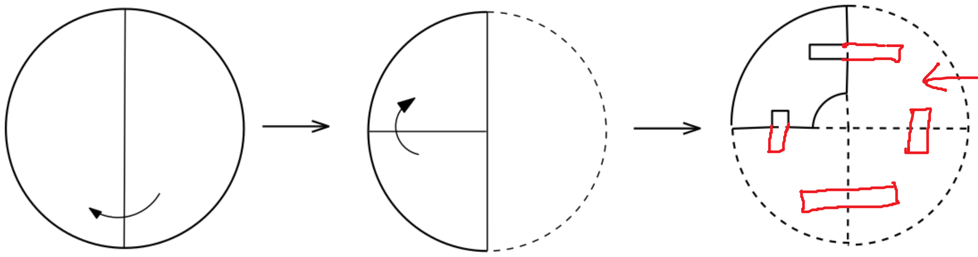
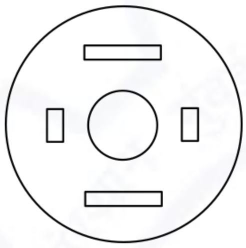
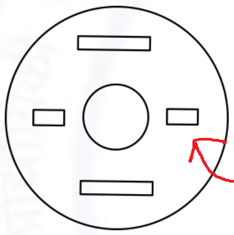
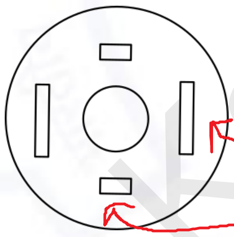
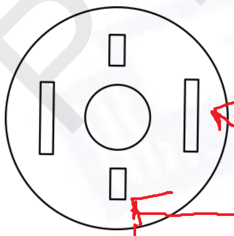
General Aptitude (GA)

Q.1 – Q.5 Multiple Choice Question (MCQ), carry ONE mark each (for each wrong answer: – 1/3).

Q.1	**The ratio of boys to girls in a class is 7 to 3.** **Among the options below, an acceptable value for the total number of students in the class is:**
(A)	21
(B)	37
(C)	50
(D)	73

Q.2	**A polygon is convex if, for every pair of points, P and Q belonging to the polygon, the line segment PQ lies completely inside or on the polygon.** **Which one of the following is <u>NOT</u> a convex polygon?**
(A)	
(B)	
(C)	
(D)	

Q.3	**Consider the following sentences:** **(i) Everybody in the class is prepared for the exam.** **(ii) Babu invited Danish to his home because he enjoys playing chess.** **Which of the following is the CORRECT observation about the above two sentences?**
(A)	(i) is grammatically correct and (ii) is unambiguous
(B)	(i) is grammatically incorrect and (ii) is unambiguous
(C)	(i) is grammatically correct and (ii) is ambiguous
(D)	(i) is grammatically incorrect and (ii) is ambiguous

Q.4	 **A circular sheet of paper is folded along the lines in the directions shown. The paper, after being punched in the final folded state as shown and unfolded in the reverse order of folding, will look like _______.** Looks like this, when unfolded ⇒ op(A) ✓
(A)	 Solution: Imagine structure when it will be unfolding
(B)	 ← Incorrect Because
(C)	 ← Incorrect Because
(D)	 ← Incorrect Because

Q.5	**_____ is to *surgery* as *writer* is to ________** **Which one of the following options maintains a similar logical relation in the above sentence?**
(A)	Plan, outline
(B)	Hospital, library
(C)	Doctor, book
(D)	Medicine, grammar

Q. 6 – Q.10 Multiple Choice Question (MCQ), carry TWO marks each (for each wrong answer: – 2/3).

Q.6	**We have 2 rectangular sheets of paper, M and N, of dimensions 6 cm x 1 cm each. Sheet M is rolled to form an open cylinder by bringing the short edges of the sheet together. Sheet N is cut into equal square patches and assembled to form the largest possible closed cube. Assuming the ends of the cylinder are closed, the ratio of the volume of the cylinder to that of the cube is ________**
(A)	$\frac{\pi}{2}$
(B)	$\frac{3}{\pi}$
(C)	$\frac{9}{\pi}$
(D)	3π

Q.7

Items	**Cost (₹)**	**Profit %**	**Marked Price (₹)**
P	**5,400**	**---**	**5,860**
Q	**---**	**25**	**10,000**

Details of prices of two items P and Q are presented in the above table. The ratio of cost of item P to cost of item Q is 3:4. Discount is calculated as the difference between the marked price and the selling price. The profit percentage is calculated as the ratio of the difference between selling price and cost, to the cost (Profit % $= \frac{\text{Selling price} - \text{Cost}}{\text{Cost}} \times 100$).

The discount on item Q, as a percentage of its marked price, is ______

(A) 25

(B) 12.5

(C) 10

(D) 5

Q.8 **There are five bags each containing identical sets of ten distinct chocolates. One chocolate is picked from each bag.**

The probability that at least two chocolates are identical is ___________

(A) 0.3024

(B) 0.4235

(C) 0.6976

(D) 0.8125

Q.9	**Given below are two statements 1 and 2, and two conclusions I and II.** **Statement 1: All bacteria are microorganisms.** **Statement 2: All pathogens are microorganisms.** **Conclusion I: Some pathogens are bacteria.** **Conclusion II: All pathogens are not bacteria.** **Based on the above statements and conclusions, which one of the following options is logically CORRECT?**
(A)	Only conclusion I is correct
(B)	Only conclusion II is correct
(C)	Either conclusion I or II is correct.
(D)	Neither conclusion I nor II is correct.

Q.10	**Some people suggest anti-obesity measures (AOM) such as displaying calorie information in restaurant menus. Such measures sidestep addressing the core problems that cause obesity: poverty and income inequality.** **Which one of the following statements summarizes the passage?**
(A)	The proposed AOM addresses the core problems that cause obesity.
(B)	If obesity reduces, poverty will naturally reduce, since obesity causes poverty.
(C)	AOM are addressing the core problems and are likely to succeed.
(D)	AOM are addressing the problem superficially.

Mathematics (MA)

Q.1 – Q.14 Multiple Choice Question (MCQ), carry ONE mark each (for each wrong answer: – 1/3).

Q.1	**Let A be a 3×4 matrix and B be a 4×3 matrix with real entries such that AB is non-singular. Consider the following statements:** **P: Nullity of A is 0.** **Q: BA is a non-singular matrix.** **Then**
(A)	both P and Q are TRUE
(B)	P is TRUE and Q is FALSE
(C)	P is FALSE and Q is TRUE
(D)	both P and Q are FALSE

Q.2	**Let $f(z) = u(x,y) + i\,v(x,y)$ for $z = x + iy \in \mathbb{C}$, where x and y are real numbers, be a non-constant analytic function on the complex plane $\mathbb{C}$. Let u_x, v_x and u_y, v_y denote the first order partial derivatives of $u(x,y) = Re(f(z))$ and $v(x,y) = Im(f(z))$ with respect to real variables x and y, respectively. Consider the following two functions defined on $\mathbb{C}$:** $$g_1(z) = u_x(x,y) - i\,u_y(x,y) \ \text{ for } z = x+iy \in \mathbb{C},$$ $$g_2(z) = v_x(x,y) + i\,v_y(x,y) \ \text{ for } z = x+iy \in \mathbb{C}.$$ **Then**
(A)	both $g_1(z)$ and $g_2(z)$ are analytic in $\mathbb{C}$
(B)	$g_1(z)$ is analytic in $\mathbb{C}$ and $g_2(z)$ is NOT analytic in $\mathbb{C}$
(C)	$g_1(z)$ is NOT analytic in $\mathbb{C}$ and $g_2(z)$ is analytic in $\mathbb{C}$
(D)	neither $g_1(z)$ nor $g_2(z)$ is analytic in $\mathbb{C}$

Q.3	**Let $T(z) = \frac{az+b}{cz+d}$, $ad - bc \neq 0$, be the Möbius transformation which maps the points $z_1 = 0$, $z_2 = -i$, $z_3 = \infty$ in the z-plane onto the points $w_1 = 10$, $w_2 = 5 - 5i$, $w_3 = 5 + 5i$ in the w-plane, respectively. Then the image of the set $S = \{z \in \mathbb{C} : Re(z) < 0\}$ under the map $w = T(z)$ is**
(A)	$\{w \in \mathbb{C} : \lvert w\rvert < 5\}$
(B)	$\{w \in \mathbb{C} : \lvert w\rvert > 5\}$
(C)	$\{w \in \mathbb{C} : \lvert w - 5\rvert < 5\}$
(D)	$\{w \in \mathbb{C} : \lvert w - 5\rvert > 5\}$

Q.4	**Let R be the row reduced echelon form of a 4×4 real matrix A and let the third column of R be $\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$. Consider the following statements:** **P: If $\begin{bmatrix} \alpha \\ \beta \\ \gamma \\ 0 \end{bmatrix}$ is a solution of $A\mathrm{x} = 0$, then $\gamma = 0$.** **Q: For all $\mathrm{b} \in \mathbb{R}^4$, $rank[A\|\ \mathrm{b}] = rank[R\|\ \mathrm{b}]$.** **Then**
(A)	both P and Q are TRUE
(B)	P is TRUE and Q is FALSE
(C)	P is FALSE and Q is TRUE
(D)	both P and Q are FALSE

Q.5	**The eigenvalues of the boundary value problem** $$\frac{d^2y}{dx^2} + \lambda y = 0, \qquad x \in (0, \pi), \ \ \lambda > 0,$$ $$y(0) = 0, \qquad y(\pi) - \frac{dy}{dx}(\pi) = 0,$$ **are given by**
(A)	$\lambda = (n\pi)^2, \qquad n = 1,2,3, \dots$
(B)	$\lambda = n^2, \qquad n = 1,2,3, \dots$
(C)	$\lambda = k_n^2$, where $k_n, n = 1,2,3, \dots$ are the roots of $k - \tan(k\pi) = 0$
(D)	$\lambda = k_n^2$, where $k_n, n = 1,2,3, \dots$ are the roots of $k + \tan(k\pi) = 0$

Q.6	**The family of surfaces given by $u = xy + f(x^2 - y^2)$, where $f: \mathbb{R} \to \mathbb{R}$ is a differentiable function, satisfies**
(A)	$y\dfrac{\partial u}{\partial x} + x\dfrac{\partial u}{\partial y} = x^2 + y^2$
(B)	$x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = x^2 + y^2$
(C)	$y\dfrac{\partial u}{\partial x} + x\dfrac{\partial u}{\partial y} = x^2 - y^2$
(D)	$x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = x^2 - y^2$

Q.7	**The function $u(x,t)$ satisfies the initial value problem** $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2},\ x \in \mathbb{R},\ t > 0,$ $u(x,0) = 0,\ \frac{\partial u}{\partial t}(x,0) = 4xe^{-x^2}$. **Then $u(5,5)$ is**
(A)	$1 - \dfrac{1}{e^{100}}$
(B)	$1 - e^{100}$
(C)	$1 - \dfrac{1}{e^{10}}$
(D)	$1 - e^{10}$

Q.8	**Consider the fixed-point iteration** $$x_{n+1} = \varphi(x_n),\ \ n \geq 0,$$ **with** $\varphi(x) = 3 + (x-3)^3,\ \ x \in (2.5,\ 3.5)$, **and the initial approximation** $x_0 = 3.25$. **Then, the order of convergence of the fixed-point iteration method is**
(A)	1
(B)	2
(C)	3
(D)	4

Q.9	**Let** $\{e_n : n = 1, 2, 3, \dots\}$ **be an orthonormal basis of a complex Hilbert space** H**. Consider the following statements:** **P: There exists a bounded linear functional** $f : H \to \mathbb{C}$ **such that** $f(e_n) = \frac{1}{n}$ **for** $n = 1, 2, 3, \dots$. **Q: There exists a bounded linear functional** $g : H \to \mathbb{C}$ **such that** $g(e_n) = \frac{1}{\sqrt{n}}$ **for** $n = 1, 2, 3, \dots$. **Then**
(A)	both P and Q are TRUE
(B)	P is TRUE and Q is FALSE
(C)	P is FALSE and Q is TRUE
(D)	both P and Q are FALSE

Q.10	Let $f: \left(\frac{-\pi}{2}, \frac{\pi}{2}\right) \to \mathbb{R}$ be given by $f(x) = \frac{\pi}{2} + x - \tan^{-1}x$. Consider the following statements: P: $\lvert f(x) - f(y)\rvert < \lvert x - y\rvert$ for all $x, y \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$. Q: f has a fixed point. Then
(A)	both P and Q are TRUE
(B)	P is TRUE and Q is FALSE
(C)	P is FALSE and Q is TRUE
(D)	both P and Q are FALSE

Q.11	Consider the following statements: P: $d_1(x, y) = \left\lvert \log\left(\frac{x}{y}\right)\right\rvert$ is a metric on $(0, 1)$. Q: $d_2(x, y) = \begin{cases} \lvert x\rvert + \lvert y\rvert, & \text{if } x \neq y, \\ 0, & \text{if } x = y, \end{cases}$ is a metric on $(0, 1)$. Then
(A)	both P and Q are TRUE
(B)	P is TRUE and Q is FALSE
(C)	P is FALSE and Q is TRUE
(D)	both P and Q are FALSE

Q.12	**Let $f: \mathbb{R}^3 \to \mathbb{R}$ be a twice continuously differentiable scalar field such that $div(\nabla f) = 6$. Let S be the surface $x^2 + y^2 + z^2 = 1$ and $\hat{n}$ be unit outward normal to S. Then the value of $\iint_S (\nabla f \cdot \hat{n})\, dS$ is**
(A)	2π
(B)	4π
(C)	6π
(D)	8π

Q.13	**Consider the following statements:** **P: Every compact metrizable topological space is separable.** **Q: Every Hausdorff topology on a finite set is metrizable.** **Then**
(A)	both P and Q are TRUE
(B)	P is TRUE and Q is FALSE
(C)	P is FALSE and Q is TRUE
(D)	both P and Q are FALSE

Q.14	**Consider the following topologies on the set $\mathbb{R}$ of all real numbers:** $T_1 = \{U \subset \mathbb{R} : 0 \notin U \text{ or } U = \mathbb{R}\}$, $T_2 = \{U \subset \mathbb{R} : 0 \in U \text{ or } U = \emptyset\}$, $T_3 = T_1 \cap T_2$. **Then the closure of the set $\{1\}$ in $(\mathbb{R}, T_3)$ is**
(A)	$\{1\}$
(B)	$\{0,1\}$
(C)	$\mathbb{R}$
(D)	$\mathbb{R}\setminus\{0\}$

Mathematics (MA)

Q.15 – Q.25 Numerical Answer Type (NAT), carry ONE mark each (no negative marks).

Q.15	**Let $f: \mathbb{R}^2 \to \mathbb{R}$ be differentiable. Let $D_u f(0,0)$ and $D_v f(0,0)$ be the directional derivatives of f at $(0,0)$ in the directions of the unit vectors $u = \left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right)$ and $v = \left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$, respectively. If $D_u f(0,0) = \sqrt{5}$ and $D_v f(0,0) = \sqrt{2}$, then $\frac{\partial f}{\partial x}(0,0) + \frac{\partial f}{\partial y}(0,0) =$ ______ .**

Q.16	**Let Γ denote the boundary of the square region R with vertices $(0,0)$, $(2,0)$, $(2,2)$ and $(0,2)$ oriented in the counter-clockwise direction. Then** $$\oint_\Gamma (1-y^2)\,dx + x\,dy = \_\_\_\_\_\_\_\_.$$

Q.17	**The number of 5-Sylow subgroups in the symmetric group S_5 of degree 5 is ___ .**

Q.18	**Let I be the ideal generated by $x^2 + x + 1$ in the polynomial ring $R = \mathbb{Z}_3[x]$, where $\mathbb{Z}_3$ denotes the ring of integers modulo 3. Then the number of units in the quotient ring R/I is ________ .**

Q.19	**Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation such that** $$T\left(\begin{bmatrix}1\\1\\1\end{bmatrix}\right) = \begin{bmatrix}1\\-1\\1\end{bmatrix},\ T^2\left(\begin{bmatrix}1\\1\\1\end{bmatrix}\right) = \begin{bmatrix}1\\1\\1\end{bmatrix},\text{ and } T^2\left(\begin{bmatrix}1\\1\\2\end{bmatrix}\right) = \begin{bmatrix}1\\1\\1\end{bmatrix}.$$ **Then the rank of T is _____ .**

Q.20	**Let $y(x)$ be the solution of the following initial value problem** $$x^2\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 6y = 0, \quad x > 0,$$ $$y(2) = 0, \quad \frac{dy}{dx}(2) = 4.$$ **Then $y(4) =$ __________ .**

Q.21	**Let** $$f(x) = x^4 + 2x^3 - 11x^2 - 12x + 36 \quad \text{for } x \in \mathbb{R}.$$ **The order of convergence of the Newton-Raphson method** $$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n \geq 0,$$ **with $x_0 = 2.1$, for finding the root $\alpha = 2$ of the equation $f(x) = 0$ is** ________ **.**

Q.22	**If the polynomial** $$p(x) = \alpha + \beta(x+2) + \gamma(x+2)(x+1) + \delta(x+2)(x+1)x$$ **interpolates the data**

x	-2	-1	0	1	2
$f(x)$	2	-1	8	5	-34

then $\alpha + \beta + \gamma + \delta =$ __________ .

Q.23	**Consider the Linear Programming Problem P:** $$\text{Maximize } 2x_1 + 3x_2$$ **subject to** $$2x_1 + x_2 \le 6,$$ $$-x_1 + x_2 \le 1,$$ $$x_1 + x_2 \le 3,$$ $$x_1 \ge 0 \text{ and } x_2 \ge 0.$$ **Then the optimal value of the dual of P is equal to ________ .**

Q.24	**Consider the Linear Programming Problem P:** $$\text{Minimize } 2x_1 - 5x_2$$ **subject to** $$2x_1 + 3x_2 + s_1 = 12,$$ $$-x_1 + x_2 + s_2 = 1,$$ $$-x_1 + 2x_2 + s_3 = 3,$$ $$x_1 \ge 0,\ x_2 \ge 0,\ s_1 \ge 0,\ s_2 \ge 0, \text{ and } s_3 \ge 0.$$ **If $\begin{bmatrix} x_1 \\ 2 \\ s_1 \\ s_2 \\ s_3 \end{bmatrix}$ is a basic feasible solution of P, then $x_1 + s_1 + s_2 + s_3 =$________ .**

Q.25	**Let H be a complex Hilbert space. Let $u, v \in H$ be such that $\langle u, v \rangle = 2$. Then** $$\frac{1}{2\pi}\int_0^{2\pi} \left\| u + e^{it}v \right\|^2 e^{it}\, dt = \text{________}.$$

Q.26 – Q.43 Multiple Choice Question (MCQ), carry TWO mark each (for each wrong answer: – 2/3).

Q.26	**Let $\mathbb{Z}$ denote the ring of integers. Consider the subring $R = \{a + b\sqrt{-17} : \ a, b \in \mathbb{Z}\}$ of the field $\mathbb{C}$ of complex numbers.** **Consider the following statements:** **P: $2 + \sqrt{-17}$ is an irreducible element.** **Q: $2 + \sqrt{-17}$ is a prime element.** **Then**
(A)	both P and Q are TRUE
(B)	P is TRUE and Q is FALSE
(C)	P is FALSE and Q is TRUE
(D)	both P and Q are FALSE

Q.27	**Consider the second-order partial differential equation (PDE)** $\frac{\partial^2 u}{\partial x^2} + 4\frac{\partial^2 u}{\partial x \partial y} + (x^2 + 4y^2)\frac{\partial^2 u}{\partial y^2} = \sin(x+y).$ **Consider the following statements:** **P: The PDE is parabolic on the ellipse $\frac{x^2}{4} + y^2 = 1$.** **Q: The PDE is hyperbolic inside the ellipse $\frac{x^2}{4} + y^2 = 1$.** **Then**
(A)	both P and Q are TRUE
(B)	P is TRUE and Q is FALSE
(C)	P is FALSE and Q is TRUE
(D)	both P and Q are FALSE

Q.28	**If $u(x,y)$ is the solution of the Cauchy problem** $$x\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=1, \qquad u(x,0)=-x^2, \quad x>0,$$ **then $u(2,1)$ is equal to**
(A)	$1-2\,e^{-2}$
(B)	$1+4\,e^{-2}$
(C)	$1-4\,e^{-2}$
(D)	$1+2\,e^{-2}$

Q.29	**Let $y(t)$ be the solution of the initial value problem** $$\frac{d^2y}{dt^2}+a\frac{dy}{dt}+b\,y=f(t), \qquad a>0, \qquad b>0, \qquad a\neq b, \qquad a^2-4b=0,$$ $$y(0)=0, \quad \frac{dy}{dt}(0)=0,$$ **obtained by the method of Laplace transform. Then**
(A)	$y(t)=\int_0^t \tau\, e^{\frac{-a\tau}{2}} f(t-\tau)\,d\tau$
(B)	$y(t)=\int_0^t e^{\frac{-a\tau}{2}} f(t-\tau)\,d\tau$
(C)	$y(t)=\int_0^t \tau\, e^{\frac{-b\tau}{2}} f(t-\tau)\,d\tau$
(D)	$y(t)=\int_0^t e^{\frac{-b\tau}{2}} f(t-\tau)\,d\tau$

Mathematics (MA)

Q.30	**The critical point of the differential equation** $$\frac{d^2y}{dt^2} + 2\,\alpha\,\frac{dy}{dt} + \beta^2 y = 0,\ \ \alpha > \beta > 0,$$ **is a**
(A)	node and is asymptotically stable
(B)	spiral point and is asymptotically stable
(C)	node and is unstable
(D)	saddle point and is unstable

Q.31	**The initial value problem** $$\frac{dy}{dt} = f(t,y),\ \ t > 0,\ \ y(0) = 1,$$ **where** $f(t,y) = -10\,y$**, is solved by the following Euler method** $$y_{n+1} = y_n + h\,f(t_n, y_n),\ \ n \geq 0,$$ **with step-size** h**. Then** $y_n \to 0$ **as** $n \to \infty$**, provided**
(A)	$0 < h < 0.2$
(B)	$0.3 < h < 0.4$
(C)	$0.4 < h < 0.5$
(D)	$0.5 < h < 0.55$

Q.32	**Consider the Linear Programming Problem P:** $$\text{Maximize } c_1x_1 + c_2x_2$$ **subject to** $$a_{11}x_1 + a_{12}x_2 \le b_1,$$ $$a_{21}x_1 + a_{22}x_2 \le b_2,$$ $$a_{31}x_1 + a_{32}x_2 \le b_3,$$ $x_1 \ge 0$ **and** $x_2 \ge 0$**, where** a_{ij}, b_i **and** c_j **are real numbers** $(i = 1, 2, 3; j = 1, 2)$. **Let** $\begin{bmatrix} p \\ q \end{bmatrix}$ **be a feasible solution of** P **such that** $p\,c_1 + q\,c_2 = 6$ **and let all feasible solutions** $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ **of** P **satisfy** $-5 \le c_1x_1 + c_2x_2 \le 12$. **Then, which one of the following statements is NOT true?**
(A)	P has an optimal solution
(B)	The feasible region of P is a bounded set
(C)	If $\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$ is a feasible solution of the dual of P, then $b_1y_1 + b_2y_2 + b_3y_3 \ge 6$
(D)	The dual of P has at least one feasible solution

Q.33	**Let** $L^2[-1, 1]$ **be the Hilbert space of real valued square integrable functions on** $[-1, 1]$ **equipped with the norm** $\|f\| = \left(\int_{-1}^{1} |f(x)|^2\, dx\right)^{1/2}$. **Consider the subspace** $M = \{f \in L^2[-1, 1] \,:\, \int_{-1}^{1} f(x)dx = 0\}$. **For** $f(x) = x^2$**, define** $d = \inf\,\{\|f - g\| : g \in M\}$. **Then**
(A)	$d = \frac{\sqrt{2}}{3}$
(B)	$d = \frac{2}{3}$
(C)	$d = \frac{3}{\sqrt{2}}$
(D)	$d = \frac{3}{2}$

Q.34	**Let $C[0,1]$ be the Banach space of real valued continuous functions on $[0,1]$ equipped with the supremum norm. Define $T: C[0,1] \to C[0,1]$ by** $$(Tf)(x) = \int_0^x x\, f(t)\, dt.$$ **Let $R(T)$ denote the range space of T. Consider the following statements:** **P: T is a bounded linear operator.** **Q: $T^{-1}: R(T) \to C[0,1]$ exists and is bounded.** **Then**
(A)	both P and Q are TRUE
(B)	P is TRUE and Q is FALSE
(C)	P is FALSE and Q is TRUE
(D)	both P and Q are FALSE

Q.35	**Let $\ell^1 = \{x = (x(1), x(2), \ldots, x(n), \ldots) \mid \sum_{n=1}^{\infty} \lvert x(n)\rvert < \infty\}$ be the sequence space equipped with the norm $\lVert x\rVert = \sum_{n=1}^{\infty} \lvert x(n)\rvert$. Consider the subspace** $$X = \left\{x \in \ell^1 : \sum_{n=1}^{\infty} n\,\lvert x(n)\rvert < \infty\right\},$$ **and the linear transformation $T: X \to \ell^1$ given by** **$(Tx)(n) = n\, x(n)$ for $n = 1, 2, 3, \ldots$. Then**
(A)	T is closed but NOT bounded
(B)	T is bounded
(C)	T is neither closed nor bounded
(D)	T^{-1} exists and is an open map

Q.36	**Let $f_n: [0, 10] \to \mathbb{R}$ be given by $f_n(x) = n\, x^3\, e^{-nx}$ for $n = 1, 2, 3, \dots$.** **Consider the following statements:** **P: (f_n) is equicontinuous on $[0, 10]$.** **Q: $\sum_{n=1}^{\infty} f_n$ does NOT converge uniformly on $[0, 10]$.** **Then**
(A)	both P and Q are TRUE
(B)	P is TRUE and Q is FALSE
(C)	P is FALSE and Q is TRUE
(D)	both P and Q are FALSE

Q.37	**Let $f: \mathbb{R}^2 \to \mathbb{R}$ be given by** $f(x,y) = \begin{cases} \sqrt{x^2+y^2}\ \sin(y^2/x) & \text{if } x \neq 0, \\ 0 & \text{if } x = 0. \end{cases}$ **Consider the following statements:** **P: f is continuous at $(0, 0)$ but f is NOT differentiable at $(0, 0)$.** **Q: The directional derivative $D_u f(0, 0)$ of f at $(0, 0)$ exists in the direction of every unit vector $u \in \mathbb{R}^2$.** **Then**
(A)	both P and Q are TRUE
(B)	P is TRUE and Q is FALSE
(C)	P is FALSE and Q is TRUE
(D)	both P and Q are FALSE

Q.38	**Let V be the solid region in $\mathbb{R}^3$ bounded by the paraboloid $y = (x^2 + z^2)$ and the plane $y = 4$. Then the value of $\iiint_V 15\sqrt{x^2 + z^2}\, dV$ is**
(A)	$128\,\pi$
(B)	$64\,\pi$
(C)	$28\,\pi$
(D)	$256\,\pi$

Q.39	**Let $f : \mathbb{R}^2 \to \mathbb{R}$ be given by $f(x, y) = 4xy - 2x^2 - y^4$. Then f has**
(A)	a point of local maximum and a saddle point
(B)	a point of local minimum and a saddle point
(C)	a point of local maximum and a point of local minimum
(D)	two saddle points

Q.40	**The equation $xy - z\log y + e^{xz} = 1$ can be solved in a neighborhood of the point $(0, 1, 1)$ as $y = f(x, z)$ for some continuously differentiable function f. Then**
(A)	$\nabla f(0, 1) = (2, 0)$
(B)	$\nabla f(0, 1) = (0, 2)$
(C)	$\nabla f(0, 1) = (0, 1)$
(D)	$\nabla f(0, 1) = (1, 0)$

Q.41	**Consider the following topologies on the set $\mathbb{R}$ of all real numbers.** **T_1 is the upper limit topology having all sets $(a, b]$ as basis.** **$T_2 = \{U \subset \mathbb{R} : \mathbb{R}\setminus U \text{ is finite}\} \cup \{\emptyset\}$.** **$T_3$ is the standard topology having all sets (a, b) as basis.** **Then**
(A)	$T_2 \subset T_3 \subset T_1$
(B)	$T_1 \subset T_2 \subset T_3$
(C)	$T_3 \subset T_2 \subset T_1$
(D)	$T_2 \subset T_1 \subset T_3$

Q.42	**Let $\mathbb{R}$ denote the set of all real numbers. Consider the following topological spaces.** **$X_1 = (\mathbb{R}, T_1)$, where T_1 is the upper limit topology having all sets $(a, b]$ as basis.** **$X_2 = (\mathbb{R}, T_2)$, where $T_2 = \{U \subset \mathbb{R} : \mathbb{R}\setminus U \text{ is finite}\} \cup \{\emptyset\}$.** **Then**
(A)	both X_1 and X_2 are connected
(B)	X_1 is connected and X_2 is NOT connected
(C)	X_1 is NOT connected and X_2 is connected
(D)	neither X_1 nor X_2 is connected

Q.43	**Let $\langle\cdot,\ \cdot\rangle: \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ be an inner product on the vector space $\mathbb{R}^n$ over $\mathbb{R}$. Consider the following statements:** **P: $\|\langle u, v\rangle\| \le \frac{1}{2}(\langle u, u\rangle + \langle v, v\rangle)$ for all $u, v \in \mathbb{R}^n$.** **Q: If $\langle u, v\rangle = \langle 2u, -v\rangle$ for all $v \in \mathbb{R}^n$, then $u = 0$.** **Then**
(A)	both P and Q are TRUE
(B)	P is TRUE and Q is FALSE
(C)	P is FALSE and Q is TRUE
(D)	both P and Q are FALSE

Mathematics (MA)

Q.44 -Q.55 Numerical Answer Type (NAT), carry TWO mark each (no negative marks).

Q.44	**Let G be a group of order 5^4 with center having 5^2 elements. Then the number of conjugacy classes in G is _________ .**

Q.45	**Let F be a finite field and $F^\times$ be the group of all nonzero elements of F under multiplication. If $F^\times$ has a subgroup of order 17, then the smallest possible order of the field F is ________ .**

Q.46	**Let $R = \{z = x + iy \in \mathbb{C} : 0 < x < 1 \text{ and } -11\pi < y < 11\pi\}$ and Γ be the positively oriented boundary of R. Then the value of the integral $$\frac{1}{2\pi i}\int_\Gamma \frac{e^z\, dz}{e^z - 2}$$ is ________ .**

Q.47	**Let $D = \{z \in \mathbb{C} : \lvert z\rvert < 2\pi\}$ and $f : D \to \mathbb{C}$ be the function defined by $$f(z) = \begin{cases} \dfrac{3z^2}{(1-\cos z)} & \text{if } z \neq 0, \\ 6 & \text{if } z = 0. \end{cases}$$ If $f(z) = \sum_{n=0}^{\infty} a_n z^n$ for $z \in D$, then $6\,a_2 =$ __________.**

Q.48	**The number of zeroes (counting multiplicity) of $P(z) = 3z^5 + 2i\,z^2 + 7i\,z + 1$ in the annular region $\{z \in \mathbb{C} : 1 < \lvert z\rvert < 7\}$ is ________ .**

Q.49	**Let A be a square matrix such that $\det(xI - A) = x^4(x-1)^2(x-2)^3$, where $\det(M)$ denotes the determinant of a square matrix M. If $\operatorname{rank}(A^2) < \operatorname{rank}(A^3) = \operatorname{rank}(A^4)$, then the geometric multiplicity of the eigenvalue 0 of A is ________ .**

Mathematics (MA)

Q.50	**If $y = \sum_{k=0}^{\infty} a_k x^k$, $(a_0 \neq 0)$ is the power series solution of the differential equation $\frac{d^2y}{dx^2} - 24\,x^2 y = 0$, then $\frac{a_4}{a_0} =$ ________.**

Q.51	**If $u(x,t) = A\,e^{-t}\sin x$ solves the following initial boundary value problem** $$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < \pi, \qquad t > 0,$$ $$u(0,t) = u(\pi,t) = 0, \qquad t > 0,$$ $$u(x,0) = \begin{cases} 60, & 0 < x \le \frac{\pi}{2}, \\ 40, & \frac{\pi}{2} < x < \pi, \end{cases}$$ **then $\pi A =$ _______ .**

Q.52	**Let $V = \{p \,:\, p(x) = a_0 + a_1 x + a_2 x^2,\; a_0, a_1, a_2 \in \mathbb{R}\}$ be the vector space of all polynomials of degree at most 2 over the real field $\mathbb{R}$. Let $T: V \to V$ be the linear operator given by** $$T(p) = (p(0) - p(1)) + \big(p(0) + p(1)\big)\,x + p(0)\,x^2.$$ **Then the sum of the eigenvalues of T is _____ .**

Q.53	**The quadrature formula** $$\int_0^2 x\,f(x)\,dx \approx \alpha\,f(0) + \beta\,f(1) + \gamma\,f(2)$$ **is exact for all polynomials of degree ≤ 2. Then $2\beta - \gamma =$ _______ .**

Q.54	**For each $x \in (0,1]$, consider the decimal representation $x = \cdot\, d_1 d_2 d_3 \cdots d_n \cdots$. Define $f: [0,1] \to \mathbb{R}$ by $f(x) = 0$ if x is rational and $f(x) = 18\,n$ if x is irrational, where n is the number of zeroes immediately after the decimal point up to the first nonzero digit in the decimal representation of x. Then the Lebesgue integral $\int_0^1 f(x)\,dx =$ _______.**

Q.55	Let $\tilde{x} = \begin{bmatrix} 11/3 \\ 2/3 \\ 0 \end{bmatrix}$ be an optimal solution of the following Linear Programming Problem P: $$\text{Maximize } 4x_1 + x_2 - 3x_3$$ subject to $$2x_1 + 4x_2 + ax_3 \le 10,$$ $$x_1 - x_2 + bx_3 \le 3,$$ $$2x_1 + 3x_2 + 5x_3 \le 11,$$ $$x_1 \ge 0, x_2 \ge 0 \text{ and } x_3 \ge 0, \text{ where } a, b \text{ are real numbers.}$$ If $\tilde{y} = \begin{bmatrix} p \\ q \\ r \end{bmatrix}$ is an optimal solution of the dual of P, then $p + q + r =$ ________ (round off to two decimal places).

END OF THE QUESTION PAPER

GATE 2021 Answer Key for Mathematics (MA)

Graduate Aptitude Test in Engineering (GATE 2021)

Answer Keys and Marks for Subject/Paper: Mathematics (MA)

Q. No.	Session	Question Type MCQ/MSQ/NAT	Section Name	Answer Key/Range	Marks	Negative Marks
1	5	MCQ	GA	C	1	1/3
2	5	MCQ	GA	A	1	1/3
3	5	MCQ	GA	C	1	1/3
4	5	MCQ	GA	A	1	1/3
5	5	MCQ	GA	C	1	1/3
6	5	MCQ	GA	C	2	2/3
7	5	MCQ	GA	C	2	2/3
8	5	MCQ	GA	C	2	2/3
9	5	MCQ	GA	D	2	2/3
10	5	MCQ	GA	D	2	2/3
1	5	MCQ	MA	D	1	1/3
2	5	MCQ	MA	B	1	1/3
3	5	MCQ	MA	C	1	1/3
4	5	MCQ	MA	D	1	1/3
5	5	MCQ	MA	C	1	1/3
6	5	MCQ	MA	A	1	1/3
7	5	MCQ	MA	A	1	1/3
8	5	MCQ	MA	C	1	1/3
9	5	MCQ	MA	B	1	1/3
10	5	MCQ	MA	D	1	1/3

NEW

GATE 2021 Answer Key for Mathematics (MA)

Q. No.	Session	Question Type MCQ/MSQ/NAT	Section Name	Answer Key/Range	Marks	Negative Marks
11	5	MCQ	MA	A	1	1/3
12	5	MCQ	MA	D	1	1/3
13	5	MCQ	MA	A	1	1/3
14	5	MCQ	MA	C	1	1/3
15	5	NAT	MA	4 to 4	1	0
16	5	NAT	MA	12 to 12	1	0
17	5	NAT	MA	6 to 6	1	0
18	5	NAT	MA	6 to 6	1	0
19	5	NAT	MA	2 to 2	1	0
20	5	NAT	MA	32 to 32	1	0
21	5	NAT	MA	1 to 1	1	0
22	5	NAT	MA	1 to 1	1	0
23	5	NAT	MA	8 to 8	1	0
24	5	NAT	MA	5 to 5	1	0
25	5	NAT	MA	2 to 2	1	0
26	5	MCQ	MA	B	2	2/3
27	5	MCQ	MA	A	2	2/3
28	5	MCQ	MA	C	2	2/3
29	5	MCQ	MA	A	2	2/3
30	5	MCQ	MA	A	2	2/3
31	5	MCQ	MA	A	2	2/3
32	5	MCQ	MA	B	2	2/3
33	5	MCQ	MA	A	2	2/3

GATE 2021 Answer Key for Mathematics (MA)

Q. No.	Session	Question Type MCQ/MSQ/NAT	Section Name	Answer Key/Range	Marks	Negative Marks	
34	5	MCQ	MA	B	2	2/3	
35	5	MCQ	MA	A	2	2/3	
36	5	MCQ	MA	B	2	2/3	
37	5	MCQ	MA	A	2	2/3	
38	5	MCQ	MA	A	2	2/3	
39	5	MCQ	MA	A	2	2/3	
40	5	MCQ	MA	A	2	2/3	
41	5	MCQ	MA	A	2	2/3	
42	5	MCQ	MA	C	2	2/3	
43	5	MCQ	MA	A	2	2/3	
44	5	NAT	MA	145 to 145	2	0	
45	5	NAT	MA	103 to 103	2	0	
46	5	NAT	MA	11 to 11	2	0	
47	5	NAT	MA	3 to 3	2	0	
48	5	NAT	MA	4 to 4	2	0	
49	5	NAT	MA	MTA	2	0	NEW
50	5	NAT	MA	2 to 2	2	0	
51	5	NAT	MA	200 to 200	2	0	
52	5	NAT	MA	1 to 1	2	0	
53	5	NAT	MA	2 to 2	2	0	
54	5	NAT	MA	2 to 2	2	0	
55	5	NAT	MA	3.14 to 3.18	2	0	

MTA means Marks to All

GATE 2020 Que. Paper With Answer Key

GATE 2020

Graduate Aptitude Test in Engineering 2020

IIT Delhi
Organising Institute

MA: Mathematics

GA - General Aptitude

Q1 - Q5 carry one mark each.

Q.No. 1 Rajiv Gandhi Khel Ratna Award was conferred_____Mary Kom, a six-time world champion in boxing, recently in a ceremony_____the Rashtrapati Bhawan (the President's official residence) in New Delhi.

(A) with, at

(B) on, in

(C) on, at

(D) to, at

Q.No. 2 Despite a string of poor performances, the chances of K. L. Rahul's selection in the team are_____.

(A) slim

(B) bright

(C) obvious

(D) uncertain

Q.No. 3 Select the word that fits the analogy:

Cover : Uncover :: Associate : _____

____

(A) Unassociate

(B) Inassociate

(C) Misassociate

(D) Dissociate

Q.No. 4 Hit by floods, the kharif (summer sown) crops in various parts of the country have been affected. Officials believe that the loss in production of the kharif crops can be recovered in the output of the rabi (winter sown) crops so that the country can achieve its food-grain production target of 291 million tons in the crop year 2019-20 (July-June). They are hopeful that good rains in July-August will help the soil retain moisture for a longer period, helping winter sown crops such as wheat and pulses during the November-February period.

Which of the following statements can be inferred from the given passage?

(A) Officials declared that the food-grain production target will be met due to good rains.

(B) Officials want the food-grain production target to be met by the November-February period.

(C) Officials feel that the food-grain production target cannot be met due to floods.

(D) Officials hope that the food-grain production target will be met due to a good rabi produce.

Q.No. 5 The difference between the sum of the first $2n$ natural numbers and the sum of the first n odd natural numbers is _____.

(A) $n^2 - n$

(B) $n^2 + n$

(C) $2n^2 - n$

(D) $2n^2 + n$

Q6 - Q10 carry two marks each.

Q.No. 6 Repo rate is the rate at which Reserve Bank of India (RBI) lends commercial banks, and reverse repo rate is the rate at which RBI borrows money from commercial banks.

Which of the following statements can be inferred from the above passage?

(A) Decrease in repo rate will increase cost of borrowing and decrease lending by commercial banks.

(B) Increase in repo rate will decrease cost of borrowing and increase lending by commercial banks.

(C) Increase in repo rate will decrease cost of borrowing and decrease lending by commercial banks.

(D) Decrease in repo rate will decrease cost of borrowing and increase lending by commercial banks.

Q.No. 7 P, Q, R, S, T, U, V, and W are seated around a circular table.

I. S is seated opposite to W.
II. U is seated at the second place to the right of R.
III. T is seated at the third place to the left of R.
IV. V is a neighbour of S.

Which of the following must be true?

(A) P is a neighbour of R.

(B) Q is a neighbour of R.

(C) P is not seated opposite to Q.

(D) R is the left neighbour of S.

Q.No. 8 The distance between Delhi and Agra is 233 km. A car P started travelling from Delhi to Agra and another car Q started from Agra to Delhi along the same road 1 hour after the car P started. The two cars crossed each other 75 minutes after the car Q started. Both cars were travelling at constant speed. The speed of car P was 10 km/hr more than the speed of car Q. How many kilometers the car Q had travelled when the cars crossed each other?

(A) 66.6

(B) 75.2

(C) 88.2

(D) 116.5

Q.No. 9 For a matrix $M = [m_{ij}]$; $i, j = 1,2,3,4$, the diagonal elements are all zero and $m_{ij} = -m_{ji}$. The minimum number of elements required to fully specify the matrix is_____.

(A) 0

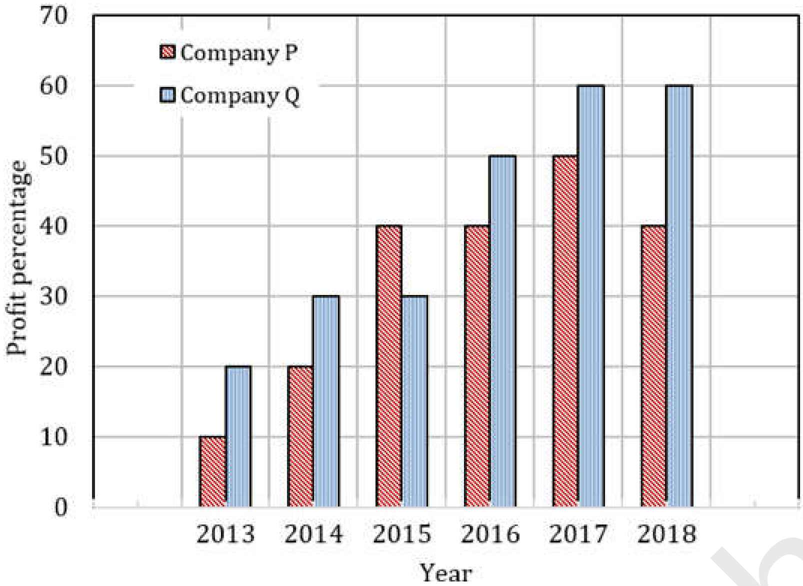
(B) 6

(C) 12

(D) 16

Q.No. 10

The profit shares of two companies P and Q are shown in the figure. If the two companies have invested a fixed and equal amount every year, then the ratio of the total revenue of company P to the total revenue of company Q, during 2013 - 2018 is_____.



(A) 15 : 17
(B) 16 : 17
(C) 17 : 15
(D) 17 : 16

MA: Mathematics

Q1 - Q25 carry one mark each.

Q.No. 1 Suppose that $\mathfrak{J}_1$ and $\mathfrak{J}_2$ are topologies on X induced by metrics d_1 and d_2, respectively, such that $\mathfrak{J}_1 \subseteq \mathfrak{J}_2$. Then which of the following statements is TRUE?

(A) If a sequence converges in (X, d_2) then it converges in (X, d_1)
(B) If a sequence converges in (X, d_1) then it converges in (X, d_2)
(C) Every open ball in (X, d_1) is an open ball in (X, d_2)
(D) The map $x \mapsto x$ from (X, d_1) to (X, d_2) is continuous

Q.No. 2 Let $D = [-1, 1] \times [-1, 1]$. If the function $f: D \to \mathbb{R}$ is defined by

$$f(x,y) = \begin{cases} \frac{x^2-y^2}{(x^2+y^2)^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases},$$

then

(A) f is continuous at $(0,0)$
(B) both the first order partial derivatives of f exist at $(0,0)$
(C) $\iint_D |f(x,y)|^{\frac{1}{2}}\, dx\, dy$ is finite
(D) $\iint_D |f(x,y)|\ \ dx\, dy$ is finite

Q.No. 3

The initial value problem

$$y' = y^{\frac{3}{5}}, \qquad y(0) = b$$

has

(A) a unique solution if $b = 0$

(B) no solution if $b = 1$

(C) infinitely many solutions if $b = 2$

(D) a unique solution if $b = 1$

Q.No. 4 Consider the following statements:

I : $\log(|z|)$ is harmonic on $\mathbb{C} \setminus \{0\}$

II : $\log(|z|)$ has a harmonic conjugate on $\mathbb{C} \setminus \{0\}$

Then

(A) both I and II are true

(B) I is true but II is false

(C) I is false but II is true

(D) both I and II are false

Q.No. 5 Let G and H be defined by

$$G = \mathbb{C} \setminus \{z = x + iy \in \mathbb{C} : \ x \le 0, y = 0\},$$

$$H = \mathbb{C} \setminus \{z = x + iy \in \mathbb{C} : x \in \mathbb{Z}, \ x \le 0, \ y = 0\}.$$

Suppose $f: G \to \mathbb{C}$ and $g: H \to \mathbb{C}$ are analytic functions. Consider the following statements:

I : $\int_\gamma f\, dz$ is independent of paths γ in G joining $-i$ and i

II : $\int_\gamma g\, dz$ is independent of paths γ in H joining $-i$ and i

Then

(A) both I and II are true

(B) I is true but II is false

(C) I is false but II is true

(D) both I and II are false

Q.No. 6 Let $f(z) = e^{1/z}$, $z \in \mathbb{C} \setminus \{0\}$ and let, for $n \in \mathbb{N}$,

$$R_n = \left\{ z = x + iy \in \mathbb{C} : |x| < \tfrac{1}{n}, \ |y| < \tfrac{1}{n} \right\} \setminus \{0\}.$$

If for a subset S of $\mathbb{C}$, $\overline{S}$ denotes the closure of S in $\mathbb{C}$, then

(A) $\overline{f(R_{n+1})} \neq \overline{f(R_n)}$

(B) $\overline{f(R_n)} \setminus \overline{f(R_{n+1})} = \overline{f(R_n \setminus R_{n+1})}$

(C) $\overline{f(\cap_{n=1}^{\infty} R_n)} = \cap_{n=1}^{\infty} \overline{f(R_n)}$

(D) $\overline{f(R_n)} = \overline{f(R_{n+1})}$

Q.No. 7 Suppose that

$$U = \mathbb{R}^2 \setminus \{(x,y) \in \mathbb{R}^2 : x,y \in \mathbb{Q}\},$$

$$V = \mathbb{R}^2 \setminus \left\{(x,y) \in \mathbb{R}^2 : x > 0, y = \frac{1}{x}\right\}.$$

Then, with respect to the Euclidean metric on $\mathbb{R}^2$,

(A) both U and V are disconnected
(B) U is disconnected but V is connected
(C) U is connected but V is disconnected
(D) both U and V are connected

Q.No. 8 If (D1) and (D2) denote the dual problems of the linear programming problems (P1) and (P2), respectively, where

(P1) : minimize $x_1 - 2x_2$ subject to $-x_1 + x_2 = 10,\ x_1, x_2 \geq 0$,

(P2) : minimize $x_1 - 2x_2$ subject to $-x_1 + x_2 = 10, x_1 - x_2 = 10, x_1, x_2 \geq 0$,

then

(A) both (D1) and (D2) are infeasible
(B) (P2) is infeasible and (D2) is feasible
(C) (D1) is infeasible and (D2) is feasible but unbounded
(D) (P1) is feasible but unbounded and (D1) is feasible

Q.No. 9 If $(4,0)$ and $\left(0,-\frac{1}{2}\right)$ are critical points of the function

$$f(x,y) = 5 - (\alpha+\beta)x^2 + \beta y^2 + (\alpha+1)y^3 + x^3,$$

where $\alpha, \beta \in \mathbb{R}$, then

(A) $\left(4,-\frac{1}{2}\right)$ is a point of local maxima of f
(B) $\left(4,-\frac{1}{2}\right)$ is a saddle point of f
(C) $\alpha = 4,\ \beta = 2$
(D) $\left(4,-\frac{1}{2}\right)$ is a point of local minima of f

Q.No. 10 Consider the iterative scheme

$$x_n = \frac{x_{n-1}}{2} + \frac{3}{x_{n-1}}, \qquad n \geq 1,$$

with initial point $x_0 > 0$. Then the sequence $\{x_n\}$

(A) converges only if $x_0 > 1$
(B) converges only if $x_0 < 3$
(C) converges for any x_0
(D) does not converge for any x_0

Q.No. 11 Let $C[0,1]$ denote the space of all real-valued continuous functions on $[0,1]$ equipped with the supremum norm $\|\cdot\|_\infty$. Let $T: C[0,1] \to C[0,1]$ be the linear operator defined by

$$T(f)(x) = \int_0^x e^{-y} f(y)dy.$$

Then

(A) $\|T\| = 1$

(B) $I - T$ is not invertible

(C) T is surjective

(D) $\| I + T\| = 1 + \|T\|$

Q.No. 12 Suppose that M is a 5×5 matrix with real entries and $p(x) = \det(xI - M)$. Then

(A) $p(0) = \det(M)$

(B) every eigenvalue of M is real if $p(1) + p(2) = 0 = p(2) + p(3)$

(C) M^{-1} is necessarily a polynomial in M of degree 4 if M is invertible

(D) M is not invertible if $M^2 - 2M = 0$

Q.No. 13 Let $C[0,1]$ denote the space of all real-valued continuous functions on $[0,1]$ equipped with the supremum norm $\|\cdot\|_\infty$. Let $f \in C[0,1]$ be such that

$$|f(x) - f(y)| \le M|x - y|, \text{ for all } x, y \in [0,1] \text{ and for some } M > 0.$$

For $n \in \mathbb{N}$, let $f_n(x) = f\left(x^{1+\frac{1}{n}}\right)$. If $S = \{ f_n : n \in \mathbb{N} \}$, then

(A) the closure of S is compact

(B) S is closed and bounded

(C) S is bounded but not totally bounded

(D) S is compact

Q.No. 14 Let $K : \mathbb{R} \times (0, \infty) \to \mathbb{R}$ be a function such that the solution of the initial value problem $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, $u(x,0) = f(x)$, $x \in \mathbb{R}$, $t > 0$, is given by

$$u(x,t) = \int_{\mathbb{R}} K(x-y,t) f(y)\, dy$$

for all bounded continuous functions f. Then the value of $\int_{\mathbb{R}} K(x,t)dx$ is________

Q.No. 15 The number of cyclic subgroups of the quaternion group

$$Q_8 = \langle a, b \mid a^4 = 1,\ a^2 = b^2,\ ba = a^3 b\rangle$$

is ______________

Q.No. 16 The number of elements of order 3 in the symmetric group S_6 is ___________

Q.No. 17 Let F be the field with 4096 elements. The number of proper subfields of F is

____

Q.No. 18 If (x_1^*, x_2^*) is an optimal solution of the linear programming problem,

$$\text{minimize } x_1 + 2x_2$$
$$\text{subject to}$$
$$\begin{aligned} 4x_1 - x_2 &\geq 8 \\ 2x_1 + x_2 &\geq 10 \\ -x_1 + x_2 &\leq 7 \\ x_1, x_2 &\geq 0 \end{aligned}$$

and $(\lambda_1^*, \lambda_2^*, \lambda_3^*)$ is an optimal solution of its dual problem, then $\sum_{i=1}^{2} x_i^* + \sum_{j=1}^{3} \lambda_j^*$ is equal to ____________________(correct up to one decimal place)

Q.No. 19 Let $a, b, c \in \mathbb{R}$ be such that the quadrature rule

$$\int_{-1}^{1} f(x)dx \approx af(-1) + bf(0) + cf'(1)$$

is exact for all polynomials of degree less than or equal to 2. Then b is equal to____________ (rounded off to two decimal places)

Q.No. 20 Let $f(x) = x^4$ and let $p(x)$ be the interpolating polynomial of f at nodes $1, 2$ and 3. Then $p(0)$ is equal to__________

Q.No. 21 For $n \geq 2$, define the sequence $\{x_n\}$ by

$$x_n = \frac{1}{2\pi} \int_{0}^{\frac{\pi}{2}} \tan^{\frac{1}{n}} t \ dt.$$

Then the sequence $\{x_n\}$ converges to ______________ (correct up to two decimal places)

Q.No. 22

Let

$$L^2[0,10] = \left\{f:[0,10] \to \mathbb{R} : f \text{ is Lebesgue measurable and } \int_0^{10} f^2 dx < \infty\right\}$$

equipped with the norm $\|f\| = \left(\int_0^{10} f^2 dx\right)^{\frac{1}{2}}$ and let T be the linear functional on $L^2[0,10]$ given by

$$T(f) = \int_0^2 f(x)dx - \int_3^{10} f(x)dx.$$

Then $\|T\|$ is equal to __________

Q.No. 23 If $\{x_{13}, x_{22}, x_{23} = 10, x_{31}, x_{32}, x_{34}\}$ is the set of basic variables of a balanced transportation problem seeking to minimize cost of transportation from origins to destinations, where the cost matrix is,

	D_1	D_2	D_3	D_4	Availability
O_1	6	2	-1	0	10
O_2	4	2	2	3	λ+5
O_3	3	1	2	1	3λ
Demand	10	$\mu - 5$	$\mu + 5$	15	

and $\lambda, \mu \in \mathbb{R}$, then x_{32} is equal to _______________

Q.No. 24 Let $\mathbb{Z}_{225}$ be the ring of integers modulo 225. If x is the number of prime ideals and y is the number of nontrivial units in $\mathbb{Z}_{225}$, then $x + y$ is equal to _________

Q.No. 25 Let $u(x,t)$ be the solution of

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = 0, \;\; u(x,0) = f(x), \qquad \frac{\partial u}{\partial t}(x,0) = 0, \;\; x \in \mathbb{R}, t > 0,$$

where f is a twice continuously differentiable function. If $f(-2) = 4, f(0) = 0$, and $u(2,2) = 8$, then the value of $u(1,3)$ is _________

Q26 - Q55 carry two marks each.

Q.No. 26 Let $\{e_n\}_{n=1}^{\infty}$ be an orthonormal basis for a separable Hilbert space H with the inner product $\langle \cdot , \cdot \rangle$. Define

$$f_n = e_n - \frac{1}{n+1} e_{n+1} \text{ for } n \in \mathbb{N}.$$

Then

(A) the closure of the span $\{f_n : n \in \mathbb{N}\}$ equals H

(B) $f = 0$ if $\langle f, f_n \rangle = \langle f, e_n \rangle$ for all $n \in \mathbb{N}$

(C) $\{f_n\}_{n=1}^{\infty}$ is an orthogonal subset of H

(D) there does not exist nonzero $f \in H$ such that $\langle f, e_2 \rangle = \langle f, f_2 \rangle$

Q.No. 27 Suppose V is a finite dimensional non-zero vector space over $\mathbb{C}$ and $T: V \to V$ is a linear transformation such that $\text{Range}(T) = \text{Nullspace}(T)$. Then which of the following statements is FALSE?

(A) The dimension of V is even

(B) 0 is the only eigenvalue of T

(C) Both 0 and 1 are eigenvalues of T

(D) $T^2 = 0$

Q.No. 28 Let $P \in M_{m\times n}(\mathbb{R})$. Consider the following statements:

I : If $XPY = 0$ for all $X \in M_{1\times m}(\mathbb{R})$ and $Y \in M_{n\times 1}(\mathbb{R})$, then $P = 0$.

II : If $m = n$, P is symmetric and $P^2 = 0$, then $P = 0$.

Then

(A) both I and II are true

(B) I is true but II is false

(C) I is false but II is true

(D) both I and II are false

Q.No. 29 For $n \in \mathbb{N}$, let $T_n: (l^1, \|\cdot\|_1) \to (l^\infty, \|\cdot\|_\infty)$ and $T: (l^1, \|\cdot\|_1) \to (l^\infty, \|\cdot\|_\infty)$ be the bounded linear operators defined by

$$T_n(x_1, x_2, \ldots) = (y_1, y_2, \ldots), \text{ where } y_j = \begin{cases} x_j, & j \le n \\ x_n, & j > n \end{cases}$$

and

$$T(x_1, x_2, \ldots) = (x_1, x_2, \ldots).$$

Then

(A) $\|T_n\|$ does not converge to $\|T\|$ as $n \to \infty$

(B) $\|T_n - T\|$ converges to zero as $n \to \infty$

(C) for all $x \in l^1$, $\|T_n(x) - T(x)\|$ converges to zero as $n \to \infty$

(D) for each non-zero $x \in l^1$, there exists a continuous linear functional g on l^∞ such that $g(T_n(x))$ does not converge to $g(T(x))$ as $n \to \infty$

Q.No. 30 Let $P(\mathbb{R})$ denote the power set of $\mathbb{R}$, equipped with the metric

$$d(U, V) = \sup_{x\in\mathbb{R}} |\chi_U(x) - \chi_V(x)|,$$

where χ_U and χ_V denote the characteristic functions of the subsets U and V, respectively, of $\mathbb{R}$. The set $\{\{m\} : m \in \mathbb{Z}\}$ in the metric space $(P(\mathbb{R}), d)$ is

(A) bounded but not totally bounded

(B) totally bounded but not compact

(C) compact

(D) not bounded

Q.No. 31 Let $f:\mathbb{R}\to\mathbb{R}$ be defined by

$$f(x)=\sum_{n=0}^{\infty}\frac{1}{2^n}\chi_{(n,n+1]}(x),$$

where $\chi_{(n,n+1]}$ is the characteristic function of the interval $(n,n+1]$. For $\alpha\in\mathbb{R}$, let $S_\alpha=\{x\in\mathbb{R}: f(x)>\alpha\}$. Then

(A) $S_{\frac{1}{2}}$ is open

(B) $S_{\frac{\sqrt{5}}{2}}$ is not measurable

(C) S_0 is closed

(D) $S_{\frac{1}{\sqrt{2}}}$ is measurable

Q.No. 32 For $n\in\mathbb{N}$, let $f_n, g_n: (0,1)\to\mathbb{R}$ be functions defined by

$$f_n(x)=x^n \text{ and } g_n(x)=x^n(1-x).$$

Then

(A) $\{f_n\}$ converges uniformly but $\{g_n\}$ does not converge uniformly

(B) $\{g_n\}$ converges uniformly but $\{f_n\}$ does not converge uniformly

(C) both $\{f_n\}$ and $\{g_n\}$ converge uniformly

(D) neither $\{f_n\}$ nor $\{g_n\}$ converge uniformly

Q.No. 33 Let u be a solution of the differential equation $y'+xy=0$ and let $\phi=u\psi$ be a solution of the differential equation $y''+2xy'+(x^2+2)y=0$ satisfying $\phi(0)=1$ and $\phi'(0)=0$. Then $\phi(x)$ is

(A) $(\cos^2 x)e^{-\frac{x^2}{2}}$

(B) $(\cos x)e^{-\frac{x^2}{2}}$

(C) $(1+x^2)e^{-\frac{x^2}{2}}$

(D) $(\cos x)e^{-x^2}$

Q.No. 34 For $n\in\mathbb{N}\cup\{0\}$, let y_n be a solution of the differential equation

$$xy''+(1-x)y'+n\,y=0$$

satisfying $y_n(0)=1$. For which of the following functions $w(x)$, the integral

$$\int_0^{\infty} y_p(x)\,y_q(x)\,w(x)dx, \qquad (p\neq q)$$

is equal to zero?

(A) e^{-x^2}

(B) e^{-x}

(C) xe^{-x^2}

(D) xe^{-x}

Q.No. 35 Suppose that

$$X = \{(0,0)\} \cup \left\{ \left(x, \sin\frac{1}{x}\right) : x \in \mathbb{R} \setminus \{0\} \right\}$$

and

$$Y = \{(0,0)\} \cup \left\{ \left(x, x\sin\frac{1}{x}\right) : x \in \mathbb{R} \setminus \{0\} \right\}$$

are metric spaces with metrics induced by the Euclidean metric of $\mathbb{R}^2$. Let B_X and B_Y be the open unit balls around $(0,0)$ in X and Y, respectively. Consider the following statements:

I : The closure of B_X in X is compact.

II : The closure of B_Y in Y is compact.

Then

(A) both I and II are true

(B) I is true but II is false

(C) I is false but II is true

(D) both I and II are false

Q.No. 36 If $f: \mathbb{C} \setminus \{0\} \to \mathbb{C}$ is a function such that $f(z) = f\left(\frac{z}{|z|}\right)$ and its restriction to the unit circle is continuous, then

(A) f is continuous but not necessarily analytic

(B) f is analytic but not necessarily a constant function

(C) f is a constant function

(D) $\lim_{z\to 0} f(z)$ exists

Q.No. 37 For a subset S of a topological space, let $\text{Int}(S)$ and $\bar{S}$ denote the interior and closure of S, respectively. Then which of the following statements is TRUE?

(A) If S is open, then $S = \text{Int}(\bar{S})$

(B) If the boundary of S is empty, then S is open

(C) If the boundary of S is empty, then S is not closed

(D) If $\bar{S} \setminus S$ is a proper subset of the boundary of S, then S is open

Q.No. 38 Suppose $\mathfrak{I}_1, \mathfrak{I}_2$ and $\mathfrak{I}_3$ are the smallest topologies on $\mathbb{R}$ containing S_1, S_2 and S_3, respectively, where

$$S_1 = \left\{ \left(a, a + \frac{\pi}{n}\right) : a \in \mathbb{Q}, n \in \mathbb{N} \right\},$$

$$S_2 = \{ (a,b) : a < b, \quad a, b \in \mathbb{Q} \},$$

$$S_3 = \{ (a,b) : a < b, \quad a, b \in \mathbb{R} \}.$$

Then

(A) $\mathfrak{I}_3 \supsetneq \mathfrak{I}_1$

(B) $\mathfrak{I}_3 \supsetneq \mathfrak{I}_2$

(C) $\mathfrak{I}_1 = \mathfrak{I}_2$

(D) $\mathfrak{I}_1 \supsetneq \mathfrak{I}_2$

Q.No. 39 Let $M = \begin{bmatrix} \alpha & 3 & 0 \\ \beta & 3 & 1 \\ 0 & 1 & 2 \end{bmatrix}$. Consider the following statements:

I: There exists a lower triangular matrix L such that $M = LL^t$, where L^t denotes transpose of L.

II: Gauss-Seidel method for $Mx = b$ ($b \in \mathbb{R}^3$) converges for any initial choice $x_0 \in \mathbb{R}^3$.

Then

(A) I is not true when $\alpha > \frac{9}{2},\ \beta = 3$

(B) II is not true when $\alpha > \frac{9}{2},\ \beta = -1$

(C) II is not true when $\alpha = 4,\ \beta = \frac{3}{2}$

(D) I is true when $\alpha = 5,\ \beta = 3$

Q.No. 40 Let I and J be the ideals generated by $\{5, \sqrt{10}\}$ and $\{4, \sqrt{10}\}$ in the ring $\mathbb{Z}[\sqrt{10}\,] = \{a + b\sqrt{10} \mid a, b \in \mathbb{Z}\}$, respectively. Then

(A) both I and J are maximal ideals

(B) I is a maximal ideal but J is not a prime ideal

(C) I is not a maximal ideal but J is a prime ideal

(D) neither I nor J is a maximal ideal

Q.No. 41 Suppose V is a finite dimensional vector space over $\mathbb{R}$. If W_1, W_2 and W_3 are subspaces of V, then which of the following statements is TRUE?

(A) If $W_1 + W_2 + W_3 = V$ then
$\text{span}(W_1 \cup W_2) \cup \text{span}(W_2 \cup W_3) \cup \text{span}(W_3 \cup W_1) = V$

(B) If $W_1 \cap W_2 = \{0\}$ and $W_1 \cap W_3 = \{0\}$, then $W_1 \cap (W_2 + W_3) = \{0\}$

(C) If $W_1 + W_2 = W_1 + W_3$, then $W_2 = W_3$

(D) If $W_1 \neq V$, then $\text{span}(V \setminus W_1) = V$

Q.No. 42 Let $\alpha, \beta \in \mathbb{R}, \alpha \neq 0$. The system

$$\begin{gathered} x_1 - 2x_2 + \alpha x_3 = 8 \\ x_1 - x_2 + x_4 = \beta \\ x_1, x_2, x_3, x_4 \geq 0 \end{gathered}$$

has NO basic feasible solution if

(A) $\alpha < 0,\ \beta > 8$

(B) $\alpha > 0,\ 0 < \beta < 8$

(C) $\alpha > 0,\ \beta < 0$

(D) $\alpha < 0,\ \beta < 8$

Q.No. 43 Let $0 < p < 1$ and let

$$X = \left\{ f: \mathbb{R} \to \mathbb{R} \text{ is continuous and } \int_{\mathbb{R}} |f(x)|^p \, dx < \infty \right\}.$$

For $f \in X$, define

$$|f|_p = \left(\int_{\mathbb{R}} |f(x)|^p dx \right)^{\frac{1}{p}}.$$

Then

(A) $|\cdot|_p$ defines a norm on X

(B) $|f+g|_p \le |f|_p + |g|_p$ for all $f, g \in X$

(C) $|f+g|_p^p \le |f|_p^p + |g|_p^p$ for all $f, g \in X$

(D) if f_n converges to f pointwise on $\mathbb{R}$, then $\lim_{n\to\infty} |f_n|_p = |f|_p$

Q.No. 44 Suppose that ϕ_1 and ϕ_2 are linearly independent solutions of the differential equation

$$2x^2y'' - (x + x^2)y' + (x^2 - 2)y = 0,$$

and $\phi_1(0) = 0$. Then the smallest positive integer n such that

$$\lim_{x\to 0} x^n \frac{\phi_2(x)}{\phi_1(x)} = 0$$

is __________

Q.No. 45 Suppose that $f(z) = \prod_{n=1}^{17} \left(z - \frac{\pi}{n}\right)$, $z \in \mathbb{C}$ and $\gamma(t) = e^{2it}, t \in [0, 2\pi]$. If

$$\int_{\gamma} \frac{f'(z)}{f(z)} dz = \alpha \pi i,$$

then the value of α is equal to ______________

Q.No. 46 If $\gamma(t) = \frac{1}{2} e^{3\pi i\, t}$, $t \in [0, 2]$ and

$$\int_{\gamma} \frac{1}{z^2(e^z - 1)} \, dz = \beta \, \pi i,$$

then β is equal to__________(correct up to one decimal place)

Q.No. 47 Let $K = \mathbb{Q}\left(\sqrt{3 + 2\sqrt{2}},\ \omega\right)$, where ω is a primitive cube root of unity. Then the degree of extension of K over $\mathbb{Q}$ is _____________________

Q.No. 48 Let $\alpha \in \mathbb{R}$. If $(3,0,0,\beta)$ is an optimal solution of the linear programming problem

$$\text{minimize } x_1 + x_2 + x_3 - \alpha x_4$$

subject to

$$\begin{gathered} 2x_1 - x_2 + x_3 = 6 \\ -x_1 + x_2 + x_4 = 3 \\ x_1, x_2, x_3, x_4 \geq 0 \end{gathered}$$

then the maximum value of $\beta - \alpha$ is _______________

Q.No. 49 Suppose that $T: \mathbb{R}^4 \to \mathbb{R}[x]$ is a linear transformation over $\mathbb{R}$ satisfying

$$T(-1,1,1,1) = x^2 + 2x^4, \ T(1,2,3,4) = 1 - x^2,$$

$$T(2,-1,-1,0) = x^3 - x^4.$$

Then the coefficient of x^4 in $T(-3,5,6,6)$ is ________________

Q.No. 50 Let $\vec{F}(x,y,z) = (2x - 2y\cos x)\,\hat{\imath} + (2y - y^2 \sin x)\,\hat{\jmath} + 4z\,\hat{k}$ and let S be the surface of the tetrahedron bounded by the planes

$$x = 0, y = 0, z = 0 \text{ and } x + y + z = 1.$$

If $\hat{n}$ is the unit outward normal to the tetrahedron, then the value of

$$\iint_S \vec{F} \cdot \hat{n}\, dS$$

is______________________(rounded off to two decimal places)

Q.No. 51 Let $\vec{F} = (x + 2y)e^z\,\hat{\imath} + (ye^z + x^2)\,\hat{\jmath} + y^2 z\,\hat{k}$ and let S be the surface

$x^2 + y^2 + z = 1, \ z \geq 0$. If $\hat{n}$ is a unit normal to S and

$$\left| \iint_S (\nabla \times \vec{F}) \cdot \hat{n}\, dS \right| = \alpha\pi.$$

Then α is equal to ________

Q.No. 52 Let G be a non-cyclic group of order 57. Then the number of elements of order 3 in G is ___________

Q.No. 53

The coefficient of $(x-1)^5$ in the Taylor expansion about $x = 1$ of the function

$$F(x) = \int_1^x \frac{\log_e t}{t-1}\, dt, \quad 0 < x < 2$$

is ______________(correct up to two decimal places)

Q.No. 54 Let $u(x, y)$ be the solution of the initial value problem

$$\frac{\partial u}{\partial x} + (\sqrt{u})\frac{\partial u}{\partial y} = 0, \ u(x,0) = 1 + x^2.$$

Then the value of $u(0,1)$ is __________(rounded off to three decimal places)

Q.No. 55 The value of

$$\lim_{n\to\infty} \int_0^1 nx^n e^{x^2} dx$$

is___________ (rounded off to three decimal places)

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Answer Key - MA: Mathematics

Q.No.	Session	Que.Type	Sec. Name	Key	Marks
1	3	MCQ	GA	C	1
2	3	MCQ	GA	B	1
3	3	MCQ	GA	D	1
4	3	MCQ	GA	D	1
5	3	MCQ	GA	B	1
6	3	MCQ	GA	D	2
7	3	MCQ	GA	C	2
8	3	MCQ	GA	B	2
9	3	MCQ	GA	B	2
10	3	MCQ	GA	B	2
1	3	MCQ	MA	A	1
2	3	MCQ	MA	C	1
3	3	MCQ	MA	D	1
4	3	MCQ	MA	B	1
5	3	MCQ	MA	B	1
6	3	MCQ	MA	A or D	1
7	3	MCQ	MA	C	1
8	3	MCQ	MA	A	1
9	3	MCQ	MA	B	1
10	3	MCQ	MA	C	1
11	3	MCQ	MA	D	1
12	3	MCQ	MA	C	1
13	3	MCQ	MA	A	1
14	3	NAT	MA	1 to 1	1
15	3	NAT	MA	5 to 5	1
16	3	NAT	MA	80 to 80	1
17	3	NAT	MA	5 to 5	1
18	3	NAT	MA	5.5 to 5.5	1
19	3	NAT	MA	1.70 to 1.80	1
20	3	NAT	MA	36 to 36	1
21	3	NAT	MA	0.25 to 0.25	1
22	3	NAT	MA	3 to 3	1
23	3	NAT	MA	5 to 5	1
24	3	NAT	MA	121 to 121	1
25	3	NAT	MA	10 to 10	1
26	3	MCQ	MA	A	2
27	3	MCQ	MA	C	2
28	3	MCQ	MA	A	2
29	3	MCQ	MA	C	2
30	3	MCQ	MA	A	2

31	3	MCQ	MA	D	2
32	3	MCQ	MA	B	2
33	3	MCQ	MA	B	2
34	3	MCQ	MA	B	2
35	3	MCQ	MA	C	2
36	3	MCQ	MA	A	2
37	3	MCQ	MA	B	2
38	3	MCQ	MA	C	2
39	3	MCQ	MA	D	2
40	3	MCQ	MA	B	2
41	3	MCQ	MA	D	2
42	3	MCQ	MA	D	2
43	3	MCQ	MA	C	2
44	3	NAT	MA	3 to 3	2
45	3	NAT	MA	56 to 56	2
46	3	NAT	MA	0.5 to 0.5	2
47	3	NAT	MA	4 to 4	2
48	3	NAT	MA	7 to 7	2
49	3	NAT	MA	5 to 5	2
50	3	NAT	MA	1.30 to 1.40	2
51	3	NAT	MA	2 to 2	2
52	3	NAT	MA	38 to 38	2
53	3	NAT	MA	0.04 to 0.04	2
54	3	NAT	MA	1.610 to 1.625	2
55	3	NAT	MA	2.710 to 2.725	2

GATE 2019 General Aptitude (GA) Set-8

Q. 1 – Q. 5 carry one mark each.

Q.1 The fishermen, _______ the flood victims owed their lives, were rewarded by the government.

(A) whom (B) to which (C) to whom (D) that

Q.2 Some students were not involved in the strike.

If the above statement is true, which of the following conclusions is/are logically necessary?

1. Some who were involved in the strike were students.
2. No student was involved in the strike.
3. At least one student was involved in the strike.
4. Some who were not involved in the strike were students.

(A) 1 and 2 (B) 3 (C) 4 (D) 2 and 3

Q.3 The radius as well as the height of a circular cone increases by 10%. The percentage increase in its volume is ______.

(A) 17.1 (B) 21.0 (C) 33.1 (D) 72.8

Q.4 Five numbers 10, 7, 5, 4 and 2 are to be arranged in a sequence from left to right following the directions given below:
1. No two odd or even numbers are next to each other.
2. The second number from the left is exactly half of the left-most number.
3. The middle number is exactly twice the right-most number.

Which is the second number from the right?

(A) 2 (B) 4 (C) 7 (D) 10

Q.5 Until Iran came along, India had never been ________________ in kabaddi.

(A) defeated (B) defeating (C) defeat (D) defeatist

GATE 2019 General Aptitude (GA) Set-8

Q. 6 – Q. 10 carry two marks each.

Q.6 Since the last one year, after a 125 basis point reduction in repo rate by the Reserve Bank of India, banking institutions have been making a demand to reduce interest rates on small saving schemes. Finally, the government announced yesterday a reduction in interest rates on small saving schemes to bring them on par with fixed deposit interest rates.

Which one of the following statements can be inferred from the given passage?

(A) Whenever the Reserve Bank of India reduces the repo rate, the interest rates on small saving schemes are also reduced

(B) Interest rates on small saving schemes are always maintained on par with fixed deposit interest rates

(C) The government sometimes takes into consideration the demands of banking institutions before reducing the interest rates on small saving schemes

(D) A reduction in interest rates on small saving schemes follow only after a reduction in repo rate by the Reserve Bank of India

Q.7 In a country of 1400 million population, 70% own mobile phones. Among the mobile phone owners, only 294 million access the Internet. Among these Internet users, only half buy goods from e-commerce portals. What is the percentage of these buyers in the country?

(A) 10.50 (B) 14.70 (C) 15.00 (D) 50.00

Q.8 The nomenclature of Hindustani music has changed over the centuries. Since the medieval period *dhrupad* styles were identified as *baanis*. Terms like *gayaki* and *baaj* were used to refer to vocal and instrumental styles, respectively. With the institutionalization of music education the term *gharana* became acceptable. *Gharana* originally referred to hereditary musicians from a particular lineage, including disciples and grand disciples.

Which one of the following pairings is NOT correct?

(A) *dhrupad*, *baani*
(B) *gayaki*, vocal
(C) *baaj*, institution
(D) *gharana*, lineage

Q.9 Two trains started at 7AM from the same point. The first train travelled north at a speed of 80km/h and the second train travelled south at a speed of 100 km/h. The time at which they were 540 km apart is ______ AM.

(A) 9 (B) 10 (C) 11 (D) 11.30

Q.10 "I read somewhere that in ancient times the prestige of a kingdom depended upon the number of taxes that it was able to levy on its people. It was very much like the prestige of a head-hunter in his own community."

Based on the paragraph above, the prestige of a head-hunter depended upon ___________

(A) the prestige of the kingdom
(B) the prestige of the heads
(C) the number of taxes he could levy
(D) the number of heads he could gather

END OF THE QUESTION PAPER

Q. 1 – Q. 25 carry one mark each.

Q.1 For a balanced transportation problem with three sources and three destinations where costs, availabilities and demands are all finite and positive, which one of the following statements is **FALSE**?

(A) The transportation problem does not have unbounded solution

(B) The number of non-basic variables of the transportation problem is 4

(C) The dual variables of the transportation problem are unrestricted in sign

(D) The transportation problem has at most 5 basic feasible solutions

Q.2 Let $f:[a,b]\to\mathbb{R}$ (the set of all real numbers) be any function which is twice differentiable in (a,b) with only one root α in (a,b). Let $f'(x)$ and $f''(x)$ denote the first and second order derivatives of $f(x)$ with respect to x. If α is a simple root and is computed by the Newton-Raphson method, then the method converges if

(A) $|f(x)\,f''(x)|<|f'(x)|^2$, for all $x\in(a,b)$

(B) $|f(x)\,f'(x)|<|f''(x)|$, for all $x\in(a,b)$

(C) $|f'(x)\,f''(x)|<|f(x)|^2$, for all $x\in(a,b)$

(D) $|f(x)\,f''(x)|<|f'(x)|$, for all $x\in(a,b)$

Q.3 Let $f:\mathbb{C}\to\mathbb{C}$ (the set of all complex numbers) be defined by

$$f(x+iy)=x^3+3xy^2+i\left(y^3+3x^2y\right),\quad i=\sqrt{-1}.$$

Let $f'(z)$ denote the derivative of f with respect to z.

Then which one of the following statements is TRUE?

(A) $f'(1+i)$ exists and $|f'(1+i)|=3\sqrt{5}$

(B) f is analytic at the origin

(C) f is not differentiable at i

(D) f is differentiable at 1

Q.4 The partial differential equation

$$\left(x^2+y^2-1\right)\frac{\partial^2 u}{\partial x^2}+2\frac{\partial^2 u}{\partial x\partial y}+\left(x^2+y^2-1\right)\frac{\partial^2 u}{\partial y^2}=0$$

is

(A) parabolic in the region $x^2+y^2>2$

(B) hyperbolic in the region $x^2+y^2>2$

(C) elliptic in the region $0<x^2+y^2<2$

(D) hyperbolic in the region $0<x^2+y^2<2$

Q.5 If

$$u_n = \int_1^n e^{-t^2}\, dt, \quad n = 1, 2, 3, \dots,$$

then which one of the following statements is TRUE?

(A) Both the sequence $\{u_n\}_{n=1}^{\infty}$ and the series $\sum_{n=1}^{\infty} u_n$ are convergent

(B) Both the sequence $\{u_n\}_{n=1}^{\infty}$ and the series $\sum_{n=1}^{\infty} u_n$ are divergent

(C) The sequence $\{u_n\}_{n=1}^{\infty}$ is convergent but the series $\sum_{n=1}^{\infty} u_n$ is divergent

(D) $\lim_{n\to\infty} u_n = \frac{2}{e}$

Q.6 Let $\Gamma = \{(x, y, z) \in \mathbb{R}^3 : -1 < x < 1, -1 < y < 1, -1 < z < 1\}$ and $\phi : \Gamma \to \mathbb{R}$ be a function whose all second order partial derivatives exist and are continuous. If ϕ satisfies the Laplace equation $\nabla^2 \phi = 0$ for all $(x, y, z) \in \Gamma$, then which one of the following statements is TRUE in Γ?

($\mathbb{R}$ is the set of all real numbers, and $\mathbb{R}^3 = \{(x, y, z) : x, y, z \in \mathbb{R}\}$)

(A) $\vec{\nabla}\phi$ is solenoidal but not irrotational

(B) $\vec{\nabla}\phi$ is irrotational but not solenoidal

(C) $\vec{\nabla}\phi$ is both solenoidal and irrotational

(D) $\vec{\nabla}\phi$ is neither solenoidal nor irrotational

Q.7 Let $X = \{(x_1, x_2, ...) :\ x_i \in \mathbb{R}$ and only finitely many x_i's are non-zero$\}$ and $d : X \times X \to \mathbb{R}$ be a metric on X defined by

$$d(x, y) = \sup_{i \in \mathbb{N}} |x_i - y_i| \text{ for } x = (x_1, x_2, ...), y = (y_1, y_2, ...) \text{ in } X.$$

($\mathbb{R}$ is the set of all real numbers and $\mathbb{N}$ is the set of all natural numbers)

Consider the following statements:

$P : (X, d)$ is a complete metric space.

$Q :$ The set $\{x \in X : d(\underline{0}, x) \leq 1\}$ is compact, where $\underline{0}$ is the zero element of X.

Which of the above statements is/are TRUE?

(A) Both P and Q (B) P only (C) Q only (D) Neither P nor Q

Q.8 Consider the following statements:

I. The set $\mathbb{Q} \times \mathbb{Z}$ is uncountable.

II. The set $\{f : f$ is a function from $\mathbb{N}$ to $\{0, 1\}\}$ is uncountable.

III. The set $\{\sqrt{p} : p$ is a prime number$\}$ is uncountable.

IV. For any infinite set, there exists a bijection from the set to one of its proper subsets.

($\mathbb{Q}$ is the set of all rational numbers, $\mathbb{Z}$ is the set of all integers and $\mathbb{N}$ is the set of all natural numbers)

Which of the above statements are TRUE?

(A) I and IV only (B) II and IV only (C) II and III only (D) I, II and IV only

Q.9 Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x, y) = x^6 - 2x^2y - x^4y + 2y^2.$$

($\mathbb{R}$ is the set of all real numbers and $\mathbb{R}^2 = \{(x, y) : x, y \in \mathbb{R}\}$)

Which one of the following statements is TRUE?

(A) f has a local maximum at origin

(B) f has a local minimum at origin

(C) f has a saddle point at origin

(D) The origin is not a critical point of f

Q.10 Let $\{a_n\}_{n=0}^{\infty}$ be any sequence of real numbers such that $\sum_{n=0}^{\infty}|a_n|^2 < \infty$. If the radius of convergence of $\sum_{n=0}^{\infty} a_n x^n$ is r, then which one of the following statements is necessarily TRUE?

(A) $r \geq 1$ or r is infinite

(B) $r < 1$

(C) $r = \left(\sum_{n=0}^{\infty}|a_n|^2\right)^{\frac{1}{2}}$

(D) $r = \sum_{n=0}^{\infty}|a_n|^2$

Q.11 Let T_1 be the co-countable topology on $\mathbb{R}$ (the set of real numbers) and T_2 be the co-finite topology on $\mathbb{R}$.

Consider the following statements:

I. In $(\mathbb{R}, T_1)$, the sequence $\left\{\frac{1}{n}\right\}_{n=1}^{\infty}$ converges to 0.

II. In $(\mathbb{R}, T_2)$, the sequence $\left\{\frac{1}{n}\right\}_{n=1}^{\infty}$ converges to 0.

III. In $(\mathbb{R}, T_1)$, there is no sequence of rational numbers which converges to $\sqrt{3}$.

IV. In $(\mathbb{R}, T_2)$, there is no sequence of rational numbers which converges to $\sqrt{3}$.

Which of the above statements are TRUE?

(A) I and II only
(B) II and III only
(C) III and IV only
(D) I and IV only

Q.12 Let X and Y be normed linear spaces, and let $T : X \to Y$ be any bijective linear map with closed graph. Then which one of the following statements is TRUE?

(A) The graph of T is equal to $X \times Y$

(B) T^{-1} is continuous

(C) The graph of T^{-1} is closed

(D) T is continuous

Q.13 Let $g:\mathbb{R}^2 \to \mathbb{R}^2$ be a function defined by $g(x,y)=(e^x \cos y,\, e^x \sin y)$ and $(a,b)=g\left(1,\frac{\pi}{3}\right)$.

($\mathbb{R}$ is the set of all real numbers and $\mathbb{R}^2=\{(x,y): x,y\in\mathbb{R}\}$)

Which one of the following statements is TRUE?

(A) g is injective

(B) If h is the continuous inverse of g, defined in some neighbourhood of $(a,b)\in\mathbb{R}^2$, such that $h(a,b)=\left(1,\frac{\pi}{3}\right)$, then the Jacobian of h at (a,b) is e^2

(C) If h is the continuous inverse of g, defined in some neighbourhood of $(a,b)\in\mathbb{R}^2$, such that $h(a,b)=\left(1,\frac{\pi}{3}\right)$, then the Jacobian of h at (a,b) is e^{-2}

(D) g is surjective

Q.14 Let

$$u_n=\frac{n!}{1.3.5...(2n-1)},\ n\in\mathbb{N} \text{ (the set of all natural numbers).}$$

Then $\lim_{n\to\infty} u_n$ is equal to _____ .

Q.15 If the differential equation

$$\frac{dy}{dx}=\sqrt{x^2+y^2},\ y(1)=2$$

is solved using the Euler's method with step-size $h=0.1$, then $y(1.2)$ is equal to _____ (round off to 2 places of decimal).

Q.16 Let f be any polynomial function of degree at most 2 over $\mathbb{R}$ (the set of all real numbers). If the constants a and b are such that

$$\frac{df}{dx}=a\,f(x)+2\,f(x+1)+b\,f(x+2),\ \text{for all } x\in\mathbb{R},$$

then $4a + 3b$ is equal to _____ (round off to 2 places of decimal).

Q.17 Let L denote the value of the line integral $\oint_C (3x-4x^2y)dx+(4xy^2+2y)dy$, where C, a circle of radius 2 with centre at origin of the xy-plane, is traversed once in the anti-clockwise direction. Then $\frac{L}{\pi}$ is equal to _____ .

Q.18 The temperature $T:\mathbb{R}^3\setminus\{(0,0,0)\}\to\mathbb{R}$ at any point $P(x,y,z)$ is inversely proportional to the square of the distance of P from the origin. If the value of the temperature T at the point $R(0,0,1)$ is $\sqrt{3}$, then the rate of change of T at the point $Q(1,1,2)$ in the direction of $\overrightarrow{QR}$ is equal to _____ (round off to 2 places of decimal).

($\mathbb{R}$ is the set of all real numbers, $\mathbb{R}^3=\{(x,y,z):x,y,z\in\mathbb{R}\}$ and $\mathbb{R}^3\setminus\{(0,0,0)\}$ denotes $\mathbb{R}^3$ excluding the origin)

Q.19 Let f be a continuous function defined on $[0,2]$ such that $f(x)\geq 0$ for all $x\in[0,2]$. If the area bounded by $y=f(x)$, $x=0, y=0$ and $x=b$ is $\sqrt{3+b^2}-\sqrt{3}$, where $b\in(0,2]$, then $f(1)$ is equal to _____ (round off to 1 place of decimal).

Q.20 If the characteristic polynomial and minimal polynomial of a square matrix A are $(\lambda-1)(\lambda+1)^4(\lambda-2)^5$ and $(\lambda-1)(\lambda+1)(\lambda-2)$, respectively, then the rank of the matrix $A+I$ is _____, where I is the identity matrix of appropriate order.

Q.21 Let ω be a primitive complex cube root of unity and $i=\sqrt{-1}$. Then the degree of the field extension $\mathbb{Q}(i,\sqrt{3},\omega)$ over $\mathbb{Q}$ (the field of rational numbers) is _____.

Q.22 Let

$$\alpha = \int_C \frac{e^{i\pi z}\,dz}{2z^2 - 5z + 2}, \quad C : \cos t + i \sin t,\ 0 \le t \le 2\pi,\ i = \sqrt{-1}.$$

Then the greatest integer less than or equal to $|\alpha|$ is _____.

Q.23 Consider the system:

$$\begin{aligned} 3x_1 + x_2 + 2x_3 - x_4 &= a, \\ x_1 + x_2 + x_3 - 2x_4 &= 3, \\ x_1, x_2, x_3, x_4 &\ge 0. \end{aligned}$$

If $x_1 = 1,\ x_2 = b,\ x_3 = 0,\ x_4 = c$ is a basic feasible solution of the above system (where a, b and c are real constants), then $a + b + c$ is equal to _____.

Q.24 Let $f : \mathbb{C} \to \mathbb{C}$ be a function defined by $f(z) = z^6 - 5z^4 + 10$. Then the number of zeros of f in $\{z \in \mathbb{C} : |z| < 2\}$ is _____.

($\mathbb{C}$ is the set of all complex numbers)

Q.25 Let

$$\ell^2 = \{x = (x_1, x_2, \ldots) : x_i \in \mathbb{C},\ \sum_{i=1}^{\infty} |x_i|^2 < \infty\}$$

be a normed linear space with the norm

$$\|x\|_2 = \left(\sum_{i=1}^{\infty} |x_i|^2 \right)^{\frac{1}{2}}.$$

Let $g : \ell^2 \to \mathbb{C}$ be the bounded linear functional defined by

$$g(x) = \sum_{n=1}^{\infty} \frac{x_n}{3^n} \text{ for all } x = (x_1, x_2, \ldots) \in \ell^2.$$

Then $\left(\sup\{|g(x)| : \|x\|_2 \le 1\} \right)^2$ is equal to _____ (round off to 3 places of decimal).

($\mathbb{C}$ is the set of all complex numbers).

Q. 26 – Q. 55 carry two marks each.

Q.26 For the linear programming problem (LPP):

$$\text{Maximize } Z = 2x_1 + 4x_2$$
$$\text{subject to } -x_1 + 2x_2 \le 4,$$
$$3x_1 + \beta x_2 \le 6,$$
$$x_1, x_2 \ge 0, \ \beta \in \mathbb{R},$$

($\mathbb{R}$ is the set of all real numbers)

consider the following statements:

I. The LPP always has a finite optimal value for any $\beta \ge 0$.

II. The dual of the LPP may be infeasible for some $\beta \ge 0$.

III. If for some β, the point (1,2) is feasible to the dual of the LPP, then $Z \le 16$, for any feasible solution (x_1, x_2) of the LPP.

IV. If for some β, x_1 and x_2 are the basic variables in the optimal table of the LPP with $x_1 = \frac{1}{2}$, then the optimal value of dual of the LPP is 10.

Then which of the above statements are TRUE?

(A) I and III only
(B) I, III and IV only
(C) III and IV only
(D) II and IV only

Q.27 Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x,y) = \begin{cases} (x^2+y^2)\sin\left(\dfrac{1}{x^2+y^2}\right), & \text{if } (x,y) \ne (0,0) \\ 0, & \text{if } (x,y) = (0,0). \end{cases}$$

Consider the following statements:

I. The partial derivatives $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ exist at (0, 0) but are unbounded in any neighbourhood of (0, 0).

II. f is continuous but not differentiable at (0, 0).

III. f is not continuous at (0, 0).

IV. f is differentiable at (0, 0).

($\mathbb{R}$ is the set of all real numbers and $\mathbb{R}^2 = \{(x,y) : x, y \in \mathbb{R}\}$)

Which of the above statements is/are TRUE?

(A) I and II only (B) I and IV only (C) IV only (D) III only

Q.28 Let $K=\left[k_{i,j}\right]_{i,j=1}^{\infty}$ be an infinite matrix over $\mathbb{C}$ (the set of all complex numbers) such that

(i) for each $i\in\mathbb{N}$ (the set of all natural numbers), the i^{th} row $\left(k_{i,1},k_{i,2},...\right)$ of K is in ℓ^{∞} and

(ii) for every $x=\left(x_1,x_2,...\right)\in\ell^1$, $\sum_{j=1}^{\infty}k_{i,j}\,x_j$ is summable for all $i\in\mathbb{N}$, and $(y_1,y_2,...)\in\ell^1$, where $y_i=\sum_{j=1}^{\infty}k_{i,j}\,x_j$.

Let the set of all rows of K be denoted by E . Consider the following statements:

P: E is a bounded set in ℓ^{∞}.

Q: E is a dense set in ℓ^{∞}.

$$\left(\ell^1=\{(x_1,x_2,...):x_i\in\mathbb{C},\ \sum_{i=1}^{\infty}|x_i|<\infty\}\right)$$
$$\left(\ell^{\infty}=\{(x_1,x_2,...):x_i\in\mathbb{C},\ \sup_{i\in\mathbb{N}}|x_i|<\infty\}\right)$$

Which of the above statements is/are TRUE?

(A) Both P and Q (B) P only (C) Q only (D) Neither P nor Q

Q.29 Consider the following heat conduction problem for a finite rod

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}-x\,e^t-2t,\quad t>0,\quad 0<x<\pi,$$

with the boundary conditions $u(0,t)=-t^2$, $u(\pi,t)=-\pi e^t-t^2$, $t>0$ and the initial condition $u(x,0)=\sin x-\sin^3 x-x$, $0\le x\le\pi$. If $v(x,t)=u(x,t)+x\,e^t+t^2$, then which one of the following is CORRECT?

(A) $v(x,t)=\frac{1}{4}\left(e^{-t}\ \sin x+e^{-9t}\ \sin 3x\right)$

(B) $v(x,t)=\frac{1}{4}\left(7e^{-t}\ \sin x-e^{-9t}\ \sin 3x\right)$

(C) $v(x,t)=\frac{1}{4}\left(e^{-t}\ \sin x+e^{-3t}\ \sin 3x\right)$

(D) $v(x,t)=\frac{1}{4}\left(3e^{-t}\ \sin x-e^{-3t}\ \sin 3x\right)$

Q.30 Let $f:\mathbb{C}\to\mathbb{C}$ be non-zero and analytic at all points in $\mathbb{Z}$.

If $F(z)=\pi f(z)\cot(\pi z)$ for $z\in\mathbb{C}\setminus\mathbb{Z}$, then the residue of F at $n\in\mathbb{Z}$ is _____ .

($\mathbb{C}$ is the set of all complex numbers, $\mathbb{Z}$ is the set of all integers and $\mathbb{C}\setminus\mathbb{Z}$ denotes the set of all complex numbers excluding integers)

(A) $\pi f(n)$ (B) $f(n)$ (C) $\dfrac{f(n)}{\pi}$ (D) $\left(\dfrac{df}{dz}\right)_{z=n}$

Q.31 Let the general integral of the partial differential equation

$$(2xy-1)\frac{\partial z}{\partial x}+\left(z-2x^2\right)\frac{\partial z}{\partial y}=2(x-yz)$$

be given by $F(u,v)=0$, where $F:\mathbb{R}^2\to\mathbb{R}$ is a continuously differentiable function.

($\mathbb{R}$ is the set of all real numbers and $\mathbb{R}^2=\{(x,y):x,y\in\mathbb{R}\}$)

Then which one of the following is TRUE?

(A) $u=x^2+y^2+z,\ v=xz+y$ (B) $u=x^2+y^2-z,\ v=xz-y$

(C) $u=x^2-y^2+z,\ v=yz+x$ (D) $u=x^2+y^2-z,\ v=yz-x$

Q.32 Consider the following statements:

I. If $\mathbb{Q}$ denotes the additive group of rational numbers and $f:\mathbb{Q}\to\mathbb{Q}$ is a non-trivial homomorphism, then f is an isomorphism.

II. Any quotient group of a cyclic group is cyclic.

III. If every subgroup of a group G is a normal subgroup, then G is abelian.

IV. Every group of order 33 is cyclic.

Which of the above statements are TRUE?

(A) II and IV only (B) II and III only

(C) I, II and IV only (D) I, III and IV only

Q.33 A solution of the Dirichlet problem

$$\nabla^2 u(r,\theta)=0,\ \ 0<r<1,\ -\pi\le\theta\le\pi,$$
$$u(1,\theta)=|\theta|,\ \ -\pi\le\theta\le\pi,$$

is given by

(A) $u(r,\theta)=\frac{\pi}{2}+\sum_{n=1}^{\infty}\left[\frac{(-1)^n-1}{n^2}\right]r^n\cos(n\theta)$

(B) $u(r,\theta)=\frac{2}{\pi}\sum_{n=1}^{\infty}\left[\frac{(-1)^{n+1}}{n^2}\right]r^n\cos(n\theta)$

(C) $u(r,\theta)=\frac{\pi}{2}+\frac{2}{\pi}\sum_{n=1}^{\infty}\left[\frac{(-1)^n-1}{n^2}\right]r^n\cos(n\theta)$

(D) $u(r,\theta)=\frac{\pi}{2}-\frac{2}{\pi}\sum_{n=1}^{\infty}\left[\frac{(-1)^n+1}{n^2}\right]r^n\cos(n\theta)$

Q.34 Consider the subspace $Y=\{(x,x):x\in\mathbb{C}\}$ of the normed linear space $\left(\mathbb{C}^2,\|\ \|_\infty\right)$. If ϕ is a bounded linear functional on Y, defined by $\phi(x,x)=x$, then which one of the following sets is equal to

$$\left\{\psi(1,0):\psi \text{ is a norm preserving extension of } \phi \text{ to } \left(\mathbb{C}^2,\|\ \|_\infty\right)\right\}.$$

($\mathbb{C}$ is the set of all complex numbers, $\mathbb{C}^2=\{(x,y):x,y\in\mathbb{C}\}$ and $\|(x_1,x_2)\|_\infty=\sup\{|x_1|,|x_2|\}$)

(A) $\{1\}$ (B) $\left[\frac{1}{2},\frac{3}{2}\right]$ (C) $[1,\infty)$ (D) $[0,1]$

Q.35 Consider the following statements:

I. The ring $\mathbb{Z}[\sqrt{-1}]$ is a unique factorization domain.

II. The ring $\mathbb{Z}[\sqrt{-5}]$ is a principal ideal domain.

III. In the polynomial ring $\mathbb{Z}_2[x]$, the ideal generated by x^3+x+1 is a maximal ideal.

IV. In the polynomial ring $\mathbb{Z}_3[x]$, the ideal generated by x^6+1 is a prime ideal.

($\mathbb{Z}$ denotes the set of all integers, $\mathbb{Z}_n$ denotes the set of all integers modulo n, for any positive integer n)

Which of the above statements are TRUE?

(A) I, II and III only
(B) I and III only
(C) I, II and IV only
(D) II and III only

Q.36 Let M be a 3×3 real symmetric matrix with eigenvalues 0, 2 and a with the respective eigenvectors $u=(4,b,c)^T$, $v=(-1,2,0)^T$ and $w=(1,1,1)^T$.

Consider the following statements:

I. $a+b-c=10$.

II. The vector $x=\left(0,\frac{3}{2},\frac{1}{2}\right)^T$ satisfies $Mx=v+w$.

III. For any $d\in\text{span}\{u,v,w\}$, $Mx=d$ has a solution.

IV. The trace of the matrix M^2+2M is 8.

(y^T denotes the transpose of the vector y)

Which of the above statements are TRUE?

(A) I, II and III only (B) I and II only

(C) II and IV only (D) III and IV only

Q.37 Consider the region

$$\Omega=\left\{x+iy:\ -1\le x\le 2,\ \frac{-\pi}{3}\le y\le\frac{\pi}{3}\right\},\ i=\sqrt{-1}$$

in the complex plane. The transformation $x+iy\mapsto e^{x+iy}$ maps the region Ω onto the region $S\subset\mathbb{C}$ (the set of all complex numbers). Then the area of the region S is equal to

(A) $\frac{\pi}{3}\left(e^4-e^{-2}\right)$ (B) $\frac{\pi}{4}\left(e^4+e^{-2}\right)$

(C) $\frac{2\pi}{3}\left(e^4-e^{-2}\right)$ (D) $\frac{\pi}{6}\left(e^4-e^{-2}\right)$

Q.38 Consider the sequence $\{g_n\}_{n=1}^{\infty}$ of functions, where $g_n(x)=\frac{x}{1+nx^2}$, $x\in\mathbb{R}$, $n\in\mathbb{N}$ and $g_n'(x)$ is the derivative of $g_n(x)$ with respect to x.

($\mathbb{R}$ is the set of all real numbers, $\mathbb{N}$ is the set of all natural numbers).

Then which one of the following statements is TRUE?

(A) $\{g_n\}_{n=1}^{\infty}$ does **NOT** converge uniformly on $\mathbb{R}$

(B) $\{g_n'\}_{n=1}^{\infty}$ converges uniformly on any closed interval which does **NOT** contain 1

(C) $\{g_n'\}_{n=1}^{\infty}$ converges point-wise to a continuous function on $\mathbb{R}$

(D) $\{g_n'\}_{n=1}^{\infty}$ converges uniformly on any closed interval which does **NOT** contain 0

Q.39 Consider the boundary value problem (BVP)

$$\frac{d^2y}{dx^2}+\alpha\, y(x)=0,\ \alpha\in\mathbb{R}\text{ (the set of all real numbers)},$$

with the boundary conditions $y(0)=0, y(\pi)=k$ (k is a non-zero real number).

Then which one of the following statements is TRUE?

(A) For $\alpha=1$, the BVP has infinitely many solutions

(B) For $\alpha=1$, the BVP has a unique solution

(C) For $\alpha=-1,\ k<0$, the BVP has a solution $y(x)$ such that $y(x)>0$ for all $x\in(0,\pi)$

(D) For $\alpha=-1,\ k>0$, the BVP has a solution $y(x)$ such that $y(x)>0$ for all $x\in(0,\pi)$

Q.40 Consider the ordered square I_0^2, the set $[0,1]\times[0,1]$ with the dictionary order topology. Let the general element of I_0^2 be denoted by $x\times y$, where $x,y\in[0,1]$. Then the closure of the subset

$$S=\left\{x\times\frac{3}{4} : 0<a<x<b<1\right\}\text{ in } I_0^2$$

is

(A) $S\cup\left((a,b]\times\{0\}\right)\cup\left([a,b)\times\{1\}\right)$

(B) $S\cup\left([a,b)\times\{0\}\right)\cup\left((a,b]\times\{1\}\right)$

(C) $S\cup\left((a,b)\times\{0\}\right)\cup\left((a,b)\times\{1\}\right)$

(D) $S\cup\left((a,b]\times\{0\}\right)$

Q.41 Let P_2 be the vector space of all polynomials of degree at most 2 over $\mathbb{R}$ (the set of real numbers). Let a linear transformation $T:P_2\to P_2$ be defined by

$$T\left(a+bx+cx^2\right)=(a+b)+(b-c)\,x+(a+c)\,x^2.$$

Consider the following statements:

I. The null space of T is $\left\{\alpha\left(-1+x+x^2\right):\alpha\in\mathbb{R}\right\}$.

II. The range space of T is spanned by the set $\left\{1+x^2, 1+x\right\}$.

III. $T(T(1+x))=1+x^2$.

IV. If M is the matrix representation of T with respect to the standard basis $\left\{1,x,x^2\right\}$ of P_2, then the trace of the matrix M is 3.

Which of the above statements are TRUE?

(A) I and II only

(B) I, III and IV only

(C) I, II and IV only

(D) II and IV only

Q.42 Let T_1 and T_2 be two topologies defined on $\mathbb{N}$ (the set of all natural numbers), where T_1 is the topology generated by $\mathcal{B} = \{\{2n-1, 2n\} : n \in \mathbb{N}\}$ and T_2 is the discrete topology on $\mathbb{N}$.

Consider the following statements :

I. In $(\mathbb{N}, T_1)$, every infinite subset has a limit point.

II. The function $f : (\mathbb{N}, T_1) \to (\mathbb{N}, T_2)$ defined by

$$f(n) = \begin{cases} \frac{n}{2}, & \text{if } n \text{ is even} \\ \frac{n+1}{2}, & \text{if } n \text{ is odd} \end{cases}$$

is a continuous function.

Which of the above statements is/are TRUE?

(A) Both I and II (B) I only
(C) II only (D) Neither I nor II

Q.43 Let $1 \le p < q < \infty$. Consider the following statements:

I. $\ell^p \subset \ell^q$

II. $L^p[0,1] \subset L^q[0,1]$,

where $\ell^p = \{(x_1, x_2, \ldots) : x_i \in \mathbb{R}, \sum_{i=1}^{\infty} |x_i|^p < \infty\}$ and

$$L^p[0,1] = \left\{ f : [0,1] \to \mathbb{R} : f \text{ is } \mu\text{-measurable}, \int_{[0,1]} |f|^p \, d\mu < \infty, \text{where } \mu \text{ is the Lebesgue measure} \right\}$$

($\mathbb{R}$ is the set of all real numbers)

Which of the above statements is/are TRUE?

(A) Both I and II (B) I only (C) II only (D) Neither I nor II

Q.44 Consider the differential equation

$$t\frac{d^2y}{dt^2}+2\frac{dy}{dt}+t\,y=0,\ t>0,\ y(0+)=1,\ \left(\frac{dy}{dt}\right)_{t=0+}=0.$$

If $Y(s)$ is the Laplace transform of $y(t)$, then the value of $Y(1)$ is _____ (round off to 2 places of decimal).

(Here, the inverse trigonometric functions assume principal values only)

Q.45 Let R be the region in the xy-plane bounded by the curves $y=x^2,\ y=4x^2,\ xy=1$ and $xy=5$.

Then the value of the integral $\iint_R \frac{y^2}{x}\,dy\,dx$ is equal to _____.

Q.46 Let V be the vector space of all 3×3 matrices with complex entries over the real field. If

$$W_1=\{A\in V : A=\bar{A}^T\} \text{ and } W_2=\{A\in V : \text{trace of } A=0\},$$

then the dimension of W_1+W_2 is equal to _____.

($\bar{A}^T$ denotes the conjugate transpose of A)

Q.47 The number of elements of order 15 in the additive group $\mathbb{Z}_{60}\times\mathbb{Z}_{50}$ is _____ .

($\mathbb{Z}_n$ denotes the group of integers modulo n, under the operation of addition modulo n, for any positive integer n)

Q.48 Consider the following cost matrix of assigning four jobs to four persons:

Jobs

Persons	J_1	J_2	J_3	J_4
P_1	5	8	6	10
P_2	2	5	4	8
P_3	6	7	6	9
P_4	6	9	8	10

Then the minimum cost of the assignment problem subject to the constraint that job J_4 is assigned to person P_2, is _____ .

Q.49 Let $y:[-1,1]\to\mathbb{R}$ with $y(1)=1$ satisfy the Legendre differential equation

$$\left(1-x^2\right)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+6y=0 \text{ for } |x|<1.$$

Then the value of $\int_{-1}^{1} y(x)\left(x+x^2\right)dx$ is equal to _____ (round off to 2 places of decimal).

Q.50 Let $\mathbb{Z}_{125}$ be the ring of integers modulo 125 under the operations of addition modulo 125 and multiplication modulo 125. If m is the number of maximal ideals of $\mathbb{Z}_{125}$ and n is the number of non-units of $\mathbb{Z}_{125}$, then $m+n$ is equal to _____ .

Q.51 The maximum value of the error term of the composite Trapezoidal rule when it is used to evaluate the definite integral

$$\int_{0.2}^{1.4}\left(\sin x-\log_e x\right)dx$$

with 12 sub-intervals of equal length, is equal to _____ (round off to 3 places of decimal).

Q.52 By the Simplex method, the optimal table of the linear programming problem:

$$\begin{aligned} &\text{Maximize } Z=\alpha x_1+3x_2 \\ &\text{subject to } \beta x_1+x_2+x_3=8, \\ &\qquad 2x_1+x_2+x_4=\gamma, \\ &\qquad x_1,x_2,x_3,x_4\ge 0, \end{aligned}$$

where α,β,γ are real constants, is

$c_j \to$	α	3	0	0	
Basic variable	x_1	x_2	x_3	x_4	Solution
x_2	1	0	2	-1	6
x_1	0	1	-1	1	2
z_j-c_j	0	0	2	1	-

Then the value of $\alpha+\beta+\gamma$ is _____ .

Q.53 Consider the inner product space P_2 of all polynomials of degree at most 2 over the field of real numbers with the inner product $\langle f, g\rangle = \int_0^1 f(t)\,g(t)\,dt$ for $f, g \in P_2$.

Let $\{f_0, f_1, f_2\}$ be an orthogonal set in P_2, where $f_0 = 1$, $f_1 = t + c_1$, $f_2 = t^2 + c_2 f_1 + c_3$ and c_1, c_2, c_3 are real constants. Then the value of $2c_1 + c_2 + 3c_3$ is equal to _____ .

Q.54 Consider the system of linear differential equations

$$\frac{dx_1}{dt} = 5x_1 - 2x_2,$$
$$\frac{dx_2}{dt} = 4x_1 - x_2,$$

with the initial conditions $x_1(0) = 0,\ x_2(0) = 1$.

Then $\log_e\left(x_2(2) - x_1(2)\right)$ is equal to _____ .

Q.55 Consider the differential equation

$$x(1+x^2)\frac{d^2y}{dx^2} - 9\frac{dy}{dx} + 7y = 0.$$

The sum of the roots of the indicial equation of the Frobenius series solution for the above differential equation in a neighborhood of $x = 0$ is equal to _____.

END OF THE QUESTION PAPER

Q.No.	Type	Section	Key/Range	Marks
1	MCQ	GA	C	1
2	MCQ	GA	C	1
3	MCQ	GA	C	1
4	MCQ	GA	C	1
5	MCQ	GA	A	1
6	MCQ	GA	C	2
7	MCQ	GA	A	2
8	MCQ	GA	C	2
9	MCQ	GA	B	2
10	MCQ	GA	D	2
1	MCQ	MA	D	1
2	MCQ	MA	A	1
3	MCQ	MA	D	1
4	MCQ	MA	D	1
5	MCQ	MA	C	1
6	MCQ	MA	C	1
7	MCQ	MA	D	1
8	MCQ	MA	B	1
9	MCQ	MA	C	1
10	MCQ	MA	A	1
11	MCQ	MA	B	1
12	MCQ	MA	C	1
13	MCQ	MA	C	1

Q.No.	Type	Section	Key/Range	Marks
14	NAT	MA	0 to 0	1
15	NAT	MA	2.40 to 2.50	1
16	NAT	MA	-7.5 to -7.5	1
17	NAT	MA	31.90 to 32.10	1
18	NAT	MA	0.21 to 0.23	1
19	NAT	MA	0.5 to 0.5	1
20	NAT	MA	6 to 6	1
21	NAT	MA	4 to 4	1
22	NAT	MA	2 to 2	1
23	NAT	MA	7 to 7	1
24	NAT	MA	4 to 4	1
25	NAT	MA	0.125 to 0.125	1
26	MCQ	MA	B	2
27	MCQ	MA	B	2
28	MCQ	MA	B	2
29	MCQ	MA	A	2
30	MCQ	MA	B	2
31	MCQ	MA	A	2
32	MCQ	MA	C	2
33	MCQ	MA	C	2
34	MCQ	MA	D	2
35	MCQ	MA	B	2
36	MCQ	MA	B	2

Q.No.	Type	Section	Key/Range	Marks
37	MCQ	MA	A	2
38	MCQ	MA	D	2
39	MCQ	MA	D	2
40	MCQ	MA	A	2
41	MCQ	MA	C	2
42	MCQ	MA	A	2
43	MCQ	MA	B	2
44	NAT	MA	0.76 to 0.83	2
45	NAT	MA	12 to 12	2
46	NAT	MA	17 to 17	2
47	NAT	MA	48 to 48	2
48	NAT	MA	27 to 27	2
49	NAT	MA	0.25 to 0.30	2
50	NAT	MA	26 to 26	2
51	NAT	MA	0.022 to 0.028	2
52	NAT	MA	14.5 to 16.5	2
53	NAT	MA	-3 to -3	2
54	NAT	MA	1.95 to 2.05	2
55	NAT	MA	10 to 10	2

Q. 1 – Q. 5 carry one mark each.

Q.1 "The dress _________ her so well that they all immediately _________ her on her appearance."

The words that best fill the blanks in the above sentence are

(A) complemented, complemented
(B) complimented, complemented
(C) complimented, complimented
(D) complemented, complimented

Q.2 "The judge's standing in the legal community, though shaken by false allegations of wrongdoing, remained _________."

The word that best fills the blank in the above sentence is

(A) undiminished (B) damaged (C) illegal (D) uncertain

Q.3 Find the missing group of letters in the following series:
BC, FGH, LMNO, _____

(A) UVWXY (B) TUVWX (C) STUVW (D) RSTUV

Q.4 The perimeters of a circle, a square and an equilateral triangle are equal. Which one of the following statements is true?

(A) The circle has the largest area.
(B) The square has the largest area.
(C) The equilateral triangle has the largest area.
(D) All the three shapes have the same area.

Q.5 The value of the expression $\frac{1}{1+\log_u vw} + \frac{1}{1+\log_v wu} + \frac{1}{1+\log_w uv}$ is _________.

(A) -1 (B) 0 (C) 1 (D) 3

Q. 6 – Q. 10 carry two marks each.

Q.6 Forty students watched films A, B and C over a week. Each student watched either only one film or all three. Thirteen students watched film A, sixteen students watched film B and nineteen students watched film C. How many students watched all three films?

(A) 0 (B) 2 (C) 4 (D) 8

Q.7 A wire would enclose an area of 1936 m^2, if it is bent into a square. The wire is cut into two pieces. The longer piece is thrice as long as the shorter piece. The long and the short pieces are bent into a square and a circle, respectively. Which of the following choices is closest to the sum of the areas enclosed by the two pieces in square meters?

(A) 1096 (B) 1111 (C) 1243 (D) 2486

Q.8 A contract is to be completed in 52 days and 125 identical robots were employed, each operational for 7 hours a day. After 39 days, five-seventh of the work was completed. How many additional robots would be required to complete the work on time, if each robot is now operational for 8 hours a day?

(A) 50 (B) 89 (C) 146 (D) 175

Q.9 A house has a number which needs to be identified. The following three statements are given that can help in identifying the house number.

i. If the house number is a multiple of 3, then it is a number from 50 to 59.
ii. If the house number is NOT a multiple of 4, then it is a number from 60 to 69.
iii. If the house number is NOT a multiple of 6, then it is a number from 70 to 79.

What is the house number?

(A) 54 (B) 65 (C) 66 (D) 76

Q.10 An unbiased coin is tossed six times in a row and four different such trials are conducted. One trial implies six tosses of the coin. If H stands for head and T stands for tail, the following are the observations from the four trials:
(1) HTHTHT (2) TTHHHT (3) HTTHHT (4) HHHT__ __.

Which statement describing the last two coin tosses of the fourth trial has the highest probability of being correct?

(A) Two T will occur.
(B) One H and one T will occur.
(C) Two H will occur.
(D) One H will be followed by one T.

END OF THE QUESTION PAPER

Q.1 – Q.25 carry one mark each

Q.1 The principal value of $(-1)^{(-2i/\pi)}$ is

(A) e^2 (B) e^{2i} (C) e^{-2i} (D) e^{-2}

Q.2 Let $f : \mathbb{C} \to \mathbb{C}$ be an entire function with $f(0) = 1$, $f(1) = 2$ and $f'(0) = 0$. If there exists $M > 0$ such that $|f''(z)| \leq M$ for all $z \in \mathbb{C}$, then $f(2) =$

(A) 2 (B) 5 (C) $2 + 5i$ (D) $5 + 2i$

Q.3 In the Laurent series expansion of $f(z) = \dfrac{1}{z(z-1)}$ valid for $|z - 1| > 1$, the coefficient of $\dfrac{1}{z-1}$ is

(A) -2 (B) -1 (C) 0 (D) 1

Q.4 Let X and Y be metric spaces, and let $f : X \to Y$ be a continuous map. For any subset S of X, which one of the following statements is true?

(A) If S is open, then $f(S)$ is open

(B) If S is connected, then $f(S)$ is connected

(C) If S is closed, then $f(S)$ is closed

(D) If S is bounded, then $f(S)$ is bounded

Q.5 The general solution of the differential equation

$$xy' = y + \sqrt{x^2 + y^2} \quad \text{for } x > 0$$

is given by (with an arbitrary positive constant k)

(A) $ky^2 = x + \sqrt{x^2 + y^2}$

(B) $kx^2 = x + \sqrt{x^2 + y^2}$

(C) $kx^2 = y + \sqrt{x^2 + y^2}$

(D) $ky^2 = y + \sqrt{x^2 + y^2}$

Q.6 Let $p_n(x)$ be the polynomial solution of the differential equation

$$\frac{d}{dx}[(1-x^2)y'] + n(n+1)y = 0$$

with $p_n(1) = 1$ for $n = 1, 2, 3, \ldots$. If

$$\frac{d}{dx}[p_{n+2}(x) - p_n(x)] = \alpha_n p_{n+1}(x),$$

then α_n is

(A) $2n$ (B) $2n+1$ (C) $2n+2$ (D) $2n+3$

Q.7 In the permutation group S_6, the number of elements of order 8 is

(A) 0 (B) 1 (C) 2 (D) 4

Q.8 Let R be a commutative ring with 1 (unity) which is not a field. Let $I \subset R$ be a proper ideal such that every element of R not in I is invertible in R. Then the number of maximal ideals of R is

(A) 1 (B) 2 (C) 3 (D) infinite

Q.9 Let $f : \mathbb{R} \to \mathbb{R}$ be a twice continuously differentiable function. The order of convergence of the secant method for finding root of the equation $f(x) = 0$ is

(A) $\frac{1+\sqrt{5}}{2}$ (B) $\frac{2}{1+\sqrt{5}}$ (C) $\frac{1+\sqrt{5}}{3}$ (D) $\frac{3}{1+\sqrt{5}}$

Q.10 The Cauchy problem $uu_x + yu_y = x$ with $u(x, 1) = 2x$, when solved using its characteristic equations with an independent variable t, is found to admit of a solution in the form

$$x = \frac{3}{2}se^t - \frac{1}{2}se^{-t}, \quad y = e^t, \quad u = f(s,t).$$

Then $f(s,t) =$

(A) $\frac{3}{2}se^t + \frac{1}{2}se^{-t}$ (B) $\frac{1}{2}se^t + \frac{3}{2}se^{-t}$ (C) $\frac{1}{2}se^t - \frac{3}{2}se^{-t}$ (D) $\frac{3}{2}se^t - \frac{1}{2}se^{-t}$

Q.11 An urn contains four balls, each ball having equal probability of being white or black. Three black balls are added to the urn. The probability that five balls in the urn are black is

(A) $2/7$ (B) $3/8$ (C) $1/2$ (D) $5/7$

Q.12 For a linear programming problem, which one of the following statements is FALSE?

(A) If a constraint is an equality, then the corresponding dual variable is unrestricted in sign

(B) Both primal and its dual can be infeasible

(C) If primal is unbounded, then its dual is infeasible

(D) Even if both primal and dual are feasible, the optimal values of the primal and the dual can differ

Q.13 Let $A = \begin{bmatrix} a & 2f & 0 \\ 2f & b & 3f \\ 0 & 3f & c \end{bmatrix}$, where a, b, c, f are real numbers and $f \neq 0$. The geometric multiplicity of the largest eigenvalue of A equals ________ .

Q.14 Consider the subspaces

$$\begin{aligned} W_1 &= \{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_1 = x_2 + 2x_3\} \\ W_2 &= \{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_1 = 3x_2 + 2x_3\} \end{aligned}$$

of $\mathbb{R}^3$. Then the dimension of $W_1 + W_2$ equals ________ .

Q.15 Let V be the real vector space of all polynomials of degree less than or equal to 2 with real coefficients. Let $T : V \to V$ be the linear transformation given by

$$T(p) = 2p + p' \quad \text{for } p \in V,$$

where p' is the derivative of p. Then the number of nonzero entries in the Jordan canonical form of a matrix of T equals ________ .

Q.16 Let $I = [2, 3)$, J be the set of all rational numbers in the interval $[4, 6]$, K be the Cantor (ternary) set, and let $L = \{7 + x : x \in K\}$. Then the Lebesgue measure of the set $I \cup J \cup L$ equals ________ .

Q.17 Let $u(x, y, z) = x^2 - 2y + 4z^2$ for $(x, y, z) \in \mathbb{R}^3$. Then the directional derivative of u in the direction $\frac{3}{5}\hat{i} - \frac{4}{5}\hat{k}$ at the point $(5, 1, 0)$ is ________ .

Q.18 If the Laplace transform of $y(t)$ is given by $Y(s) = L(y(t)) = \frac{5}{2(s-1)} - \frac{2}{s-2} + \frac{1}{2(s-3)}$, then $y(0) + y'(0) =$ ________ .

Q.19 The number of regular singular points of the differential equation

$$[(x-1)^2 \sin x]y'' + [\cos x \sin(x-1)]y' + (x-1)y = 0$$

in the interval $\left[0, \frac{\pi}{2}\right]$ is equal to ________ .

Q.20 Let F be a field with 7^6 elements and let K be a subfield of F with 49 elements. Then the dimension of F as a vector space over K is ________ .

Q.21 Let $C([0,1])$ be the real vector space of all continuous real valued functions on $[0,1]$, and let T be the linear operator on $C([0,1])$ given by

$$(Tf)(x) = \int_0^1 \sin(x+y) f(y)\, dy, \quad x \in [0,1].$$

Then the dimension of the range space of T equals ________ .

Q.22 Let $a \in (-1,1)$ be such that the quadrature rule

$$\int_{-1}^{1} f(x)\, dx \simeq f(-a) + f(a)$$

is exact for all polynomials of degree less than or equal to 3. Then $3a^2 =$ ________ .

Q.23 Let X and Y have joint probability density function given by

$$f_{X,Y}(x,y) = \begin{cases} 2, & 0 \le x \le 1-y,\ 0 \le y \le 1 \\ 0, & \text{otherwise.} \end{cases}$$

If f_Y denotes the marginal probability density function of Y, then $f_Y(1/2) =$ ________ .

Q.24 Let the cumulative distribution function of the random variable X be given by

$$F_X(x) = \begin{cases} 0, & x < 0, \\ x, & 0 \le x < 1/2, \\ (1+x)/2, & 1/2 \le x < 1, \\ 1, & x \ge 1. \end{cases}$$

Then $\mathbb{P}(X = 1/2) =$ ________ .

Q.25 Let $\{X_j\}$ be a sequence of independent Bernoulli random variables with $\mathbb{P}(X_j = 1) = 1/4$ and let $Y_n = \frac{1}{n}\sum_{j=1}^{n} X_j^2$. Then Y_n converges, in probability, to ________ .

Q.26 – Q.55 carry two marks each

Q.26 Let Γ be the circle given by $z = 4e^{i\theta}$, where θ varies from 0 to 2π. Then

$$\oint_{\Gamma} \frac{e^z}{z^2 - 2z} dz =$$

(A) $2\pi i(e^2 - 1)$ (B) $\pi i(1 - e^2)$ (C) $\pi i(e^2 - 1)$ (D) $2\pi i(1 - e^2)$

Q.27 The image of the half plane $\mathrm{Re}(z) + \mathrm{Im}(z) > 0$ under the map $w = \dfrac{z-1}{z+i}$ is given by

(A) $\mathrm{Re}(w) > 0$ (B) $\mathrm{Im}(w) > 0$ (C) $|w| > 1$ (D) $|w| < 1$

Q.28 Let $D \subset \mathbb{R}^2$ denote the closed disc with center at the origin and radius 2. Then

$$\iint_D e^{-(x^2+y^2)}\, dxdy =$$

(A) $\pi(1 - e^{-4})$ (B) $\dfrac{\pi}{2}(1 - e^{-4})$ (C) $\pi(1 - e^{-2})$ (D) $\dfrac{\pi}{2}(1 - e^{-2})$

Q.29 Consider the polynomial $p(X) = X^4 + 4$ in the ring $\mathbb{Q}[X]$ of polynomials in the variable X with coefficients in the field $\mathbb{Q}$ of rational numbers. Then

(A) the set of zeros of $p(X)$ in $\mathbb{C}$ forms a group under multiplication

(B) $p(X)$ is reducible in the ring $\mathbb{Q}[X]$

(C) the splitting field of $p(X)$ has degree 3 over $\mathbb{Q}$

(D) the splitting field of $p(X)$ has degree 4 over $\mathbb{Q}$

Q.30 Which one of the following statements is true?

(A) Every group of order 12 has a non-trivial proper normal subgroup

(B) Some group of order 12 does not have a non-trivial proper normal subgroup

(C) Every group of order 12 has a subgroup of order 6

(D) Every group of order 12 has an element of order 12

Q.31 For an odd prime p, consider the ring $\mathbb{Z}[\sqrt{-p}] = \{a + b\sqrt{-p} : a, b \in \mathbb{Z}\} \subseteq \mathbb{C}$. Then the element 2 in $\mathbb{Z}[\sqrt{-p}]$ is

(A) a unit (B) a square (C) a prime (D) irreducible

Q.32 Consider the following two statements:

P: The matrix $\begin{bmatrix} 0 & 5 \\ 0 & 7 \end{bmatrix}$ has infinitely many LU factorizations, where L is lower triangular with each diagonal entry 1 and U is upper triangular.

Q: The matrix $\begin{bmatrix} 0 & 0 \\ 2 & 5 \end{bmatrix}$ has no LU factorization, where L is lower triangular with each diagonal entry 1 and U is upper triangular.

Then which one of the following options is correct?

(A) P is TRUE and Q is FALSE

(B) Both P and Q are TRUE

(C) P is FALSE and Q is TRUE

(D) Both P and Q are FALSE

Q.33 If the characteristic curves of the partial differential equation $xu_{xx} + 2x^2u_{xy} = u_x - 1$ are $\mu(x, y) = c_1$ and $\nu(x, y) = c_2$, where c_1 and c_2 are constants, then

(A) $\mu(x, y) = x^2 - y,\ \nu(x, y) = y$

(B) $\mu(x, y) = x^2 + y,\ \nu(x, y) = y$

(C) $\mu(x, y) = x^2 + y,\ \nu(x, y) = x^2$

(D) $\mu(x, y) = x^2 - y,\ \nu(x, y) = x^2$

Q.34 Let $f : X \to Y$ be a continuous map from a Hausdorff topological space X to a metric space Y. Consider the following two statements:

P: f is a closed map and the inverse image $f^{-1}(y) = \{x \in X : f(x) = y\}$ is compact for each $y \in Y$.

Q: For every compact subset $K \subset Y$, the inverse image $f^{-1}(K)$ is a compact subset of X.

Which one of the following is true?

(A) Q implies P but P does NOT imply Q

(B) P implies Q but Q does NOT imply P

(C) P and Q are equivalent

(D) neither P implies Q nor Q implies P

Q.35 Let X denote $\mathbb{R}^2$ endowed with the usual topology. Let Y denote $\mathbb{R}$ endowed with the co-finite topology. If Z is the product topological space $Y \times Y$, then

(A) the topology of X is the same as the topology of Z

(B) the topology of X is strictly coarser (weaker) than that of Z

(C) the topology of Z is strictly coarser (weaker) than that of X

(D) the topology of X cannot be compared with that of Z

Q.36 Consider $\mathbb{R}^n$ with the usual topology for $n = 1, 2, 3$. Each of the following options gives topological spaces X and Y with respective induced topologies. In which option is X homeomorphic to Y?

(A) $X = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 = 1\}$, $\quad Y = \{(x, y, z) \in \mathbb{R}^3 : z = 0, x^2 + y^2 \neq 0\}$

(B) $X = \{(x, y) \in \mathbb{R}^2 : y = \sin(1/x), 0 < x \leq 1\} \cup \{(x, y) \in \mathbb{R}^2 : x = 0, -1 \leq y \leq 1\}$, $Y = [0, 1] \subseteq \mathbb{R}$

(C) $X = \{(x, y) \in \mathbb{R}^2 : y = x\sin(1/x), 0 < x \leq 1\}$, $\quad Y = [0, 1] \subseteq \mathbb{R}$

(D) $X = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 = 1\}$, $\quad Y = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 = z^2 \neq 0\}$

Q.37 Let $\{X_i\}$ be a sequence of independent Poisson(λ) variables and let $W_n = \frac{1}{n}\sum_{i=1}^{n} X_i$. Then the limiting distribution of $\sqrt{n}(W_n - \lambda)$ is the normal distribution with zero mean and variance given by

(A) 1 (B) $\sqrt{\lambda}$ (C) λ (D) λ^2

Q.38 Let $X_1, X_2, \ldots, X_n$ be independent and identically distributed random variables with probability density function given by

$$f_X(x;\theta) = \begin{cases} \theta e^{-\theta(x-1)}, & x \geq 1, \\ 0 & \text{otherwise.} \end{cases}$$

Also, let $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$. Then the maximum likelihood estimator of θ is

(A) $1/\overline{X}$ (B) $(1/\overline{X}) - 1$ (C) $1/(\overline{X} - 1)$ (D) $\overline{X}$

Q.39 Consider the Linear Programming Problem (LPP):

Maximize $\alpha x_1 + x_2$

Subject to $2x_1 + x_2 \leq 6,$

$-x_1 + x_2 \leq 1,$

$x_1 + x_2 \leq 4,$

$x_1 \geq 0, x_2 \geq 0,$

where α is a constant. If $(3, 0)$ is the only optimal solution, then

(A) $\alpha < -2$ (B) $-2 < \alpha < 1$ (C) $1 < \alpha < 2$ (D) $\alpha > 2$

Q.40 Let $M_2(\mathbb{R})$ be the vector space of all 2×2 real matrices over the field $\mathbb{R}$. Define the linear transformation $S : M_2(\mathbb{R}) \to M_2(\mathbb{R})$ by $S(X) = 2X + X^T$, where X^T denotes the transpose of the matrix X. Then the trace of S equals ________ .

Q.41 Consider $\mathbb{R}^3$ with the usual inner product. If d is the distance from $(1, 1, 1)$ to the subspace $\text{span}\{(1, 1, 0),\ (0, 1, 1)\}$ of $\mathbb{R}^3$, then $3d^2 =$ ________ .

Q.42 Consider the matrix $A = I_9 - 2u^T u$ with $u = \frac{1}{3}[1, 1, 1, 1, 1, 1, 1, 1, 1]$, where I_9 is the 9×9 identity matrix and u^T is the transpose of u. If λ and μ are two distinct eigenvalues of A, then $|\lambda - \mu| =$ ________ .

Q.43 Let $f(z) = z^3 e^{z^2}$ for $z \in \mathbb{C}$ and let Γ be the circle $z = e^{i\theta}$, where θ varies from 0 to 4π. Then

$$\frac{1}{2\pi i}\oint_{\Gamma} \frac{f'(z)}{f(z)}dz = \underline{\hspace{3cm}}.$$

Q.44 Let S be the surface of the solid

$$V = \{(x, y, z) : 0 \leq x \leq 1,\ 0 \leq y \leq 2,\ 0 \leq z \leq 3\}.$$

Let $\hat{n}$ denote the unit outward normal to S and let

$$\vec{F}(x, y, z) = x\hat{i} + y\hat{j} + z\hat{k}, \quad (x, y, z) \in V.$$

Then the surface integral $\iint_S \vec{F} \cdot \hat{n}\, dS$ equals 18.

Q.45 Let A be a 3×3 matrix with real entries. If three solutions of the linear system of differential equations $\dot{x}(t) = Ax(t)$ are given by

$$\begin{bmatrix} e^t - e^{2t} \\ -e^t + e^{2t} \\ e^t + e^{2t} \end{bmatrix}, \quad \begin{bmatrix} -e^{2t} - e^{-t} \\ e^{2t} - e^{-t} \\ e^{2t} + e^{-t} \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} e^{-t} + 2e^t \\ e^{-t} - 2e^t \\ -e^{-t} + 2e^t \end{bmatrix},$$

then the sum of the diagonal entries of A is equal to 2.

Q.46 If $y_1(x) = e^{-x^2}$ is a solution of the differential equation

$$xy'' + \alpha y' + \beta x^3 y = 0$$

for some real numbers α and β, then $\alpha\beta =$ 4.

Q.47 Let $L^2([0,1])$ be the Hilbert space of all real valued square integrable functions on $[0,1]$ with the usual inner product. Let ϕ be the linear functional on $L^2([0,1])$ defined by

$$\phi(f) = \int_{1/4}^{3/4} 3\sqrt{2} f\, d\mu,$$

where μ denotes the Lebesgue measure on $[0,1]$. Then $\|\phi\| =$ 3.

Q.48 Let U be an orthonormal set in a Hilbert space H and let $x \in H$ be such that $\|x\| = 2$. Consider the set

$$E = \left\{ u \in U : |\langle x, u \rangle| \geq \frac{1}{4} \right\}.$$

Then the maximum possible number of elements in E is 64.

Q.49 If $p(x) = 2 - (x+1) + x(x+1) - \beta x(x+1)(x-\alpha)$ interpolates the points (x, y) in the table

x	-1	0	1	2
y	2	1	2	-7

then $\alpha + \beta =$ ________ .

Q.50 If $\sin(\pi x) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\pi x)$ for $0 < x < 1$, then $(a_0 + a_1)\pi =$ ________ .

Q.51 For $n = 1, 2, \ldots$, let $f_n(x) = \dfrac{2nx^{n-1}}{1+x}$, $x \in [0, 1]$. Then $\lim_{n\to\infty} \int_0^1 f_n(x)dx =$ ________ .

Q.52 Let X_1, X_2, X_3, X_4 be independent exponential random variables with mean $1, 1/2, 1/3, 1/4$, respectively. Then $Y = \min(X_1, X_2, X_3, X_4)$ has exponential distribution with mean equal to ________ .

Q.53 Let X be the number of heads in 4 tosses of a fair coin by Person 1 and let Y be the number of heads in 4 tosses of a fair coin by Person 2. Assume that all the tosses are independent. Then the value of $\mathbb{P}(X = Y)$ correct up to three decimal places is________.

Q.54 Let X_1 and X_2 be independent geometric random variables with the same probability mass function given by $\mathbb{P}(X = k) = p(1-p)^{k-1}$, $k = 1, 2, \ldots$. Then the value of $\mathbb{P}(X_1 = 2 | X_1 + X_2 = 4)$ correct up to three decimal places is ________ .

Q.55 A certain commodity is produced by the manufacturing plants P_1 and P_2 whose capacities are 6 and 5 units, respectively. The commodity is shipped to markets M_1, M_2, M_3 and M_4 whose requirements are 1, 2, 3 and 5 units, respectively. The transportation cost per unit from plant P_i to market M_j is as follows:

	M_1	M_2	M_3	M_4	
P_1	1	3	5	8	6
P_2	2	5	6	7	5
	1	2	3	5	

Then the optimal cost of transportation is ________ .

END OF THE QUESTION PAPER

Graduate Aptitude Test in Engineering 2017

Question Paper Name: Mathematics 5th Feb 2017
Subject Name: Mathematics
Duration: 180
Total Marks: 100

Organizing Institute:
Indian Institute of Technology Roorkee

Question Number : 1 **Correct : 1 Wrong : 0**

Consider the vector space $V = \{a_0 + a_1x + a_2x^2 : a_i \in \mathbb{R} \text{ for } i = 0,1,2\}$ of polynomials of degree at most 2. Let $f : V \to \mathbb{R}$ be a linear functional such that $f(1+x) = 0$, $f(1-x^2)=0$ and $f(x^2-x)=2$. Then $f(1+x+x^2)$ equals _______.

Question Number : 2 **Correct : 1 Wrong : 0**

Let A be a 7×7 matrix such that $2A^2 - A^4 = I$, where I is the identity matrix. If A has two distinct eigenvalues and each eigenvalue has geometric multiplicity 3, then the total number of nonzero entries in the Jordan canonical form of A equals _______.

Question Number : 3 **Correct : 1 Wrong : -0.33**

Let $f(z) = (x^2 + y^2) + i\,2xy$ and $g(z) = 2xy + i(y^2 - x^2)$ for $z = x + i\,y \in \mathbb{C}$. Then, in the complex plane $\mathbb{C}$,

(A) f is analytic and g is NOT analytic

(B) f is NOT analytic and g is analytic

(C) neither f nor g is analytic

(D) both f and g are analytic

Question Number : 4 **Correct : 1 Wrong : 0**

If $\sum_{n=-\infty}^{\infty} a_n(z-2)^n$ is the Laurent series of the function $f(z)=\dfrac{z^4+z^3+z^2}{(z-2)^3}$ for $z \in \mathbb{C}\setminus\{2\}$, then a_{-2} equals _______.

Question Number : 5 **Correct : 1 Wrong : -0.33**

Let $f_n : [0,1] \to \mathbb{R}$ be given by $f_n(x) = \dfrac{2x^2}{x^2 + (1-2nx)^2}$, $n = 1, 2, \ldots$. Then the sequence (f_n)

(A) converges uniformly on $[0,1]$

(B) does NOT converge uniformly on $[0,1]$ but has a subsequence that converges uniformly on $[0,1]$

(C) does NOT converge pointwise on $[0,1]$

(D) converges pointwise on $[0,1]$ but does NOT have a subsequence that converges uniformly on $[0,1]$

Question Number : 6 **Correct : 1 Wrong : 0**

Let $C : x^2 + y^2 = 9$ be the circle in $\mathbb{R}^2$ oriented positively. Then

$\dfrac{1}{\pi}\oint_C \left(3y - e^{\cos x^2}\right)dx + \left(7x + \sqrt{y^4 + 11}\right)dy$ equals ________.

Question Number : 7 **Correct : 1 Wrong : -0.33**

Consider the following statements:

(P): There exists an unbounded subset of $\mathbb{R}$ whose Lebesgue measure is equal to 5.

(Q): If $f : \mathbb{R} \to \mathbb{R}$ is continuous and $g : \mathbb{R} \to \mathbb{R}$ is such that $f = g$ almost everywhere on $\mathbb{R}$, then g must be continuous almost everywhere on $\mathbb{R}$.

Which of the above statements hold TRUE?

(A) Both P and Q

(B) Only P

(C) Only Q

(D) Neither P nor Q

Question Number : 8 **Correct : 1 Wrong : -0.33**

If $x^3 y^2$ is an integrating factor of $(6y^2 + axy)dx + (6xy + bx^2)dy = 0$, where $a, b \in \mathbb{R}$, then

(A) $3a - 5b = 0$

(B) $2a - b = 0$

(C) $3a + 5b = 0$

(D) $2a + b = 0$

Question Number : 9 **Correct : 1 Wrong : 0**

If $x(t)$ and $y(t)$ are the solutions of the system $\frac{dx}{dt}=y$ and $\frac{dy}{dt}=-x$ with the initial conditions $x(0)=1$ and $y(0)=1$, then $x(\pi/2)+y(\pi/2)$ equals _______.

Question Number : 10 **Correct : 1 Wrong : -0.33**

If $y=3e^{2x}+e^{-2x}-\alpha x$ is the solution of the initial value problem

$$\frac{d^2y}{dx^2}+\beta y=4\alpha x,\ y(0)=4 \text{ and } \frac{dy}{dx}(0)=1, \text{ where } \alpha,\beta\in\mathbb{R},$$

then

(A) $\alpha=3$ and $\beta=4$

(B) $\alpha=1$ and $\beta=2$

(C) $\alpha=3$ and $\beta=-4$

(D) $\alpha=1$ and $\beta=-2$

Question Number : 11 **Correct : 1 Wrong : 0**

Let G be a non-abelian group of order 125. Then the total number of elements in $Z(G)=\{x\in G : gx=xg \text{ for all } g\in G\}$ equals _______.

Question Number : 12 **Correct : 1 Wrong : 0**

Let F_1 and F_2 be subfields of a finite field F consisting of 2^9 and 2^6 elements, respectively. Then the total number of elements in $F_1\cap F_2$ equals _______.

Question Number : 13 **Correct : 1 Wrong : 0**

Consider the normed linear space $\mathbb{R}^2$ equipped with the norm given by $\|(x,y)\| = |x| + |y|$ and the subspace $X = \{(x,y) \in \mathbb{R}^2 \,:\, x = y\}$. Let f be the linear functional on X given by $f(x,y) = 3x$. If $g(x,y) = \alpha x + \beta y$, $\alpha, \beta \in \mathbb{R}$, is a Hahn-Banach extension of f on $\mathbb{R}^2$, then $\alpha - \beta$ equals _______.

Question Number : 14 **Correct : 1 Wrong : -0.33**

For $n \in \mathbb{Z}$, define $c_n = \frac{1}{\sqrt{2\pi}} \int_{-\pi}^{\pi} e^{i(n-i)x}\, dx$, where $i^2 = -1$. Then $\sum_{n \in \mathbb{Z}} |c_n|^2$ equals

(A) $\cosh(\pi)$ (B) $\sinh(\pi)$ (C) $\cosh(2\pi)$ (D) $\sinh(2\pi)$

Question Number : 15 **Correct : 1 Wrong : 0**

If the fourth order divided difference of $f(x) = \alpha x^4 + 5x^3 + 3x + 2$, $\alpha \in \mathbb{R}$, at the points $0.1, 0.2, 0.3, 0.4, 0.5$ is 5, then α equals _______.

Question Number : 16 **Correct : 1 Wrong : 0**

If the quadrature rule $\int_0^2 f(x)\, dx \approx c_1 f(0) + 3 f(c_2)$, where $c_1, c_2 \in \mathbb{R}$, is exact for all polynomials of degree ≤ 1, then $c_1 + 3c_2$ equals _______.

Question Number : 17 **Correct : 1 Wrong : -0.33**

If $u(x,y)=1+x+y+f(xy)$, where $f:\mathbb{R}^2\to\mathbb{R}$ is a differentiable function, then u satisfies

(A) $x\frac{\partial u}{\partial x}-y\frac{\partial u}{\partial y}=x^2-y^2$

(B) $x\frac{\partial u}{\partial x}-y\frac{\partial u}{\partial y}=0$

(C) $x\frac{\partial u}{\partial x}-y\frac{\partial u}{\partial y}=x-y$

(D) $y\frac{\partial u}{\partial x}-x\frac{\partial u}{\partial y}=x-y$

Question Number : 18 **Correct : 1 Wrong : -0.33**

The partial differential equation $x\frac{\partial^2 u}{\partial x^2}+(x-y)\frac{\partial^2 u}{\partial x\,\partial y}-y\frac{\partial^2 u}{\partial y^2}+\frac{1}{4}\left(\frac{\partial u}{\partial y}-\frac{\partial u}{\partial x}\right)=0$ is

(A) hyperbolic along the line $x+y=0$

(B) elliptic along the line $x-y=0$

(C) elliptic along the line $x+y=0$

(D) parabolic along the line $x+y=0$

Question Number : 19 **Correct : 1 Wrong : -0.33**

Let X and Y be topological spaces and let $f:X\to Y$ be a continuous surjective function. Which one of the following statements is TRUE?

(A) If X is separable, then Y is separable

(B) If X is first countable, then Y is first countable

(C) If X is Hausdorff, then Y is Hausdorff

(D) If X is regular, then Y is regular

Question Number : 20 **Correct : 1 Wrong : -0.33**

Consider the topology $\mathcal{T}=\{U\subseteq\mathbb{Z}:\ \mathbb{Z}\setminus U \text{ is finite or } 0\notin U\}$ on $\mathbb{Z}$. Then, the topological space $(\mathbb{Z},\mathcal{T})$ is

(A) compact but NOT connected

(B) connected but NOT compact

(C) both compact and connected

(D) neither compact nor connected

Question Number : 21 **Correct : 1 Wrong : -0.33**

Let $F(x)$ be the distribution function of a random variable X. Consider the functions:

$$G_1(x) = (F(x))^3, \ x \in \mathbb{R},$$

$$G_2(x) = 1 - (1 - F(x))^5, \ x \in \mathbb{R}.$$

Which of the above functions are distribution functions?

(A) Neither G_1 nor G_2

(B) Only G_1

(C) Only G_2

(D) Both G_1 and G_2

Question Number : 22 **Correct : 1 Wrong : -0.33**

Let $X_1, X_2, \ldots, X_n$ $(n \geq 2)$ be independent and identically distributed random variables with finite variance σ^2 and let $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$. Then the covariance between $\bar{X}$ and $X_1 - \bar{X}$ is

(A) 0

(B) $-\sigma^2$

(C) $\frac{-\sigma^2}{n}$

(D) $\frac{\sigma^2}{n}$

Question Number : 23 **Correct : 1 Wrong : 0**

Let $X_1, X_2, \ldots, X_n$ $(n \geq 2)$ be a random sample from a $N(\mu, \sigma^2)$ population, where $\sigma^2 = 144$. The smallest n such that the length of the shortest 95% confidence interval for μ will not exceed 10 is _______.

Question Number : 24 **Correct : 1 Wrong : -0.33**

Consider the linear programming problem (LPP):

Maximize $4x_1 + 6x_2$

Subject to $x_1 + x_2 \leq 8$,

$2x_1 + 3x_2 \geq 18$,

$x_1 \geq 6$, x_2 is unrestricted in sign.

Then the LPP has

(A) no optimal solution
(B) only one basic feasible solution and that is optimal
(C) more than one basic feasible solution and a unique optimal solution
(D) infinitely many optimal solutions

Question Number : 25 **Correct : 1 Wrong : -0.33**

For a linear programming problem (LPP) and its dual, which one of the following is NOT TRUE?

(A) The dual of the dual is primal
(B) If the primal LPP has an unbounded objective function, then the dual LPP is infeasible
(C) If the primal LPP is infeasible, then the dual LPP must have unbounded objective function
(D) If the primal LPP has a finite optimal solution, then the dual LPP also has a finite optimal solution

Question Number : 26 **Correct : 2 Wrong : 0**

If U and V are the null spaces of $\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 2 & 3 & 2 \\ 0 & 1 & 2 & 1 \end{bmatrix}$, respectively, then the dimension of the subspace $U+V$ equals _______.

Question Number : 27 **Correct : 2 Wrong : -0.66**

Given two $n \times n$ matrices A and B with entries in $\mathbb{C}$, consider the following statements:

(P): If A and B have the same minimal polynomial, then A is similar to B.

(Q): If A has n distinct eigenvalues, then there exists $u \in \mathbb{C}^n$ such that $u, Au, \ldots, A^{n-1}u$ are linearly independent.

Which of the above statements hold TRUE?

(A) Both P and Q

(B) Only P

(C) Only Q

(D) Neither P nor Q

Question Number : 28 **Correct : 2 Wrong : 0**

Let $A = (a_{ij})$ be a 10×10 matrix such that $a_{ij} = 1$ for $i \neq j$ and $a_{ii} = \alpha + 1$, where $\alpha > 0$. Let λ and μ be the largest and the smallest eigenvalues of A, respectively. If $\lambda + \mu = 24$, then α equals _______.

Question Number : 29 **Correct : 2 Wrong : 0**

Let C be the simple, positively oriented circle of radius 2 centered at the origin in the complex plane. Then

$$\frac{2}{\pi i} \int_C \left(z e^{(1/z)} + \tan\left(\frac{z}{2}\right) + \frac{1}{(z-1)(z-3)^2} \right) dz$$ equals _______.

Question Number : 30 **Correct : 2 Wrong : -0.66**

Let $\operatorname{Re}(z)$ and $\operatorname{Im}(z)$, respectively, denote the real part and the imaginary part of a complex number z. Let $T:\mathbb{C}\cup\{\infty\}\to\mathbb{C}\cup\{\infty\}$ be the bilinear transformation such that $T(6)=0$, $T(3-3i)=i$ and $T(0)=\infty$. Then, the image of $D=\{z\in\mathbb{C}:|z-3|<3\}$ under the mapping $w=T(z)$ is

(A) $\{w\in\mathbb{C}: \operatorname{Im}(w)<0\}$

(B) $\{w\in\mathbb{C}: \operatorname{Re}(w)<0\}$

(C) $\{w\in\mathbb{C}: \operatorname{Im}(w)>0\}$

(D) $\{w\in\mathbb{C}: \operatorname{Re}(w)>0\}$

Question Number : 31 **Correct : 2 Wrong : -0.66**

Let (x_n) and (y_n) be two sequences in a complete metric space (X,d) such that $d(x_n,x_{n+1})\le\frac{1}{n^2}$ and $d(y_n,y_{n+1})\le\frac{1}{n}$ for all $n\in\mathbb{N}$. Then

(A) both (x_n) and (y_n) converge

(B) (x_n) converges but (y_n) need NOT converge

(C) (y_n) converges but (x_n) need NOT converge

(D) neither (x_n) nor (y_n) converges

Question Number : 32 **Correct : 2 Wrong : 0**

Let $f:[0,1]\to\mathbb{R}$ be given by $f(x)=0$ if x is rational, and if x is irrational then $f(x)=9^n$, where n is the number of zeroes immediately after the decimal point in the decimal representation of x. Then the Lebesgue integral $\int_0^1 f(x)\,dx$ equals _______.

Question Number : 33 **Correct : 2 Wrong : -0.66**

Let $f:\mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x,y)=\begin{cases} \sin\left(\dfrac{y^2}{x}\right)\sqrt{x^2+y^2}, & x\neq 0, \\ 0, & x=0. \end{cases}$

Then, at $(0,0)$,

(A) f is continuous and the directional derivative of f does NOT exist in some direction

(B) f is NOT continuous and the directional derivatives of f exist in all directions

(C) f is NOT differentiable and the directional derivatives of f exist in all directions

(D) f is differentiable

Question Number : 34 **Correct : 2 Wrong : 0**

Let D be the region in $\mathbb{R}^2$ bounded by the parabola $y^2=2x$ and the line $y=x$. Then $\iint_D 3xy\, dx\, dy$ equals _______.

Question Number : 35 **Correct : 2 Wrong : -0.66**

Let $y_1(x)=x^3$ and $y_2(x)=x^2|x|$ for $x\in\mathbb{R}$.

Consider the following statements:

(P): $y_1(x)$ and $y_2(x)$ are linearly independent solutions of $x^2\dfrac{d^2y}{dx^2}-4x\dfrac{dy}{dx}+6y=0$ on $\mathbb{R}$.

(Q): The Wronskian $y_1(x)\dfrac{dy_2}{dx}(x)-y_2(x)\dfrac{dy_1}{dx}(x)=0$ for all $x\in\mathbb{R}$.

Which of the above statements hold TRUE?

(A) Both P and Q

(B) Only P

(C) Only Q

(D) Neither P nor Q

Question Number : 36 **Correct : 2 Wrong : 0**

Let α and β with $\alpha > \beta$ be the roots of the indicial equation of

$(x^2-1)^2\dfrac{d^2y}{dx^2}+(x+1)\dfrac{dy}{dx}-y=0$ at $x=-1$. Then $\alpha-4\beta$ equals _______.

Question Number : 37 **Correct : 2 Wrong : 0**

Let S_9 be the group of all permutations of the set $\{1,2,3,4,5,6,7,8,9\}$. Then the total number of elements of S_9 that commute with $\tau=(1\ 2\ 3)(4\ 5\ 6\ 7)$ in S_9 equals _______.

Question Number : 38 **Correct : 2 Wrong : 0**

Let $\mathbb{Q}[x]$ be the ring of polynomials over $\mathbb{Q}$. Then the total number of maximal ideals in the quotient ring $\mathbb{Q}[x]/\left(x^4-1\right)$ equals _______.

Question Number : 39 **Correct : 2 Wrong : -0.66**

Let $\{e_n : n\in\mathbb{N}\}$ be an orthonormal basis of a Hilbert space H. Let $T:H\to H$ be given by $Tx=\sum_{n=1}^{\infty}\frac{1}{n}\langle x,e_n\rangle e_n$. For each $n\in\mathbb{N}$, define $T_n:H\to H$ by $T_nx=\sum_{j=1}^{n}\frac{1}{j}\langle x,e_j\rangle e_j$.

Then

(A) $\|T_n-T\|\to 0$ as $n\to\infty$

(B) $\|T_n-T\|\not\to 0$ as $n\to\infty$ but for each $x\in H$, $\|T_nx-Tx\|\to 0$ as $n\to\infty$

(C) for each $x\in H$, $\|T_nx-Tx\|\to 0$ as $n\to\infty$ but the sequence $\left(\|T_n\|\right)$ is unbounded

(D) there exist $x,\ y\in H$ such that $\langle T_nx,\ y\rangle \not\to \langle Tx,\ y\rangle$ as $n\to\infty$

Question Number : 40 **Correct : 2 Wrong : -0.66**

Consider the subspace $V = \left\{(x_n) \in \ell^2 \ : \ \sum_{n=1}^{\infty} |x_n| < \infty\right\}$ of the Hilbert space ℓ^2 of all square summable real sequences. For $n \in \mathbb{N}$, define $T_n : V \to \mathbb{R}$ by $T_n((x_k)) = \sum_{i=1}^{n} x_i$.

Consider the following statements:

(P): $\{T_n : n \in \mathbb{N}\}$ is pointwise bounded on V.

(Q): $\{T_n : n \in \mathbb{N}\}$ is uniformly bounded on $\{x \in V : \|x\|_2 = 1\}$.

Which of the above statements hold TRUE?

(A) Both P and Q
(B) Only P
(C) Only Q
(D) Neither P nor Q

Question Number : 41 **Correct : 2 Wrong : 0**

Let $p(x)$ be the polynomial of degree at most 2 that interpolates the data $(-1,2)$, $(0,1)$ and $(1,2)$. If $q(x)$ is a polynomial of degree at most 3 such that $p(x)+q(x)$ interpolates the data $(-1,2)$, $(0,1)$, $(1,2)$ and $(2,11)$, then $q(3)$ equals _______.

Question Number : 42 **Correct : 2 Wrong : -0.66**

Let J be the Jacobi iteration matrix of the linear system $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ -4 & 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

Consider the following statements:

(P): One of the eigenvalues of J lies in the interval [2, 3].

(Q): The Jacobi iteration converges for the above system.

Which of the above statements hold TRUE?

(A) Both P and Q
(B) Only P
(C) Only Q
(D) Neither P nor Q

Question Number : 43 **Correct : 2 Wrong : 0**

Let $u(x,y)$ be the solution of $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=4u$ satisfying the condition $u(x,y)=1$ on the circle $x^2+y^2=1$. Then $u(2,2)$ equals _______.

Question Number : 44 **Correct : 2 Wrong : -0.66**

Let $u(r,\theta)$ be the bounded solution of the following boundary value problem in polar coordinates:

$$r^2\frac{\partial^2 u}{\partial r^2}+r\frac{\partial u}{\partial r}+\frac{\partial^2 u}{\partial \theta^2}=0, \quad 0<r<2 \quad \text{and} \quad 0\le\theta\le 2\pi,$$
$$u(2,\theta)=\cos^2\theta, \quad 0\le\theta\le 2\pi.$$

Then $u(1,\pi/2)+u(1,\pi/4)$ equals

(A) 1 (B) $\frac{9}{8}$ (C) $\frac{7}{8}$ (D) $\frac{3}{8}$

Question Number : 45 **Correct : 2 Wrong : -0.66**

Let $\mathcal{T}_u$ and $\mathcal{T}_d$ denote the usual topology and the discrete topology on $\mathbb{R}$, respectively. Consider the following three topologies:

$\mathcal{T}_1$ = Usual topology on $\mathbb{R}^2=\mathbb{R}\times\mathbb{R}$,

$\mathcal{T}_2$ = Topology generated by the basis $\{U\times V : U\in\mathcal{T}_d, V\in\mathcal{T}_u\}$ on $\mathbb{R}\times\mathbb{R}$,

$\mathcal{T}_3$ = Dictionary order topology on $\mathbb{R}\times\mathbb{R}$.

Then

(A) $\mathcal{T}_3 \subsetneqq \mathcal{T}_1 \subseteq \mathcal{T}_2$

(B) $\mathcal{T}_1 \subsetneqq \mathcal{T}_2 \subsetneqq \mathcal{T}_3$

(C) $\mathcal{T}_3 \subseteq \mathcal{T}_2 \subsetneqq \mathcal{T}_1$

(D) $\mathcal{T}_1 \subsetneqq \mathcal{T}_2 = \mathcal{T}_3$

Question Number : 46 **Correct : 2 Wrong : 0**

Let X be a random variable with probability mass function

$p(n)=\left(\frac{3}{4}\right)^{n-1}\left(\frac{1}{4}\right)$ for $n=1,2,\ldots$. Then $E(X-3\mid X>3)$ equals _______.

Question Number : 47 **Correct : 2 Wrong : 0**

Let X and Y be independent and identically distributed random variables with probability mass function $p(n) = 2^{-n}$, $n = 1, 2, \ldots$.
Then $P(X \geq 2Y)$ equals (rounded to 2 decimal places) _______.

Question Number : 48 **Correct : 2 Wrong : 0**

Let $X_1, X_2, \ldots$ be a sequence of independent and identically distributed Poisson random variables with mean 4. Then $\lim_{n\to\infty} P\left(4 - \frac{2}{\sqrt{n}} < \frac{1}{n}\sum_{i=1}^{n} X_i < 4 + \frac{2}{\sqrt{n}}\right)$ equals _______.

Question Number : 49 **Correct : 2 Wrong : 0**

Let X and Y be independent and identically distributed exponential random variables with probability density function

$$f(x) = \begin{cases} e^{-x}, & x > 0, \\ 0, & \text{otherwise.} \end{cases}$$

Then $P(\max(X, Y) < 2)$ equals (rounded to 2 decimal places) _______.

Question Number : 50 **Correct : 2 Wrong : 0**

Let E and F be any two events with $P(E) = 0.4$, $P(F) = 0.3$ and $P(F \mid E) = 3\, P(F \mid E^c)$.
Then $P(E \mid F)$ equals (rounded to 2 decimal places) _______.

Question Number : 51 Correct : 2 Wrong : -0.66

Let $X_1, X_2, \ldots, X_m$ $(m \geq 2)$ be a random sample from a binomial distribution with parameters $n = 1$ and p, $p \in (0,1)$, and let $\bar{X} = \frac{1}{m}\sum_{i=1}^{m} X_i$. Then a uniformly minimum variance unbiased estimator for $p(1-p)$ is

(A) $\frac{m}{m-1}\bar{X}(1-\bar{X})$

(B) $\bar{X}(1-\bar{X})$

(C) $\frac{m-1}{m}\bar{X}(1-\bar{X})$

(D) $\frac{1}{m}\bar{X}(1-m\bar{X})$

Question Number : 52 Correct : 2 Wrong : 0

Let $X_1, X_2, \ldots, X_9$ be a random sample from a $N(0, \sigma^2)$ population. For testing $H_0 : \sigma^2 = 2$ against $H_1 : \sigma^2 = 1$, the most powerful test rejects H_0 if $\sum_{i=1}^{9} X_i^2 < c$, where c is to be chosen such that the level of significance is 0.1. Then the power of this test equals _______.

Question Number : 53 Correct : 2 Wrong : -0.66

Let $X_1, X_2, \ldots, X_n$ $(n \geq 2)$ be a random sample from a $N(\theta, \theta)$ population, where $\theta > 0$, and let $W = \frac{1}{n}\sum_{i=1}^{n} X_i^2$. Then the maximum likelihood estimator of θ is

(A) $\frac{1}{2}+\frac{1}{2}\sqrt{1-4W}$

(B) $\frac{1}{2}+\frac{1}{2}\sqrt{1+4W}$

(C) $\frac{-1}{2}+\frac{1}{2}\sqrt{1-4W}$

(D) $\frac{-1}{2}+\frac{1}{2}\sqrt{1+4W}$

Question Number : 54 **Correct : 2 Wrong : 0**

Consider the following transportation problem. The entries inside the cells denote per unit cost of transportation from the origins to the destinations.

		Destination 1	2	3	Supply
Origin	1	4	3	6	20
	2	7	10	5	30
	3	8	9	7	50
	Demand	10	30	60	

The optimal cost of transportation equals _______.

Question Number : 55 **Correct : 2 Wrong : 0**

Consider the linear programming problem (LPP):

$$\text{Maximize } k\,x_1 + 5\,x_2$$
$$\text{Subject to } x_1 + x_2 \le 1,$$
$$2\,x_1 + 3\,x_2 \le 1,$$
$$x_1,\ x_2 \ge 0.$$

If $x^* = (x_1^*, x_2^*)$ is an optimal solution of the above LPP with $k = 2$, then the largest value of k (rounded to 2 decimal places) for which x^* remains optimal equals _______.

Question Number : 56 **Correct : 1 Wrong : -0.33**

The ninth and the tenth of this month are Monday and Tuesday ____________.

(A) figuratively (B) retrospectively (C) respectively (D) rightfully

Question Number : 57 **Correct : 1 Wrong : -0.33**

It is __________ to read this year's textbook ___________the last year's.

(A) easier, than (B) most easy, than (C) easier, from (D) easiest, from

Question Number : 58 **Correct : 1 Wrong : -0.33**

A rule states that in order to drink beer, one must be over 18 years old. In a bar, there are 4 people. P is 16 years old, Q is 25 years old, R is drinking milkshake and S is drinking a beer. What must be checked to ensure that the rule is being followed?

(A) Only P's drink
(B) Only P's drink and S's age
(C) Only S's age
(D) Only P's drink, Q's drink and S's age

Question Number : 59 **Correct : 1 Wrong : -0.33**

Fatima starts from point P, goes North for 3 km, and then East for 4 km to reach point Q. She then turns to face point P and goes 15 km in that direction. She then goes North for 6 km. How far is she from point P, and in which direction should she go to reach point P?

(A) 8 km, East (B) 12 km, North (C) 6 km, East (D) 10 km, North

Question Number : 60 **Correct : 1 Wrong : -0.33**

500 students are taking one or more courses out of Chemistry, Physics, and Mathematics. Registration records indicate course enrolment as follows: Chemistry (329), Physics (186), Mathematics (295), Chemistry and Physics (83), Chemistry and Mathematics (217), and Physics and Mathematics (63). How many students are taking all 3 subjects?

(A) 37 (B) 43 (C) 47 (D) 53

Question Number : 61 **Correct : 2 Wrong : -0.66**

"If you are looking for a history of India, or for an account of the rise and fall of the British Raj, or for the reason of the cleaving of the subcontinent into two mutually antagonistic parts and the effects this mutilation will have in the respective sections, and ultimately on Asia, you will not find it in these pages; for though I have spent a lifetime in the country, I lived too near the seat of events, and was too intimately associated with the actors, to get the perspective needed for the impartial recording of these matters."

Which of the following statements best reflects the author's opinion?

(A) An intimate association does not allow for the necessary perspective.
(B) Matters are recorded with an impartial perspective.
(C) An intimate association offers an impartial perspective.
(D) Actors are typically associated with the impartial recording of matters.

Question Number : 62 **Correct : 2 Wrong : -0.66**

Each of P, Q, R, S, W, X, Y and Z has been married at most once. X and Y are married and have two children P and Q. Z is the grandfather of the daughter S of P. Further, Z and W are married and are parents of R. Which one of the following must necessarily be FALSE?

(A) X is the mother-in-law of R
(B) P and R are not married to each other
(C) P is a son of X and Y
(D) Q cannot be married to R

Question Number : 63 **Correct : 2 Wrong : -0.66**

1200 men and 500 women can build a bridge in 2 weeks. 900 men and 250 women will take 3 weeks to build the same bridge. How many men will be needed to build the bridge in one week?

(A) 3000 (B) 3300 (C) 3600 (D) 3900

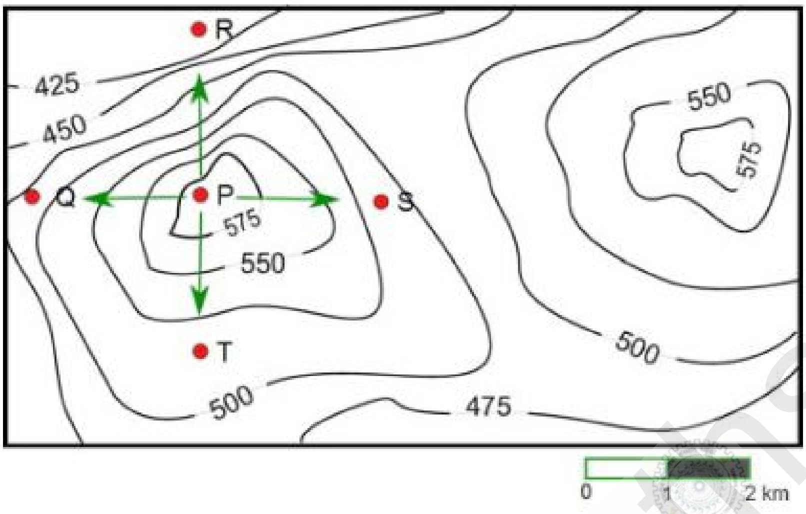
Question Number : 64 **Correct : 2 Wrong : -0.66**

The number of 3-digit numbers such that the digit 1 is never to the immediate right of 2 is

(A) 781 (B) 791 (C) 881 (D) 891

Question Number : 65 Correct : 2 Wrong : -0.66

A contour line joins locations having the same height above the mean sea level. The following is a contour plot of a geographical region. Contour lines are shown at 25 m intervals in this plot.



Which of the following is the steepest path leaving from P?

(A) P to Q (B) P to R (C) P to S (D) P to T

Q. No.	Type	Section	Key	Marks
1	NAT	MA	1 to 1	1
2	NAT	MA	8 to 8	1
3	MCQ	MA	B	1
4	NAT	MA	48 to 48	1
5	MCQ	MA	D	1
6	NAT	MA	36 to 36	1
7	MCQ	MA	B	1
8	MCQ	MA	A	1
9	NAT	MA	0 to 0	1
10	MCQ	MA	C	1
11	NAT	MA	5 to 5	1
12	NAT	MA	8 to 8	1
13	NAT	MA	0 to 0	1
14	MCQ	MA	D	1
15	NAT	MA	5 to 5	1
16	NAT	MA	1 to 1	1
17	MCQ	MA	C	1
18	MCQ	MA	D	1
19	MCQ	MA	A	1
20	MCQ	MA	A	1
21	MCQ	MA	D	1
22	MCQ	MA	A	1
23	NAT	MA	23 to 23	1
24	MCQ	MA	B	1
25	MCQ	MA	C	1
26	NAT	MA	3 to 3	2
27	MCQ	MA	C	2
28	NAT	MA	7 to 7	2
29	NAT	MA	3 to 3	2
30	MCQ	MA	D	2
31	MCQ	MA	B	2
32	NAT	MA	9 to 9	2
33	MCQ	MA	C	2
34	NAT	MA	2 to 2	2
35	MCQ	MA	A	2
36	NAT	MA	2 to 2	2

37	NAT	MA	24 to 24	2
38	NAT	MA	3 to 3	2
39	MCQ	MA	A	2
40	MCQ	MA	B	2
41	NAT	MA	24 to 24	2
42	MCQ	MA	B	2
43	NAT	MA	64 to 64	2
44	MCQ	MA	C	2
45	MCQ	MA	D	2
46	NAT	MA	4 to 4	2
47	NAT	MA	0.27 to 0.30	2
48	NAT	MA	0.67 to 0.70	2
49	NAT	MA	0.73 to 0.77	2
50	NAT	MA	0.65 to 0.68	2
51	MCQ	MA	A	2
52	NAT	MA	0.49 to 0.51	2
53	MCQ	MA	D	2
54	NAT	MA	590 to 590	2
55	NAT	MA	3.32 to 3.34	2
56	MCQ	GA	C	1
57	MCQ	GA	A	1
58	MCQ	GA	B	1
59	MCQ	GA	A	1
60	MCQ	GA	D	1
61	MCQ	GA	A	2
62	MCQ	GA	D	2
63	MCQ	GA	C	2
64	MCQ	GA	C	2
65	MCQ	GA	B	2

Q. 1 – Q. 5 carry one mark each.

Q.1 An apple costs Rs. 10. An onion costs Rs. 8.

Select the most suitable sentence with respect to grammar and usage.

(A) The price of an apple is greater than an onion.

(B) The price of an apple is more than onion.

(C) The price of an apple is greater than that of an onion.

(D) Apples are more costlier than onions.

Q.2 The Buddha said, "Holding on to anger is like grasping a hot coal with the intent of throwing it at someone else; you are the one who gets burnt."

Select the word below which is closest in meaning to the word underlined above.

(A) burning (B) igniting (C) clutching (D) flinging

Q.3 **M** has a son **Q** and a daughter **R**. He has no other children. **E** is the mother of **P** and daughter-in-law of **M**. How is **P** related to **M**?

(A) **P** is the son-in-law of **M**.

(B) **P** is the grandchild of **M**.

(C) **P** is the daughter-in law of **M**.

(D) **P** is the grandfather of **M**.

Q.4 The number that least fits this set: (324, 441, 97 and 64) is ________.

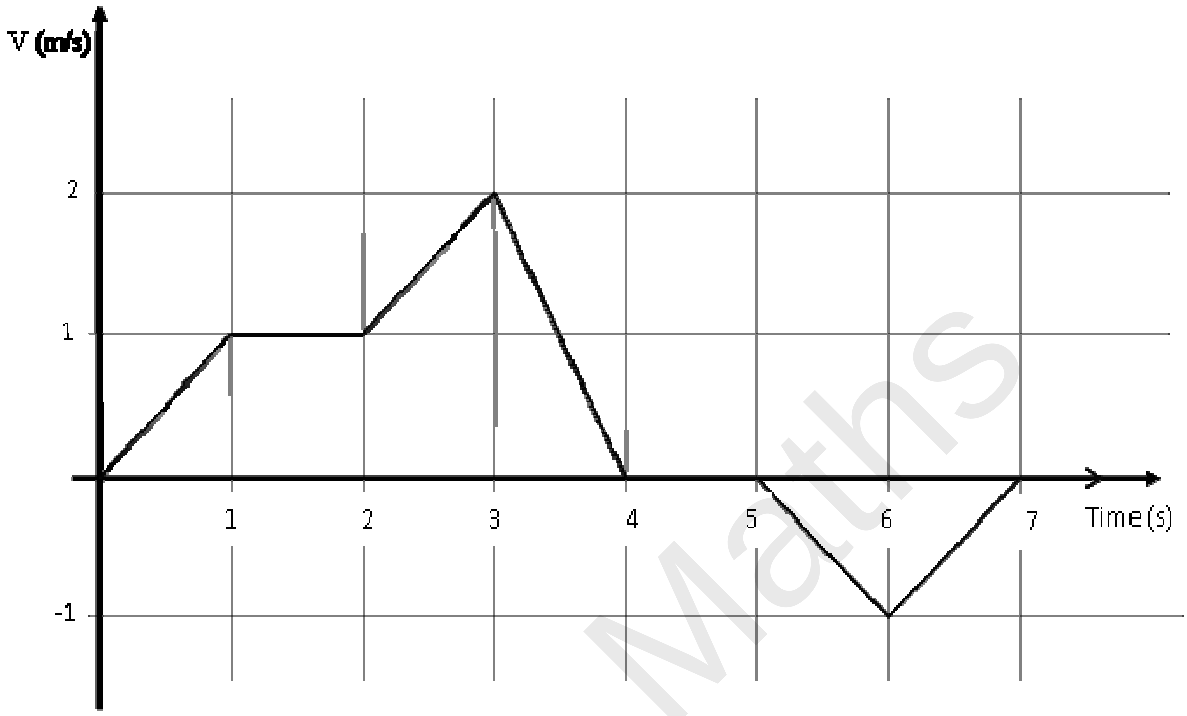
(A) 324 (B) 441 (C) 97 (D) 64

Q.5 It takes 10 s and 15 s, respectively, for two trains travelling at different constant speeds to completely pass a telegraph post. The length of the first train is 120 m and that of the second train is 150 m. The magnitude of the difference in the speeds of the two trains (in m/s) is ____________.

(A) 2.0 (B) 10.0 (C) 12.0 (D) 22.0

Q. 6 – Q. 10 carry two marks each.

Q.6 The velocity V of a vehicle along a straight line is measured in m/s and plotted as shown with respect to time in seconds. At the end of the 7 seconds, how much will the odometer reading increase by (in m)?



(A) 0 (B) 3 (C) 4 (D) 5

Q.7 The overwhelming number of people infected with rabies in India has been flagged by the World Health Organization as a source of concern. It is estimated that inoculating 70% of pets and stray dogs against rabies can lead to a significant reduction in the number of people infected with rabies.

Which of the following can be logically inferred from the above sentences?

(A) The number of people in India infected with rabies is high.

(B) The number of people in other parts of the world who are infected with rabies is low.

(C) Rabies can be eradicated in India by vaccinating 70% of stray dogs.

(D) Stray dogs are the main source of rabies worldwide.

Q.8 A flat is shared by four first year undergraduate students. They agreed to allow the oldest of them to enjoy some extra space in the flat. Manu is two months older than Sravan, who is three months younger than Trideep. Pavan is one month older than Sravan. Who should occupy the extra space in the flat?

(A) Manu (B) Sravan (C) Trideep (D) Pavan

Q.9 Find the area bounded by the lines 3x+2y=14, 2x-3y=5 in the first quadrant.

(A) 14.95 (B) 15.25 (C) 15.70 (D) 20.35

Q.10 A straight line is fit to a data set ($\ln x$, y). This line intercepts the abscissa at $\ln x = 0.1$ and has a slope of -0.02. What is the value of y at $x = 5$ from the fit?

(A) -0.030 (B) -0.014 (C) 0.014 (D) 0.030

END OF THE QUESTION PAPER

List of Symbols, Notations and Data

i.i.d. : independent and identically distributed

$N(\mu, \sigma^2)$: Normal distribution with mean μ and variance σ^2, $\mu \in (-\infty, \infty)$, $\sigma > 0$

$E(X)$: Expected value (mean) of the random variable X

$\Phi(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{t} e^{-\frac{x^2}{2}} dx$

$[x]$: the greatest integer less than or equal to x

$\mathbb{Z}$: Set of integers

$\mathbb{Z}_n$: Set of integers modulo n

$\mathbb{R}$: Set of real numbers

$\mathbb{C}$: Set of complex numbers

$\mathbb{R}^n$: n- dimensional Euclidean space

Usual metric d on $\mathbb{R}^n$ is given by $d\big((x_1, x_2, \dots, x_n), (y_1, y_2, \dots, y_n)\big) = \left(\sum_{i=1}^{n} (x_i - y_i)^2\right)^{1/2}$

ℓ_2 : Normed linear space of all square-summable real sequences

$C[0,1]$: Set of all real valued continuous functions on the interval $[0,1]$

$\overline{B(0,1)} := \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1\}$

M^* : Conjugate transpose of the matrix M

M^T : Transpose of the matrix M

Id : Identity matrix of appropriate order

$\mathcal{R}(M)$: Range space of M

$\mathcal{N}(M)$: Null space of M

$W^{\perp}$: Orthogonal complement of the subspace W

Q. 1 – Q. 25 carry one mark each.

Q.1 Let $\{X, Y, Z\}$ be a basis of $\mathbb{R}^3$. Consider the following statements P and Q:

(P) : $\{X + Y, Y + Z, X - Z\}$ is a basis of $\mathbb{R}^3$.

(Q) : $\{X + Y + Z, X + 2Y - Z, X - 3Z\}$ is a basis of $\mathbb{R}^3$.

Which of the above statements hold TRUE?

(A) both P and Q
(B) only P
(C) only Q
(D) Neither P nor Q

Q.2 Consider the following statements P and Q:

(P) : If $M = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{bmatrix}$, then M is singular .

(Q) : Let S be a diagonalizable matrix. If T is a matrix such that $S + 5\,T = Id$, then T is diagonalizable.

Which of the above statements hold TRUE?

(A) both P and Q
(B) only P
(C) only Q
(D) Neither P nor Q

Q.3 Consider the following statements P and Q:

(P) : If M is an $n \times n$ complex matrix, then $\mathcal{R}(M) = (\mathcal{N}(M^*))^{\perp}$.

(Q) : There exists a unitary matrix with an eigenvalue λ such that $|\lambda| < 1$.

Which of the above statements hold TRUE?

(A) both P and Q
(B) only P
(C) only Q
(D) Neither P nor Q

Q.4 Consider a real vector space V of dimension n and a non-zero linear transformation $T : V \to V$. If dimension$(T(V)) < n$ and $T^2 = \lambda T$, for some $\lambda \in \mathbb{R}\setminus\{0\}$, then which of the following statements is TRUE?

(A) determinant$(T) = |\lambda|^n$

(B) There exists a non-trivial subspace V_1 of V such that $T(X) = 0$ for all $X \in V_1$

(C) T is invertible

(D) λ is the only eigenvalue of T

Q.5 Let $S = [0,1) \cup [2,3]$ and $f : S \to \mathbb{R}$ be a strictly increasing function such that $f(S)$ is connected. Which of the following statements is TRUE?

(A) f has exactly one discontinuity

(B) f has exactly two discontinuities

(C) f has infinitely many discontinuities

(D) f is continuous

Q.6 Let $a_1 = 1$ and $a_n = a_{n-1} + 4,\ n \geq 2$. Then,

$$\lim_{n \to \infty} \left[\frac{1}{a_1 a_2} + \frac{1}{a_2 a_3} + \cdots + \frac{1}{a_{n-1} a_n}\right]$$

is equal to ___________________

Q.7 Maximum $\{x + y : (x, y) \in \overline{B(0,1)}\}$ is equal to _______________

Q.8 Let $a, b, c, d \in \mathbb{R}$ such that $c^2 + d^2 \neq 0$. Then, the Cauchy problem

$$a\,u_x + b\,u_y = e^{x+y}, \qquad x, y \in \mathbb{R},$$
$$u(x, y) = 0 \ \text{ on } c\,x + d\,y = 0$$

has a unique solution if

(A) $a\,c + b\,d \neq 0$

(B) $a\,d - b\,c \neq 0$

(C) $a\,c - b\,d \neq 0$

(D) $a\,d + b\,c \neq 0$

Q.9 Let $u(x,t)$ be the d'Alembert's solution of the initial value problem for the wave equation

$$u_{tt} - c^2 u_{xx} = 0$$
$$u(x,0) = f(x), \; u_t(x,0) = g(x),$$

where c is a positive real number and f, g are smooth odd functions. Then, $u(0,1)$ is equal to ________

Q.10 Let the probability density function of a random variable X be

$$f(x) = \begin{cases} x & 0 \le x < \frac{1}{2} \\ c(2x-1)^2 & \frac{1}{2} < x \le 1 \\ 0 & \text{otherwise.} \end{cases}$$

Then, the value of c is equal to ________________

Q.11 Let V be the set of all solutions of the equation $y'' + a\ y' + b\ y = 0$ satisfying $y(0) = y(1)$, where a, b are positive real numbers. Then, dimension(V) is equal to ______________

Q.12 Let $y'' + p(x)\, y' + q(x) y = 0, x \in (-\infty, \infty)$, where $p(x)$ and $q(x)$ are continuous functions. If $y_1(x) = sin\,(x) - 2\, cos\,(x)$ and $y_2(x) = 2\, sin\,(x) + cos\,(x)$ are two linearly independent solutions of the above equation, then $|\, 4\, p(0) + 2\, q(1)\, |$ is equal to _____________

Q.13 Let $P_n(x)$ be the Legendre polynomial of degree n and $I = \int_{-1}^{1} x^k\ P_n(x) dx$, where k is a non-negative integer. Consider the following statements P and Q:

(P) : $I = 0$ if $k < n$.

(Q) : $I = 0$ if $n - k$ is an odd integer.

Which of the above statements hold TRUE?

(A) both P and Q
(B) only P
(C) only Q
(D) Neither P nor Q

Q.14 Consider the following statements P and Q:

(P) : $x^2y'' + x\,y' + \left(x^2 - \frac{1}{4}\right)y = 0$ has two linearly independent Frobenius series solutions near $x = 0$.

(Q) : $x^2y'' + 3\,sin(x)\,y' + y = 0$ has two linearly independent Frobenius series solutions near $x = 0$.

Which of the above statements hold TRUE?

(A) both P and Q (B) only P
(C) only Q (D) Neither P nor Q

Q.15 Let the polynomial x^4 be approximated by a polynomial of degree ≤ 2, which interpolates x^4 at $x = -1, 0$ and 1. Then, the maximum absolute interpolation error over the interval $[-1, 1]$ is equal to ________________

Q.16 Let (z_n) be a sequence of distinct points in $D(0,1) = \{z \in \mathbb{C} : |z| < 1\}$ with $\lim_{n\to\infty} z_n = 0$. Consider the following statements P and Q:

(P) : There exists a unique analytic function f on $D(0,1)$ such that $f(z_n) = sin(z_n)$ for all n.

(Q) : There exists an analytic function f on $D(0,1)$ such that $f(z_n) = 0$ if n is even and $f(z_n) = 1$ if n is odd.

Which of the above statements hold TRUE?

(A) both P and Q (B) only P
(C) only Q (D) Neither P nor Q

Q.17 Let $(\mathbb{R}, \tau)$ be a topological space with the cofinite topology. Every infinite subset of $\mathbb{R}$ is

(A) Compact but NOT connected
(B) Both compact and connected
(C) NOT compact but connected
(D) Neither compact nor connected

Q.18 Let $c_0 = \{(x_n) : x_n \in \mathbb{R}, x_n \to 0\}$ and $M = \{(x_n) \in c_0 : x_1 + x_2 + \cdots + x_{10} = 0\}$. Then, dimension$\left({}^{c_0}/_{M}\right)$ is equal to ________

Q.19 Consider $(\mathbb{R}^2, \|\cdot\|_\infty)$, where $\|(x,y)\|_\infty = \text{maximum}\{|x|, |y|\}$. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x,y) = \frac{x+y}{2}$ and $\tilde{f}$ the norm preserving linear extension of f to $(\mathbb{R}^3, \|\cdot\|_\infty)$. Then, $\tilde{f}(1,1,1)$ is equal to ________

Q.20 $f : [0,1] \to [0,1]$ is called a shrinking map if $|f(x) - f(y)| < |x - y|$ for all $x, y \in [0,1]$ and a contraction if there exists an $\alpha < 1$ such that $|f(x) - f(y)| \le \alpha|x - y|$ for all $x, y \in [0,1]$.

Which of the following statements is TRUE for the function $f(x) = x - \frac{x^2}{2}$?

(A) f is both a shrinking map and a contraction

(B) f is a shrinking map but NOT a contraction

(C) f is NOT a shrinking map but a contraction

(D) f is Neither a shrinking map nor a contraction

Q.21 Let $\mathbb{M}$ be the set of all $n \times n$ real matrices with the usual norm topology. Consider the following statements P and Q:

(P) : The set of all symmetric positive definite matrices in $\mathbb{M}$ is connected.

(Q) : The set of all invertible matrices in $\mathbb{M}$ is compact.

Which of the above statements hold TRUE?

(A) both P and Q (B) only P

(C) only Q (D) Neither P nor Q

Q.22 Let $X_1, X_2, X_3, \dots, X_n$ be a random sample from the following probability density function for $0 < \mu < \infty,\ 0 < \alpha < 1$,

$$f(x;\ \mu, \alpha) = \begin{cases} \frac{1}{\Gamma(\alpha)}\ (x-\mu)^{\alpha-1} e^{-(x-\mu)}; & x > \mu \\ 0 & \text{otherwise.} \end{cases}$$

Here α and μ are unknown parameters. Which of the following statements is TRUE?

(A) Maximum likelihood estimator of only μ exists

(B) Maximum likelihood estimator of only α exists

(C) Maximum likelihood estimators of both μ and α exist

(D) Maximum likelihood estimator of Neither μ nor α exists

Q.23 Suppose X and Y are two random variables such that $aX + bY$ is a normal random variable for all $a, b \in \mathbb{R}$. Consider the following statements P, Q, R and S:

(P) : X is a standard normal random variable.

(Q) : The conditional distribution of X given Y is normal.

(R) : The conditional distribution of X given $X + Y$ is normal.

(S) : $X - Y$ has mean 0.

Which of the above statements ALWAYS hold TRUE?

(A) both P and Q (B) both Q and R

(C) both Q and S (D) both P and S

Q.24 Consider the following statements P and Q:

(P) : If H is a normal subgroup of order 4 of the symmetric group S_4, then ${S_4}/{H}$ is abelian.

(Q) : If $Q = \{\pm 1,\ \pm i,\ \pm j,\ \pm k\}$ is the quaternion group, then ${Q}/{\{-1,1\}}$ is abelian.

Which of the above statements hold TRUE?

(A) both P and Q (B) only P

(C) only Q (D) Neither P nor Q

Q.25 Let F be a field of order 32. Then the number of non-zero solutions $(a, b) \in F \times F$ of the equation $x^2 + x\,y + y^2 = 0$ is equal to ____________

Q. 26 – Q. 55 carry two marks each.

Q.26 Let $\gamma = \{z \in \mathbb{C} : \ |z| = 2\}$ be oriented in the counter-clockwise direction. Let

$$I = \frac{1}{2\,\pi\, i} \oint_{\gamma} z^7 \cos\left(\frac{1}{z^2}\right) dz.$$

Then, the value of I is equal to ________________

Q.27 Let $u(x,y) = x^3 + a\,x^2\,y + b\,x\,y^2 + 2\,y^3$ be a harmonic function and $v(x,y)$ its harmonic conjugate. If $v(0,0) = 1$, then $|a + b + v(1,1)|$ is equal to __________

Q.28 Let γ be the triangular path connecting the points (0,0), (2,2) and (0,2) in the counter-clockwise direction in $\mathbb{R}^2$. Then

$$I = \oint_{\gamma} \sin(x^3) dx + 6\,x\,y\,dy$$

is equal to ______________

Q.29 Let y be the solution of

$$y' + y = |x|, \qquad x \in \mathbb{R}$$
$$y(-1) = 0.$$

Then $y(1)$ is equal to

(A) $\frac{2}{e} - \frac{2}{e^2}$

(B) $\frac{2}{e} - 2\,e^2$

(C) $2 - \frac{2}{e}$

(D) $2 - 2\,e$

Q.30 Let X be a random variable with the following cumulative distribution function:

$$F(x) = \begin{cases} 0 & x < 0 \\ x^2 & 0 \le x < \frac{1}{2} \\ \frac{3}{4} & \frac{1}{2} \le x < 1 \\ 1 & x \ge 1. \end{cases}$$

Then $P\left(\frac{1}{4} < X < 1\right)$ is equal to ____________

Q.31 Let γ be the curve which passes through $(0,1)$ and intersects each curve of the family $y = c\,x^2$ orthogonally. Then γ also passes through the point

(A) $(\sqrt{2}, 0)$ (B) $(0, \sqrt{2})$

(C) $(1,1)$ (D) $(-1,1)$

Q.32 Let $S(x) = a_0 + \sum_{n=1}^{\infty}(a_n \cos(n\,x) + b_n \sin(n\,x))$ be the Fourier series of the $2\,\pi$ periodic function defined by $f(x) = x^2 + 4\sin(x)\cos(x)$, $-\pi \leq x \leq \pi$. Then

$$\left|\sum_{n=0}^{\infty} a_n - \sum_{n=1}^{\infty} b_n\right|$$

is equal to ______________________

Q.33 Let $y(t)$ be a continuous function on $[0, \infty)$. If

$$y(t) = t\left(1 - 4\int_0^t y(x)dx\right) + 4\int_0^t x\,y(x)\,dx,$$

then $\int_0^{\pi/2} y(t)\,dt$ is equal to ____________________

Q.34 Let $S_n = \sum_{k=1}^{n}\frac{1}{k}$ and $I_n = \int_1^n \frac{x-[x]}{x^2}\,dx$. Then, $S_{10} + I_{10}$ is equal to

(A) $\ln 10 + 1$ (B) $\ln 10 - 1$

(C) $\ln 10 - \frac{1}{10}$ (D) $\ln 10 + \frac{1}{10}$

Q.35 For any $(x, y) \in \mathbb{R}^2 \setminus \overline{B(0,1)}$, let

$$f(x,y) = \text{distance}\left((x,y), \overline{B(0,1)}\right)$$
$$= \text{infimum}\left\{\sqrt{(x-x_1)^2 + (y-y_1)^2} \,:\, (x_1, y_1) \in \overline{B(0,1)}\right\}.$$

Then, $||\ \nabla f\ (3,4)\ ||$ is equal to ________________

Q.36 Let $f(x) = \left(\int_0^x e^{-t^2}\,dt\right)^2$ and $g(x) = \int_0^1 \frac{e^{-x^2(1+t^2)}}{1+t^2}\,dt$. Then $f'(\sqrt{\pi}) + g'(\sqrt{\pi})$ is equal to _______________________________

Q.37 Let $M = \begin{bmatrix} a & b & c \\ b & d & e \\ c & e & f \end{bmatrix}$ be a real matrix with eigenvalues 1, 0 and 3. If the eigenvectors corresponding to 1 and 0 are $(1,1,1)^T$ and $(1,-1,0)^T$ respectively, then the value of $3f$ is equal to ____________

Q.38 Let $M = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ and $e^M = Id + M + \frac{1}{2!}M^2 + \frac{1}{3!}M^3 + \cdots$. If $e^M = [b_{ij}]$, then

$$\frac{1}{e}\sum_{i=1}^{3}\sum_{j=1}^{3} b_{ij}$$

is equal to ________________

Q.39 Let the integral $I = \int_0^4 f(x)dx$, where $f(x) = \begin{cases} x & 0 \le x \le 2 \\ 4-x & 2 \le x \le 4. \end{cases}$

Consider the following statements P and Q:

(P) : If I_2 is the value of the integral obtained by the composite trapezoidal rule with two equal sub-intervals, then I_2 is exact.

(Q) : If I_3 is the value of the integral obtained by the composite trapezoidal rule with three equal sub-intervals, then I_3 is exact.

Which of the above statements hold TRUE?

(A) both P and Q (B) only P

(C) only Q (D) Neither P nor Q

Q.40 The difference between the least two eigenvalues of the boundary value problem

$$y'' + \lambda y = 0, \qquad 0 < x < \pi$$
$$y(0) = 0, \qquad y'(\pi) = 0,$$

is equal to ____________________

Q.41 The number of roots of the equation $x^2 - \cos(x) = 0$ in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ is equal to __________

Q.42 For the fixed point iteration $x_{k+1} = g(x_k)$, $k = 0, 1, 2, \ldots \ldots$, consider the following statements P and Q:

(P) : If $g(x) = 1 + \frac{2}{x}$ then the fixed point iteration converges to 2 for all $x_0 \in [1, 100]$.

(Q) : If $g(x) = \sqrt{2 + x}$ then the fixed point iteration converges to 2 for all $x_0 \in [0, 100]$.

Which of the above statements hold TRUE?

(A) both P and Q
(B) only P
(C) only Q
(D) Neither P nor Q

Q.43 Let $T : \ell_2 \rightarrow \ell_2$ be defined by

$$T\big((x_1, x_2, \cdots, x_n, \cdots)\big) = (x_2 - x_1, x_3 - x_2, \cdots, x_{n+1} - x_n, \cdots).$$

Then

(A) $\|T\| = 1$
(B) $\|T\| > 2$ but bounded
(C) $1 < \|T\| \leq 2$
(D) $\|T\|$ is unbounded

Q.44 Minimize $w = x + 2y$ subject to

$$\begin{aligned} 2x + y &\geq 3 \\ x + y &\geq 2 \\ x \geq 0, y &\geq 0. \end{aligned}$$

Then, the minimum value of w is equal to ______________________

Q.45 Maximize $w = 11x - z$ subject to

$$\begin{aligned} 10x + y - z &\leq 1 \\ 2x - 2y + z &\leq 2 \\ x, y, z &\geq 0. \end{aligned}$$

Then, the maximum value of w is equal to ______________________

Q.46 Let $X_1, X_2, X_3, \dots$ be a sequence of i.i.d. random variables with mean 1. If N is a geometric random variable with the probability mass function $P(N = k) = \frac{1}{2^k}$; $k = 1,2,3, \dots$ and it is independent of the X_i's, then $E(X_1 + X_2 + \cdots + X_N)$ is equal to ________________

Q.47 Let X_1 be an exponential random variable with mean 1 and X_2 a gamma random variable with mean 2 and variance 2. If X_1 and X_2 are independently distributed, then $P(X_1 < X_2)$ is equal to ____________________

Q.48 Let $X_1, X_2, X_3, \dots$ be a sequence of i.i.d. uniform (0,1) random variables. Then, the value of

$$\lim_{n\to\infty} P(-\ln(1 - X_1) - \cdots - \ln(1 - X_n) \geq n)$$

is equal to ________________

Q.49 Let X be a standard normal random variable. Then, $P\left(X < 0 \,\middle|\, |[X]| = 1\right)$ is equal to

(A) $\frac{\Phi(1)-\frac{1}{2}}{\Phi(2)-\frac{1}{2}}$

(B) $\frac{\Phi(1)+\frac{1}{2}}{\Phi(2)+\frac{1}{2}}$

(C) $\frac{\Phi(1)-\frac{1}{2}}{\Phi(2)+\frac{1}{2}}$

(D) $\frac{\Phi(1)+1}{\Phi(2)+1}$

Q.50 Let $X_1, X_2, X_3, \dots, X_n$ be a random sample from the probability density function

$$f(x) = \begin{cases} \theta\, \alpha\, e^{-\alpha x} + (1-\theta) 2\, \alpha\, e^{-2\alpha x}; & x \geq 0 \\ 0 & \text{otherwise,} \end{cases}$$

where $\alpha > 0$, $0 \leq \theta \leq 1$ are parameters. Consider the following testing problem: $H_0: \theta = 1, \alpha = 1$ versus $H_1: \theta = 0, \alpha = 2$.

Which of the following statements is TRUE?

(A) Uniformly Most Powerful test does NOT exist

(B) Uniformly Most Powerful test is of the form $\sum_{i=1}^{n} X_i > c$, for some $0 < c < \infty$

(C) Uniformly Most Powerful test is of the form $\sum_{i=1}^{n} X_i < c$, for some $0 < c < \infty$

(D) Uniformly Most Powerful test is of the form $c_1 < \sum_{i=1}^{n} X_i < c_2$, for some $0 < c_1 < c_2 < \infty$

Q.51 Let $X_1, X_2, X_3, \ldots$ be a sequence of i.i.d. $N(\mu, 1)$ random variables. Then,

$$\lim_{n\to\infty} \frac{\sqrt{\pi}}{2n} \sum_{i=1}^{n} E(|X_i - \mu|)$$

is equal to ______________________

Q.52 Let $X_1, X_2, X_3, \ldots, X_n$ be a random sample from uniform $[1, \theta]$, for some $\theta > 1$. If $X_{(n)} =$ Maximum $(X_1, X_2, X_3, \ldots, X_n)$, then the UMVUE of θ is

(A) $\frac{n+1}{n} X_{(n)} + \frac{1}{n}$

(B) $\frac{n+1}{n} X_{(n)} - \frac{1}{n}$

(C) $\frac{n}{n+1} X_{(n)} + \frac{1}{n}$

(D) $\frac{n}{n+1} X_{(n)} + \frac{n+1}{n}$

Q.53 Let $x_1 = x_2 = x_3 = 1$, $x_4 = x_5 = x_6 = 2$ be a random sample from a Poisson random variable with mean θ, where $\theta \in \{1, 2\}$. Then, the maximum likelihood estimator of θ is equal to ________________

Q.54 The remainder when 98! is divided by 101 is equal to ______________________

Q.55 Let G be a group whose presentation is

$$G = \{\, x, y \mid x^5 = y^2 = e, \qquad x^2 y = y\, x\}.$$

Then G is isomorphic to

(A) $\mathbb{Z}_5$

(B) $\mathbb{Z}_{10}$

(C) $\mathbb{Z}_2$

(D) $\mathbb{Z}_{30}$

END OF THE QUESTION PAPER

Q. No	Type	Section	Key	Marks
1	MCQ	GA	C	1
2	MCQ	GA	C	1
3	MCQ	GA	B	1
4	MCQ	GA	C ; D	1
5	MCQ	GA	A	1
6	MCQ	GA	D	2
7	MCQ	GA	A	2
8	MCQ	GA	C	2
9	MCQ	GA	B	2
10	MCQ	GA	A	2
1	MCQ	MA	C	1
2	MCQ	MA	C	1
3	MCQ	MA	B	1
4	MCQ	MA	B	1
5	MCQ	MA	D	1
6	NAT	MA	0.24 : 0.26	1
7	NAT	MA	1.39 : 1.43	1
8	MCQ	MA	A	1
9	NAT	MA	-0.1 : 0.1	1
10	NAT	MA	5.2 : 5.3	1
11	NAT	MA	0.9 : 1.1	1
12	NAT	MA	1.95 : 2.05	1
13	MCQ	MA	A	1
14	MCQ	MA	B	1
15	NAT	MA	0.22 : 0.28	1
16	MCQ	MA	B	1
17	MCQ	MA	B	1
18	NAT	MA	0.9 : 1.1	1
19	NAT	MA	0.9 : 1.1	1
20	MCQ	MA	B ; D	1
21	MCQ	MA	B	1
22	MCQ	MA	D	1
23	MCQ	MA	B	1
24	MCQ	MA	C	1
25	NAT	MA	-0.1 : 0.1	1
26	NAT	MA	0.039 : 0.043	2
27	NAT	MA	9.9 : 10.1	2
28	NAT	MA	15.9 : 16.1	2
29	MCQ	MA	A	2
30	NAT	MA	0.65 : 0.71	2
31	MCQ	MA	A	2
32	NAT	MA	1.9 : 2.1	2
33	NAT	MA	0.45 : 0.55	2
34	MCQ	MA	A	2
35	NAT	MA	0.9 : 1.1	2
36	NAT	MA	-0.1 : 0.1	2
37	NAT	MA	6.9 : 7.1	2
38	NAT	MA	5.4 : 5.6	2
39	MCQ	MA	B	2

40	NAT	MA	1.9 : 2.1	2
41	NAT	MA	1.9 : 2.1	2
42	MCQ	MA	A	2
43	MCQ	MA	C	2
44	NAT	MA	1.9 : 2.1	2
45	NAT	MA	1.2 : 1.3	2
46	NAT	MA	1.9 : 2.1	2
47	NAT	MA	0.7 : 0.8	2
48	NAT	MA	0.4 : 0.6	2
49	MCQ	MA	A	2
50	MCQ	MA	C	2
51	NAT	MA	0.69 : 0.73	2
52	MCQ	MA	B	2
53	NAT	MA	1.9 : 2.1	2
54	NAT	MA	49.9 : 50.1	2
55	MCQ	MA	C	2

List of Symbols, Notations and Data

$B(n,p)$: Binomial distribution with n trials and success probability p; $n \in \{1,2,\ldots\}$ and $p \in (0,1)$

$U(a,b)$: Uniform distribution on the interval $(a,b), -\infty < a < b < \infty$

$N(\mu,\sigma^2)$: Normal distribution with mean μ and variance σ^2, $\mu \in (-\infty,\infty)$, $\sigma > 0$

$P(A)$: Probability of the event A

Poisson(λ): Poisson distribution with mean $\lambda, \lambda > 0$

$E(X)$: Expected value (mean) of the random variable X

If $Z \sim N(0,1)$, then $P(Z \le 1.96) = 0.975$ and $P(Z \le 0.54) = 0.7054$

$\mathbb{Z}$: Set of integers

$\mathbb{Q}$: Set of rational numbers

$\mathbb{R}$: Set of real numbers

$\mathbb{C}$: Set of complex numbers

$\mathbb{Z}_n$: The cyclic group of order n

$\mathbb{F}[x]$: Polynomial ring over the field $\mathbb{F}$

$C[0,1]$: Set of all real valued continuous functions on the interval $[0,1]$

$C^1[0,1]$: Set of all real valued continuously differentiable functions on the interval $[0,1]$

ℓ_2 : Normed space of all square-summable real sequences

$L^2[0,1]$: Space of all square-Lebesgue integrable real valued functions on the interval $[0,1]$

$(C[0,1], \|\ \|_2)$: The space $C[0,1]$ with $\|f\|_2 = \left(\int_0^1 |f(x)|^2\,dx\right)^{\frac{1}{2}}$

$(C[0,1], \|\ \|_\infty)$: The space $C[0,1]$ with $\|f\|_\infty = \sup\{|f(x)| : x \in [0,1]\}$

$V^\perp$: The orthogonal complement of V in an inner product space

$\mathbb{R}^n$: n-dimensional Euclidean space

Usual metric d on $\mathbb{R}^n$ is given by $d\big((x_1,x_2,\ldots,x_n),(y_1,y_2,\ldots,y_n)\big) = \left(\sum_{i=1}^n (x_i - y_i)^2\right)^{1/2}$

I_n: The $n \times n$ identity matrix (I : the identity matrix when order is NOT specified)

$o(g)$: The order of the element g of a group

Q. 1 – Q. 25 carry one mark each.

Q.1 Let $T : \mathbb{R}^4 \to \mathbb{R}^4$ be a linear map defined by

$$T(x, y, z, w) = (x + z, 2x + y + 3z, 2y + 2z, w).$$

Then the rank of T is equal to _________

Q.2 Let M be a 3×3 matrix and suppose that $1, 2$ and 3 are the eigenvalues of M. If

$$M^{-1} = \frac{M^2}{\alpha} - M + \frac{11}{\alpha} I_3$$

for some scalar $\alpha \neq 0$, then α is equal to ___________

Q.3 Let M be a 3×3 singular matrix and suppose that 2 and 3 are eigenvalues of M. Then the number of linearly independent eigenvectors of $M^3 + 2M + I_3$ is equal to __________

Q.4 Let M be a 3×3 matrix such that $M \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 6 \\ -3 \\ 0 \end{pmatrix}$ and suppose that $M^3 \begin{pmatrix} 1 \\ -1/2 \\ 0 \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}$ for some $\alpha, \beta, \gamma \in \mathbb{R}$. Then $|\,\alpha\,|$ is equal to _______

Q.5 Let $f : [0, \infty) \to \mathbb{R}$ be defined by

$$f(x) = \int_0^x \sin^2(t^2)\, dt.$$

Then the function f is

(A) uniformly continuous on $[0, 1)$ but NOT on $(0, \infty)$
(B) uniformly continuous on $(0, \infty)$ but NOT on $[0, 1)$
(C) uniformly continuous on both $[0, 1)$ and $(0, \infty)$
(D) neither uniformly continuous on $[0, 1)$ nor uniformly continuous on $(0, \infty)$

Q.6 Consider the power series $\sum_{n=0}^{\infty} a_n z^n$, where $a_n = \begin{cases} \frac{1}{3^n} & \text{if } n \text{ is even} \\ \frac{1}{5^n} & \text{if } n \text{ is odd.} \end{cases}$

The radius of convergence of the series is equal to __________

Q.7 Let $C = \{ z \in \mathbb{C} : |z - i| = 2\}$. Then $\frac{1}{2\pi} \oint_C \frac{z^2 - 4}{z^2 + 4}\, dz$ is equal to ____________

Q.8 Let $X \sim B(5, \frac{1}{2})$ and $Y \sim U(0,1)$. Then $\frac{P(X+Y \leq 2)}{P(X+Y \geq 5)}$ is equal to ___________

Q.9 Let the random variable X have the distribution function

$$F(x) = \begin{cases} 0 & \text{if } \quad x < 0 \\ \frac{x}{2} & \text{if } 0 \le x < 1 \\ \frac{3}{5} & \text{if } 1 \le x < 2 \\ \frac{1}{2} + \frac{x}{8} & \text{if } 2 \le x < 3 \\ 1 & \text{if } \quad x \ge 3. \end{cases}$$

Then $P(2 \le X < 4)$ is equal to ___________

Q.10 Let X be a random variable having the distribution function

$$F(x) = \begin{cases} 0 & \text{if } \quad x < 0 \\ \frac{1}{4} & \text{if } 0 \le x < 1 \\ \frac{1}{3} & \text{if } 1 \le x < 2 \\ \frac{1}{2} & \text{if } 2 \le x < \frac{11}{3} \\ 1 & \text{if } \quad x \ge \frac{11}{3}. \end{cases}$$

Then $E(X)$ is equal to _________

Q.11 In an experiment, a fair die is rolled until two sixes are obtained in succession. The probability that the experiment will end in the fifth trial is equal to

(A) $\frac{125}{6^5}$ (B) $\frac{150}{6^5}$ (C) $\frac{175}{6^5}$ (D) $\frac{200}{6^5}$

Q.12 Let $x_1 = 2.2,\ x_2 = 4.3,\ x_3 = 3.1,\ x_4 = 4.5,\ x_5 = 1.1$ and $x_6 = 5.7$ be the observed values of a random sample of size 6 from a $U(\theta - 1,\ \theta + 4)$ distribution, where $\theta \in (0, \infty)$ is unknown. Then a maximum likelihood estimate of θ is equal to

(A) 1.8 (B) 2.3 (C) 3.1 (D) 3.6

Q.13 Let $\Omega = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 < 1\}$ be the open unit disc in $\mathbb{R}^2$ with boundary $\partial\Omega$. If $u(x, y)$ is the solution of the Dirichlet problem

$$u_{xx} + u_{yy} = 0 \qquad \text{in } \Omega$$
$$u(x, y) = 1 - 2y^2 \qquad \text{on } \partial\Omega,$$

then $u\left(\frac{1}{2}, 0\right)$ is equal to

(A) -1 (B) $\frac{-1}{4}$ (C) $\frac{1}{4}$ (D) 1

Q.14 Let $c \in \mathbb{Z}_3$ be such that $\frac{\mathbb{Z}_3[X]}{\langle X^3 + c\,X+1 \rangle}$ is a field. Then c is equal to __________

Q.15 Let $V = C^1[0,1]$, $X = (C[0,1], \|\ \ \|_\infty)$ and $Y = (C[0,1], \|\ \ \|_2)$. Then V is

(A) dense in X but NOT in Y
(B) dense in Y but NOT in X
(C) dense in both X and Y
(D) neither dense in X nor dense in Y

Q.16 Let $T : (C[0,1], \|\ \ \|_\infty) \to \mathbb{R}$ be defined by $T(f) = \int_0^1 2xf(x)\,dx$ for all $f \in C[0,1]$. Then $\|T\|$ is equal to __________

Q.17 Let τ_1 be the usual topology on $\mathbb{R}$. Let τ_2 be the topology on $\mathbb{R}$ generated by $\mathcal{B} = \{[a,b) \subset \mathbb{R} : -\infty < a < b < \infty\}$. Then the set $\{x \in \mathbb{R} : 4 \sin^2 x \le 1\} \cup \left\{\frac{\pi}{2}\right\}$ is

(A) closed in $(\mathbb{R}, \tau_1)$ but NOT in $(\mathbb{R}, \tau_2)$
(B) closed in $(\mathbb{R}, \tau_2)$ but NOT in $(\mathbb{R}, \tau_1)$
(C) closed in both $(\mathbb{R}, \tau_1)$ and $(\mathbb{R}, \tau_2)$
(D) neither closed in $(\mathbb{R}, \tau_1)$ nor closed in $(\mathbb{R}, \tau_2)$

Q.18 Let X be a connected topological space such that there exists a non-constant continuous function $f : X \to \mathbb{R}$, where $\mathbb{R}$ is equipped with the usual topology. Let $f(X) = \{f(x) : x \in X\}$. Then

(A) X is countable but $f(X)$ is uncountable
(B) $f(X)$ is countable but X is uncountable
(C) both $f(X)$ and X are countable
(D) both $f(X)$ and X are uncountable

Q.19 Let d_1 and d_2 denote the usual metric and the discrete metric on $\mathbb{R}$, respectively. Let $f : (\mathbb{R}, d_1) \to (\mathbb{R}, d_2)$ be defined by $f(x) = x$, $x \in \mathbb{R}$. Then

(A) f is continuous but f^{-1} is NOT continuous
(B) f^{-1} is continuous but f is NOT continuous
(C) both f and f^{-1} are continuous
(D) neither f nor f^{-1} is continuous

Q.20 If the trapezoidal rule with single interval $[0,1]$ is exact for approximating the integral $\int_0^1 (x^3 - c\,x^2)dx$, then the value of c is equal to ________

Q.21 Suppose that the Newton-Raphson method is applied to the equation $2x^2 + 1 - e^{x^2} = 0$ with an initial approximation x_0 sufficiently close to zero. Then, for the root $x = 0$, the order of convergence of the method is equal to _________

Q.22 The minimum possible order of a homogeneous linear ordinary differential equation with real constant coefficients having $x^2 \sin(x)$ as a solution is equal to __________

Q.23 The Lagrangian of a system in terms of polar coordinates (r, θ) is given by

$$L = \frac{1}{2} m \dot{r}^2 + \frac{1}{2} m \left(\dot{r}^2 + r^2\dot{\theta}^2\right) - m\, g\, r\, (1 - \cos(\theta)),$$

where m is the mass, g is the acceleration due to gravity and $\dot{s}$ denotes the derivative of s with respect to time. Then the equations of motion are

(A) $2\,\ddot{r} = r\,\dot{\theta}^2 - g\,(1 - \cos(\theta)),\ \frac{d}{dt}(r^2\dot{\theta}) = -g\,r\,\sin(\theta)$

(B) $2\,\ddot{r} = r\,\dot{\theta}^2 + g\,(1 - \cos(\theta)),\ \frac{d}{dt}(r^2\dot{\theta}) = -g\,r\,\sin(\theta)$

(C) $2\,\ddot{r} = r\,\dot{\theta}^2 - g\,(1 - \cos(\theta)),\ \frac{d}{dt}(r^2\dot{\theta}) = g\,r\,\sin(\theta)$

(D) $2\,\ddot{r} = r\,\dot{\theta}^2 + g\,(1 - \cos(\theta)),\ \frac{d}{dt}(r^2\dot{\theta}) = g\,r\,\sin(\theta)$

Q.24 If $y(x)$ satisfies the initial value problem

$$(x^2 + y)dx = x\,dy, \qquad y(1) = 2,$$

then $y(2)$ is equal to __________

Q.25 It is known that Bessel functions $J_n(x)$, for $n \geq 0$, satisfy the identity

$$e^{\frac{x}{2}\left(t-\frac{1}{t}\right)} = J_0(x) + \sum_{n=1}^{\infty} J_n(x)\left(t^n + \frac{(-1)^n}{t^n}\right)$$

for all $t > 0$ and $x \in \mathbb{R}$. The value of $J_0\left(\frac{\pi}{3}\right) + 2\sum_{n=1}^{\infty} J_{2n}\left(\frac{\pi}{3}\right)$ is equal to __________

Q. 26 – Q. 55 carry two marks each.

Q.26 Let X and Y be two random variables having the joint probability density function

$$f(x,y) = \begin{cases} 2 & \text{if } 0 < x < y < 1 \\ 0 & \text{otherwise.} \end{cases}$$

Then the conditional probability $P\left(X \leq \frac{2}{3} \mid Y = \frac{3}{4}\right)$ is equal to

(A) $\frac{5}{9}$ (B) $\frac{2}{3}$ (C) $\frac{7}{9}$ (D) $\frac{8}{9}$

Q.27 Let $\Omega = (0,1]$ be the sample space and let $P(\cdot)$ be a probability function defined by

$$P((0,x]) = \begin{cases} \frac{x}{2} & \text{if } 0 \leq x < \frac{1}{2} \\ x & \text{if } \frac{1}{2} \leq x \leq 1. \end{cases}$$

Then $P\left(\left\{\frac{1}{2}\right\}\right)$ is equal to __________

Q.28 Let X_1, X_2 and X_3 be independent and identically distributed random variables with $E(X_1) = 0$ and $E(X_1^2) = \frac{15}{4}$. If $\psi : (0,\infty) \to (0,\infty)$ is defined through the conditional expectation

$$\psi(t) = E(X_1^2 \mid X_1^2 + X_2^2 + X_3^2 = t), \; t > 0 ,$$

then $E(\psi((X_1 + X_2)^2))$ is equal to __________

Q.29 Let $X \sim \text{Poisson}(\lambda)$, where $\lambda > 0$ is unknown. If $\delta(X)$ is the unbiased estimator of $g(\lambda) = e^{-\lambda}(3\lambda^2 + 2\lambda + 1)$, then $\sum_{k=0}^{\infty} \delta(k)$ is equal to ___________

Q.30 Let $X_1, \dots, X_n$ be a random sample from $N(\mu, 1)$ distribution, where $\mu \in \{0, \frac{1}{2}\}$. For testing the null hypothesis $H_0: \mu = 0$ against the alternative hypothesis $H_1: \mu = \frac{1}{2}$, consider the critical region

$$R = \left\{(x_1, x_2, \dots, x_n) : \sum_{i=1}^{n} x_i > c\right\},$$

where c is some real constant. If the critical region R has size 0.025 and power 0.7054, then the value of the sample size n is equal to ___________

Q.31 Let X and Y be independently distributed central chi-squared random variables with degrees of freedom $m\ (\geq 3)$ and $n\ (\geq 3)$, respectively. If $E\left(\frac{X}{Y}\right) = 3$ and $m + n = 14$, then $E\left(\frac{Y}{X}\right)$ is equal to

(A) $\frac{2}{7}$ (B) $\frac{3}{7}$ (C) $\frac{4}{7}$ (D) $\frac{5}{7}$

Q.32 Let $X_1, X_2, \ldots$ be a sequence of independent and identically distributed random variables with $P(X_1 = 1) = \frac{1}{4}$ and $P(X_1 = 2) = \frac{3}{4}$. If $\overline{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$, for $n = 1, 2, \ldots$, then $\lim_{n\to\infty} P(\overline{X}_n \leq 1.8)$ is equal to __________

Q.33 Let $u(x, y) = 2f(y)\cos(x - 2y)$, $(x, y) \in \mathbb{R}^2$, be a solution of the initial value problem

$$2u_x + u_y = u$$
$$u(x, 0) = \cos(x).$$

Then $f(1)$ is equal to

(A) $\frac{1}{2}$ (B) $\frac{e}{2}$ (C) e (D) $\frac{3e}{2}$

Q.34 Let $u(x,t)$, $x \in \mathbb{R}$, $t \geq 0$, be the solution of the initial value problem

$$u_{tt} = u_{xx}$$
$$u(x, 0) = x$$
$$u_t(x, 0) = 1.$$

Then $u(2,2)$ is equal to ________

Q.35 Let $W = \text{Span}\left\{\frac{1}{\sqrt{2}}(0,0,1,1),\ \frac{1}{\sqrt{2}}(1,-1,0,0)\right\}$ be a subspace of the Euclidean space $\mathbb{R}^4$. Then the square of the distance from the point $(1,1,1,1)$ to the subspace W is equal to ________

Q.36 Let $T : \mathbb{R}^4 \to \mathbb{R}^4$ be a linear map such that the null space of T is

$$\{(x,y,z,w) \in \mathbb{R}^4 : x+y+z+w=0\}$$

and the rank of $(T - 4\,I_4)$ is 3. If the minimal polynomial of T is $x(x-4)^{\alpha}$, then α is equal to _______

Q.37 Let M be an invertible Hermitian matrix and let $x, y \in \mathbb{R}$ be such that $x^2 < 4y$. Then

(A) both $M^2 + x\,M + y\,I$ and $M^2 - x\,M + y\,I$ are singular
(B) $M^2 + x\,M + y\,I$ is singular but $M^2 - x\,M + y\,I$ is non-singular
(C) $M^2 + x\,M + y\,I$ is non-singular but $M^2 - x\,M + y\,I$ is singular
(D) both $M^2 + x\,M + y\,I$ and $M^2 - x\,M + y\,I$ are non-singular

Q.38 Let $G = \{e, x, x^2, x^3, y, xy, x^2y, x^3y\}$ with $o(x) = 4, o(y) = 2$ and $xy = yx^3$. Then the number of elements in the center of the group G is equal to

(A) 1 (B) 2 (C) 4 (D) 8

Q.39 The number of ring homomorphisms from $\mathbb{Z}_2 \times \mathbb{Z}_2$ to $\mathbb{Z}_4$ is equal to __________

Q.40 Let $p(x) = 9\,x^5 + 10\,x^3 + 5\,x + 15$ and $q(x) = x^3 - x^2 - x - 2$ be two polynomials in $\mathbb{Q}\,[x]$. Then, over $\mathbb{Q}$,

(A) $p(x)$ and $q(x)$ are both irreducible
(B) $p(x)$ is reducible but $q(x)$ is irreducible
(C) $p(x)$ is irreducible but $q(x)$ is reducible
(D) $p(x)$ and $q(x)$ are both reducible

Q.41 Consider the linear programming problem

$$\begin{aligned} &\text{Maximize } 3\,x + 9\,y, \\ &\text{subject to} \quad 2\,y - x \le 2 \\ &\qquad\qquad\quad 3\,y - x \ge 0 \\ &\qquad\qquad\quad 2\,x + 3\,y \le 10 \\ &\qquad\qquad\quad x, y \ge 0. \end{aligned}$$

Then the maximum value of the objective function is equal to ______

Q.42 Let $S = \{\,(x, \sin\frac{1}{x}) : 0 < x \le 1\,\}$ and $T = S \cup \{(0,0)\}$. Under the usual metric on $\mathbb{R}^2$,

(A) S is closed but T is NOT closed
(B) T is closed but S is NOT closed
(C) both S and T are closed
(D) neither S nor T is closed

Q.43 Let $H = \left\{ (x_n) \in \ell_2 : \sum_{n=1}^{\infty} \frac{x_n}{n} = 1 \right\}$. Then H

(A) is bounded
(B) is closed
(C) is a subspace
(D) has an interior point

Q.44 Let V be a closed subspace of $L^2[0,1]$ and let $f, g \in L^2[0,1]$ be given by $f(x) = x$ and $g(x) = x^2$. If $V^{\perp} = \text{Span}\{f\}$ and Pg is the orthogonal projection of g on V, then $(g - Pg)(x),\ x \in [0,1]$, is

(A) $\frac{3}{4}x$
(B) $\frac{1}{4}x$
(C) $\frac{3}{4}x^2$
(D) $\frac{1}{4}x^2$

Q.45 Let $p(x)$ be the polynomial of degree at most 3 that passes through the points $(-2,12), (-1,1)$, $(0,2)$ and $(2,-8)$. Then the coefficient of x^3 in $p(x)$ is equal to _________

Q.46 If, for some $\alpha, \beta \in \mathbb{R}$, the integration formula

$$\int_0^2 p(x)dx = p(\alpha) + p(\beta)$$

holds for all polynomials $p(x)$ of degree at most 3, then the value of $3(\alpha - \beta)^2$ is equal to _____

Q.47 Let $y(t)$ be a continuous function on $[0, \infty)$ whose Laplace transform exists. If $y(t)$ satisfies

$$\int_0^t (1 - \cos(t - \tau))\, y(\tau)\, d\tau = t^4,$$

then $y(1)$ is equal to _______

Q.48 Consider the initial value problem

$$x^2 y'' - 6y = 0, \qquad y(1) = \alpha,\ y'(1) = 6.$$

If $y(x) \to 0$ as $x \to 0^+$, then α is equal to __________

Q.49 Define $f_1, f_2 : [0,1] \to \mathbb{R}$ by

$$f_1(x) = \sum_{n=1}^{\infty} \frac{x \sin(n^2 x)}{n^2} \quad \text{and} \quad f_2(x) = \sum_{n=1}^{\infty} x^2(1 - x^2)^{n-1}.$$

Then

(A) f_1 is continuous but f_2 is NOT continuous
(B) f_2 is continuous but f_1 is NOT continuous
(C) both f_1 and f_2 are continuous
(D) neither f_1 nor f_2 is continuous

Q.50 Consider the unit sphere $S = \{(x,y,z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 1\}$ and the unit normal vector $\hat{n} = (x,y,z)$ at each point (x,y,z) on S. The value of the surface integral

$$\iint_S \left\{ \left(\frac{2x}{\pi} + \sin(y^2)\right) x + \left(e^z - \frac{y}{\pi}\right) y + \left(\frac{2z}{\pi} + \sin^2 y\right) z \right\} d\sigma$$

is equal to _______

Q.51 Let $D = \{(x, y) \in \mathbb{R}^2 : 1 \le x \le 1000,\ 1 \le y \le 1000\}$. Define

$$f(x, y) = \frac{x\,y}{2} + \frac{500}{x} + \frac{500}{y}.$$

Then the minimum value of f on D is equal to ________

Q.52 Let $\mathbb{D} = \{ z \in \mathbb{C} : |z| < 1 \}$. Then there exists a non-constant analytic function f on $\mathbb{D}$ such that for all $n = 2, 3, 4, \ldots$

(A) $f\left(\frac{\sqrt{-1}}{n}\right) = 0$

(B) $f\left(\frac{1}{n}\right) = 0$

(C) $f\left(1 - \frac{1}{n}\right) = 0$

(D) $f\left(\frac{1}{2} - \frac{1}{n}\right) = 0$

Q.53 Let $\sum_{n=-\infty}^{\infty} a_n z^n$ be the Laurent series expansion of $f(z) = \frac{1}{2\,z^2 - 13\,z + 15}$ in the annulus $\frac{3}{2} < |z| < 5$. Then $\frac{a_1}{a_2}$ is equal to _________

Q.54 The value of $\frac{i}{4-\pi} \int_{|z|=4} \frac{dz}{z\,\cos(z)}$ is equal to __________

Q.55 Suppose that among all continuously differentiable functions $y(x)$, $x \in \mathbb{R}$, with $y(0) = 0$ and $y(1) = \frac{1}{2}$, the function $y_0(x)$ minimizes the functional

$$\int_0^1 \left(e^{-(y'-x)} + (1+y)y'\right)dx.$$

Then $y_0\left(\frac{1}{2}\right)$ is equal to

(A) 0

(B) $\frac{1}{8}$

(C) $\frac{1}{4}$

(D) $\frac{1}{2}$

END OF THE QUESTION PAPER

GATE 2014: General Instructions during Examination

1. Total duration of the GATE examination is **180** minutes.
2. The clock will be set at the server. The countdown timer at the top right corner of screen will display the remaining time available for you to complete the examination. When the timer reaches zero, the examination will end by itself. You need not terminate the examination or submit your paper.
3. Any useful data required for your paper can be viewed by clicking on the **Useful Common Data** button that appears on the screen.
4. Use the scribble pad provided to you for any rough work. Submit the scribble pad at the end of the examination.
5. You are allowed to use a non-programmable type calculator, however, sharing of calculators is not allowed.
6. The Question Palette displayed on the right side of screen will show the status of each question using one of the following symbols:

You have not visited the question yet.

You have not answered the question.

You have answered the question.

You have NOT answered the question, but have marked the question for review.

You have answered the question, but marked it for review.

The **Marked for Review** status for a question simply indicates that you would like to look at that question again. *If a question is answered, but marked for review, then the answer will be considered for evaluation unless the status is modified by the candidate.*

<u>Navigating to a Question :</u>

7. To answer a question, do the following:
    a. Click on the question number in the Question Palette to go to that question directly.
    b. Select an answer for a multiple choice type question by clicking on the bubble placed before the 4 choices, namely A, B, C and D. Use the virtual numeric keypad to enter a number as answer for a numerical type question.
    c. Click on **Save & Next** to save your answer for the current question and then go to the next question.
    d. Click on **Mark for Review & Next** to save your answer for the current question and also mark it for review, and then go to the next question.

Caution: Note that your answer for the current question will not be saved, if you navigate to another question directly by clicking on a question number without saving the answer to the previous question.

You can view all the questions by clicking on the **Question Paper** button. This feature is provided, so that if you want you can just see the entire question paper at a glance.

Answering a Question :

8. Procedure for answering a multiple choice (MCQ) type question:
    a. Choose one answer from the 4 options (A,B,C,D) given below the question, click on the bubble placed before the chosen option.
    b. To deselect your chosen answer, click on the bubble of the chosen option again or click on the **Clear Response** button.
    c. To change your chosen answer, click on the bubble of another option.
    d. To save your answer, you MUST click on the **Save & Next** button.

9. Procedure for answering a numerical answer type question:
    a. To enter a number as your answer, use the virtual numerical keypad.
    b. A fraction (e.g. -0.3 or -.3) can be entered as an answer with or without '0' before the decimal point. As many as four decimal points, e.g. 12.5435 or 0.003 or -932.6711 or 12.82 can be entered.
    c. To clear your answer, click on the **Clear Response** button.
    d. To save your answer, you MUST click on the **Save & Next** button
10. To mark a question for review, click on the **Mark for Review & Next** button. *If an answer is selected (for MCQ) or entered (for numerical answer type) for a question that is* **Marked for Review***, that answer will be considered in the evaluation unless the status is modified by the candidate.*

11. To change your answer to a question that has already been answered, first select that question for answering and then follow the procedure for answering that type of question.
12. Note that ONLY Questions for which answers are ***saved*** or ***marked for review after answering*** will be considered for evaluation.

Choosing a Section :

13. Sections in this question paper are displayed on the top bar of the screen. Questions in a Section can be viewed by clicking on the name of that Section. The Section you are currently viewing will be highlighted.
14. A checkbox is displayed for every optional Section, if any, in the Question Paper. To select the optional Section for answering, click on the checkbox for that Section.
15. If the checkbox for an optional Section is not selected, the **Save & Next** button and the **Mark for Review & Next** button will NOT be enabled for that Section. You will

only be able to see questions in this Section, but you will not be able to answer questions in the Section.

16. After clicking the **Save & Next** button for the last question in a Section, you will automatically be taken to the first question of the next Section in sequence.
17. You can move the mouse cursor over the name of a Section to view the answering status for that Section.

Changing the Optional Section :

18. After answering the chosen optional Section, partially or completely, you can change the optional Section by selecting the checkbox for a new Section that you want to attempt. A warning message will appear along with a table showing the number of questions answered in each of the previously chosen optional Sections and a checkbox against each of these Sections. Click on a checkbox against a Section that you want to reset and then click on the **RESET** button. Note that RESETTING a Section will DELETE all the answers for questions in that Section. Hence, if you think that you may want to select this Section again later, you will have to note down your answers for questions in that Section. If you do not want to reset the Section and want to continue answering the previously chosen optional Section, then click on the **BACK** button.

19. If you deselect the checkbox for an optional Section in the top bar, the following warning message will appear: "Deselecting the checkbox will DELETE all the answers for questions in this Section. Do you want to deselect this Section?" If you want to deselect, click on the **RESET** button. If you do not want to deselect, click on the **BACK** button.

20. You can shuffle between different Sections or change the optional Sections any number of times.

GATE 2014 Examination

MA: Mathematics

Duration: **180 minutes** *Maximum Marks*: **100**

Read the following instructions carefully.

1. To login, enter your Registration Number and password provided to you. Kindly go through the various symbols used in the test and understand their meaning before you start the examination.
2. Once you login and after the start of the examination, you can view all the questions in the question paper, by clicking on the **View All Questions** button in the screen.
3. This question paper consists of **2 sections**, General Aptitude (GA) for **15 marks** and the subject specific GATE paper for **85 marks**. Both these sections are compulsory.
   The GA section consists of **10** questions. Question numbers 1 to 5 are of 1-mark each, while question numbers 6 to 10 are of 2-mark each.
   The subject specific GATE paper section consists of **55** questions, out of which question numbers 1 to 25 are of 1-mark each, while question numbers 26 to 55 are of 2-mark each.
4. Depending upon the GATE paper, there may be useful common data that may be required for answering the questions. If the paper has such useful data, the same can be viewed by clicking on the **Useful Common Data** button that appears at the top, right hand side of the screen.
5. The computer allotted to you at the examination center runs specialized software that permits only one answer to be selected for multiple-choice questions using a mouse and to enter a suitable number for the numerical answer type questions using the virtual keyboard and mouse.
6. Your answers shall be updated and saved on a server periodically and also at the end of the examination. The examination will **stop automatically** at the end of **180 minutes**.
7. In each paper a candidate can answer a total of 65 questions carrying 100 marks.
8. The question paper may consist of questions of **multiple choice type** (MCQ) and **numerical answer type**.
9. Multiple choice type questions will have four choices against A, B, C, D, out of which only **ONE** is the correct answer. The candidate has to choose the correct answer by clicking on the bubble (○) placed before the choice.
10. For numerical answer type questions, each question will have a numerical answer and there will not be any choices**. For these questions, the answer should be entered**by using the virtual keyboard that appears on the monitor and the mouse.
11. All questions that are not attempted will result in zero marks. However, wrong answers for multiple choice type questions (MCQ) will result in **NEGATIVE** marks. For all MCQ questions a wrong answer will result in deduction of⅓ marks for a 1-mark question and ⅔ marks for a 2-mark question.
12. There is **NO NEGATIVE MARKING** for questions of **NUMERICAL ANSWER TYPE**.
13. Non-programmable type Calculator is allowed. Charts, graph sheets, and mathematical tables are **NOT** allowed in the Examination Hall. You must use the Scribble pad provided to you at the examination centre for all your rough work. The Scribble Pad has to be returned at the end of the examination.

Declaration by the candidate:

“I have read and understood all the above instructions. I have also read and understood clearly the instructions given on the admit card and shall follow the same. I also understand that in case I am found to violate any of these instructions, my candidature is liable to be cancelled. I also confirm that at the start of the examination all the computer hardware allotted to me are in proper working condition”.

Q. 1 – Q. 5 carry one mark each.

Q.1 A student is required to demonstrate a high level of comprehension of the subject, especially in the social sciences.

The word closest in meaning to comprehension is

(A) understanding (B) meaning (C) concentration (D) stability

Q.2 Choose the most appropriate word from the options given below to complete the following sentence.

One of his biggest ______ was his ability to forgive.

(A) vice (B) virtues (C) choices (D) strength

Q.3 Rajan was not happy that Sajan decided to do the project on his own. On observing his unhappiness, Sajan explained to Rajan that he preferred to work independently.

Which one of the statements below is logically valid and can be inferred from the above sentences?

(A) Rajan has decided to work only in a group.
(B) Rajan and Sajan were formed into a group against their wishes.
(C) Sajan had decided to give in to Rajan's request to work with him.
(D) Rajan had believed that Sajan and he would be working together.

Q.4 If $y = 5x^2 + 3$, then the tangent at $x = 0, y = 3$

(A) passes through $x = 0, y = 0$ (B) has a slope of +1
(C) is parallel to the x-axis (D) has a slope of −1

Q.5 A foundry has a fixed daily cost of Rs 50,000 whenever it operates and a variable cost of Rs 800Q, where Q is the daily production in tonnes. What is the cost of production in Rs per tonne for a daily production of 100 tonnes?

Q. 6 – Q. 10 carry two marks each.

Q.6 Find the odd one in the following group: ALRVX, EPVZB, ITZDF, OYEIK

(A) ALRVX (B) EPVZB (C) ITZDF (D) OYEIK

Q.7 Anuj, Bhola, Chandan, Dilip, Eswar and Faisal live on different floors in a six-storeyed building (the ground floor is numbered 1, the floor above it 2, and so on). Anuj lives on an even-numbered floor. Bhola does not live on an odd numbered floor. Chandan does not live on any of the floors below Faisal's floor. Dilip does not live on floor number 2. Eswar does not live on a floor immediately above or immediately below Bhola. Faisal lives three floors above Dilip. Which of the following floor-person combinations is correct?

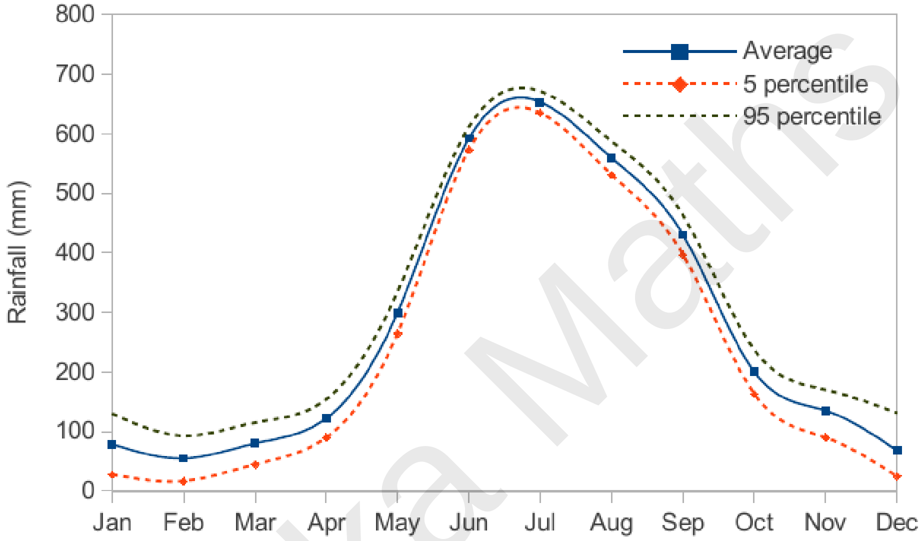
	Anuj	Bhola	Chandan	Dilip	Eswar	Faisal
(A)	6	2	5	1	3	4
(B)	2	6	5	1	3	4
(C)	4	2	6	3	1	5
(D)	2	4	6	1	3	5

Q.8 The smallest angle of a triangle is equal to two thirds of the smallest angle of a quadrilateral. The ratio between the angles of the quadrilateral is 3:4:5:6. The largest angle of the triangle is twice its smallest angle. What is the sum, in degrees, of the second largest angle of the triangle and the largest angle of the quadrilateral?

Q.9 One percent of the people of country X are taller than 6 ft. Two percent of the people of country Y are taller than 6 ft. There are thrice as many people in country X as in country Y. Taking both countries together, what is the percentage of people taller than 6 ft?

(A) 3.0 (B) 2.5 (C) 1.5 (D) 1.25

Q.10 The monthly rainfall chart based on 50 years of rainfall in Agra is shown in the following figure. Which of the following are true? (k percentile is the value such that k percent of the data fall below that value)



(i) On average, it rains more in July than in December
(ii) Every year, the amount of rainfall in August is more than that in January
(iii) July rainfall can be estimated with better confidence than February rainfall
(iv) In August, there is at least 500 mm of rainfall

(A) (i) and (ii) (B) (i) and (iii)
(C) (ii) and (iii) (D) (iii) and (iv)

END OF THE QUESTION PAPER

Symbols and Notation Used

$\mathbb{R}$	The set of all real numbers
$\mathbb{C}$	The set of all complex numbers
$\mathbb{Q}$	The set of all rational numbers
$\mathbb{N}$	The set of all positive integers
$\mathbb{Z}$	The set of all integers
$\mathbb{Z}_n$	The cyclic group of order n
$\text{Re}(z)$	Real part of a complex number z
$\text{Im}(z)$	Imaginary part of a complex number z
P(E)	Probability of an event E
$E[X]$	Expectation of a random variable X
$\text{Var}(X)$	Variance of a random variable X
$M_n(\mathbb{R})$	The set of all $n \times n$ matrices with real entries
$GL_2(\mathbb{R})$	The set of all 2×2 matrices with determinant 1
$C[a, b]$	The set of all real valued continuous functions on the real interval $[a, b]$
$C^n[a, b]$	The set of all real valued n-times continuously differentiable functions on the real interval $[a, b]$
ℓ^p	The space of all p-summable sequences
$L^p[a, b]$	The space of all p-integrable functions on the interval $[a, b]$
$\langle \cdot\, , \cdot \rangle$	An inner product
$\dot{\theta}$	Derivative of θ with respect to t

Q. 1 – Q. 25 carry one mark each.

Q.1 The function $f(z) = |z|^2 + i\,\bar{z} + 1$ is differentiable at

(A) i (B) 1 (C) $-i$ (D) no point in $\mathbb{C}$

Q.2 The radius of convergence of the power series $\sum_{n=0}^{\infty} 4^{(-1)^n n} z^{2n}$ is ____________

Q.3 Let E_1 and E_2 be two non empty subsets of a normed linear space X and let $E_1 + E_2 := \{x + y \in X : x \in E_1 \text{ and } y \in E_2\}$. Then which of the following statements is **FALSE**:

(A) If E_1 and E_2 are convex, then $E_1 + E_2$ is convex

(B) If E_1 or E_2 is open, then $E_1 + E_2$ is open

(C) $E_1 + E_2$ must be closed if E_1 and E_2 are closed

(D) If E_1 is closed and E_2 is compact, then $E_1 + E_2$ is closed

Q.4 Let $y(x)$ be the solution to the initial value problem $\frac{dy}{dx} = \sqrt{y + 2x}$ subject to $y(1.2) = 2.$ Using the Euler method with the step size $h = 0.05$, the approximate value of $y(1.3)$, correct to two decimal places, is ____________________

Q.5 Let $\alpha \in \mathbb{R}$. If αx is the polynomial which interpolates the function $f(x) = \sin \pi x$ on $[-1,1]$ at all the zeroes of the polynomial $4x^3 - 3x$, then α is ___________

Q.6 If $u(x,t)$ is the D'Alembert's solution to the wave equation $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$, $x \in \mathbb{R},\ t > 0,$ with the condition $u(x,0) = 0$ and $\frac{\partial u}{\partial t}(x,0) = \cos x$, then $u\left(0, \frac{\pi}{4}\right)$ is _______________________

Q.7 The solution to the integral equation $\varphi(x) = x + \int_0^x \sin(x - \xi)\varphi(\xi)\,d\xi$ is

(A) $x^2 + \frac{x^3}{3}$ (B) $x - \frac{x^3}{3!}$ (C) $x + \frac{x^3}{3!}$ (D) $x^2 - \frac{x^3}{3!}$

Q.8 The general solution to the ordinary differential equation $x^2\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}+\left(4x^2-\dfrac{9}{25}\right)y=0$ in terms of Bessel's functions, $J_\nu(x)$, is

(A) $y(x)=c_1J_{3/5}(2x)+c_2J_{-3/5}(2x)$

(B) $y(x)=c_1J_{3/10}(x)+c_2J_{-3/10}(x)$

(C) $y(x)=c_1J_{3/5}(x)+c_2J_{-3/5}(x)$

(D) $y(x)=c_1J_{3/10}(2x)+c_2J_{-3/10}(2x)$

Q.9 The inverse Laplace transform of $\dfrac{2s^2-4}{(s-3)(s^2-s-2)}$ is

(A) $(1+t)e^{-t}+\dfrac{7}{2}e^{-3t}$

(B) $\dfrac{e^t}{3}+te^{-t}+2t$

(C) $\dfrac{7}{2}e^{3t}-\dfrac{e^{-t}}{6}-\dfrac{4}{3}e^{2t}$

(D) $\dfrac{7}{2}e^{-3t}-\dfrac{e^{t}}{6}-\dfrac{4}{3}e^{-2t}$

Q.10 If $X_1,\ X_2$ is a random sample of size 2 from an $\mathcal{N}(0,1)$ population, then $\dfrac{(X_1+X_2)^2}{(X_1-X_2)^2}$ follows

(A) $\chi^2_{(2)}$ (B) $F_{2,2}$ (C) $F_{2,1}$ (D) $F_{1,1}$

Q.11 Let $Z\sim\mathcal{N}(0,1)$ be a random variable. Then the value of $E\left[\max\{Z,0\}\right]$ is

(A) $\dfrac{1}{\sqrt{\pi}}$ (B) $\sqrt{\dfrac{2}{\pi}}$ (C) $\dfrac{1}{\sqrt{2\pi}}$ (D) $\dfrac{1}{\pi}$

Q.12 The number of non-isomorphic groups of order 10 is ____________

Q.13 Let a, b, c, d be real numbers with $a < c < d < b$. Consider the ring $C[a,b]$ with pointwise addition and multiplication. If $S = \{ f \in C[a,b] : f(x) = 0 \text{ for all } x \in [c,d] \}$, then

(A) S is NOT an ideal of $C[a,b]$

(B) S is an ideal of $C[a,b]$ but NOT a prime ideal of $C[a,b]$

(C) S is a prime ideal of $C[a,b]$ but NOT a maximal ideal of $C[a,b]$

(D) S is a maximal ideal of $C[a,b]$

Q.14 Let R be a ring. If $R[x]$ is a principal ideal domain, then R is necessarily a

(A) Unique Factorization Domain

(B) Principal Ideal Domain

(C) Euclidean Domain

(D) Field

Q.15 Consider the group homomorphism $\varphi: M_2(\mathbb{R}) \to \mathbb{R}$ given by $\varphi(A) = \text{trace}(A)$. The kernel of φ is isomorphic to which of the following groups?

(A) $M_2(\mathbb{R}) / \{A \in M_2(\mathbb{R}) : \varphi(A) = 0\}$

(B) $\mathbb{R}^2$

(C) $\mathbb{R}^3$

(D) $GL_2(\mathbb{R})$

Q.16 Let X be a set with at least two elements. Let τ and τ' be two topologies on X such that $\tau' \neq \{\phi, X\}$. Which of the following conditions is necessary for the identity function $id : (X, \tau) \to (X, \tau')$ to be continuous?

(A) $\tau \subseteq \tau'$

(B) $\tau' \subseteq \tau$

(C) no conditions on τ and τ'

(D) $\tau \cap \tau' = \{\phi, X\}$

Q.17 Let $A \in M_3(\mathbb{R})$ be such that $\det(A - I) = 0$, where I denotes the 3×3 identity matrix. If the $\text{trace}(A) = 13$ and $\det(A) = 32$, then the sum of squares of the eigenvalues of A is ______

Q.18 Let V denote the vector space $C^5[a,b]$ over $\mathbb{R}$ and $W = \left\{ f \in V : \frac{d^4 f}{dt^4} + 2\frac{d^2 f}{dt^2} - f = 0 \right\}$. Then

(A) $\dim(V) = \infty$ and $\dim(W) = \infty$

(B) $\dim(V) = \infty$ and $\dim(W) = 4$

(C) $\dim(V) = 6$ and $\dim(W) = 5$

(D) $\dim(V) = 5$ and $\dim(W) = 4$

Q.19 Let V be a real inner product space of dimension 10. Let $x, y \in V$ be non-zero vectors such that $\langle x, y \rangle = 0$. Then the dimension of $\{x\}^{\perp} \cap \{y\}^{\perp}$ is ________________________

Q.20 Consider the following linear programming problem:

Minimize $x_1 + x_2$

Subject to:

$$2x_1 + x_2 \geq 8$$
$$2x_1 + 5x_2 \geq 10$$
$$x_1, \ x_2 \geq 0$$

The optimal value to this problem is ________________________

Q.21 Let

$$f(x) := \begin{cases} -3\pi & \text{if } -\pi < x \leq 0 \\ 3\pi & \text{if } 0 < x < \pi \end{cases}$$

be a periodic function of period 2π. The coefficient of $\sin 3x$ in the Fourier series expansion of $f(x)$ on the interval $[-\pi, \ \pi]$ is ________________________

Q.22 For the sequence of functions

$$f_n(x) = \frac{1}{x^2} \sin\left(\frac{1}{nx}\right), \quad x \in [1, \infty),$$

consider the following quantities expressed in terms of Lebesgue integrals

I: $\lim_{n \to \infty} \int_1^{\infty} f_n(x)dx$ **II:** $\int_1^{\infty} \lim_{n \to \infty} f_n(x)dx$.

Which of the following is **TRUE?**

(A) The limit in **I** does not exist

(B) The integrand in **II** is not integrable on $[1, \infty)$

(C) Quantities **I** and **II** are well-defined, but **I** ≠ **II**

(D) Quantities **I** and **II** are well-defined and **I** = **II**

Q.23 Which of the following statements about the spaces ℓ^p and $L^p[0,1]$ is **TRUE?**

(A) $\ell^3 \subset \ell^7$ and $L^6[0,1] \subset L^9[0,1]$

(B) $\ell^3 \subset \ell^7$ and $L^9[0,1] \subset L^6[0,1]$

(C) $\ell^7 \subset \ell^3$ and $L^6[0,1] \subset L^9[0,1]$

(D) $\ell^7 \subset \ell^3$ and $L^9[0,1] \subset L^6[0,1]$

Q.24 The maximum modulus of e^{z^2} on the set $S = \{z \in \mathbb{C} : \ 0 \leq Re(z) \leq 1, \ 0 \leq Im(z) \leq 1\}$ is

(A) $2/e$ (B) e (C) $e+1$ (D) e^2

Q.25 Let d_1, d_2 and d_3 be metrics on a set X with at least two elements. Which of the following is **NOT** a metric on X?

(A) $\min\{d_1, 2\}$ (B) $\max\{d_2, 2\}$

(C) $\frac{d_3}{1+d_3}$ (D) $\frac{d_1+d_2+d_3}{3}$

Q. 26 – Q. 55 carry two marks each.

Q.26 Let $\Omega = \{z \in \mathbb{C} : Im(z) > 0\}$ and let C be a smooth curve lying in Ω with initial point $-1+2i$ and final point $1+2i$. The value of $\int_C \frac{1+2z}{1+z}\,dz$ is

(A) $4-\frac{1}{2}\ln 2+i\frac{\pi}{4}$ (B) $-4+\frac{1}{2}\ln 2+i\frac{\pi}{4}$

(C) $4+\frac{1}{2}\ln 2-i\frac{\pi}{4}$ (D) $4-\frac{1}{2}\ln 2+i\frac{\pi}{2}$

Q.27 If $a \in \mathbb{C}$ with $|a|<1$, then the value of

$$\frac{\left(1-|a|^2\right)}{\pi}\int_\Gamma \frac{|dz|}{|z+a|^2},$$

where Γ is the simple closed curve $|z| = 1$ taken with the positive orientation, is _________

Q.28 Consider $C[-1,1]$ equipped with the supremum norm given by $\|f\|_\infty = \sup\{|f(t)| : t \in [-1, 1]\}$ for $f \in C[-1, 1]$. Define a linear functional T on $C[-1,1]$ by $T(f) = \int_{-1}^{0} f(t)dt - \int_{0}^{1} f(t)dt$ for all $f \in C[-1,1]$. Then the value of $\|T\|$ is _______

Q.29 Consider the vector space $C[0,1]$ over $\mathbb{R}$. Consider the following statements:

P: If the set $\{t f_1, t^2 f_2, t^3 f_3\}$ is linearly independent, then the set $\{f_1, f_2, f_3\}$ is linearly independent, where $f_1, f_2, f_3 \in C[0,1]$ and t^n represents the polynomial function $t \mapsto t^n$, $n \in \mathbb{N}$

Q: If $F: C[0,1] \to \mathbb{R}$ is given by $F(x) = \int_0^1 x(t^2)\,dt$ for each $x \in C[0,1]$, then F is a linear map.

Which of the above statements hold **TRUE**?

(A) Only **P** (B) Only **Q** (C) Both **P** and **Q** (D) Neither **P** nor **Q**

Q.30 Using the Newton-Raphson method with the initial guess $x^{(0)} = 6$, the approximate value of the real root of $x\log_{10} x = 4.77$, after the second iteration, is ____________________

Q.31 Let the following discrete data be obtained from a curve $y = y(x)$:

x:	0	0.25	0.5	0.75	1.0
y:	1	0.9896	0.9589	0.9089	0.8415

Let S be the solid of revolution obtained by rotating the above curve about the x-axis between $x = 0$ and $x = 1$ and let V denote its volume. The approximate value of V, obtained using Simpson's $\frac{1}{3}$ rule, is ______________

Q.32 The integral surface of the first order partial differential equation

$$2y(z-3)\frac{\partial z}{\partial x}+(2x-z)\frac{\partial z}{\partial y}=y(2x-3)$$

passing through the curve $x^2+y^2 = 2x,\ z = 0$ is

(A) $x^2+y^2-z^2-2x+4z=0$

(B) $x^2+y^2-z^2-2x+8z=0$

(C) $x^2+y^2+z^2-2x+16z=0$

(D) $x^2+y^2+z^2-2x+8z=0$

Q.33 The boundary value problem, $\dfrac{d^2\varphi}{dx^2}+\lambda\varphi=x;\ \varphi(0)=0$ and $\dfrac{d\varphi}{dx}(1)=0$, is converted into the integral equation $\varphi(x)=g(x)+\lambda\int_0^1 k(x,\xi)\varphi(\xi)d\xi$, where the kernel $k(x,\xi)=\begin{cases}\xi, & 0<\xi<x\\ x, & x<\xi<1\end{cases}$

Then $g\left(\dfrac{2}{3}\right)$ is ______________

Q.34 If $y_1(x) = x$ is a solution to the differential equation $(1-x^2)\dfrac{d^2y}{dx^2}-2x\dfrac{dy}{dx}+2y=0$, then its general solution is

(A) $y(x)=c_1x+c_2\left(x\ln\left|1+x^2\right|-1\right)$

(B) $y(x)=c_1x+c_2\left(\ln\left|\dfrac{1-x}{1+x}\right|+1\right)$

(C) $y(x)=c_1x+c_2\left(\dfrac{x}{2}\ln\left|1-x^2\right|+1\right)$

(D) $y(x)=c_1x+c_2\left(\dfrac{x}{2}\ln\left|\dfrac{1+x}{1-x}\right|-1\right)$

Q.35 The solution to the initial value problem $\frac{d^2y}{dt^2}+2\frac{dy}{dt}+5y=3e^{-t}\sin t$, $y(0)=0$ and $\frac{dy}{dt}(0)=3$, is

(A) $y(t)=e^{t}(\sin t+\sin 2t)$

(B) $y(t)=e^{-t}(\sin t+\sin 2t)$

(C) $y(t)=3e^{t}\sin t$

(D) $y(t)=3e^{-t}\sin t$

Q.36 The time to failure, in months, of light bulbs manufactured at two plants A and B obey the exponential distribution with means 6 and 2 months respectively. Plant B produces four times as many bulbs as plant A does. Bulbs from these plants are indistinguishable. They are mixed and sold together. Given that a bulb purchased at random is working after 12 months, the probability that it was manufactured at plant A is ______

Q.37 Let X, Y be continuous random variables with joint density function

$$f_{X,Y}(x,y)=\begin{cases} e^{-y}(1-e^{-x}) & \text{if } 0<x<y<\infty \\ e^{-x}(1-e^{-y}) & \text{if } 0<y\le x<\infty \end{cases}$$

The value of $E\left[X+Y\right]$ is ____________________

Q.38 Let $X=[0,1)\cup(1,2)$ be the subspace of $\mathbb{R}$, where $\mathbb{R}$ is equipped with the usual topology. Which of the following is **FALSE?**

(A) There exists a non-constant continuous function $f\colon X\to\mathbb{Q}$

(B) X is homeomorphic to $(-\infty,-3)\cup[0,\infty)$

(C) There exists an onto continuous function $f:\left[0,1\right]\to\overline{X}$, where $\overline{X}$ is the closure of X in $\mathbb{R}$

(D) There exists an onto continuous function $f:\left[0,1\right]\to X$

Q.39 Let $X=\begin{bmatrix}2&0&-3\\3&-1&-3\\0&0&-1\end{bmatrix}$. A matrix P such that $P^{-1}XP$ is a diagonal matrix, is

(A) $\begin{bmatrix}1&1&1\\0&1&1\\1&1&0\end{bmatrix}$

(B) $\begin{bmatrix}-1&1&1\\0&1&1\\1&1&0\end{bmatrix}$

(C) $\begin{bmatrix}1&-1&1\\0&1&1\\1&1&0\end{bmatrix}$

(D) $\begin{bmatrix}-1&-1&1\\0&-1&1\\1&1&0\end{bmatrix}$

Q.40 Using the Gauss-Seidel iteration method with the initial guess $\left\{x_1^{(0)}=3.5, x_2^{(0)}=2.25, x_3^{(0)}=1.625\right\}$, the second approximation $\left\{x_1^{(2)}, x_2^{(2)}, x_3^{(2)}\right\}$ for the solution to the system of equations

$$2x_1 - x_2 = 7$$
$$-x_1 + 2x_2 - x_3 = 1$$
$$-x_2 + 2x_3 = 1,$$

is

(A) $x_1^{(2)}=5.3125,\ x_2^{(2)}=4.4491,\ x_3^{(2)}=2.1563$

(B) $x_1^{(2)}=5.3125,\ x_2^{(2)}=4.3125,\ x_3^{(2)}=2.6563$

(C) $x_1^{(2)}=5.3125,\ x_2^{(2)}=4.4491,\ x_3^{(2)}=2.6563$

(D) $x_1^{(2)}=5.4991,\ x_2^{(2)}=4.4491,\ x_3^{(2)}=2.1563$

Q.41 The fourth order Runge-Kutta method given by

$$u_{j+1}=u_j+\frac{h}{6}\left[K_1+2K_2+2K_3+K_4\right],\quad j=0,1,2,\ldots,$$

is used to solve the initial value problem $\frac{du}{dt}=u$, $u(0) = \alpha$.

If $u(1) = 1$ is obtained by taking the step size $h = 1$, then the value of K_4 is _______________

Q.42 A particle P of mass m moves along the cycloid $x = (\theta - \sin\theta)$ and $y = (1 + \cos\theta)$, $0 \le \theta \le 2\pi$. Let g denote the acceleration due to gravity. Neglecting the frictional force, the Lagrangian associated with the motion of the particle P is:

(A) $m(1-\cos\theta)\,\dot{\theta}^2 - mg\,(1+\cos\theta)$

(B) $m(1+\cos\theta)\,\dot{\theta}^2 + mg\,(1+\cos\theta)$

(C) $m(1+\cos\theta)\,\dot{\theta}^2 + mg\,(1-\cos\theta)$

(D) $m(\theta-\sin\theta)\,\dot{\theta}^2 - mg\,(1+\cos\theta)$

Q.43 Suppose that X is a population random variable with probability density function

$$f(x;\theta) = \begin{cases} \theta x^{\theta-1} & \text{if } 0 < x < 1 \\ 0 & \text{otherwise,} \end{cases}$$

where θ is a parameter. In order to test the null hypothesis $H_0: \theta = 2$, against the alternative hypothesis $H_1: \theta = 3$, the following test is used: Reject the null hypothesis if $X_1 \ge \frac{1}{2}$ and accept otherwise, where X_1 is a random sample of size 1 drawn from the above population. Then the power of the test is ______

Q.44 Suppose that $X_1, X_2, \ldots, X_n$ is a random sample of size n drawn from a population with probability density function

$$f(x;\theta) = \begin{cases} \frac{x}{\theta^2} e^{-\frac{x}{\theta}} & \text{if } x > 0 \\ 0 & \text{otherwise,} \end{cases}$$

where θ is a parameter such that $\theta > 0$. The maximum likelihood estimator of θ is

(A) $\dfrac{\sum_{i=1}^{n} X_i}{n}$

(B) $\dfrac{\sum_{i=1}^{n} X_i}{n-1}$

(C) $\dfrac{\sum_{i=1}^{n} X_i}{2n}$

(D) $\dfrac{2\sum_{i=1}^{n} X_i}{n}$

Q.45 Let $\vec{F}$ be a vector field defined on $\mathbb{R}^2 \setminus \{(0,0)\}$ by $\vec{F}(x,y) = \frac{y}{x^2+y^2}\hat{i} - \frac{x}{x^2+y^2}\hat{j}$. Let $\gamma, \alpha: [0,1] \to \mathbb{R}^2$ be defined by

$$\gamma(t) = (8\cos 2\pi t,\ 17\sin 2\pi t) \quad \text{and} \quad \alpha(t) = (26\cos 2\pi t,\ -10\sin 2\pi t).$$

If $3\int_{\alpha} \vec{F}\cdot d\vec{r} - 4\int_{\gamma} \vec{F}\cdot d\vec{r} = 2m\pi$, then m is ________________________

Q.46 Let $g: \mathbb{R}^3 \to \mathbb{R}^3$ be defined by $g(x,y,z) = (3y+4z,\ 2x-3z,\ x+3y)$ and let $S = \{(x,y,z) \in \mathbb{R}^3 : 0 \le x \le 1,\ 0 \le y \le 1, 0 \le z \le 1\}$. If

$$\iiint_{g(S)} (2x+y-2z)\,dx\,dy\,dz = \alpha \iiint_{S} z\ dx\,dy\,dz,$$

then α is ______________________

Q.47 Let $T_1, T_2 : \mathbb{R}^5 \to \mathbb{R}^3$ be linear transformations such that $rank(T_1) = 3$ and $nullity(T_2) = 3$. Let $T_3 : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation such that $T_3 \circ T_1 = T_2$. Then $rank(T_3)$ is __________

Q.48 Let $\mathbb{F}_3$ be the field of 3 elements and let $\mathbb{F}_3 \times \mathbb{F}_3$ be the vector space over $\mathbb{F}_3$. The number of distinct linearly dependent sets of the form $\{u, v\}$, where $u, v \in \mathbb{F}_3 \times \mathbb{F}_3 \setminus \{(0,0)\}$ and $u \neq v$ is ______________

Q.49 Let $\mathbb{F}_{125}$ be the field of 125 elements. The number of non-zero elements $\alpha \in \mathbb{F}_{125}$ such that $\alpha^5 = \alpha$ is ________________________

Q.50 The value of $\iint_R xy\ dx\ dy$, where R is the region in the first quadrant bounded by the curves $y = x^2$, $y + x = 2$ and x = 0 is ______________

Q.51 Consider the heat equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < \pi, \qquad t > 0,$$

with the boundary conditions $u(0,t) = 0$, $u(\pi,t) = 0$ for $t > 0$, and the initial condition $u(x,0) = \sin x$. Then $u\left(\frac{\pi}{2}, 1\right)$ is ____________

Q.52 Consider the partial order in $\mathbb{R}^2$ given by the relation $(x_1, y_1) < (x_2, y_2)$ EITHER if $x_1 < x_2$ OR if $x_1 = x_2$ and $y_1 < y_2$. Then in the order topology on $\mathbb{R}^2$ defined by the above order

(A) [0,1] x {1} is compact but [0,1] x [0,1] is NOT compact

(B) [0,1] x [0,1] is compact but [0,1] x {1} is NOT compact

(C) both $[0,\ 1] \times [0,\ 1]$ and [0,1] x {1} are compact

(D) both $[0,\ 1] \times [0,\ 1]$ and [0,1] x {1} are NOT compact.

Q.53 Consider the following linear programming problem:

Minimize: $x_1 + x_2 + 2x_3$

Subject to

$x_1 + 2x_2 \geq 4$

$x_2 + 7x_3 \leq 5$

$x_1 - 3x_2 + 5x_3 = 6$

$x_1,\ x_2 \geq 0,\ x_3$ is unrestricted

The dual to this problem is:

Maximize: $4y_1 + 5y_2 + 6y_3$

Subject to

$y_1 + y_3 \leq 1$

$2y_1 + y_2 - 3y_3 \leq 1$

$7y_2 + 5y_3 = 2$

and further subject to:

(A) $y_1 \geq 0,\ y_2 \leq 0$ and y_3 is unrestricted

(B) $y_1 \geq 0,\ y_2 \geq 0$ and y_3 is unrestricted

(C) $y_1 \geq 0,\ y_3 \leq 0$ and y_2 is unrestricted

(D) $y_3 \geq 0,\ y_2 \leq 0$ and y_1 is unrestricted

Q.54 Let $X = C^1[0,1]$. For each $f \in X$, define

$$p_1(f) := \sup \{|f(t)| : t \in [0,1]\}$$
$$p_2(f) := \sup \{|f'(t)| : t \in [0,1]\}$$
$$p_3(f) := p_1(f) + p_2(f).$$

Which of the following statements is **TRUE?**

(A) (X, p_1) is a Banach space

(B) (X, p_2) is a Banach space

(C) (X, p_3) is NOT a Banach space

(D) (X, p_3) does NOT have denumerable basis

Q.55 If the power series $\sum_{n=0}^{\infty} a_n (z+3-i)^n$ converges at $5i$ and diverges at $-3i$, then the power series

(A) converges at $-2+5i$ and diverges at $2-3i$

(B) converges at $2-3i$ and diverges at $-2+5i$

(C) converges at both $2-3i$ and $-2+5i$

(D) diverges at both $2-3i$ and $-2+5i$

END OF THE QUESTION PAPER

GATE 2014
Answer Keys for MA - Mathematics

Section	Q. No.	Key / Range	Marks
GA	1	A	1
GA	2	B	1
GA	3	D	1
GA	4	C	1
GA	5	1300 to 1300	1
GA	6	D	2
GA	7	B	2
GA	8	180 to 180	2
GA	9	D	2
GA	10	B	2
MA	1	C	1
MA	2	0.49 to 0.51	1
MA	3	C	1
MA	4	2.2 to 2.22	1
MA	5	0.46 to 0.48	1
MA	6	0.69 to 0.72	1
MA	7	C	1
MA	8	A	1
MA	9	C	1
MA	10	D	1
MA	11	C	1
MA	12	1.99 to 2.01	1
MA	13	B	1
MA	14	D	1
MA	15	C	1
MA	16	B	1
MA	17	80.99 to 81.01	1
MA	18	B	1
MA	19	7.99 to 8.01	1
MA	20	4.24 to 4.26	1
MA	21	3.99 to 4.01	1
MA	22	D	1
MA	23	B	1

Section	Q. No.	Key / Range	Marks
MA	24	B	1
MA	25	B	1
MA	26	A	2
MA	27	1.99 to 2.01	2
MA	28	1.99 to 2.01	2
MA	29	B	2
MA	30	6.07 to 6.09	2
MA	31	2.81 to 2.83	2
MA	32	A	2
MA	33	-0.29 to -0.27	2
MA	34	D	2
MA	35	B	2
MA	36	0.92 to 0.94	2
MA	37	3.99 to 4.01	2
MA	38	D	2
MA	39	A	2
MA	40	B	2
MA	41	1.01 to 1.03	2
MA	42	A	2
MA	43	0.86 to 0.88	2
MA	44	C	2
MA	45	6.99 to 7.01	2
MA	46	74.99 to 75.01	2
MA	47	1.99 to 2.01	2
MA	48	3.99 to 4.01	2
MA	49	3.99 to 4.01	2
MA	50	0.36 to 0.38	2
MA	51	0.36 to 0.38	2
MA	52	D	2
MA	53	A	2
MA	54	D	2
MA	55	A	2

MA:MATHEMATICS

Duration: Three Hours *Maximum Marks*:100

Please read the following instructions carefully:

General Instructions:

1. Total duration of examination is 180 minutes (3 hours).

2. The clock will be set at the server. The countdown timer in the top right corner of screen will display the remaining time available for you to complete the examination. When the timer reaches zero, the examination will end by itself. You will not be required to end or submit your examination.

3. The Question Palette displayed on the right side of screen will show the status of each question using one of the following symbols:

You have not visited the question yet.

You have not answered the question.

You have answered the question.

You have NOT answered the question, but have marked the question for review.

You have answered the question, but marked it for review.

The Marked for Review status for a question simply indicates that you would like to look at that question again. ***If a question is answered and Marked for Review, your answer for that question will be considered in the evaluation.***

Navigating to a Question

4. To answer a question, do the following:
   a. Click on the question number in the Question Palette to go to that question directly.
   b. Select an answer for a multiple choice type question. Use the virtual numeric keypad to enter a number as answer for a numerical type question.
   c. Click on **Save and Next** to save your answer for the current question and then go to the next question.
   d. Click on **Mark for Review and Next** to save your answer for the current question, mark it for review, and then go to the next question.
   e. Caution: Note that your answer for the current question will not be saved, if you navigate to another question directly by clicking on its question number.

5. You can view all the questions by clicking on the **Question Paper** button. Note that the options for multiple choice type questions will not be shown.

Answering a Question

6. Procedure for answering a multiple choice type question:
   a. To select your answer, click on the button of one of the options
   b. To deselect your chosen answer, click on the button of the chosen option again or click on the **Clear Response** button
   c. To change your chosen answer, click on the button of another option
   d. To save your answer, you MUST click on the **Save and Next** button
   e. To mark the question for review, click on the **Mark for Review and Next** button. ***If an answer is selected for a question that is Marked for Review, that answer will be considered in the evaluation.***

7. Procedure for answering a numerical answer type question:
   a. To enter a number as your answer, use the virtual numerical keypad
   b. A fraction (eg.,-0.3 or -.3) can be entered as an answer with or without '0' before the decimal point
   c. To clear your answer, click on the **Clear Response** button
   d. To save your answer, you MUST click on the **Save and Next** button
   e. To mark the question for review, click on the **Mark for Review and Next** button. ***If an answer is entered for a question that is Marked for Review, that answer will be considered in the evaluation.***

8. To change your answer to a question that has already been answered, first select that question for answering and then follow the procedure for answering that type of question.

9. Note that ONLY Questions for which answers are saved or marked for review after answering will be considered for evaluation.

Paper specific instructions:

1. There are a total of 65 questions carrying 100 marks. Questions are of multiple choice type or numerical answer type. A multiple choice type question will have four choices for the answer with only **one** correct choice. For numerical answer type questions, the answer is a number and no choices will be given**. A number as the answer should be entered** using the virtual keyboard on the monitor.

2. Questions Q.1 – Q.25 carry 1mark each. Questions Q.26 – Q.55 carry 2marks each. The 2marks questions include two pairs of common data questions and two pairs of linked answer questions. The answer to the second question of the linked answer questions depends on the answer to the first question of the pair. If the first question in the linked pair is wrongly answered or is not attempted, then the answer to the second question in the pair will not be evaluated.

3. Questions Q.56 – Q.65 belong to General Aptitude (GA) section and carry a total of 15 marks. Questions Q.56 – Q.60 carry 1mark each, and questions Q.61 – Q.65 carry 2marks each.

4. Questions not attempted will result in zero mark. Wrong answers for multiple choice type questions will result in **NEGATIVE** marks. For all 1 mark questions, ⅓ mark will be deducted for each wrong answer. For all 2 marks questions, ⅔ mark will be deducted for each wrong answer. However, in the case of the linked answer question pair, there will be negative marks only for wrong answer to the first question and no negative marks for wrong answer to the second question. There is no negative marking for questions of numerical answer type.

5. Calculator is allowed. Charts, graph sheets or tables are **NOT** allowed in the examination hall.

6. Do the rough work in the Scribble Pad provided.

USEFUL DATA FOR MA: MATHEMATICS

Notations and Symbols used

$\mathbb{R}$: The set of all real numbers.

$\mathbb{Z}$: The set of all integers.

$\mathbb{C}$: The set of all complex numbers.

$\mathbb{N}$: The set of all positive integers.

$\mathbb{Q}$: The set of all rational numbers.

$\mathbb{Z}_n$: The cyclic group of order n.

S_n: The group of permutations of the set $\{1, 2, \dots, n\}$.

X^t: Transpose of the matrix X.

$\text{Re}(z)$: Real part of the complex number z.

$\text{Im}(z)$: Imaginary part of the complex number z.

$\overline{A}$: Closure of the set A.

A°: Interior of the set A.

$\langle a \rangle$: Ideal generated by an element a

$P(E)$: Probability of the event E.

$E[X]$: Expectation of the random variable X.

$\text{Var}(X)$: Variance of the random variable X.

$\log x$: Natural logarithm of the positive real number x.

Q. 1 – Q. 25 carry one mark each.

Q.1 The possible set of eigen values of a 4×4 skew-symmetric orthogonal real matrix is

(A) $\{\pm i\}$ (B) $\{\pm i, \ \pm 1\}$ (C) $\{\pm 1\}$ (D) $\{0, \pm i\}$

Q.2 The coefficient of $(z-\pi)^2$ in the Taylor series expansion of

$$f(z) = \begin{cases} \dfrac{\sin z}{z-\pi} & \text{if } z \neq \pi \\ -1 & \text{if } z = \pi \end{cases}$$

around π is

(A) $\frac{1}{2}$ (B) $-\frac{1}{2}$ (C) $\frac{1}{6}$ (D) $-\frac{1}{6}$

Q.3 Consider $\mathbb{R}^2$ with the usual topology. Which of the following statements are **TRUE** for all $A, B \subseteq \mathbb{R}^2$?

P: $\overline{A \cup B} = \overline{A} \cup \overline{B}$.
Q: $\overline{A \cap B} = \overline{A} \cap \overline{B}$.
R: $(A \cup B)^\circ = A^\circ \cup B^\circ$.
S: $(A \cap B)^\circ = A^\circ \cap B^\circ$.

(A) P and R only (B) P and S only (C) Q and R only (D) Q and S only

Q.4 Let $f: \mathbb{R} \to \mathbb{R}$ be a continuous function with $f(1) = 5$ and $f(3) = 11$. If $g(x) = \int_1^3 f(x+t)dt$ then $g'(0)$ is equal to ______

Q.5 Let P be a 2×2 complex matrix such that trace$(P) = 1$ anddet$(P) = -6$. Then, trace$(P^4 - P^3)$ is ______

Q.6 Suppose that R is a unique factorization domain and that $a, b \in R$ are distinct irreducible elements. Which of the following statements is **TRUE**?

(A) The ideal $\langle 1 + a \rangle$ is a prime ideal
(B) The ideal $\langle a + b \rangle$ is a prime ideal
(C) The ideal $\langle 1 + ab \rangle$ is a prime ideal
(D) The ideal $\langle a \rangle$ is not necessarily a maximal ideal

Q.7 Let X be a compact Hausdorff topological space and let Y be a topological space. Let $f: X \to Y$ be a bijective continuous mapping. Which of the following is **TRUE**?

(A) f is a closed mapbut not necessarily an open map
(B) f is an open map but not necessarily a closed map
(C) f is both an open map and a closed map
(D) f need not be an open map or a closed map

Q.8 Consider the linear programming problem:

$$\begin{aligned} \text{Maximize} \quad & x + \frac{3}{2}y \\ \text{subject to} \quad & 2x + 3y \leq 16, \\ & x + 4y \leq 18, \\ & x \geq 0, y \geq 0. \end{aligned}$$

If S denotes the set of all solutions of the above problem, then

(A) S is empty (B) S is a singleton
(C) S is a line segment (D) S has positive area

Q.9 Which of the following groups has a proper subgroup that is **NOT** cyclic?

(A) $\mathbb{Z}_{15} \times \mathbb{Z}_{77}$
(B) S_3
(C) $(\mathbb{Z}, +)$
(D) $(\mathbb{Q}, +)$

Q.10 The value of the integral

$$\int_0^\infty \int_x^\infty \left(\frac{1}{y}\right) e^{-y/2}\, dy dx$$

is ______

Q.11 Suppose the random variable U has uniform distribution on $[0,1]$ and $X = -2 \log U$. The density of X is

(A) $f(x) = \begin{cases} e^{-x} \text{if } x > 0 \\ 0 \quad \text{otherwise} \end{cases}$

(B) $f(x) = \begin{cases} 2e^{-2x} \text{if } x > 0 \\ 0 \quad \text{otherwise} \end{cases}$

(C) $f(x) = \begin{cases} \frac{1}{2} e^{-\frac{x}{2}} \text{if } x > 0 \\ 0 \quad \text{otherwise} \end{cases}$

(D) $f(x) = \begin{cases} 1/2 & \text{if } x \in [0,2] \\ 0 & \text{otherwise} \end{cases}$

Q.12 Let f be an entire function on $\mathbb{C}$ such that $|f(z)| \leq 100 \log|z|$ for each z with $|z| \geq 2$. If $f(i) = 2i$, then $f(1)$

(A) must be 2
(B) must be $2i$
(C) must be i
(D) cannot be determined from the given data

Q.13 The number of group homomorphisms from $\mathbb{Z}_3$ to $\mathbb{Z}_9$ is ______

Q.14 Let $u(x,t)$ be the solution to the wave equation

$$\frac{\partial^2 u}{\partial x^2}(x,t) = \frac{\partial^2 u}{\partial t^2}(x,t), \qquad u(x,0) = \cos(5\pi x), \qquad \frac{\partial u}{\partial t}(x,0) = 0.$$

Then, the value of $u(1,1)$ is ______

Q.15 Let $f(x) = \sum_{n=1}^{\infty} \frac{\sin(nx)}{n^2}$. Then

(A) $\lim_{x\to 0} f(x) = 0$
(B) $\lim_{x\to 0} f(x) = 1$
(C) $\lim_{x\to 0} f(x) = \pi^2/6$
(D) $\lim_{x\to 0} f(x)$ does not exist

Q.16 Suppose X is a random variable with $P(X = k) = (1 - p)^k p$ for $k \in \{0,1,2, ...\}$ and some $p \in (0,1)$. For the hypothesis testing problem

$$H_0: p = \frac{1}{2} H_1: p \neq \frac{1}{2}$$

consider the test "Reject H_0 if $X \leq A$ or if $X \geq B$", where$A < B$ are given positive integers. The type-I error of this test is

(A) $1 + 2^{-B} - 2^{-A}$
(B) $1 - 2^{-B} + 2^{-A}$
(C) $1 + 2^{-B} - 2^{-A-1}$
(D) $1 - 2^{-B} + 2^{-A-1}$

Q.17 Let G be a group of order 231. The number of elements of order 11 in G is ______

Q.18 Let $f: \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $f(x, y) = (e^{x+y}, e^{x-y})$. The area of the image of the region $\{(x, y) \in \mathbb{R}^2: 0 < x, y < 1\}$ under the mapping f is

(A) 1 (B) $e - 1$ (C) e^2 (D) $e^2 - 1$

Q.19 Which of the following is a field?

(A) $\mathbb{C}[x]/\langle x^2 + 2\rangle$

(B) $\mathbb{Z}[x]/\langle x^2 + 2\rangle$

(C) $\mathbb{Q}[x]/\langle x^2 - 2\rangle$

(D) $\mathbb{R}[x]/\langle x^2 - 2\rangle$

Q.20 Let$x_0 = 0$. Define $x_{n+1} = \cos x_n$ for every $n \geq 0$. Then

(A) $\{x_n\}$ is increasing and convergent
(B) $\{x_n\}$ is decreasing and convergent
(C) $\{x_n\}$ is convergent and $x_{2n} < \lim_{m\to\infty} x_m < x_{2n+1}$ for every $n \in \mathbb{N}$
(D) $\{x_n\}$ is not convergent

Q.21 Let C be the contour $|z| = 2$ oriented in the anti-clockwise direction. The value of the integral $\oint_C z e^{3/z}\, dz$ is

(A) $3\pi i$ (B) $5\pi i$ (C) $7\pi i$ (D) $9\pi i$

Q.22 For each $\lambda > 0$, let X_λ be a random variable with exponential density $\lambda e^{-\lambda x}$ on$(0, \infty)$. Then, $\mathrm{Var}(\log X_\lambda)$

(A) is strictly increasing in λ
(B) is strictly decreasing in λ
(C) does not depend on λ
(D) first increases and then decreases in λ

Q.23 Let $\{a_n\}$ be the sequence of consecutive positive solutions of the equation $\tan x = x$ and let $\{b_n\}$ be the sequence of consecutive positive solutions of the equation $\tan \sqrt{x} = x$. Then

(A) $\sum_{n=1}^{\infty} \frac{1}{a_n}$ converges but $\sum_{n=1}^{\infty} \frac{1}{b_n}$ diverges

(B) $\sum_{n=1}^{\infty} \frac{1}{a_n}$ diverges but $\sum_{n=1}^{\infty} \frac{1}{b_n}$ converges

(C) Both $\sum_{n=1}^{\infty} \frac{1}{a_n}$ and $\sum_{n=1}^{\infty} \frac{1}{b_n}$ converge

(D) Both $\sum_{n=1}^{\infty} \frac{1}{a_n}$ and $\sum_{n=1}^{\infty} \frac{1}{b_n}$ diverge

Q.24 Let f be an analytic function on $\overline{D} = \{z \in \mathbb{C}: |z| \leq 1\}$. Assume that $|f(z)| \leq 1$ for each $z \in \overline{D}$. Then, which of the following is **NOT** a possible value of $(e^f)''(0)$?

(A) 2 (B) 6 (C) $\frac{7}{9}e^{1/9}$ (D) $\sqrt{2} + i\sqrt{2}$

Q.25 The number of non-isomorphic abelian groups of order 24 is ______

Q. 26 to Q. 55 carry two marks each.

Q.26 Let V be the real vector space of all polynomials in one variable with real coefficients and having degree at most 20. Define the subspaces

$$W_1 = \left\{p \in V : p(1) = 0, \quad p\left(\frac{1}{2}\right) = 0, \quad p(5) = 0, \quad p(7) = 0\right\},$$
$$W_2 = \left\{p \in V : p\left(\frac{1}{2}\right) = 0, \quad p(3) = 0, \quad p(4) = 0, \quad p(7) = 0\right\}.$$

Then the dimension of $W_1 \cap W_2$ is ______

Q.27 Let $f, g : [0,1] \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} x & \text{if } x = \frac{1}{n} \text{ for } n \in \mathbb{N} \\ 0 & \text{otherwise} \end{cases}$$

and

$$g(x) = \begin{cases} 1 & \text{if } x \in \mathbb{Q} \cap [0,1] \\ 0 & \text{otherwise.} \end{cases}$$

Then

(A) Both f and g are Riemann integrable
(B) f is Riemann integrable and g is Lebesgue integrable
(C) g is Riemann integrable and f is Lebesgue integrable
(D) Neither f nor g is Riemann integrable

Q.28 Consider the following linear programming problem:

$$\begin{array}{ll} \text{Maximize} & x + 3y + 6z - w \\ \text{subject to} & 5x + y + 6z + 7w \leq 20, \\ & 6x + 2y + 2z + 9w \leq 40, \\ & x \geq 0,\ y \geq 0,\ z \geq 0,\ w \geq 0. \end{array}$$

Then the optimal value is ______

Q.29 Suppose X is a real-valued random variable. Which of the following values **CANNOT** be attained by $E[X]$ and $E[X^2]$, respectively?

(A) 0 and 1 (B) 2 and 3 (C) $\frac{1}{2}$ and $\frac{1}{3}$ (D) 2 and 5

Q.30 Which of the following subsets of $\mathbb{R}^2$ is **NOT** compact?

(A) $\{(x,y) \in \mathbb{R}^2 :\ -1 \le x \le 1,\ y = \sin x\}$

(B) $\{(x,y) \in \mathbb{R}^2 :\ -1 \le y \le 1,\ y = x^8 - x^3 - 1\}$

(C) $\{(x,y) \in \mathbb{R}^2 :\ y = 0,\ \sin(e^{-x}) = 0\}$

(D) $\left\{(x,y) \in \mathbb{R}^2 :\ x > 0,\ y = \sin\left(\frac{1}{x}\right)\right\} \cap \left\{(x,y) \in \mathbb{R}^2 :\ x > 0,\ y = \frac{1}{x}\right\}$

Q.31 Let Mbe the real vector space of 2×3 matrices with real entries. Let $T: M \to M$ be defined by

$$T\left(\begin{bmatrix} x_1 & x_2 & x_3 \\ x_4 & x_5 & x_6 \end{bmatrix}\right) = \begin{bmatrix} -x_6 & x_4 & x_1 \\ x_3 & x_5 & x_2 \end{bmatrix}.$$

The determinant of T is ______

Q.32 Let $\mathcal{H}$ be a Hilbert space and let $\{e_n : n \ge 1\}$ be an orthonormal basis of $\mathcal{H}$. Suppose $T: \mathcal{H} \to \mathcal{H}$ is a bounded linear operator. Which of the following **CANNOT** be true?

(A) $T(e_n) = e_1$ for all $n \ge 1$

(B) $T(e_n) = e_{n+1}$ for all $n \ge 1$

(C) $T(e_n) = \sqrt{\frac{n+1}{n}}\, e_n$ for all $n \ge 1$

(D) $T(e_n) = e_{n-1}$ for all $n \ge 2$ and $T(e_1) = 0$

Q.33 The value of the limit

$$\lim_{n\to\infty} \frac{2^{-n^2}}{\sum_{k=n+1}^{\infty} 2^{-k^2}}$$

is

(A) 0 (B) some $c \in (0,1)$ (C) 1 (D) ∞

Q.34 Let $f: \mathbb{C}\setminus\{3i\} \to \mathbb{C}$ be defined by $f(z) = \frac{z-i}{iz+3}$. Which of the following statements about f is **FALSE**?

(A) f is conformal on $\mathbb{C}\setminus\{3i\}$
(B) f maps circles in $\mathbb{C}\setminus\{3i\}$ onto circles in $\mathbb{C}$
(C) All the fixed points of f are in the region $\{z \in \mathbb{C} : \mathrm{Im}(z) > 0\}$
(D) There is no straight line in $\mathbb{C}\setminus\{3i\}$ which is mapped onto a straight line in $\mathbb{C}$ by f

Q.35 The matrix $A = \begin{bmatrix} 1 & 2 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 3 \end{bmatrix}$ can be decomposed uniquely into the product $A = LU$, where $L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}$ and $U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$. The solution of the system $LX = [1 \quad 2 \quad 2]^t$ is

(A) $[1 \quad 1 \quad 1]^t$ (B) $[1 \quad 1 \quad 0]^t$ (C) $[0 \quad 1 \quad 1]^t$ (D) $[1 \quad 0 \quad 1]^t$

Q.36 Let $S = \left\{x \in \mathbb{R} : x \geq 0,\ \sum_{n=1}^{\infty} x^{\sqrt{n}} < \infty\right\}$. Then the supremum of S is

(A) 1 (B) $\frac{1}{e}$ (C) 0 (D) ∞

Q.37 The image of the region $\{z \in \mathbb{C} : \operatorname{Re}(z) > \operatorname{Im}(z) > 0\}$ under the mapping $z \mapsto e^{z^2}$ is

(A) $\{w \in \mathbb{C} : \operatorname{Re}(w) > 0, \operatorname{Im}(w) > 0\}$ (B) $\{w \in \mathbb{C} : \operatorname{Re}(w) > 0, \operatorname{Im}(w) > 0, |w| > 1\}$

(C) $\{w \in \mathbb{C} : |w| > 1\}$ (D) $\{w \in \mathbb{C} : \operatorname{Im}(w) > 0, |w| > 1\}$

Q.38 Which of the following groups contains a unique normal subgroup of order four?

(A) $\mathbb{Z}_2 \oplus \mathbb{Z}_4$ (B) The dihedral group, D_4, of order eight

(C) The quaternion group, Q_8 (D) $\mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_2$

Q.39 Let B be a real symmetric positive-definite $n \times n$ matrix. Consider the inner product on $\mathbb{R}^n$ defined by $\langle X, Y \rangle = Y^t BX$. Let A be an $n \times n$ real matrix and let $T: \mathbb{R}^n \to \mathbb{R}^n$ be the linear operator defined by $T(X) = AX$ for all $X \in \mathbb{R}^n$. If S is the adjoint of T, then $S(X) = CX$ for all $X \in \mathbb{R}^n$, where C is the matrix

(A) $B^{-1}A^tB$ (B) BA^tB^{-1} (C) $B^{-1}AB$ (D) A^t

Q.40 Let X be an arbitrary random variable that takes values in $\{0, 1, \dots, 10\}$. The minimum and maximum possible values of the variance of X are

(A) 0 and 30 (B) 1 and 30 (C) 0 and 25 (D) 1 and 25

Q.41 Let M be the space of all 4×3 matrices with entries in the finite field of three elements. Then the number of matrices of rank three in M is

(A) $(3^4 - 3)(3^4 - 3^2)(3^4 - 3^3)$
(B) $(3^4 - 1)(3^4 - 2)(3^4 - 3)$
(C) $(3^4 - 1)(3^4 - 3)(3^4 - 3^2)$
(D) $3^4(3^4 - 1)(3^4 - 2)$

Q.42 Let V be a vector space of dimension $m \geq 2$. Let $T: V \to V$ be a linear transformation such that $T^{n+1} = 0$ and $T^n \neq 0$ for some $n \geq 1$. Then which of the following is necessarily **TRUE**?

(A) Rank $(T^n) \leq$ Nullity (T^n) (B) trace $(T) \neq 0$
(C) T is diagonalizable (D) $n = m$

Q.43 Let X be a convex region in the plane bounded by straight lines. Let X have 7 vertices. Suppose $f(x, y) = ax + by + c$ has maximum value M and minimum value N on X and $N < M$. Let $S = \{P : P \text{ is a vertex of } X \text{ and } N < f(P) < M\}$. If S has n elements, then which of the following statements is **TRUE**?

(A) n cannot be 5 (B) n can be 2
(C) n cannot be 3 (D) n can be 4

Q.44 Which of the following statements are **TRUE**?

P: If $f \in L^1(\mathbb{R})$, then f is continuous.
Q: If $f \in L^1(\mathbb{R})$ and $\lim_{|x| \to \infty} f(x)$ exists, then the limit is zero.
R: If $f \in L^1(\mathbb{R})$, then f is bounded.
S: If $f \in L^1(\mathbb{R})$ is uniformly continuous, then $\lim_{|x| \to \infty} f(x)$ exists and equals zero.

(A) Q and S only (B) P and R only (C) P and Q only (D) R and S only

Q.45 Let u be a real valued harmonic function on $\mathbb{C}$. Let $g: \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$g(x,y) = \int_0^{2\pi} u\big(e^{i\theta}(x+iy)\big) \sin\theta \, d\theta.$$

Which of the following statements is **TRUE**?

(A) g is a harmonic polynomial
(B) g is a polynomial but not harmonic
(C) g is harmonic but not a polynomial
(D) g is neither harmonic nor a polynomial

Q.46 Let $S = \{z \in \mathbb{C} : |z| = 1\}$ with the induced topology from $\mathbb{C}$ and let $f: [0,2] \to S$ be defined as $f(t) = e^{2\pi i t}$. Then, which of the following is **TRUE**?

(A) K is closed in $[0,2] \Longrightarrow f(K)$ is closed in S
(B) U is open in $[0,2] \Longrightarrow f(U)$ is open in S
(C) $f(X)$ is closed in $S \Longrightarrow X$ is closed in $[0,2]$
(D) $f(Y)$ is open in $S \Longrightarrow Y$ is open in $[0,2]$

Q.47 Assume that all the zeros of the polynomial $a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ have negative real parts. If $u(t)$ is any solution to the ordinary differential equation

$$a_n \frac{d^n u}{dt^n} + a_{n-1}\frac{d^{n-1}u}{dt^{n-1}} + \cdots + a_1 \frac{du}{dt} + a_0 u = 0,$$

then $\lim_{t\to\infty} u(t)$ is equal to

(A) 0 (B) 1 (C) $n-1$ (D) ∞

Common Data Questions

Common Data for Questions 48 and 49:

Let c_{00} be the vector space of all complex sequences having finitely many non-zero terms. Equip c_{00} with the inner product $\langle x, y\rangle = \sum_{n=1}^{\infty} x_n \overline{y_n}$ for all $x = (x_n)$ and $y = (y_n)$ in c_{00}. Define $f: c_{00} \to \mathbb{C}$ by $f(x) = \sum_{n=1}^{\infty} \frac{x_n}{n}$. Let N be the kernel of f.

Q.48 Which of the following is **FALSE**?

(A) f is a continuous linear functional
(B) $\|f\| \le \frac{\pi}{\sqrt{6}}$
(C) There does not exist any $y \in c_{00}$ such that $f(x) = \langle x, y\rangle$ for all $x \in c_{00}$
(D) $N^{\perp} \neq \{0\}$

Q.49 Which of the following is **FALSE**?

(A) $c_{00} \neq N$
(B) N is closed
(C) c_{00} is not a complete inner product space
(D) $c_{00} = N \oplus N^{\perp}$

Common Data for Questions 50 and 51:

Let $X_1,\ X_2,\ \dots, X_n$ be an i.i.d. random sample from exponential distribution with mean μ. In other words, they have density

$$f(x) = \begin{cases} \frac{1}{\mu} e^{-x/\mu} \text{ if } x > 0 \\ 0 \quad\quad \text{otherwise.} \end{cases}$$

Q.50 Which of the following is **NOT** an unbiased estimate of μ?

(A) X_1
(B) $\frac{1}{n-1}(X_2 + X_3 + \cdots + X_n)$
(C) $n \cdot (\min\{X_1, X_2, \dots, X_n\})$
(D) $\frac{1}{n}\max\{X_1, X_2, \dots, X_n\}$

Q.51 Consider the problem of estimating μ. The m.s.e (mean square error) of the estimate

$$T(X) = \frac{X_1 + X_2 + \cdots + X_n}{n+1}$$

is

(A) μ^2 (B) $\frac{1}{n+1}\mu^2$ (C) $\frac{1}{(n+1)^2}\mu^2$ (D) $\frac{n^2}{(n+1)^2}\mu^2$

Linked Answer Questions

Statement for Linked Answer Questions 52 and 53:

Let $X = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 1\} \cup ([-1,1] \times \{0\}) \cup (\{0\} \times [-1,1])$.
Let $n_0 = \max\{k : k < \infty$, there are k distinct points $p_1, \dots, p_k \in X$ such that $X \setminus \{p_1, \dots, p_k\}$ is connected$\}$

Q.52 The value of n_0 is ______

Q.53 Let $q_1, \dots, q_{n_0+1}$ be $n_0 + 1$ distinct points and $Y = X \setminus \{q_1, \dots, q_{n_0+1}\}$. Let m be the number of connected components of Y. The maximum possible value of m is ______

Statement for Linked Answer Questions 54 and 55:

Let $W(y_1, y_2)$ be the Wrönskian of two linearly independent solutionsy_1 and y_2 of the equation $y'' + P(x)y' + Q(x)y = 0$.

Q.54 The product $W(y_1, y_2)P(x)$ equals

(A) $y_2 y_1'' - y_1 y_2''$ (B) $y_1 y_2' - y_2 y_1'$
(C) $y_1' y_2'' - y_2' y_1''$ (D) $y_2' y_1' - y_1'' y_2''$

Q.55 If $y_1 = e^{2x}$ and $y_2 = xe^{2x}$, then the value of $P(0)$ is

(A) 4 (B) -4 (C) 2 (D) -2

General Aptitude (GA) Questions

Q. 56 – Q. 60 carry one mark each.

Q.56 A number is as much greater than 75 as it is smaller than 117. The number is:

(A) 91 (B) 93 (C) 89 (D) 96

Q.57 The professor ordered to the students to go out of the class.
I: The professor; II: ordered to; III: the students to go; IV: out of the class

Which of the above underlined parts of the sentence is grammatically incorrect?

(A) I (B) II (C) III (D) IV

Q.58 Which of the following options is the closest in meaning to the word given below:

Primeval

(A) Modern (B) Historic
(C) Primitive (D) Antique

Q.59 Friendship, no matter how __________it is, has its limitations.

(A) cordial

(B) intimate

(C) secret

(D) pleasant

Q.60 Select the pair that best expresses a relationship similar to that expressed in the pair:
Medicine: Health

(A) Science: Experiment (B) Wealth: Peace
(C) Education: Knowledge (D) Money: Happiness

Q. 61 to Q. 65 carry two marks each.

Q.61 X and Y are two positive real numbers such that $2X + Y \leq 6$ and $X + 2Y \leq 8$. For which of the following values of (X, Y) the function $f(X, Y) = 3X + 6Y$ will give maximum value?

(A) (4/3, 10/3)

(B) (8/3, 20/3)

(C) (8/3, 10/3)

(D) (4/3, 20/3)

Q.62 If $|4X - 7| = 5$ then the values of $2\,|X| - |-X|$ is:

(A) 2, 1/3 (B) 1/2, 3 (C) 3/2, 9 (D) 2/3, 9

Q.63 Following table provides figures (in rupees) on annual expenditure of a firm for two years - 2010 and 2011.

Category	2010	2011
Raw material	5200	6240
Power & fuel	7000	9450
Salary & wages	9000	12600
Plant & machinery	20000	25000
Advertising	15000	19500
Research & Development	22000	26400

In 2011, which of the following two categories have registered increase by same percentage?

(A) Raw material and Salary & wages

(B) Salary & wages and Advertising

(C) Power & fuel and Advertising

(D) Raw material and Research & Development

Q.64 A firm is selling its product at Rs. 60 per unit. The total cost of production is Rs. 100 and firm is earning total profit of Rs. 500. Later, the total cost increased by 30%. By what percentage the price should be increased to maintained the same profit level.

(A) 5 (B) 10 (C) 15 (D) 30

Q.65 Abhishek is elder to Savar.
Savar is younger to Anshul.

Which of the given conclusions is logically valid and is inferred from the above statements?

(A) Abhishek is elder to Anshul

(B) Anshul is elder to Abhishek

(C) Abhishek and Anshul are of the same age

(D) No conclusion follows

END OF THE QUESTION PAPER

GATE 2013 : Answer keys for MA - Mathematics

Paper	Q.No	Key(s)/Value(s)
MA	1	A
MA	2	C
MA	3	B
MA	4	6
MA	5	78
MA	6	D
MA	7	D
MA	8	C
MA	9	D
MA	10	2
MA	11	C
MA	12	B
MA	13	3
MA	14	1
MA	15	A
MA	16	C
MA	17	10
MA	18	D
MA	19	C
MA	20	C
MA	21	D
MA	22	C
MA	23	B
MA	24	B
MA	25	3
MA	26	15
MA	27	B
MA	28	60
MA	29	B
MA	30	C
MA	31	-1
MA	32	A
MA	33	D
MA	34	C
MA	35	A

Paper	Q.No	Key(s)/Value(s)
MA	36	A
MA	37	C
MA	38	Marks to All
MA	39	A
MA	40	C
MA	41	C
MA	42	A
MA	43	D
MA	44	A
MA	45	A
MA	46	A
MA	47	A
MA	48	D
MA	49	D
MA	50	D
MA	51	B
MA	52	4
MA	53	8
MA	54	A
MA	55	B
MA	56	D
MA	57	B
MA	58	C
MA	59	B
MA	60	C
MA	61	A
MA	62	B
MA	63	D
MA	64	A
MA	65	D

MA : MATHEMATICS

Duration: Three Hours *Maximum Marks*: 100

Read the following instructions carefully.

1. Do not open the seal of the Question Booklet until you are asked to do so by the invigilator.
2. Take out the **O**ptical **R**esponse **S**heet (**ORS)** from this Question Booklet **without breaking the seal** and read the instructions printed on the **ORS** carefully.
3. On the right half of the **ORS,** using ONLY a **black ink ball point pen**, (i) darken the bubble corresponding to your test paper code and the appropriate bubble under each digit of your registration number and (ii) write your registration number, your name and name of the examination centre and put your signature at the specified location.
4. This Question Booklet contains **20** pages including blank pages for rough work. After you are permitted to open the seal, please check all pages and report discrepancies, if any, to the invigilator.
5. There are a total of 65 questions carrying 100 marks. All these questions are of objective type. Each question has only **one** correct answer. Questions must be answered on the left hand side of the **ORS** by darkening the appropriate bubble (marked A, B, C, D) using ONLY a **black ink ball point pen** against the question number. **For each question darken the bubble of the correct answer**. More than one answer bubbled against a question will be treated as an incorrect response.
6. Since bubbles darkened by the black ink ball point pen **cannot** be erased, candidates should darken the bubbles in the **ORS very carefully**.
7. Questions Q.1 – Q.25 carry 1 mark each. Questions Q.26 – Q.55 carry 2 marks each. The 2 marks questions include two pairs of common data questions and two pairs of linked answer questions. The answer to the second question of the linked answer questions depends on the answer to the first question of the pair. If the first question in the linked pair is wrongly answered or is unattempted, then the answer to the second question in the pair will not be evaluated.
8. Questions Q.56 – Q.65 belong to General Aptitude (GA) section and carry a total of 15 marks. Questions Q.56 – Q.60 carry 1 mark each, and questions Q.61 – Q.65 carry 2 marks each.
9. Unattempted questions will result in zero mark and wrong answers will result in **NEGATIVE** marks. For all 1 mark questions, ⅓ mark will be deducted for each wrong answer. For all 2 marks questions, ⅔ mark will be deducted for each wrong answer. However, in the case of the linked answer question pair, there will be negative marks only for wrong answer to the first question and no negative marks for wrong answer to the second question.
10. Calculator is allowed whereas charts, graph sheets or tables are **NOT** allowed in the examination hall.
11. Rough work can be done on the question paper itself. Blank pages are provided at the end of the question paper for rough work.
12. Before the start of the examination, write your name and registration number in the space provided below using a black ink ball point pen.

Name								
Registration Number	**MA**							

Notations and Symbols used

$\mathbb{R}$	: Set of all real numbers
$\mathbb{C}$	: Set of all complex numbers
$\mathbb{Z}$	: Set of all integers
F	: A field
$\mathbb{C}^n$	: The set of all *n*-tuples of complex numbers
F^n	: The set of all n-tuples over F
$R_1 \times R_2 \times ... \times R_n$	: Cartesian product of rings $R_1, R_2, ..., R_n$
$D_x f(x, y)$	: Partial derivative with respect to *x*.
$N(\mu, \sigma^2)$	: Normal distribution with mean μ and variance σ^2
$E(X)$	: Expectation of *X*
$Cov(X, Y)$	: Covariance between *X* and *Y*
S_n	: The group of all permutations on *n* symbols
P_n	: The set of all polynomials of degree at most *n*
C_n	: Cyclic Group of Order *n*
$Z(G)$	: Centre of the Group *G*

$i = \sqrt{-1}$

Q. 1 – Q. 25 carry one mark each.

Q.1 The straight lines $L_1: x=0$, $L_2: y=0$ and $L_3: x+y=1$ are mapped by the transformation $w=z^2$ into the curves C_1, C_2 and C_3 respectively. The angle of intersection between the curves at $w=0$ is

(A) 0 (B) $\pi/4$ (C) $\pi/2$ (D) π

Q.2 In a topological space, which of the following statements is **NOT** always true :

(A) Union of any finite family of compact sets is compact.
(B) Union of any family of closed sets is closed.
(C) Union of any family of connected sets having a non empty intersection is connected.
(D) Union of any family of dense subsets is dense.

Q.3 Consider the following statements:

P: The family of subsets $\left\{A_n = \left(-\frac{1}{n}, \frac{1}{n}\right), n=1,2,\ldots\right\}$ satisfies the finite intersection property.

Q: On an infinite set X, a metric $d: X\times X \to R$ is defined as $d(x,y)=\begin{cases}0, & x=y\\ 1, & x\neq y\end{cases}$.

The metric space (X,d) is compact.

R: In a Frechet (T_1) topological space, every finite set is closed.

S: If $f: R\to X$ is continuous, where R is given the usual topology and (X,τ) is a Hausdorff (T_2) space, then f is a one-one function.

Which of the above statements are correct?

(A) P and R (B) P and S (C) R and S (D) Q and S

Q.4 Let H be a Hilbert space and $S^{\perp}$ denote the orthogonal complement of a set $S\subseteq H$. Which of the following is **INCORRECT**?

(A) For $S_1, S_2 \subseteq H; S_1 \subseteq S_2 \Rightarrow S_1^{\perp} \subseteq S_2^{\perp}$ (B) $S\subseteq (S^{\perp})^{\perp}$

(C) $\{0\}^{\perp} = H$ (D) $S^{\perp}$ is always closed.

Q.5 Let H be a complex Hilbert space, $T: H\to H$ be a bounded linear operator and let T^* denote the adjoint of T. Which of the following statements are always **TRUE**?

P: $\forall x,y\in H, \langle Tx,y\rangle = \langle x, T^*y\rangle$ Q: $\forall x,y\in H, \langle x,Ty\rangle = \langle T^*x, y\rangle$

R: $\forall x,y\in H, \langle x,Ty\rangle = \langle x, T^*y\rangle$ S: $\forall x,y\in H, \langle Tx,Ty\rangle = \langle T^*x, T^*y\rangle$

(A) P and Q (B) P and R (C) Q and S (D) P and S

Q.6 Let $X=\{a,b,c\}$ and let $\Im=\{\phi,\{a\},\{b\},\{a,b\},X\}$ be a topology defined on X. Then which of the following statements are **TRUE**?

P: $(X,\Im)$ is a Hausdorff space. Q: $(X,\Im)$ is a regular space.

R: $(X,\Im)$ is a normal space. S: $(X,\Im)$ is a connected space.

(A) P and Q (B) Q and R (C) R and S (D) P and S

Q.7 Consider the statements

P: If X is a normed linear space and $M \subseteq X$ is a subspace, then the closure $\bar{M}$ is also a subspace of X.

Q: If X is a Banach space and $\sum x_n$ is an absolutely convergent series in X, then $\sum x_n$ is convergent.

R: Let M_1 and M_2 be subspaces of an inner product space such that $M_1 \cap M_2 = \{0\}$.Then $\forall m_1 \in M_1,\ m_2 \in M_2;\ \|m_1 + m_2\|^2 = \|m_1\|^2 + \|m_2\|^2$.

S: Let $f : X \to Y$ be a linear transformation from the Banach Space X into the Banach space Y. If f is continuous, then the graph of f is always compact.

The correct statements amongst the above are:

(A) P and R only (B) Q and R only (C) P and Q only (D) R and S only

Q.8 A continuous random variable X has the probability density function

$$f(x) = \begin{cases} \frac{3}{5}e^{-\frac{3}{5}x}, & x > 0 \\ 0, & x \le 0. \end{cases}$$

The probability density function of $Y = 3X + 2$ is

(A) $f(y) = \begin{cases} \frac{1}{5}e^{-\frac{1}{5}(y-2)}, & y > 2 \\ 0, & \text{y} \le 2 \end{cases}$ (B) $f(y) = \begin{cases} \frac{2}{5}e^{-\frac{2}{5}(y-2)}, & y > 2 \\ 0, & \text{y} \le 2 \end{cases}$

(C) $f(y) = \begin{cases} \frac{3}{5}e^{-\frac{3}{5}(y-2)}, & y > 2 \\ 0, & \text{y} \le 2 \end{cases}$ (D) $f(y) = \begin{cases} \frac{4}{5}e^{-\frac{4}{5}(y-2)}, & y > 2 \\ 0, & \text{y} \le 2 \end{cases}$

Q.9 A simple random sample of size 10 from $N(\mu, \sigma^2)$ gives 98% confidence interval (20.49, 23.51). Then the null hypothesis $H_0 :\ \mu = 20.5$ against $H_A : \mu \ne 20.5$

(A) can be rejected at 2% level of significance
(B) cannot be rejected at 5% level of significance
(C) can be rejected at 10% level of significance
(D) cannot be rejected at any level of significance

Q.10 For the linear programming problem

Maximize $z = x_1 + 2x_2 + 3x_3 - 4x_4$

Subject to

$$2x_1 + 3x_2 - x_3 - x_4 = 15$$
$$6x_1 + x_2 + x_3 - 3x_4 = 21$$
$$8x_1 + 2x_2 + 3x_3 - 4x_4 = 30$$
$$x_1,\ x_2,\ x_3,\ x_4 \ge 0,$$

$x_1 = 4,\ x_2 = 3,\ x_3 = 0,\ x_4 = 2$ is

(A) an optimal solution
(B) a degenerate basic feasible solution
(C) a non-degenerate basic feasible solution
(D) a non-basic feasible solution

Q.11 Which one of the following statements is **TRUE**?

(A) A convex set cannot have infinite many extreme points.
(B) A linear programming problem can have infinite many extreme points.
(C) A linear programming problem can have exactly two different optimal solutions.
(D) A linear programming problem can have a non-basic optimal solution.

Q.12 Let $\alpha = e^{2\pi i/5}$ and the matrix

$$M = \begin{bmatrix} 1 & \alpha & \alpha^2 & \alpha^3 & \alpha^4 \\ 0 & \alpha & \alpha^2 & \alpha^3 & \alpha^4 \\ 0 & 0 & \alpha^2 & \alpha^3 & \alpha^4 \\ 0 & 0 & 0 & \alpha^3 & \alpha^4 \\ 0 & 0 & 0 & 0 & \alpha^4 \end{bmatrix}.$$

Then the trace of the matrix $I + M + M^2$ is

(A) -5 (B) 0 (C) 3 (D) 5

Q.13 Let $V = \mathbb{C}^2$ be the vector space over the field of complex numbers and $B = \{(1,i),(i,1)\}$ be a given ordered basis of V. Then for which of the following, $B^* = \{f_1, f_2\}$ is a dual basis of B over $\mathbb{C}$?

(A) $f_1(z_1,z_2) = \frac{1}{2}(z_1 - iz_2)$, $f_2(z_1,z_2) = \frac{1}{2}(z_1 + iz_2)$

(B) $f_1(z_1,z_2) = \frac{1}{2}(z_1 + iz_2)$, $f_2(z_1,z_2) = \frac{1}{2}(iz_1 + z_2)$

(C) $f_1(z_1,z_2) = \frac{1}{2}(z_1 - iz_2)$, $f_2(z_1,z_2) = \frac{1}{2}(-iz_1 + z_2)$

(D) $f_1(z_1,z_2) = \frac{1}{2}(z_1 + iz_2)$, $f_2(z_1,z_2) = \frac{1}{2}(-iz_1 - z_2)$

Q.14 Let $R = \mathbb{Z}\times\mathbb{Z}\times\mathbb{Z}$ and $I = \mathbb{Z}\times\mathbb{Z}\times\{0\}$. Then which of the following statement is correct?

(A) I is a maximal ideal but not a prime ideal of R.
(B) I is a prime ideal but not a maximal ideal of R.
(C) I is both maximal ideal as well as a prime ideal of R.
(D) I is neither a maximal ideal nor a prime ideal of R.

Q.15 The function $u(r,\theta)$ satisfying the Laplace equation

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0, \qquad e < r < e^2$$

subject to the conditions $u(e,\theta) = 1$, $u(e^2,\theta) = 0$ is

(A) $\ln(e/r)$ (B) $\ln(e/r^2)$ (C) $\ln(e^2/r)$ (D) $\sum_{n=1}^{\infty}\left(\frac{r-e^2}{e-e^2}\right)\sin n\theta$

Q.16 The functional

$$\int_0^1 \left(y'^2 + (y+2y')y'' + kxyy' + y^2\right)dx, \; y(0)=0, y(1)=1, y'(0)=2, y'(1)=3$$

is path independent if k equals

(A) 1 (B) 2 (C) 3 (D) 4

Q.17 If a transformation $y = uv$ transforms the given differential equation $f(x)y'' - 4f'(x)y' + g(x)y = 0$ into the equation of the form $v'' + h(x)v = 0$, then u must be

(A) $1/f^2$ (B) xf (C) $1/2f$ (D) f^2

Q.18 The expression $\frac{1}{D_x^2 - D_y^2}\sin(x-y)$ is equal to

(A) $-\frac{x}{2}\cos(x-y)$ (B) $-\frac{x}{2}\sin(x-y)+\cos(x-y)$

(C) $-\frac{x}{2}\cos(x-y)+\sin(x-y)$ (D) $\frac{3x}{2}\sin(x-y)$

Q.19 The function $\phi(x)$ satisfying the integral equation

$$\int_0^x e^{x-\xi}\phi(\xi)\,d\xi = \frac{x^2}{2}$$

is

(A) $\frac{x^2}{2}$ (B) $x+\frac{x^2}{2}$ (C) $x-\frac{x^2}{2}$ (D) $1+\frac{x^2}{2}$

Q.20 Given the data:

x	1	2	3	4	5
y	–1	2	–3	4	–5

If the derivative of $y(x)$ is approximated as: $y'(x_k) \approx \frac{1}{h}(\Delta y_k + \frac{1}{2}\Delta^2 y_k - \frac{1}{4}\Delta^3 y_k)$, then the value of $y'(2)$ is

(A) 4 (B) 8 (C) 12 (D) 16

Q.21 If $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, then A^{50} is

(A) $\begin{bmatrix} 1 & 0 & 0 \\ 50 & 1 & 0 \\ 50 & 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 0 & 0 \\ 48 & 1 & 0 \\ 48 & 0 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 & 0 \\ 25 & 1 & 0 \\ 25 & 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 0 & 0 \\ 24 & 1 & 0 \\ 24 & 0 & 1 \end{bmatrix}$

Q.22 If $y = \sum_{m=0}^{\infty} c_m x^{r+m}$ is assumed to be a solution of the differential equation

$$x^2 y'' - xy' - 3(1+x^2)y = 0,$$

then the values of r are

(A) 1 and 3 (B) –1 and 3 (C) 1 and –3 (D) –1 and –3

Q.23 Let the linear transformation $T: F^2 \to F^3$ be defined by $T(x_1, x_2) = (x_1, x_1 + x_2, x_2)$. Then the nullity of T is

(A) 0 (B) 1 (C) 2 (D) 3

Q.24 The approximate eigenvalue of the matrix

$$A = \begin{bmatrix} -15 & 4 & 3 \\ 10 & -12 & 6 \\ 20 & -4 & 2 \end{bmatrix}$$

obtained after two iterations of Power method, with the initial vector $[1\ 1\ 1]^T$, is

(A) 7.768 (B) 9.468 (C) 10.548 (D) 19.468

Q.25 The root of the equation $xe^x = 1$ between 0 and 1, obtained by using two iterations of bisection method, is

(A) 0.25 (B) 0.50 (C) 0.75 (D) 0.65

Q. 26 to Q. 55 carry two marks each.

Q.26 Let $\int_C \left[\frac{1}{(z-2)^4} - \frac{(a-2)^2}{z} + 4 \right] dz = 4\pi$, where the close curve C is the triangle having vertices at i, $\left(\frac{-1-i}{\sqrt{2}}\right)$ and $\left(\frac{1-i}{\sqrt{2}}\right)$, the integral being taken in anti-clockwise direction. Then one value of a is

(A) $1+i$ (B) $2+i$ (C) $3+i$ (D) $4+i$

Q.27 The Lebesgue measure of the set $A = \left\{ 0 < x \le 1 : x \sin\left(\frac{\pi}{2x}\right) \ge 0 \right\}$ is

(A) 0 (B) 1 (C) ln2 (D) $1 - \ln\sqrt{2}$

Q.28 Which of the following statements are **TRUE**?

P : The set $\{x \in R : |\cos x| \le \frac{1}{2}\}$ is compact.

Q : The set $\{x \in R : \tan x \text{ is not differentiable}\}$ is complete.

R : The set $\{x \in R : \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} \text{ is convergent}\}$ is bounded.

S : The set $\{x \in R : f(x) = \cos x$ has a local maxima$\}$ is closed.

(A) P and Q (B) R and S (C) Q and S (D) P and S

Q.29 If a random variable X assumes only positive integral values, with the probability

$$P(X = x) = \frac{2}{3}\left(\frac{1}{3}\right)^{x-1}, x = 1, 2, 3, \ldots,$$

then $E(X)$ is

(A) 2/9 (B) 2/3 (C) 1 (D) 3/2

Q.30 The probability density function of the random variable X is

$$f(x)=\begin{cases}\frac{1}{\lambda}e^{-x/\lambda}, & x>0\\ 0, & x\le 0,\end{cases}$$

where $\lambda>0$. For testing the hypothesis $H_0:\lambda=3$ against $H_A:\lambda=5$, a test is given as "Reject H_0 if $X\ge 4.5$". The probability of type I error and power of this test are, respectively,

(A) 0.1353 and 0.4966 (B) 0.1827 and 0.379
(C) 0.2021 and 0.4493 (D) 0.2231 and 0.4066

Q.31 The order of the smallest possible non trivial group containing elements x and y such that $x^7=y^2=e$ and $yx=x^4y$ is

(A) 1 (B) 2 (C) 7 (D) 14

Q.32 The number of 5-Sylow subgroup(s) in a group of order 45 is

(A) 1 (B) 2 (C) 3 (D) 4

Q.33 The solution of the initial value problem

$$y''+2y'+10y=6\,\delta(t),\qquad y(0)=0,\quad y'(0)=0,$$

where $\delta(t)$ denotes the Dirac-delta function, is

(A) $2e^{t}\sin 3t$ (B) $6e^{t}\sin 3t$ (C) $2e^{-t}\sin 3t$ (D) $6e^{-t}\sin 3t$

Q.34 Let $\omega=\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}$, $M=\begin{pmatrix}0 & i\\ i & 0\end{pmatrix}$, $N=\begin{pmatrix}\omega & 0\\ 0 & \omega^2\end{pmatrix}$ and $G=\langle M,N\rangle$ be the group generated by the matrices M and N under matrix multiplication. Then

(A) $G/Z(G)\cong C_6$ (B) $G/Z(G)\cong S_3$ (C) $G/Z(G)\cong C_2$ (D) $G/Z(G)\cong C_4$

Q.35 The flux of the vector field $\vec{u}=x\hat{i}+y\hat{j}+z\hat{k}$ flowing out through the surface of the ellipsoid

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1,\ a>b>c>0,$$

is

(A) πabc (B) $2\pi abc$ (C) $3\pi abc$ (D) $4\pi abc$

Q.36 The integral surface satisfying the partial differential equation $\frac{\partial z}{\partial x}+z^2\frac{\partial z}{\partial y}=0$ and passing through the straight line $x=1, y=z$ is

(A) $(x-1)z+z^2=y^2$ (B) $x^2+y^2-z^2=1$
(C) $(y-z)x+x^2=1$ (D) $(x-1)z^2+z=y$

Q.37 The diffusion equation

$$\frac{\partial^2 u}{\partial x^2}=\frac{\partial u}{\partial t},\quad u=u(x,t),\quad u(0,t)=0=u(\pi,t),\quad u(x,0)=\cos x\sin 5x$$

admits the solution

(A) $\frac{e^{-36t}}{2}\left[\sin 6x+e^{20t}\sin 4x\right]$ (B) $\frac{e^{-36t}}{2}\left[\sin 4x+e^{20t}\sin 6x\right]$

(C) $\frac{e^{-20t}}{2}\left[\sin 3x+e^{15t}\sin 5x\right]$ (D) $\frac{e^{-36t}}{2}\left[\sin 5x+e^{20t}\sin x\right]$

Q.38 Let $f(x)$ and $xf(x)$ be the particular solutions of a differential equation

$$y''+R(x)y'+S(x)y=0.$$

Then the solution of the differential equation $y''+R(x)y'+S(x)y=f(x)$ is

(A) $y=\left(-\frac{x^2}{2}+\alpha x+\beta\right)f(x)$ (B) $y=\left(\frac{x^2}{2}+\alpha x+\beta\right)f(x)$

(C) $y=(-x^2+\alpha x+\beta)f(x)$ (D) $y=(x^3+\alpha x+\beta)f(x)$

Q.39 Let the Legendre equation $(1-x^2)y''-2xy'+n(n+1)y=0$ have n^{th} degree polynomial solution $y_n(x)$ such that $y_n(1)=3$. If $\int_{-1}^{1}\left(y_n^2(x)+y_{n-1}^2(x)\right)dx=\frac{144}{15}$, then n is

(A) 1 (B) 2 (C) 3 (D) 4

Q.40 The maximum value of the function $f(x,y,z)=xyz$ subject to the constraint $xy+yz+zx-a=0, a>0$ is

(A) $a^{\frac{3}{2}}$ (B) $(a/3)^{3/2}$ (C) $(3/a)^{3/2}$ (D) $(3a/2)^{3/2}$

Q.41 The functional $\int_0^1\left(y'^2+4y^2+8ye^x\right)dx,\quad y(0)=-\frac{4}{3},\quad y(1)=-\frac{4e}{3}$ possesses :

(A) strong minima on $y=-\frac{1}{3}e^x$ (B) strong minima on $y=-\frac{4}{3}e^x$

(C) weak maxima on $y=-\frac{1}{3}e^x$ (D) strong maxima on $y=-\frac{4}{3}e^x$

Q.42 A particle of mass *m* is constrained to move on a circle with radius *a* which itself is rotating about its vertical diameter with a constant angular velocity ω. Assume that the initial angular velocity is zero and g is the acceleration due to gravity. If θ be the inclination of the radius vector of the particle with the axis of rotation and $\dot{\theta}$ denotes the derivative of θ with respect to t, then the Lagrangian of this system is

(A) $\frac{1}{2}ma^2(\dot{\theta}^2+\omega^2\sin^2\theta)+mga\cos\theta$ (B) $\frac{1}{2}ma^2(\dot{\theta}^2+2\omega\sin\theta)-mga\sin\theta$

(C) $\frac{1}{2}ma^2(\dot{\theta}^2+2\omega^2\cos\theta)-mga\sin\theta$ (D) $\frac{1}{2}ma^2(\dot{\theta}^2+\omega\sin 2\theta)+mga\sin\theta$

Q.43 For the matrix

$$M = \begin{bmatrix} 2 & 3+2i & -4 \\ 3-2i & 5 & 6i \\ -4 & -6i & 3 \end{bmatrix},$$

which of the following statements are correct?

P : M is skew-Hermitian and iM is Hermitian
Q : M is Hermitian and iM is skew Hermitian
R : eigenvalues of M are real
S : eigenvalues of iM are real

(A) P and R only (B) Q and R only (C) P and S only (D) Q and S only

Q.44 Let $T : P_3 \to P_3$ be the map given by $T(p(x)) = \int_1^x p'(t)\,dt$. If the matrix of T relative to the standard bases $B_1 = B_2 = \{1, x, x^2, x^3\}$ is M and M' denotes the transpose of the matrix M, then $M + M'$ is

(A) $\begin{bmatrix} 0 & -1 & -1 & -1 \\ -1 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$ (B) $\begin{bmatrix} -1 & 0 & 0 & 2 \\ 0 & -1 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 2 & 0 & 2 & -1 \end{bmatrix}$

(C) $\begin{bmatrix} 2 & 0 & 0 & -1 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & -1 \\ -1 & 0 & -1 & 0 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & 2 & 2 & 2 \\ 2 & -1 & 0 & 0 \\ 2 & 0 & -1 & 0 \\ 2 & 0 & 0 & -1 \end{bmatrix}$

Q.45 Using Euler's method taking step size = 0.1, the approximate value of y obtained corresponding to $x = 0.2$ for the initial value problem $\frac{dy}{dx} = x^2 + y^2$ and $y(0) = 1$, is

(A) 1.322 (B) 1.122 (C) 1.222 (D) 1.110

Q.46 The following table gives the unit transportation costs, the supply at each origin and the demand of each destination for a transportation problem.

		Destination				
		D_1	D_2	D_3	D_4	Supply
	O_1	3	4	8	7	60
Origin	O_2	7	3	7	6	80
	O_3	3	9	3	4	100
Demand		40	70	50	80	

Let x_{ij} denote the number of units to be transported from origin i to destination j. If the u-v method is applied to improve the basic feasible solution given by $x_{12} = 60$, $x_{22} = 10$, $x_{23} = 50$, $x_{24} = 20$, $x_{31} = 40$ and $x_{34} = 60$, then the variables entering and leaving the basis, respectively, are

(A) x_{11} and x_{24} (B) x_{13} and x_{23} (C) x_{14} and x_{24} (D) x_{33} and x_{24}

Q.47 Consider the system of equations

$$\begin{bmatrix} 5 & -1 & 1 \\ 2 & 4 & 0 \\ 1 & 1 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 12 \\ -1 \end{bmatrix}.$$

Using Jacobi's method with the initial guess $\left[x^{(0)} \quad y^{(0)} \quad z^{(0)}\right]^T = [2.0 \quad 3.0 \quad 0.0]^T$, the approximate solution $\left[x^{(2)} \quad y^{(2)} \quad z^{(2)}\right]^T$ after two iterations, is

(A) $[2.64 \quad -1.70 \quad -1.12]^T$

(B) $[2.64 \quad -1.70 \quad 1.12]^T$

(C) $[2.64 \quad 1.70 \quad -1.12]^T$

(D) $[2.64 \quad 1.70 \quad 1.12]^T$

Common Data Questions

Common Data for Questions 48 and 49:

The optimal table for the primal linear programming problem:

Maximize $z = 6x_1 + 12x_2 + 12x_3 - 6x_4$

Subject to

$$x_1 + x_2 + x_3 = 4$$
$$x_1 + 4x_2 + x_4 = 8$$
$$x_1,\ x_2,\ x_3,\ x_4 \geq 0,$$

is

Basic variables (x_B)	x_1	x_2	x_3	x_4	RHS Constants (b)
x_3	3/4	0	1	-1/4	2
x_2	1/4	1	0	1/4	2
$z_j - c_j$	6	0	0	6	z = 48

Q.48 If y_1 and y_2 are the dual variables corresponding to the first and second primal constraints, then their values in the optimal solution of the dual problem are, respectively,

(A) 0 and 6 (B) 12 and 0 (C) 6 and 3 (D) 4 and 4

Q.49 If the right hand side of the second constraint is changed from 8 to 20, then in the optimal solution of the primal problem, the basic variables will be

(A) x_1 and x_2 (B) x_1 and x_3 (C) x_2 and x_3 (D) x_2 and x_4

Common Data for Questions 50 and 51:

Consider the Fredholm integral equation $u(x) = x + \lambda \int_0^1 x e^t u(t)\, dt$.

Q.50 The resolvent kernel $R(x,t;\lambda)$ for this integral equation is

(A) $\frac{xe^t}{1-\lambda}$ (B) $\frac{\lambda xe^t}{1+\lambda}$ (C) $\frac{xe^t}{1+\lambda^2}$ (D) $\frac{xe^t}{1-\lambda^2}$

Q.51 The solution of this integral equation is

(A) $\frac{x+1}{1-\lambda}$ (B) $\frac{x^2}{1-\lambda^2}$ (C) $\frac{x}{1+\lambda^2}$ (D) $\frac{x}{1-\lambda}$

Linked Answer Questions

Statement for Linked Answer Questions 52 and 53:
The joint probability density function of two random variables X and Y is given as

$$f(x,y)=\begin{cases}\frac{6}{5}(x+y^2), & 0\le x\le 1, 0\le y\le 1\\ 0, & \text{elsewhere}\end{cases}$$

Q.52 $E(X)$ and $E(Y)$ are, respectively,

(A) $\frac{2}{5}$ and $\frac{3}{5}$ (B) $\frac{3}{5}$ and $\frac{3}{5}$ (C) $\frac{3}{5}$ and $\frac{6}{5}$ (D) $\frac{4}{5}$ and $\frac{6}{5}$

Q.53 $Cov(X,Y)$ is

(A) -0.01 (B) 0 (C) 0.01 (D) 0.02

Statement for Linked Answer Questions 54 and 55:

Consider the functions $f(z)=\frac{z^2+\alpha z}{(z+1)^2}$ and $g(z) = \sinh(z-\frac{\pi}{2\alpha}), \alpha\neq 0$.

Q.54 The residue of $f(z)$ at its pole is equal to 1. Then the value of α is

(A) -1 (B) 1 (C) 2 (D) 3

Q.55 For the value of α obtained in Q.54, the function $g(z)$ is not conformal at a point

(A) $\frac{\pi(1+3i)}{6}$ (B) $\frac{\pi(3+i)}{6}$ (C) $\frac{2\pi}{3}$ (D) $\frac{i\pi}{2}$

General Aptitude (GA) Questions (Compulsory)

Q. 56 – Q. 60 carry one mark each.

Q.56 Choose the most appropriate word from the options given below to complete the following sentence:

Given the seriousness of the situation that he had to face, his ___ was impressive.

(A) beggary (B) nomenclature (C) jealousy (D) nonchalance

Q.57 Choose the most appropriate alternative from the options given below to complete the following sentence:

If the tired soldier wanted to lie down, he ___ the mattress out on the balcony.

(A) should take
(B) shall take
(C) should have taken
(D) will have taken

Q.58 If $(1.001)^{1259} = 3.52$ and $(1.001)^{2062} = 7.85$, then $(1.001)^{3321} =$

(A) 2.23 (B) 4.33 (C) 11.37 (D) 27.64

Q.59 One of the parts (A, B, C, D) in the sentence given below contains an ERROR. Which one of the following is **INCORRECT**?

I requested that he should be given the driving test today instead of tomorrow.

(A) requested that
(B) should be given
(C) the driving test
(D) instead of tomorrow

Q.60 Which one of the following options is the closest in meaning to the word given below?

Latitude

(A) Eligibility (B) Freedom (C) Coercion (D) Meticulousness

Q. 61 - Q. 65 carry two marks each.

Q.61 There are eight bags of rice looking alike, seven of which have equal weight and one is slightly heavier. The weighing balance is of unlimited capacity. Using this balance, the minimum number of weighings required to identify the heavier bag is

(A) 2 (B) 3 (C) 4 (D) 8

Q.62 Raju has 14 currency notes in his pocket consisting of only Rs. 20 notes and Rs. 10 notes. The total money value of the notes is Rs. 230. The number of Rs. 10 notes that Raju has is

(A) 5 (B) 6 (C) 9 (D) 10

Q.63 **One of the legacies of the Roman legions was discipline. In the legions, military law prevailed and discipline was brutal. Discipline on the battlefield kept units obedient, intact and fighting, even when the odds and conditions were against them.**

Which one of the following statements best sums up the meaning of the above passage?

(A) Thorough regimentation was the main reason for the efficiency of the Roman legions even in adverse circumstances.
(B) The legions were treated inhumanly as if the men were animals.
(C) Discipline was the armies' inheritance from their seniors.
(D) The harsh discipline to which the legions were subjected to led to the odds and conditions being against them.

Q.64 A and B are friends. They decide to meet between 1 PM and 2 PM on a given day. There is a condition that whoever arrives first will not wait for the other for more than 15 minutes. The probability that they will meet on that day is

(A) 1/4 (B) 1/16 (C) 7/16 (D) 9/16

Q.65 The data given in the following table summarizes the monthly budget of an average household.

Category	**Amount (Rs.)**
Food	4000
Clothing	1200
Rent	2000
Savings	1500
Other expenses	1800

The approximate percentage of the monthly budget **NOT** spent on savings is

(A) 10% (B) 14% (C) 81% (D) 86%

END OF THE QUESTION PAPER

GATE 2012 - Answer Key - Paper : MA

Paper	Question no.	Key
MA	1	D
MA	2	B
MA	3	A
MA	4	A
MA	5	A
MA	6	C
MA	7	C
MA	8	A
MA	9	C
MA	10	D
MA	11	D
MA	12	D
MA	13	C
MA	14	B
MA	15	C
MA	16	B
MA	17	D
MA	18	A
MA	19	C
MA	20	B
MA	21	C
MA	22	B
MA	23	A
MA	24	C
MA	25	C
MA	26	C
MA	27	D
MA	28	C
MA	29	D
MA	30	D
MA	31	B
MA	32	A
MA	33	Marks to All
MA	34	B
MA	35	D

Paper	Question no.	Key
MA	36	D
MA	37	A
MA	38	B
MA	39	B
MA	40	B
MA	41	B
MA	42	A
MA	43	B
MA	44	Marks to All
MA	45	C
MA	46	A
MA	47	C
MA	48	B
MA	49	D
MA	50	A
MA	51	D
MA	52	B
MA	53	A
MA	54	D
MA	55	A
MA	56	D
MA	57	A
MA	58	D
MA	59	B
MA	60	B
MA	61	A
MA	62	A
MA	63	A
MA	64	C
MA	65	D

MA : MATHEMATICS

Duration: Three Hours *Maximum Marks*: 100

Read the following instructions carefully.

1. Write your name and registration number in the space provided at the bottom of this page.
2. Take out the **O**ptical **R**esponse **S**heet **(ORS)** from this Question Booklet **without breaking the seal**.
3. Do not open the seal of the Question Booklet until you are asked to do so by the invigilator.
4. Write your registration number, your name and name of the examination centre at the specified locations on the right half of the **ORS**. Also, using HB pencil, darken the appropriate bubble under each digit of your registration number and the letters corresponding to your test paper code (MA).
5. This Question Booklet contains **20** pages including blank pages for rough work. After opening the seal at the specified time, please check all pages and report discrepancy, if any.
6. There are a total of 65 questions carrying 100 marks. All these questions are of objective type. Questions must be answered on the left hand side of the **ORS** by darkening the appropriate bubble (marked A, B, C, D) using HB pencil against the question number. **For each question darken the bubble of the correct answer**. In case you wish to change an answer, erase the old answer completely. More than one answer bubbled against a question will be treated as an incorrect response.
7. Questions Q.1 – Q.25 carry 1-mark each, and questions Q.26 – Q.55 carry 2-marks each.
8. Questions Q.48 – Q.51 (2 pairs) are common data questions and question pairs (Q.52, Q.53) and (Q.54, Q.55) are linked answer questions. The answer to the second question of the linked answer questions depends on the answer to the first question of the pair. If the first question in the linked pair is wrongly answered or is unattempted, then the answer to the second question in the pair will not be evaluated.
9. Questions Q.56 – Q.65 belong to General Aptitude (GA). Questions Q.56 – Q.60 carry 1-mark each, and questions Q.61 – Q.65 carry 2-marks each. The GA questions begin on a fresh page starting from page **14**.
10. Unattempted questions will result in zero mark and wrong answers will result in **NEGATIVE** marks. For Q.1 – Q.25 and Q.56 – Q.60, ⅓ mark will be deducted for each wrong answer. For Q.26 – Q.51 and Q.61 – Q.65, ⅔ mark will be deducted for each wrong answer. The question pairs (Q.52, Q.53), and (Q.54, Q.55) are questions with linked answers. There will be negative marks only for wrong answer to the first question of the linked answer question pair, i.e. for Q.52 and Q.54, ⅔ mark will be deducted for each wrong answer. There is no negative marking for Q.53 and Q.55.
11. Calculator is allowed whereas charts, graph sheets or tables are **NOT** allowed in the examination hall.
12. Rough work can be done on the question paper itself. Additionally, blank pages are provided at the end of the question paper for rough work.

Name								
Registration Number	**MA**							

Notations and Symbols used

$\mathbb{R}$	:	The set of all real numbers
$\mathbb{Z}$	:	The set of all integers
$\mathbb{C}$	:	The set of all complex numbers
$\mathbb{R}^n$	:	$\{(x_1,\ldots,x_n): x_i \in \mathbb{R} \text{ for } 1 \le i \le n\}$
l^p	:	The vector space of all scalar sequences $\{x_n\}$ such that $\sum_{i=1}^{\infty} \lvert x_i \rvert^p < \infty,\ 1 \le p < \infty$
c_{00}	:	Set of all sequences $x = \{x_n\}$ with finitely many non-zero terms
x^T	:	The transpose of the vector x
$N(\mu, \sigma^2)$	:	The normal distribution with mean μ and variance σ^2
χ_n^2	:	Chi-square distribution with n degrees of freedom
$P(E)$	:	Probability of an event E
$P(E\|F)$	:	Conditional probability of E given F
$E(X)$	:	Expectation of a random variable X
$E(X \mid Y = y)$	:	Conditional expectation of X given $Y = y$
$\exp(x)$	:	Exponential of x (that is e^x)
$\langle x, y \rangle$	:	Inner product of x and y
y'	:	$\frac{dy}{dx}$

Q. 1 – Q. 25 carry one mark each.

Q.1 The distinct eigenvalues of the matrix

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

are

(A) 0 and 1 (B) 1 and -1 (C) 1 and 2 (D) 0 and 2

Q.2 The minimal polynomial of the matrix

$$\begin{bmatrix} 3 & 3 & 0 \\ 3 & 3 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

is

(A) $x(x-1)(x-6)$ (B) $x(x-3)$ (C) $(x-3)(x-6)$ (D) $x(x-6)$

Q.3 Which of the following is the imaginary part of a possible value of $\ln(\sqrt{i})$?

(A) π (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{8}$

Q.4 Let $f:\mathbb{C}\to\mathbb{C}$ be analytic except for a simple pole at $z=0$ and let $g:\mathbb{C}\to\mathbb{C}$ be analytic.

Then, the value of $\dfrac{\operatorname{Res}_{z=0}\{f(z)\,g(z)\}}{\operatorname{Res}_{z=0} f(z)}$

is

(A) $g(0)$ (B) $g'(0)$ (C) $\lim_{z\to 0} z\,f(z)$ (D) $\lim_{z\to 0} z\,f(z)\,g(z)$

Q.5 Let $I=\oint_C (2x^2+y^2)\,dx+e^y\,dy$, where C is the boundary (oriented anticlockwise) of the region in the first quadrant bounded by $y=0,\ x^2+y^2=1$ and $x=0$. The value of I is

(A) -1 (B) $-\frac{2}{3}$ (C) $\frac{2}{3}$ (D) 1

Q.6 The series $\sum_{m=1}^{\infty} x^{\ln m}$, $x>0$, is convergent on the interval

(A) $(0, 1/e)$ (B) $(1/e,\ e)$ (C) $(0, e)$ (D) $(1, e)$

Q.7 While solving the equation $x^2-3x+1=0$ using the Newton-Raphson method with the initial guess of a root as 1, the value of the root after one iteration is

(A) 1.5 (B) 1 (C) 0.5 (D) 0

Q.8 Consider the system of equations

$$\begin{bmatrix} 5 & 2 & 1 \\ -2 & 5 & 2 \\ -1 & 2 & 8 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 13 \\ -22 \\ 14 \end{bmatrix}.$$

With the initial guess of the solution $[x_1^{(0)}, x_2^{(0)}, x_3^{(0)}]^T = [1, 1, 1]^T$, the approximate value of the solution $[x_1^{(1)}, x_2^{(1)}, x_3^{(1)}]^T$ after one iteration by the Gauss-Seidel method is

(A) $[2, \ -4.4, \ 1.625]^T$ (B) $[2, \ -4, \ -3]^T$
(C) $[2, \ 4.4, \ 1.625]^T$ (D) $[2, \ -4, \ 3]^T$

Q.9 Let y be the solution of the initial value problem

$$\frac{dy}{dx} = (y^2 + x); \ y(0) = 1.$$

Using Taylor series method of order 2 with the step size $h = 0.1$, the approximate value of $y(0.1)$ is

(A) 1.315 (B) 1.415 (C) 1.115 (D) 1.215

Q.10 The partial differential equation

$$x^2\frac{\partial^2 z}{\partial x^2} - (y^2-1)x\frac{\partial^2 z}{\partial x\partial y} + y(y-1)^2\frac{\partial^2 z}{\partial y^2} + x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 0$$

is hyperbolic in a region in the XY - plane if

(A) $x \neq 0$ and $y = 1$ (B) $x = 0$ and $y \neq 1$ (C) $x \neq 0$ and $y \neq 1$ (D) $x = 0$ and $y = 1$

Q.11 Which of the following functions is a probability density function of a random variable X ?

(A) $f(x) = \begin{cases} x(2-x), & 0 < x < 2 \\ 0, & \text{elsewhere} \end{cases}$ (B) $f(x) = \begin{cases} x(1-x), & 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$

(C) $f(x) = \begin{cases} 2xe^{-x^2}, & -1 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$ (D) $f(x) = \begin{cases} 2xe^{-x^2}, & x > 0 \\ 0, & \text{elsewhere} \end{cases}$

Q.12 Let X_1, X_2, X_3 and X_4 be independent standard normal random variables. The distribution of

$$W = \frac{1}{2}\{(X_1 - X_2)^2 + (X_3 - X_4)^2\}$$

is

(A) $N(0,1)$ (B) $N(0,2)$ (C) χ_2^2 (D) χ_4^2

Q.13 For $n \geq 1$, let $\{X_n\}$ be a sequence of independent random variables with

$$P(X_n = n) = P(X_n = -n) = \frac{1}{2n^2}, \qquad P(X_n = 0) = 1 - \frac{1}{n^2}.$$

Then, which of the following statements is **TRUE** for the sequence $\{X_n\}$?

(A) Weak Law of Large Numbers holds but Strong Law of Large Numbers does not hold
(B) Weak Law of Large Numbers does not hold but Strong Law of Large Numbers holds
(C) Both Weak Law of Large Numbers and Strong Law of Large Numbers hold
(D) Both Weak Law of Large Numbers and Strong Law of Large Numbers do not hold

Q.14 The Linear Programming Problem:

Maximize $z = x_1 + x_2$

subject to

$$x_1 + 2x_2 \le 20$$
$$x_1 + x_2 \le 15$$
$$x_2 \le 6$$
$$x_1, x_2 \ge 0$$

(A) has exactly one optimum solution
(B) has more than one optimum solutions
(C) has unbounded solution
(D) has no solution

Q.15 Consider the Primal Linear Programming Problem:

$$P: \begin{cases} \text{Maximize } z = c_1x_1 + c_2x_2 + \cdots + c_nx_n \\ \text{subject to} \\ \quad a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \le b_1 \\ \quad a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \le b_2 \\ \quad \vdots \qquad\qquad \vdots \qquad\qquad \vdots \\ \quad a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \le b_m \\ x_j \ge 0,\ j = 1, \ldots, n. \end{cases}$$

The Dual of P is

$$D: \begin{cases} \text{Minimize } z' = b_1w_1 + b_2w_2 + \cdots + b_mw_m \\ \text{subject to} \\ \quad a_{11}w_1 + a_{21}w_2 + \cdots + a_{m1}w_m \ge c_1 \\ \quad a_{12}w_1 + a_{22}w_2 + \cdots + a_{m2}w_m \ge c_2 \\ \quad \vdots \qquad\qquad \vdots \qquad\qquad \vdots \\ \quad a_{1n}w_1 + a_{2n}w_2 + \cdots + a_{mn}w_m \ge c_n \\ w_i \ge 0,\ i = 1, \ldots, m. \end{cases}$$

Which of the following statements is **FALSE**?

(A) If P has an optimal solution, then D also has an optimal solution
(B) The dual of the dual problem is a primal problem
(C) If P has an unbounded solution, then D has no feasible solution
(D) If P has no feasible solution, then D has a feasible solution

Q.16 The number of irreducible quadratic polynomials over the field of two elements $\mathbf{F}_2$ is

(A) 0 (B) 1 (C) 2 (D) 3

Q.17 The number of elements in the conjugacy class of the 3-cycle (2 3 4) in the symmetric group S_6 is

(A) 20 (B) 40 (C) 120 (D) 216

Q.18 The initial value problem

$$x\frac{dy}{dx} = y + x^2,\ x > 0;\ \ y(0) = 0,$$

has

(A) infinitely many solutions
(B) exactly two solutions
(C) a unique solution
(D) no solution

Q.19 The subspace $P = \{(x, y, z) \in \mathbb{R}^3 : z = x^2 + y^2 + 1\}$ is

(A) compact and connected
(B) compact but not connected
(C) not compact but connected
(D) neither compact nor connected

Q.20 Let $P = (0,1)$; $Q = [0,1)$; $U = (0,1]$; $S = [0,1]$, $T = \mathbb{R}$ and $A = \{P, Q, U, S, T\}$. The equivalence relation 'homeomorphism' induces which one of the following as the partition of A?

(A) $\{P,Q,U,S\}$, $\{T\}$
(B) $\{P,T\}$, $\{Q\}$, $\{U\}$, $\{S\}$
(C) $\{P,T\}$, $\{Q,U,S\}$
(D) $\{P,T\}$, $\{Q,U\}$, $\{S\}$

Q.21 Let $x = (x_1, x_2, \ldots) \in l^4$, $x \neq 0$. For which one of the following values of p, the series $\sum_{i=1}^{\infty} x_i y_i$ converges for every $y = (y_1, y_2, \ldots) \in l^p$?

(A) 1
(B) 2
(C) 3
(D) 4

Q.22 Let H be a complex Hilbert space and H^* be its dual. The mapping $\phi : H \to H^*$ defined by $\phi(y) = f_y$ where $f_y(x) = \langle x, y \rangle$ is

(A) not linear but onto
(B) both linear and onto
(C) linear but not onto
(D) neither linear nor onto

Q.23 A horizontal lever is in static equilibrium under the application of vertical forces F_1 at a distance l_1 from the fulcrum and F_2 at a distance l_2 from the fulcrum. The equilibrium for the above quantities can be obtained if

(A) $F_1 l_1 = 2F_2 l_2$
(B) $2F_1 l_1 = F_2 l_2$
(C) $F_1 l_1 = F_2 l_2$
(D) $F_1 l_1 < F_2 l_2$

Q.24 Assume F to be a twice continuously differentiable function. Let $J(y)$ be a functional of the form

$$\int_0^1 F(x, y')\, dx, \quad 0 \le x \le 1$$

defined on the set of all continuously differentiable functions y on $[0, 1]$ satisfying $y(0) = a$, $y(1) = b$. For some arbitrary constant c, a necessary condition for y to be an extremum of J is

(A) $\dfrac{\partial F}{\partial x} = c$
(B) $\dfrac{\partial F}{\partial y'} = c$
(C) $\dfrac{\partial F}{\partial y} = c$
(D) $\dfrac{\partial F}{\partial x} = 0$

Q.25 The eigenvalue λ of the following Fredholm integral equation

$$y(x) = \lambda \int_0^1 x^2 t\, y(t)\, dt,$$

is

(A) -2
(B) 2
(C) 4
(D) -4

Q. 26 to Q. 55 carry two marks each.

Q.26 The application of Gram-Schmidt process of orthonormalization to
$u_1=(1,1,0)$, $u_2=(1,0,0)$, $u_3=(1,1,1)$ yields

(A) $\frac{1}{\sqrt{2}}(1,1,0),(1,0,0),(0,0,1)$
(B) $\frac{1}{\sqrt{2}}(1,1,0),\frac{1}{\sqrt{2}}(1,-1,0),\frac{1}{\sqrt{2}}(1,1,1)$
(C) $(0,1,0),(1,0,0),(0,0,1)$
(D) $\frac{1}{\sqrt{2}}(1,1,0),\frac{1}{\sqrt{2}}(1,-1,0),(0,0,1)$

Q.27 Let $T:\mathbb{C}^3\to\mathbb{C}^3$ be defined by $T\begin{pmatrix}z_1\\z_2\\z_3\end{pmatrix}=\begin{pmatrix}z_1-iz_2\\iz_1+z_2\\z_1+z_2+iz_3\end{pmatrix}$. Then, the adjoint T^* of T is given by $T^*\begin{pmatrix}z_1\\z_2\\z_3\end{pmatrix}=$

(A) $\begin{pmatrix}z_1+iz_2\\-iz_1+z_2\\z_1+z_2-iz_3\end{pmatrix}$
(B) $\begin{pmatrix}z_1-iz_2+z_3\\-iz_1+z_2+z_3\\iz_3\end{pmatrix}$
(C) $\begin{pmatrix}z_1-iz_2+z_3\\iz_1+z_2+z_3\\-iz_3\end{pmatrix}$
(D) $\begin{pmatrix}iz_1+z_2\\z_1-iz_2\\z_1-z_2-iz_3\end{pmatrix}$

Q.28 Let $f(z)$ be an entire function such that $|f(z)|\le K|z|$, $\forall z\in\mathbb{C}$, for some $K>0$. If $f(1)=i$, the value of $f(i)$ is

(A) 1 (B) -1 (C) i (D) $-i$

Q.29 Let y be the solution of the initial value problem

$$\frac{d^2y}{dx^2}+y=6\cos 2x\,,\quad y(0)=3,\ y'(0)=1\,.$$

Let the Laplace transform of y be $F(s)$. Then, the value of $F(1)$ is

(A) $\frac{17}{5}$ (B) $\frac{13}{5}$ (C) $\frac{11}{5}$ (D) $\frac{9}{5}$

Q.30 For $0\le x\le 1$, let

$$f_n(x)=\begin{cases}\dfrac{n}{1+n}, & \text{if } x \text{ is irrational}\\ 0, & \text{if } x \text{ is rational}\end{cases}$$

and $f(x)=\lim_{n\to\infty}f_n(x)$. Then, on the interval [0, 1]

(A) f is measurable and Riemann integrable
(B) f is measurable and Lebesgue integrable
(C) f is not measurable
(D) f is not Lebesgue integrable

Q.31 If x, y and z are positive real numbers, then the minimum value of

$$x^2+8y^2+27z^2 \quad \text{where} \quad \frac{1}{x}+\frac{1}{y}+\frac{1}{z}=1$$

is

(A) 108 (B) 216 (C) 405 (D) 1048

Q.32 Let $T:\mathbb{R}^4 \to \mathbb{R}^4$ be defined by

$$T(x,y,z,w)=(x+y+5w, x+2y+w, -z+2w, 5x+y+2z).$$

The dimension of the eigenspace of T is

(A) 1 (B) 2 (C) 3 (D) 4

Q.33 Let y be a polynomial solution of the differential equation

$$(1-x^2)y''-2xy'+6y=0.$$

If $y(1)=2$, then the value of the integral $\int_{-1}^{1} y^2\,dx$ is

(A) $\frac{1}{5}$ (B) $\frac{2}{5}$ (C) $\frac{4}{5}$ (D) $\frac{8}{5}$

Q.34 The value of the integral

$$I=\int_{-1}^{1}\exp(x^2)\,dx$$

using a rectangular rule is approximated as 2. Then, the approximation error $|I-2|$ lies in the interval

(A) $(2e,3e]$ (B) $(2/3,\ 2e]$ (C) $(e/8,\ 2/3]$ (D) $(0,\ e/8]$

Q.35 The integral surface for the Cauchy problem

$$\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=1,$$

which passes through the circle $z=0,\ x^2+y^2=1$ is

(A) $x^2+y^2+2z^2+2zx-2yz-1=0$
(B) $x^2+y^2+2z^2+2zx+2yz-1=0$
(C) $x^2+y^2+2z^2-2zx-2yz-1=0$
(D) $x^2+y^2+2z^2+2zx+2yz+1=0$

Q.36 The vertical displacement $u(x,t)$ of an infinitely long elastic string is governed by the initial value problem

$$\frac{\partial^2 u}{\partial t^2}=4\frac{\partial^2 u}{\partial x^2}, \quad -\infty<x<\infty,\ t>0,$$

$$u(x,0)=-x \quad \text{and} \quad \frac{\partial u}{\partial t}(x,0)=0.$$

The value of $u(x,t)$ at $x=2$ and $t=2$ is equal to

(A) 2 (B) 4 (C) −2 (D) −4

Q.37 We have to assign four jobs I, II, III, IV to four workers A, B, C and D. The time taken by different workers (in hours) in completing different jobs is given below:

		I	II	III	IV
	A	5	3	2	8
Workers	B	7	9	2	6
	C	6	4	5	7
	D	5	7	7	8

The optimal assignment is as follows:

Job III to worker A; Job IV to worker B; Job II to worker C and Job I to worker D and hence the time taken by different workers in completing different jobs is now changed as:

		I	II	III	IV
	A	5	3	2	5
Workers	B	7	9	2	3
	C	4	2	3	2
	D	5	7	7	5

Then the minimum time (in hours) taken by the workers to complete all the jobs is

(A) 10 (B) 12 (C) 15 (D) 17

Q.38 The following table shows the information on the availability of supply to each warehouse, the requirement of each market and unit transportation cost (in rupees) from each warehouse to each market.

		Market				
		M_1	M_2	M_3	M_4	Supply
	W_1	6	3	5	4	22
Warehouse	W_2	5	9	2	7	15
	W_3	5	7	8	6	8
Requirement		7	12	17	9	

The present transportation schedule is as follows:

W_1 to M_2: 12 units; W_1 to M_3: 1 unit; W_1 to M_4: 9 units; W_2 to M_3: 15 units; W_3 to M_1: 7 units and W_3 to M_3: 1 unit. Then the minimum total transportation cost (in rupees) is

(A) 150 (B) 149 (C) 148 (D) 147

Q.39 If $\mathbb{Z}[i]$ is the ring of Gaussian integers, the quotient $\mathbb{Z}[i]/(3-i)$ is isomorphic to

(A) $\mathbb{Z}$ (B) $\mathbb{Z}/3\mathbb{Z}$ (C) $\mathbb{Z}/4\mathbb{Z}$ (D) $\mathbb{Z}/10\mathbb{Z}$

Q.40 For the rings $L = \dfrac{\mathbb{R}[x]}{\langle x^2 - x + 1\rangle}$; $M = \dfrac{\mathbb{R}[x]}{\langle x^2 + x + 1\rangle}$; $N = \dfrac{\mathbb{R}[x]}{\langle x^2 + 2x + 1\rangle}$;

which one of the following is **TRUE**?

(A) L is isomorphic to M; L is not isomorphic to N; M is not isomorphic to N
(B) M is isomorphic to N; M is not isomorphic to L; N is not isomorphic to L
(C) L is isomorphic to M; M is isomorphic to N
(D) L is not isomorphic to M; L is not isomorphic to N; M is not isomorphic to N

Q.41 The time to failure (in hours) of a component is a continuous random variable T with the probability density function

$$f(t)=\begin{cases}\frac{1}{10}e^{-\frac{t}{10}}, & t>0,\\ 0, & t\le 0.\end{cases}$$

Ten of these components are installed in a system and they work independently. Then, the probability that **NONE** of these fail before ten hours, is

(A) e^{-10} (B) $1-e^{-10}$ (C) $10e^{-10}$ (D) $1-10e^{-10}$

Q.42 Let X be the real normed linear space of all real sequences with finitely many non-zero terms, with supremum norm and $T: X \to X$ be a one to one and onto linear operator defined by

$$T(x_1,x_2,x_3\ldots)=\left(x_1,\frac{x_2}{2^2},\frac{x_3}{3^2},\ldots\right).$$

Then, which of the following is **TRUE**?

(A) T is bounded but T^{-1} is not bounded
(B) T is not bounded but T^{-1} is bounded
(C) Both T and T^{-1} are bounded
(D) Neither T nor T^{-1} is bounded

Q.43 Let $e_i=(0,\ldots,0,1,0,\ldots)$ (i.e., e_i is the vector with 1 at the i^{th} place and 0 elsewhere) for $i=1,2,\ldots$.

Consider the statements:

P: $\{f(e_i)\}$ converges for every continuous linear functional on l^2.

Q: $\{e_i\}$ converges in l^2.

Then, which of the following holds?

(A) Both P and Q are TRUE
(B) P is TRUE but Q is not TRUE
(C) P is not TRUE but Q is TRUE
(D) Neither P nor Q is TRUE

Q.44 For which subspace $X\subseteq\mathbb{R}$ with the usual topology and with $\{0,1\}\subseteq X$, will a continuous function $f: X\to\{0,1\}$ satisfying $f(0)=0$ and $f(1)=1$ exist?

(A) $X=[0,1]$ (B) $X=[-1,1]$ (C) $X=\mathbb{R}$ (D) $[0,1]\not\subset X$

Q.45 Suppose X is a finite set with more than five elements. Which of the following is **TRUE**?

(A) There is a topology on X which is T_3
(B) There is a topology on X which is T_2 but not T_3
(C) There is a topology on X which is T_1 but not T_2
(D) There is no topology on X which is T_1

Q.46 A massless wire is bent in the form of a parabola $z = r^2$ and a bead slides on it smoothly. The wire is rotated about z-axis with a constant angular acceleration α. Assume that m is the mass of the bead, ω is the initial angular velocity and g is the acceleration due to gravity. Then, the Lagrangian at any time t is

(A) $\frac{m}{2}\left[\left(\frac{dr}{dt}\right)^2(1+4r^2)+r^2(\omega+\alpha t)^2+2gr^2\right]$

(B) $\frac{m}{2}\left[\left(\frac{dr}{dt}\right)^2(1+4r^2)-r^2(\omega+\alpha t)^2+2gr^2\right]$

(C) $\frac{m}{2}\left[\left(\frac{dr}{dt}\right)^2(1+4r^2)-r^2(\omega+\alpha t)^2-2gr^2\right]$

(D) $\frac{m}{2}\left[\left(\frac{dr}{dt}\right)^2(1+4r^2)+r^2(\omega+\alpha t)^2-2gr^2\right]$

Q.47 On the interval $[0,\ 1]$, let y be a twice continuously differentiable function which is an extremal of the functional

$$J(y)=\int_0^1 \frac{\sqrt{1+2y'^2}}{x}\,dx$$

with $y(0)=1,\ y(1)=2$. Then, for some arbitrary constant c, y satisfies

(A) $y'^2(2-c^2x^2)=c^2x^2$

(B) $y'^2(2+c^2x^2)=c^2x^2$

(C) $y'^2(1-c^2x^2)=c^2x^2$

(D) $y'^2(1+c^2x^2)=c^2x^2$

Common Data Questions

Common Data for Questions 48 and 49:

Let X and Y be two continuous random variables with the joint probability density function

$$f(x,y) = \begin{cases} 2, & 0<x+y<1, \quad x>0, \quad y>0, \\ 0, & \text{elsewhere.} \end{cases}$$

Q.48 $P\left(X+Y<\frac{1}{2}\right)$ is

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{3}{4}$ (D) 1

Q.49 $E\left(X \mid Y=\frac{1}{2}\right)$ is

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) 1 (D) 2

Common Data for Questions 50 and 51:

Let $f(z)=\frac{z}{8-z^3}$, $z=x+iy$.

Q.50 $\operatorname*{Res}_{z=2} f(z)$ is

(A) $-\frac{1}{8}$ (B) $\frac{1}{8}$ (C) $-\frac{1}{6}$ (D) $\frac{1}{6}$

Q.51 The Cauchy principal value of $\int_{-\infty}^{\infty} f(x)\,dx$ is

(A) $-\frac{\pi}{6}\sqrt{3}$ (B) $-\frac{\pi}{8}\sqrt{3}$ (C) $\pi\sqrt{3}$ (D) $-\pi\sqrt{3}$

Linked Answer Questions

Statement for Linked Answer Questions 52 and 53:

Let $f_n(x)=\dfrac{x}{\{(n-1)x+1\}\{nx+1\}}$ and $s_n(x)=\sum_{j=1}^{n} f_j(x)$ for $x\in[0,1]$.

Q.52 The sequence $\{s_n\}$

(A) converges uniformly on $[0,1]$

(B) converges pointwise on $[0,1]$ but not uniformly

(C) converges pointwise for $x=0$ but not for $x\in(0,1]$

(D) does not converge for $x\in[0,1]$

Q.53 $\lim_{n\to\infty}\int_0^1 s_n(x)\,dx=1$

(A) by dominated convergence theorem

(B) by Fatou's lemma

(C) by the fact that $\{s_n\}$ converges uniformly on $[0,1]$

(D) by the fact that $\{s_n\}$ converges pointwise on $[0,1]$

Statement for Linked Answer Questions 54 and 55:

The matrix $A=\begin{bmatrix}1&1&1\\2&1&2\\1&3&2\end{bmatrix}$ can be decomposed into the product of a lower triangular matrix L and an upper triangular matrix U as $A=LU$ where

$$L=\begin{bmatrix}1&0&0\\l_{21}&1&0\\l_{31}&l_{32}&1\end{bmatrix} \text{ and } U=\begin{bmatrix}u_{11}&u_{12}&u_{13}\\0&u_{22}&u_{23}\\0&0&u_{33}\end{bmatrix}.$$

Let $x, z\in\mathbb{R}^3$ and $b=[1,\ 1,\ 1]^T$.

Q.54 The solution $z=[z_1,\ z_2,\ z_3]^T$ of the system $Lz=b$ is

(A) $[-1,\ -1,\ -2]^T$ (B) $[1,\ -1,\ 2]^T$ (C) $[1,\ -1,\ -2]^T$ (D) $[-1,\ 1,\ 2]^T$

Q.55 The solution $x=[x_1,\ x_2,\ x_3]^T$ of the system $Ux=z$ is

(A) $[2,\ 1,\ -2]^T$ (B) $[2,\ 1,\ 2]^T$ (C) $[-2,\ -1,\ -2]^T$ (D) $[-2,\ 1,\ -2]^T$

General Aptitude (GA) Questions

Q. 56 – Q. 60 carry one mark each.

Q.56 Choose the most appropriate word from the options given below to complete the following sentence:
It was her view that the country's problems had been ———————— by foreign technocrats, so that to invite them to come back would be counter-productive.

(A) identified
(B) ascertained
(C) exacerbated
(D) analysed

Q.57 There are two candidates P and Q in an election. During the campaign, 40% of the voters promised to vote for P, and rest for Q. However, on the day of election 15% of the voters went back on their promise to vote for P and instead voted for Q. 25% of the voters went back on their promise to vote for Q and instead voted for P. Suppose, P lost by 2 votes, then what was the total number of voters?

(A) 100 (B) 110 (C) 90 (D) 95

Q.58 The question below consists of a pair of related words followed by four pairs of words. Select the pair that best expresses the relation in the original pair:
Gladiator : Arena

(A) dancer : stage
(B) commuter : train
(C) teacher : classroom
(D) lawyer : courtroom

Q.59 Choose the most appropriate word from the options given below to complete the following sentence:
Under ethical guidelines recently adopted by the Indian Medical Association, human genes are to be manipulated only to correct diseases for which ———————— treatments are unsatisfactory.

(A) similar
(B) most
(C) uncommon
(D) available

Q.60 Choose the word from the options given below that is most nearly opposite in meaning to the given word:
Frequency

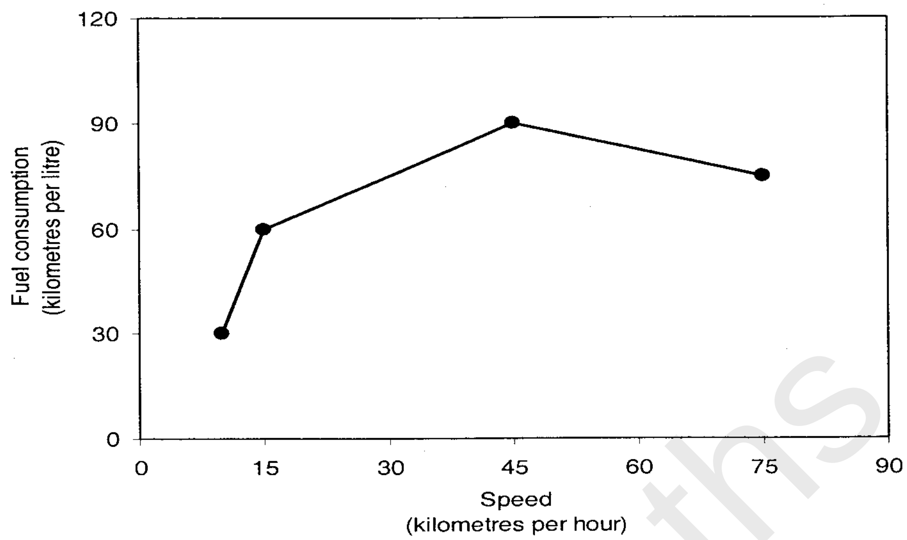
(A) periodicity
(B) rarity
(C) gradualness
(D) persistency

Q. 61 to Q. 65 carry two marks each.

Q.61 Three friends, R, S and T shared toffee from a bowl. R took $1/3^{rd}$ of the toffees, but returned four to the bowl. S took $1/4^{th}$ of what was left but returned three toffees to the bowl. T took half of the remainder but returned two back into the bowl. If the bowl had 17 toffees left, how many toffees were originally there in the bowl?

(A) 38 (B) 31 (C) 48 (D) 41

Q.62 The fuel consumed by a motorcycle during a journey while traveling at various speeds is indicated in the graph below.



The distances covered during four laps of the journey are listed in the table below

Lap	Distance (kilometres)	Average speed (kilometres per hour)
P	15	15
Q	75	45
R	40	75
S	10	10

From the given data, we can conclude that the fuel consumed per kilometre was least during the lap

(A) P (B) Q (C) R (D) S

Q.63 **The horse has played a little known but very important role in the field of medicine. Horses were injected with toxins of diseases until their blood built up immunities. Then a serum was made from their blood. Serums to fight with diphtheria and tetanus were developed this way.**

It can be inferred from the passage, that horses were

(A) given immunity to diseases
(B) generally quite immune to diseases
(C) given medicines to fight toxins
(D) given diphtheria and tetanus serums

Q.64 The sum of n terms of the series 4+44+444+…. is

(A) $(4/81)\,[10^{n+1} - 9n - 1]$
(B) $(4/81)\,[10^{n-1} - 9n - 1]$
(C) $(4/81)\,[10^{n+1} - 9n - 10]$
(D) $(4/81)\,[10^{n} - 9n - 10]$

Q.65 Given that $f(y) = |y| / y$, and q is any non-zero real number, the value of $|f(q) - f(-q)|$ is

(A) 0 (B) –1 (C) 1 (D) 2

END OF THE QUESTION PAPER

GATE 2011 - Answer Key - Paper : MA

Paper	Question no.	Key
MA	1	D
MA	2	D
MA	3	C
MA	4	A
MA	5	B
MA	6	A
MA	7	D
MA	8	D
MA	9	C
MA	10	C
MA	11	D
MA	12	C
MA	13	C
MA	14	B
MA	15	D
MA	16	B
MA	17	B
MA	18	A
MA	19	C
MA	20	D
MA	21	A
MA	22	A
MA	23	C
MA	24	B
MA	25	C
MA	26	C/D
MA	27	C
MA	28	B
MA	29	B
MA	30	B
MA	31	B
MA	32	MarkstoAll
MA	33	D
MA	34	B
MA	35	C
MA	36	C
MA	37	B
MA	38	B
MA	39	D
MA	40	A
MA	41	A
MA	42	A
MA	43	B
MA	44	D
MA	45	A

Paper	Question no.	Key
MA	46	D
MA	47	MarkstoAll
MA	48	A
MA	49	A
MA	50	C
MA	51	A
MA	52	B
MA	53	A/B
MA	54	C
MA	55	A
MA	56	C
MA	57	A
MA	58	D
MA	59	D
MA	60	B
MA	61	C
MA	62	B
MA	63	A
MA	64	C
MA	65	D

MA : MATHEMATICS

Duration: Three Hours *Maximum Marks*: 100

Read the following instructions carefully.

1. This question paper contains **24** pages including blank pages for rough work. Please check all pages and report discrepancy, if any.
2. Write your registration number, your name and name of the examination centre at the specified locations on the right half of the **Optical Response Sheet** (**ORS**).
3. Using HB pencil, darken the appropriate bubble under each digit of your registration number and the letters corresponding to your paper code.
4. All questions in this paper are of objective type.
5. Questions must be answered on the **ORS** by darkening the appropriate bubble (marked A, B, C, D) using HB pencil against the question number on the left hand side of the ORS. **For each question darken the bubble of the correct answer**. In case you wish to change an answer, erase the old answer completely. More than one answer bubbled against a question will be treated as an incorrect response.
6. There are a total of 65 questions carrying 100 marks.
7. Questions Q.1 – Q.25 will carry 1-mark each, and questions Q.26 – Q.55 will carry 2-marks each.
8. Questions Q.48 – Q.51 (2 pairs) are common data questions and question pairs (Q.52, Q.53) and (Q.54, Q.55) are linked answer questions. The answer to the second question of the linked answer questions depends on the answer to the first question of the pair. If the first question in the linked pair is wrongly answered or is un-attempted, then the answer to the second question in the pair will not be evaluated.
9. Questions Q.56 – Q.65 belong to General Aptitude (GA). Questions Q.56 – Q.60 will carry 1-mark each, and questions Q.61 – Q.65 will carry 2-marks each. The GA questions will begin on a fresh page starting from page 13.
10. Un-attempted questions will carry zero marks.
11. Wrong answers will carry **NEGATIVE** marks. For Q.1 – Q.25 and Q.56 – Q.60, ⅓ mark will be deducted for each wrong answer. For Q.26 – Q.51 and Q.61 – Q.65, ⅔ mark will be deducted for each wrong answer. The question pairs (Q.52, Q.53), and (Q.54, Q.55) are questions with linked answers. There will be negative marks only for wrong answer to the first question of the linked answer question pair i.e. for Q.52 and Q.54, ⅔ mark will be deducted for each wrong answer. There is no negative marking for Q.53 and Q.55.
12. Calculator (without data connectivity) is allowed in the examination hall.
13. Charts, graph sheets or tables are NOT allowed in the examination hall.
14. Rough work can be done on the question paper itself. Additionally, blank pages are provided at the end of the question paper for rough work.

Notations and Symbols used

$X \setminus Y$	: $\{x \in X : x \notin Y\}$
$\mathbb{N}$	: The set of all natural numbers
$\mathbb{Z}$	: The set of all integers
$\mathbb{Z}_n$	: The set of all integers modulo n
$\mathbb{Q}$	: The set of all rational numbers
$\mathbb{R}$	: The set of all real numbers
$\mathbb{R}^n$	: The set of all n-tuples of real numbers
$\mathbb{C}$	: The set of all complex numbers
$C[0,1]$	: The set of all complex valued continuous functions on $[0,1]$
$P_n[a,b]$	: The set of all polynomials of degree at most n defined on $[a,b]$
$GL(2, \mathbb{R})$	: The group of all 2×2 real invertible matrices under multiplication
S_n	: The set of all permutations on n symbols
A_n	: Alternating group on n symbols
$P(E)$	: Probability of an event E
$P(E \mid F)$	: Conditional probability of E given F
$E(X)$	: Expectation of the random variable X
$E(X \mid Y = y)$	: Conditional expectation of X given $Y = y$

Q.1 – Q.25 carry one mark each.

Q.1 Let E and F be any two events with $P(E\cup F)=0.8$, $P(E)=0.4$ and $P(E\,|\,F)=0.3$. Then $P(F)$ is

(A) $\frac{3}{7}$ (B) $\frac{4}{7}$ (C) $\frac{3}{5}$ (D) $\frac{2}{5}$

Q.2 Let X have a binomial distribution with parameters n and p, where n is an integer greater than 1 and $0<p<1$. If $P(X=0)=P(X=1)$, then the value of p is

(A) $\frac{1}{n-1}$ (B) $\frac{n}{n+1}$ (C) $\frac{1}{n+1}$ (D) $\frac{1}{1+n^{\frac{1}{n-1}}}$

Q.3 Let $u(x,y)=2x(1-y)$ for all real x and y. Then a function $v(x,y)$, so that $f(z)=u(x,y)+i\,v(x,y)$ is analytic, is

(A) $x^2-(y-1)^2$ (B) $(x-1)^2-y^2$

(C) $(x-1)^2+y^2$ (D) $x^2+(y-1)^2$

Q.4 Let $f(z)$ be analytic on $D=\{z\in\mathbb{C}:|z-1|<1\}$ such that $f(1)=1$. If $f(z)=f(z^2)$ for all $z\in D$, then which one of the following statements is **NOT** correct?

(A) $f(z)=[f(z)]^2$ for all $z\in D$ (B) $f\left(\frac{z}{2}\right)=\frac{1}{2}f(z)$ for all $z\in D$

(C) $f(z^3)=[f(z)]^3$ for all $z\in D$ (D) $f'(1)=0$

Q.5 The maximum number of linearly independent solutions of the differential equation $\frac{d^4y}{dx^4}=0$, with the condition $y(0)=1$, is

(A) 4 (B) 3 (C) 2 (D) 1

Q.6 Which one of the following sets of functions is **NOT** orthogonal (with respect to the L^2-inner product) over the given interval?

(A) $\{\sin nx : n\in\mathbb{N}\}$, $-\pi<x<\pi$ (B) $\{\cos nx : n\in\mathbb{N}\}$, $-\pi<x<\pi$

(C) $\left\{x^{2n+\frac{1}{2}} : n\in\mathbb{N}\right\}$, $-1<x<1$ (D) $\{x^{2n+1} : n\in\mathbb{N}\}$, $-1<x<1$

Q.7 If $f:[1,2]\to\mathbb{R}$ is a non-negative Riemann-integrable function such that $\int_1^2 \frac{f(x)}{\sqrt{x}}\,dx = k\int_1^2 f(x)\,dx \neq 0$, then k belongs to the interval

(A) $\left[0,\frac{1}{3}\right]$ (B) $\left(\frac{1}{3},\frac{2}{3}\right]$ (C) $\left(\frac{2}{3},1\right]$ (D) $\left(1,\frac{4}{3}\right]$

Q.8 The set $X = \mathbb{R}$ with the metric $d(x, y) = \frac{|x-y|}{1+|x-y|}$ is

(A) bounded but not compact
(B) bounded but not complete
(C) complete but not bounded
(D) compact but not complete

Q.9 Let $f(x,y) = \begin{cases} \frac{xy}{(x^2+y^2)^{5/2}}[1-\cos(x^2+y^2)], & (x,y) \neq (0,0) \\ k, & (x,y) = (0,0). \end{cases}$

Then the value of k for which $f(x,y)$ is continuous at $(0,0)$ is

(A) 0
(B) $\frac{1}{2}$
(C) 1
(D) $\frac{3}{2}$

Q.10 Let A and B be disjoint subsets of $\mathbb{R}$ and let m^* denote the Lebesgue outer measure on $\mathbb{R}$.

Consider the statements:

$P: m^*(A \cup B) = m^*(A) + m^*(B)$

Q: Both A and B are Lebesgue measurable

R: One of A and B is Lebesgue measureable

Which one of the following is correct?

(A) If P is true, then Q is true
(B) If P is NOT true, then R is true
(C) If R is true, then P is NOT true
(D) If R is true, then P is true

Q.11 Let $f: \mathbb{R} \to [0, \infty)$ be a Lebesgue measurable function and E be a Lebesgue measurable subset of $\mathbb{R}$ such that $\int_E f\, dm = 0$, where m is the Lebesgue measure on $\mathbb{R}$. Then

(A) $m(E) = 0$
(B) $\{x \in \mathbb{R} : f(x) = 0\} = E$
(C) $m(\{x \in E : f(x) \neq 0\}) = 0$
(D) $m(\{x \in E : f(x) = 0\}) = 0$

Q.12 If the nullity of the matrix $\begin{bmatrix} k & 1 & 2 \\ 1 & -1 & -2 \\ 1 & 1 & 4 \end{bmatrix}$ is 1, then the value of k is

(A) -1
(B) 0
(C) 1
(D) 2

Q.13 If a 3×3 real skew-symmetric matrix has an eigenvalue $2i$, then one of the remaining eigenvalues is

(A) $\frac{1}{2i}$
(B) $-\frac{1}{2i}$
(C) 0
(D) 1

Q.14 For the linear programming problem

$$\text{Minimize } z = x - y, \text{ subject to } 2x + 3y \le 6,\ 0 \le x \le 3,\ \ 0 \le y \le 3,$$

the number of extreme points of its feasible region and the number of basic feasible solutions respectively, are

(A) 3 and 3 (B) 4 and 4 (C) 3 and 5 (D) 4 and 5

Q.15 Which one of the following statements is correct?

(A) If a Linear Programming Problem (LPP) is infeasible, then its dual is also infeasible
(B) If an LPP is infeasible, then its dual always has unbounded solution
(C) If an LPP has unbounded solution, then its dual also has unbounded solution
(D) If an LPP has unbounded solution, then its dual is infeasible

Q.16 Which one of the following groups is simple?

(A) S_3 (B) $GL(2, \mathbb{R})$ (C) $\mathbb{Z}_2 \times \mathbb{Z}_2$ (D) A_5

Q.17 Consider the algebraic extension $E = \mathbb{Q}\left(\sqrt{2}, \sqrt{3}, \sqrt{5}\right)$ of the field $\mathbb{Q}$ of rational numbers. Then $[E : \mathbb{Q}]$, the degree of E over $\mathbb{Q}$, is

(A) 3 (B) 4 (C) 7 (D) 8

Q.18 The general solution of the partial differential equation $\dfrac{\partial^2 z}{\partial x\, \partial y} = x + y$ is of the form

(A) $\frac{1}{2}xy(x+y) + F(x) + G(y)$ (B) $\frac{1}{2}xy(x-y) + F(x) + G(y)$

(C) $\frac{1}{2}xy(x-y) + F(x)G(y)$ (D) $\frac{1}{2}xy(x+y) + F(x)G(y)$

Q.19 The numerical value obtained by applying the two-point trapezoidal rule to the integral $\displaystyle\int_0^1 \frac{\ln(1+x)}{x}\,dx$ is

(A) $\frac{1}{2}(\ln 2 + 1)$ (B) $\frac{1}{2}$ (C) $\frac{1}{2}(\ln 2 - 1)$ (D) $\frac{1}{2}\ln 2$

Q.20 Let $l_k(x)$, $k = 0, 1, \ldots, n$ denote the Lagrange's fundamental polynomials of degree n for the nodes $x_0, x_1, \ldots, x_n$. Then the value of $\displaystyle\sum_{k=0}^{n} l_k(x)$ is

(A) 0 (B) 1 (C) $x^n + 1$ (D) $x^n - 1$

Q.21 Let X and Y be normed linear spaces and $\{T_n\}$ be a sequence of bounded linear operators from X to Y. Consider the statements:

$P: \{\|T_n x\| : n \in \mathbb{N}\}$ is bounded for each $x \in X$

$Q: \{\|T_n\| : n \in \mathbb{N}\}$ is bounded

Which one of the following is correct?

(A) If P implies Q, then both X and Y are Banach spaces
(B) If P implies Q, then only one of X and Y is a Banach space
(C) If X is a Banach space, then P implies Q
(D) If Y is a Banach space, then P implies Q

Q.22 Let $X = C[0,1]$ with the norm $\|x\|_1 = \int_0^1 |x(t)|\, dt$, $x \in C[0,1]$ and $\Omega = \{f \in X' : \|f\| = 1\}$, where X' denotes the dual space of X. Let $C(\Omega)$ be the linear space of continuous functions on Ω with the norm $\|u\| = \sup_{s \in \Omega} |u(s)|$, $u \in C(\Omega)$. Then

(A) X is linearly isometric with $C(\Omega)$
(B) X is linearly isometric with a proper subspace of $C(\Omega)$
(C) there does not exist a linear isometry from X into $C(\Omega)$
(D) every linear isometry from X to $C(\Omega)$ is onto

Q.23 Let $X = \mathbb{R}$ equipped with the topology generated by open intervals of the form (a,b) and sets of the form $(a,b) \cap \mathbb{Q}$. Then which one of the following statements is correct?

(A) X is regular
(B) X is normal
(C) $X \setminus \mathbb{Q}$ is dense in X
(D) $\mathbb{Q}$ is dense in X

Q.24 Let H, T and V denote the Hamiltonian, the kinetic energy and the potential energy respectively of a mechanical system at time t. If H contains t explicitly, then $\frac{\partial H}{\partial t}$ is equal to

(A) $\frac{\partial T}{\partial t} + \frac{\partial V}{\partial t}$
(B) $\frac{\partial T}{\partial t} - \frac{\partial V}{\partial t}$
(C) $\frac{\partial V}{\partial t} - \frac{\partial T}{\partial t}$
(D) $-\frac{\partial V}{\partial t} - \frac{\partial T}{\partial t}$

Q.25 The Euler's equation for the variational problem: Minimize $I[y(x)] = \int_0^1 (2x - xy - y')y'\, dx$, is

(A) $2y'' - y = 2$
(B) $2y'' + y = 2$
(C) $y'' + 2y = 0$
(D) $2y'' - y = 0$

Q.26 – Q.55 carry two marks each.

Q.26 Let X have a binomial distribution with parameters n and p, $n=3$. For testing the hypothesis $H_0: p=\frac{2}{3}$ against $H_1: p=\frac{1}{3}$, let a test be: "Reject H_0 if $X \geq 2$ and accept H_0 if $X \leq 1$". Then the probabilities of Type I and Type II errors respectively are

(A) $\frac{20}{27}$ and $\frac{20}{27}$ (B) $\frac{7}{27}$ and $\frac{20}{27}$ (C) $\frac{20}{27}$ and $\frac{7}{27}$ (D) $\frac{7}{27}$ and $\frac{7}{27}$

Q.27 Let $I=\int_C \frac{f(z)}{(z-1)(z-2)}dz$, where $f(z)=\sin\frac{\pi z}{2}+\cos\frac{\pi z}{2}$ and C is the curve $|z|=3$ oriented anti-clockwise. Then the value of I is

(A) $4\pi i$ (B) 0 (C) $-2\pi i$ (D) $-4\pi i$

Q.28 Let $\sum_{n=-\infty}^{\infty} b_n z^n$ be the Laurent series expansion of the function $\frac{1}{z\sinh z}$, $0<|z|<\pi$. Then which one of the following is correct?

(A) $b_{-2}=1,\ b_0=-\frac{1}{6},\ b_2=\frac{7}{360}$. (B) $b_{-3}=1,\ b_{-1}=-\frac{1}{6},\ b_1=\frac{7}{360}$.

(C) $b_{-2}=0,\ b_0=-\frac{1}{6},\ b_2=\frac{7}{360}$. (D) $b_0=1,\ b_2=-\frac{1}{6},\ b_4=\frac{7}{360}$.

Q.29 Under the transformation $w=\sqrt{\frac{1-iz}{z-i}}$, the region $D=\{z\in\mathbb{C}:|z|<1\}$ is transformed to

(A) $\{z\in\mathbb{C}: 0<\arg z<\pi\}$

(B) $\{z\in\mathbb{C}: -\pi<\arg z<0$

(C) $\{z\in\mathbb{C}: 0<\arg z<\frac{\pi}{2}$ or $\pi<\arg z<\frac{3\pi}{2}\}$

(D) $\{z\in\mathbb{C}: \frac{\pi}{2}<\arg z<\pi$ or $\frac{3\pi}{2}<\arg z<2\pi\}$

Q.30 Let $y(x)$ be the solution of the initial value problem

$$y'''-y''+4y'-4y=0,\quad y(0)=y'(0)=2,\ y''(0)=0.$$

Then the value of $y(\frac{\pi}{2})$ is

(A) $\frac{1}{5}\left(4e^{\frac{\pi}{2}}-6\right)$ (B) $\frac{1}{5}\left(6e^{\frac{\pi}{2}}-4\right)$ (C) $\frac{1}{5}\left(8e^{\frac{\pi}{2}}-2\right)$ (D) $\frac{1}{5}\left(8e^{\frac{\pi}{2}}+2\right)$

Q.31 Let $y(x)$ be the solution of the initial value problem

$$x^2 y'' + xy' + y = x, \quad y(1) = y'(1) = 1.$$

Then the value of $y(e^{\frac{\pi}{2}})$ is

(A) $\frac{1}{2}(1 - e^{\frac{\pi}{2}})$ (B) $\frac{1}{2}(1 + e^{\frac{\pi}{2}})$ (C) $\frac{1}{2} + \frac{\pi}{4}$ (D) $\frac{1}{2} - \frac{\pi}{4}$

Q.32 Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation defined by $T(x, y, z) = (x + y, y + z, z - x)$. Then, an orthonormal basis for the range of T is

(A) $\left\{\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right), \left(\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)\right\}$ (B) $\left\{\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right), \left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}\right)\right\}$

(C) $\left\{\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right), \left(\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}}\right)\right\}$ (D) $\left\{\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right), \left(\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right)\right\}$

Q.33 Let $T: P_3[0,1] \to P_2[0,1]$ be defined by $(Tp)(x) = p''(x) + p'(x)$. Then the matrix representation of T with respect to the bases $\{1, x, x^2, x^3\}$ and $\{1, x, x^2\}$ of $P_3[0,1]$ and $P_2[0,1]$ respectively is

(A) $\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 2 & 2 & 0 \\ 0 & 6 & 3 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 1 & 2 & 0 \\ 0 & 0 & 2 & 6 \\ 0 & 0 & 0 & 3 \end{bmatrix}$ (C) $\begin{bmatrix} 0 & 2 & 1 & 0 \\ 6 & 2 & 0 & 0 \\ 3 & 0 & 0 & 0 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 2 & 2 \\ 3 & 6 & 0 \end{bmatrix}$

Q.34 Consider the basis $\{u_1, u_2, u_3\}$ of $\mathbb{R}^3$, where $u_1 = (1,0,0)$, $u_2 = (1,1,0)$, $u_3 = (1,1,1)$. Let $\{f_1, f_2, f_3\}$ be the dual basis of $\{u_1, u_2, u_3\}$ and f be a linear functional defined by $f(a,b,c) = a + b + c$, $(a,b,c) \in \mathbb{R}^3$. If $f = \alpha_1 f_1 + \alpha_2 f_2 + \alpha_3 f_3$, then $(\alpha_1, \alpha_2, \alpha_3)$ is

(A) (1,2,3) (B) (1,3,2) (C) (2,3,1) (D) (3,2,1)

Q.35 The following table gives the cost matrix of a transportation problem

4	5	6
3	2	2
1	1	2

The basic feasible solution given by $x_{11} = 3, x_{13} = 1, x_{23} = 6, x_{31} = 2, x_{32} = 5$ is

(A) degenerate and optimal (B) optimal but not degenerate

(C) degenerate but not optimal (D) neither degenerate nor optimal

Q.36 If z^* is the optimal value of the linear programming problem

$$\text{Maximize } z = 5x_1 + 9x_2 + 4x_3$$
$$\text{subject to } x_1 + x_2 + x_3 = 5$$
$$4x_1 + 3x_2 + 2x_3 = 12$$
$$x_1, x_2, x_3 \geq 0,$$

then

(A) $0 \leq z^* < 10$ (B) $10 \leq z^* < 20$ (C) $20 \leq z^* < 30$ (D) $30 \leq z^* < 40$

Q.37 Let G_1 be an abelian group of order 6 and $G_2 = S_3$. For $j = 1, 2$, let P_j be the statement: "G_j has a unique subgroup of order 2". Then

(A) both P_1 and P_2 hold (B) neither P_1 nor P_2 holds
(C) P_1 holds but not P_2 (D) P_2 holds but not P_1

Q.38 Let G be the group of all symmetries of the square. Then the number of conjugate classes in G is

(A) 4 (B) 5 (C) 6 (D) 7

Q.39 Consider the polynomial ring $\mathbb{Q}[x]$. The ideal of $\mathbb{Q}[x]$ generated by $x^2 - 3$ is

(A) maximal but not prime (B) prime but not maximal
(C) both maximal and prime (D) neither maximal nor prime

Q.40 Consider the wave equation $\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2}$, $0 < x < \pi$, $t > 0$, with $u(0,t) = u(\pi,t) = 0$, $u(x,0) = \sin x$ and $\frac{\partial u}{\partial t} = 0$ at $t = 0$. Then $u\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$ is

(A) 2 (B) 1 (C) 0 (D) −1

Q.41 Let $I = \int_C \frac{e^y}{x}\,dx + (e^y \ln x + x)\,dy$, where C is the positively oriented boundary of the region enclosed by $y = 1 + x^2$, $y = 2$, $x = \frac{1}{2}$. Then the value of I is

(A) $\frac{1}{8}$ (B) $\frac{5}{24}$ (C) $\frac{7}{24}$ (D) $\frac{3}{8}$

Q.42 Let $\{f_n\}$ be a sequence of real valued differentiable functions on $[a,b]$ such that $f_n(x) \to f(x)$ as $n \to \infty$ for every $x \in [a,b]$ and for some Riemann-integrable function $f : [a,b] \to \mathbb{R}$. Consider the statements

P_1 : $\{f_n\}$ converges uniformly

P_2 : $\{f_n'\}$ converges uniformly

P_3 : $\int_a^b f_n(x)\,dx \to \int_a^b f(x)\,dx$

P_4 : f is differentiable

Then which one of the following need **NOT** be true

(A) P_1 implies P_3 (B) P_2 implies P_1 (C) P_2 implies P_4 (D) P_3 implies P_1

Q.43 Let $f_n(x) = \dfrac{x^n}{1+x}$ and $g_n(x) = \dfrac{x^n}{1+nx}$ for $x \in [0,1]$ and $n \in \mathbb{N}$. Then on the interval $[0,1]$,

(A) both $\{f_n\}$ and $\{g_n\}$ converge uniformly

(B) neither $\{f_n\}$ nor $\{g_n\}$ converges uniformly

(C) $\{f_n\}$ converges uniformly but $\{g_n\}$ does not converge uniformly

(D) $\{g_n\}$ converges uniformly but $\{f_n\}$ does not converge uniformly

Q.44 Consider the power series $\sum_{n=1}^{\infty} \dfrac{x^n}{\sqrt{n}}$ and $\sum_{n=1}^{\infty} \dfrac{x^n}{n}$. Then

(A) both converge on $(-1,1]$ (B) both converge on $[-1,1)$

(C) exactly one of them converges on $(-1,1]$ (D) none of them converges on $[-1,1)$

Q.45 Let $X = \mathbb{N}$ be equipped with the topology generated by the basis consisting of sets $A_n = \{n, n+1, n+2, \ldots\}$, $n \in \mathbb{N}$. Then X is

(A) Compact and connected (B) Hausdorff and connected

(C) Hausdorff and compact (D) Neither compact nor connected

Q.46 Four weightless rods form a rhombus $PQRS$ with smooth hinges at the joints. Another weightless rod joins the midpoints E and F of PQ and PS respectively. The system is suspended from P and a weight $2W$ is attached to R. If the angle between the rods PQ and PS is 2θ, then the thrust in the rod EF is

(A) $W\tan\theta$ (B) $2W\tan\theta$ (C) $2W\cot\theta$ (D) $4W\tan\theta$

Q.47 For a continuous function $f(t)$, $0 \le t \le 1$, the integral equation $y(t) = f(t) + 3\int_0^1 ts\, y(s)\, ds$ has

(A) a unique solution if $\int_0^1 sf(s)\, ds \neq 0$

(B) no solution if $\int_0^1 sf(s)\, ds = 0$

(C) infinitely many solutions if $\int_0^1 sf(s)\, ds = 0$

(D) infinitely many solutions if $\int_0^1 sf(s)\, ds \neq 0$

Common Data Questions

Common Data for Questions 48 and 49:

Let X and Y be continuous random variables with the joint probability density function

$$f(x,y)=\begin{cases} a\,e^{-2y}, & 0<x<y<\infty, \\ 0, & \text{otherwise.} \end{cases}$$

Q.48 The value of a is

(A) 4 (B) 2 (C) 1 (D) 0.5

Q.49 The value of $E(X \mid Y=2)$ is

(A) 4 (B) 3 (C) 2 (D) 1

Common Data for Questions 50 and 51:

Let $X = \mathbb{N} \times \mathbb{Q}$ with the subspace topology of the usual topology on $\mathbb{R}^2$ and $P = \left\{ \left(n, \frac{1}{n}\right) : n \in \mathbb{N} \right\}$.

Q.50 In the space X,

(A) P is closed but not open
(B) P is open but not closed
(C) P is both open and closed
(D) P is neither open nor closed

Q.51 The boundary of P in X is

(A) an empty set (B) a singleton set (C) P (D) X

Linked Answer Questions

Statement for Linked Answer Questions 52 and 53:

For a differentiable function $f(x)$, the integral $\int_0^h f(x)\,dx$ is approximated by the formula $h[a_0 f(0)+a_1 f(h)]+h^2[b_0 f'(0)+b_1 f'(h)]$, which is exact for all polynomials of degree at most 3.

Q.52 The values of a_1 and b_1 respectively are

(A) $\frac{1}{2}$ and $-\frac{1}{12}$ (B) $-\frac{1}{12}$ and $\frac{1}{2}$ (C) $\frac{1}{2}$ and $\frac{1}{12}$ (D) $\frac{1}{12}$ and $\frac{1}{2}$

Q.53 The values of a_0 and b_0 respectively are

(A) $\frac{1}{2}$ and $\frac{1}{2}$ (B) $\frac{1}{12}$ and $-\frac{1}{12}$ (C) $\frac{1}{2}$ and $\frac{1}{12}$ (D) $\frac{1}{2}$ and $-\frac{1}{12}$

Statement for Linked Answer Questions 54 and 55:

Let $X = C[0,1]$ with the inner product $\langle x, y\rangle = \int_0^1 x(t)\overline{y(t)}\,dt, \quad x, y \in C[0,1]$,

$X_0 = \left\{ x \in X : \int_0^1 t^2 x(t)\,dt = 0 \right\}$, and $X_0^{\perp}$ be the orthogonal complement of X_0.

Q.54 Which one of the following statements is correct?

(A) Both X_0 and $X_0^{\perp}$ are complete
(B) Neither X_0 nor $X_0^{\perp}$ is complete
(C) X_0 is complete but $X_0^{\perp}$ is not complete
(D) $X_0^{\perp}$ is complete but X_0 is not complete

Q.55 Let $y(t) = t^3, t \in [0,1]$ and $x_0 \in X_0^{\perp}$ be the best approximation of y. Then $x_0(t),\ t \in [0,1]$, is

(A) $\frac{4}{5}t^2$ (B) $\frac{5}{6}t^2$ (C) $\frac{6}{7}t^2$ (D) $\frac{7}{8}t^2$

General Aptitude (GA) Questions

Q.56 – Q.60 carry one mark each.

Q.56 *Which of the following options is the closest in meaning to the word below:*
Circuitous

(A) cyclic
(B) indirect
(C) confusing
(D) crooked

Q.57 *The question below consists of a pair of related words followed by four pairs of words. Select the pair that best expresses the relation in the original pair.*
Unemployed : Worker

(A) fallow : land
(B) unaware : sleeper
(C) wit : jester
(D) renovated : house

Q.58 *Choose the most appropriate word from the options given below to complete the following sentence:*
If we manage to _____________ our natural resources, we would leave a better planet for our children.

(A) uphold
(B) restrain
(C) cherish
(D) conserve

Q.59 *Choose the most appropriate word from the options given below to complete the following sentence:*
His rather casual remarks on politics ____________ his lack of seriousness about the subject.

(A) masked
(B) belied
(C) betrayed
(D) suppressed

Q.60 25 persons are in a room. 15 of them play hockey. 17 of them play football and 10 of them play both hockey and football. Then the number of persons playing neither hockey nor football is:

(A) 2 (B) 17 (C) 13 (D) 3

Q.61 – Q.65 carry two marks each.

Q.61 **Modern warfare has changed from large scale clashes of armies to suppression of civilian populations. Chemical agents that do their work silently appear to be suited to such warfare; and regretfully, there exist people in military establishments who think that chemical agents are useful tools for their cause.**

Which of the following statements best sums up the meaning of the above passage:

(A) Modern warfare has resulted in civil strife.
(B) Chemical agents are useful in modern warfare.
(C) Use of chemical agents in warfare would be undesirable.
(D) People in military establishments like to use chemical agents in war.

Q.62 If 137 + 276 = 435 how much is 731 + 672?

(A) 534 (B) 1403 (C) 1623 (D) 1513

Q.63 5 skilled workers can build a wall in 20 days; 8 semi-skilled workers can build a wall in 25 days; 10 unskilled workers can build a wall in 30 days. If a team has 2 skilled, 6 semi-skilled and 5 unskilled workers, how long will it take to build the wall?

(A) 20 days (B) 18 days (C) 16 days (D) 15 days

Q.64 Given digits 2, 2, 3, 3, 3, 4, 4, 4, 4 how many distinct 4 digit numbers greater than 3000 can be formed?

(A) 50 (B) 51 (C) 52 (D) 54

Q.65 Hari (H), Gita (G), Irfan (I) and Saira (S) are siblings (i.e. brothers and sisters). All were born on 1st January. The age difference between any two successive siblings (that is born one after another) is less than 3 years. Given the following facts:

i. Hari's age + Gita's age > Irfan's age + Saira's age.
ii. The age difference between Gita and Saira is 1 year. However, Gita is not the oldest and Saira is not the youngest.
iii. There are no twins.

In what order were they born (oldest first)?

(A) HSIG (B) SGHI (C) IGSH (D) IHSG

END OF THE QUESTION PAPER

Space for Rough Work

Some Useful Links:

1. **Free Maths Study Materials** (https://pkalika.in/2020/04/06/free-maths-study-materials/)

2. **BSc/MSc Free Study Materials** (https://pkalika.in/2019/10/14/study-material/)

3. **MSc Entrance Exam Que. Paper:** (https://pkalika.in/2020/04/03/msc-entrance-exam-paper/)
   [JAM(MA), JAM(MS), BHU, CUCET, ...etc]

4. **PhD Entrance Exam Que. Paper:** (https://pkalika.in/que-papers-collection/)
   [CSIR-NET, GATE(MA), BHU, CUCET,IIT, NBHM, ...etc]

5. **CSIR-NET Maths Que. Paper:** (https://pkalika.in/2020/03/30/csir-net-previous-yr-papers/)
   [Upto CSIR-NET Latest Exam]

6. **Practice Que. Paper:** (https://pkalika.in/2019/02/10/practice-set-for-net-gate-set-jam/)
   [Topic-wise/Subject-wise]

7. **List of Maths Suggested Books** (https://pkalika.in/suggested-books-for-mathematics/)

8. **CSIR-NET Mathematics Details Syllabus** (https://wp.me/p6gYUB-Fc)

9. **Free Video Lectures for CSIR-NET, GATE, SET, Asst. Prof. ..etc**
   https://www.youtube.com/pkalika

10. **GATE Mathematics(MA) Solution**
    https://youtube.com/playlist?list=PLDu0JgProGz7AzPyTEks4-_fgOmwXm9-C