

* A: matrix (1. A)
 $|A - \lambda I| = 0$ Eigenvalue
 $X \rightarrow \lambda$ Eigenvector
 $AX = \lambda X$

[1]

iit KGP-
Data Science

Discrete Maths

① What is relation?

② In Equivalence relⁿ, if relⁿ is symmetric & transitive, then what is the need of reflexive relⁿ?

i.e. Symmetric: $aRb \Rightarrow bRa$ ———— ⊗

transitive: $aRb, bRc \Rightarrow aRc$ ———— ⊗

by ⊗ & ⊗, $aRb, bRa \Rightarrow aRa$

Since, Every symmetric & transitive relation need not be Reflexive relation.

e.g. - (i) empty relation R on any nonempty set. ($R = \emptyset$: no elt in R)

(ii) $X = \{0, 1, 2\}$, $R = \{(0, 0), (1, 1), (0, 1), (1, 0)\}$

↓
 symmetric & transitive
 but reflexive, $\because (2, 2) \notin R$.

* Relation - A relation b/w two sets is a collection of ordered pairs containing one object from each set.

→ A fn. is a type of relation, converse need be true.

→ A Relation (R) is the subset of the cartesian product of $X \times Y$,
 $X \times Y$ are 2 non-empty set.

③ Relation on $\mathbb{Z}$?

$R_1: a \equiv b \forall a, b \in \mathbb{Z}$

$R_2: a \equiv b \pmod{n} \forall a, b \in \mathbb{Z}$

Ring

R : comm. R unity
 $1 \neq 0$

- ① Ex of ring R doesn't have maximal ideal?
 $\langle 0 \rangle =$ maximal ideal in $\mathbb{Z}_p$
 $\mathbb{I} \neq \mathbb{Z}_p$

$\mathbb{Z}$
 $\langle n \rangle$

$R[x] \rightarrow \text{PID, UFD}$
X

Example

$\mathbb{P}_1$
 $\mathbb{Z} \rightarrow \mathbb{P}_2 \rightarrow \mathbb{P}_3$

$\mathbb{K}(x, y)$
 $R: \text{UFD} \rightarrow \text{UFD}$

③ $\mathbb{Z}_8[x]$

- Is $\langle x \rangle$ prime ideal? No
① prime ideal in $\mathbb{Z}_8[x]$?
② maximal ideal in $\mathbb{Z}_8[x]$?

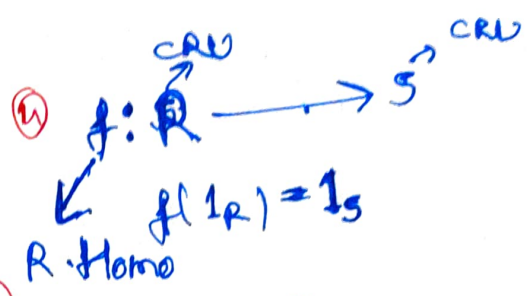
Ans @ $\frac{\mathbb{Z}_8[x]}{\langle x \rangle} \cong \mathbb{Z}_8$! Not I.D.

~~Set~~

prime but not maximal

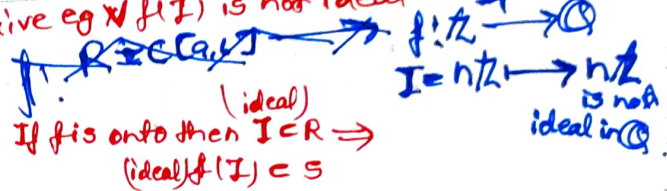
$p(x) \in \mathbb{Z}_8[x]$: Irr in $\mathbb{Z}_8$

$$\begin{aligned} 4+2+1 &= 7 \in \mathbb{Z}_8 \\ 2+3+1 &= 6 \notin \mathbb{Z}_8 \end{aligned}$$



② (Ideal) $I \subset R \rightarrow f(I)$ proper ideal in S ?

my ans: need not be an ideal.
Give eg $\forall f(I)$ is not ideal



$f(I)$

Let's try

ii) prime ideal!
 $p \in S \rightarrow f^{-1}(p)$ in R
prime or not?

~~$\mathbb{Z}_8$~~

iii) maximal ideal (M) in S
 $f^{-1}(M)$ in R
maximal @ not?

② $\rightarrow$

Band matrix -

if $a_{ij} = 0$ for $i > j + p$ and $j > i + q$
with band width $p+q+1$.

Q.1) How to use FD to solve linear pde (Heat eq)

11T
KGP

Conjugate Gradient method

Q.2) How to use FD to solve linear pde (Heat eq)

Can $\rightarrow I = \int$

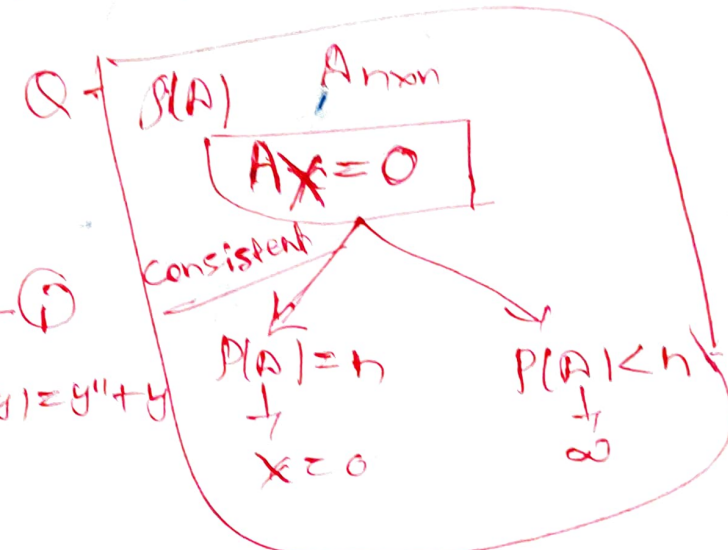
$\phi(x) =$

Q.3) $I = I(c_1, \dots, c_n)$: system of linear eq $\rightarrow$ How do you solve using iterative method for matrices

$AX = 0$

$N(A) = \{x \mid Ax = 0\}$

V be
+ $\nabla \cdot \nabla \phi \rightarrow C$



Q.5 $\frac{d^2 y}{dx^2} + y = 0$

Q.6 $y'' + y = 0$

$m^2 + 1 = 0$
 $m = \pm i$

Linear PDE soln method in CFD

Since

Null space of (1)?

$N(A) = \{y \mid y'' + y = 0\}$

$f: \mathbb{R}^2 \rightarrow \mathbb{R} \quad y'' + y' + y = f$

$I[y] = \int f(x, y(y'), y'') dx$

How do you $\frac{\partial F}{\partial y} - \frac{d}{dx} \left(\frac{\partial F}{\partial y'} \right) = 0$

Compare the convergence of iterative methods Gauss Seidel & Jacobi?

7/June/21

Stage(I) - [ITD]

Amit P.
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Linear $V = \{f: \mathbb{R} \rightarrow \mathbb{R}\}$ $V \otimes \mathbb{P}$ not?
 $\dim(V) = ?$
unclear

$V: \mathbb{R}^2 \rightarrow \mathbb{R}^2$
 $f(0) = (f(0), 0) = 0$
 $f(x) = x \cdot f(1)$
 $f(1) = 1$
 $f(n) = n$
 $f(2) = 2$

$\phi(f) = f(0,1)$
 $\text{THAT} = 1$
 $\phi(1) + \eta(1) = \dim V$
 $\phi(1)$

$f(0) = 0$
 $f(1) = 0$
 $f(2) = 0$

$f: [0,1] \rightarrow \mathbb{R}$
 $f(n) = n+2 \rightarrow$ no fixed pt.
 $T: V \rightarrow$
 $f(1) = 1$

Real

$f: [a,b] \rightarrow [a,b]$: Tell me about this fn. (non-trivial pr.)
 a, b

It has fixed pt. ? proof?
 $f(x) = x$ yes
 $f(c) = c$



~~$f(x) = x$~~ ~~$f(x) = f(x) < 0$~~

$n \sim n$

#

$$A_{2 \times 2}$$

$$\underline{\underline{A^2 = 0}}$$

$$A_2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

~~$A = I_{2 \times 2}$~~ $B_{2 \times 2}$
 $B^2 = I \rightarrow$ No. of such matrix (B)? $\rightarrow$ What is the field for entry ($\mathbb{R} | \mathbb{C}$)?

① $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ ② $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

③ $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ ④ $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

⑤ $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ ⑥ $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

~~1/2~~ 2

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \dots$$

$$a_{h1}x_1 + \dots + a_{hn}x_n = b_n$$

Downloaded from www.pkalika.in

$$Ax = b$$

has ∞ soln.
vector

~~say A is~~
~~invertible~~

$$A = U U^T$$

$n \times n$

$$U_{m \times n}$$

$$b = U_{m \times 1}$$

$$y'' + py' + qy = 0 \quad (1)$$

if given $y_1 = v_1(x), y_2 = v_2(x)$
then let $y = v(x)$
 $v'' + pv' + qv = 0$
 $v(x) = \int \frac{e^{-\int p dx}}{y^2} dx$

$$(U U^T) X = U_{m \times 1}$$

$$U Y = U$$

$A \times$

$$U^T X = Y$$

$$U^T X = 1$$

$$AX = U_{m \times 1}$$

Let $A = \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix}^T = \begin{pmatrix} 2 & 1 \\ 4 & 2 \end{pmatrix}$

$$U U^T$$

$$\rho(A) = 1$$

always

Q - Relⁿ b/w DE & I.E?

⊗ A solution to a differential equations exists does not mean that we'll be able to find it.

✓ Given, y_1 & $y_2 \rightarrow$ 2 L.I. linear diff. f's, then $\exists$ unique homo. d.e. of order 2 whose solns are y_1 & y_2 and eqn' is

$$y'' - \frac{W'(y_1, y_2)}{W(y_1, y_2)} y' + \frac{W(y_1', y_2')}{W(y_1, y_2)} y(x) = 0$$

with the help of Wronskian

27/5

1. Eqn $\rightarrow u_{xx} + u_{yy} = f(x, y)$

Q - CFD?

$(u_{xx})_{i,j} = \frac{1}{h^2} [u_{i+1,j} - 2u_{i,j} + u_{i-1,j}]$

11+9

$u_{i,j} = \frac{1}{4} [u_{i+1,j} + u_{i-1,j} + u_{i,j+1} + u_{i,j-1}]$

Banded matrix

$Ax = b$

mean of the values at the 4 neighbouring pts in the x and y directions (In Laplace eqn) elt in each

$A = \begin{pmatrix} \times & & \\ & \times & \\ & & \times \end{pmatrix}$

No. of non-zero/rows of this particular matrix (banded matrix)?

Ans - 5 (5 bold ans)

condⁿ

Under relaxation

$0 < \omega < 1$

~~$\omega > 1$~~

have them

over relaxation

$1 < \omega < 2$

What do you do in CFD?

Numerical method

ADM

2nd order linear PDE

Laplace $\rightarrow$ classification

Jury's Problem

characteristic (Hint: Boundary condⁿ on the bdy of rectangular curve (R1).

Numerically expression (Given Rectangular region)

Solve it by iterative method

Stencil for NA compact

5 point stencil

Crank-Nicolson method stencil

$$\textcircled{1} f: X \in \mathbb{R}^n \rightarrow \mathbb{R}$$

min $f(x) \in \mathbb{R}$: condition to have min. solⁿ of f .

$\textcircled{2}$ Difference b/w Hilbert space & Banach space?

what is metric?

$$d(x, y) = \sqrt{\langle x, y \rangle}$$

B.S $\rightarrow$ complete normed L.S $\|x\| = \sqrt{\langle x, x \rangle}$

$\Downarrow$ ℓ_p : B.S but only ℓ_2 is H.S.

$\textcircled{3}$ What is row rank, column rank?

$\textcircled{4}$ ∞ -dimensional V.S $\rightarrow \mathbb{R}(\mathbb{Q})$: Basis? How ∞ ?

$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$
 $\downarrow$ Column rank

$g(A, A)$

$\textcircled{5}$ $K[x]$ is vector space over K ? Yes
 If not, why?

$\textcircled{6}$ Ring which is not field? $K, F[x]$
 $\uparrow$ field

K is field
 $K[x] \neq$ field
 $K[x]$ is not a field
 $\uparrow$ (not a field)
 $H(x) = \alpha f(x) + \beta g(x)$

$\textcircled{7}$ free module?
 $\textcircled{8}$ Module?

$0 \rightarrow R \xrightarrow{\phi} R \xrightarrow{\psi} R \rightarrow 0$
 is
 free module (P.I.T.)

$\textcircled{9}$ What is projective module?

• prepare their written ^{Exam} Quest^s solⁿ, they asked from that.

- ① Is every convex set has extreme point? No ($\mathbb{R}$), Convex set?
- ② $\left| \frac{dy}{dx} \right| + |y| = 0$, $y(0) = 1$, $y \in \mathbb{R}$, solⁿ? No solⁿ
- ③ $\frac{dy}{dt} = f_1(y, z)$ } ① Critical point?
 $\frac{dz}{dt} = f_2(y, z)$ } ② Stability of system at $(0,0)$?
 } ③ What do you call this matrix, $A = \begin{bmatrix} f_{1y} & f_{1z} \\ f_{2y} & f_{2z} \end{bmatrix}$?
 Ans - Jacobian matrix.
- ④ What is the name of process for finding stability of SDEqⁿ by the above process?
 → Linearization of DEqⁿ
- ⑤ Isoelnes ?
- ⑥ Extreme point ?
- ⑦ Existence condition for solⁿ of ODE [for eq ⑤]? Peano's thm
- ⑧

→ Convex fn. - significance in optimization?

① Convex Optimization

$$\left[\begin{array}{l} f: \mathbb{R}^n \rightarrow \mathbb{R} \\ \text{s.t.} \quad h(\gamma) \leq 0 - \\ \quad \quad \forall \gamma \in X \end{array} \right]$$

Constraint Optimization?

Unconstrained " ?

$$g(\gamma) = 0 \quad \forall$$

② Significance of KKT?

IMSc

$$\frac{n!}{k=0} \quad \text{IMSc TCS (21)} \quad \frac{n!}{k=0}$$

2099

 $n!$ $\binom{n}{k}$

Q. max of

 $\binom{n}{k}$ $= \frac{n!}{k!(n-k)!}$ $k!(n-k)!$ $\frac{n!}{k!(n-k)!}$ Q. minimize $f(n)$

s.t

 $g(n) \geq 0$ ≤ 0 $= 0$ $0 \leq k \leq n$

and

 $n_i \geq 0 \forall i \in \mathbb{N}$ $0! \quad n! = 1$ $\frac{k! (n-k)!}{2} \mid n - n! \quad 0!$ $\frac{n!}{k=0}$ $\frac{n(n-1)!}{n=1}$ $\frac{n!}{(n-1)! (n-n+1)!}$

② $y'' + P(x)y' + Q(x)y = 0$ 2nd order homo eqn
basis soln?

① Zero stability?

④ characteristic poly?

$A_{n \times n}$

③ 2nd order homo. eqn

y_1 is given, $y_2(x) = ?$

$y_2(x) = v(x) \cdot y_1(x)$

variable

$$v(x) = \int \frac{e^{-\int p(x) dx}}{y_1^2} dx$$

char poly

⑤ $|A - xI|$, what is x ?

③ Cayley-Hamilton thm?

⑥ $y' = f(x, y)$, $y(x_0) = y_0$: Existence thm? Peano's thm

⑦ $(x, y) \mid |x - x_0| \leq a, |y - y_0| \leq b, a, b > 0$ Existence & Uniqueness thm?

I is max on $|x - x_0| \leq h, \forall h < a$

Name of above thm - Picard's

Singular soln?

⑧ $y = x^k \sum_{n=0}^{\infty} c_n x^n$ ⑨ Soln of DEqn about (near) singular point? using Frobenius method.

⑩ What is indicial eqn?

① Coeff of lowest degree of $x = 0$ [in ascending (↑) power of x]
Coeff. of highest degree of $x = 0$ [in descending (↓) power of x]

① $(X) = \sin(y)$ Is it linear?

[14]
Sum 0 =

⑧ $\ln(4)$
x & y
Write pros for swapping
a no.?

DIAT
23/7/21

② $y' = \sin x + \cos x$

$$\int dy = \int \sin x + \cos x dx$$

$$= \sin x - \int \frac{1}{x^2} \cdot x dx$$

④ Numerical methods to solve DEs?

$$y' = f$$

① Define Non-linear DE?

NIT R

② Define Linear D.E?

Project

$$y' = f(x, y)$$

③ Stochastic Process?

④ Random variable?

⑤ $P_0(x)y'' + P_1(x)y' + P_2(x)y = 0$ — ①

Singular point?

⑥ Convex set? If $a, b \in X$

$$\lambda a + (1-\lambda)b \in X$$

⑦ $L[f(t)] = \int_0^\infty e^{-st} f(t) dt = F(s)$

Define Laplace transform (L, T) ?

⑧ Use of L.T?

⑨ abstract defn of open set? (related topology)

⑩ Closed set?

IIT Roorkee (Project) - Measure Theory [15]

2. $f \geq 0$

$\int_E f dx = 0 \Rightarrow f = 0$?

countable subset of E

Q- If $A \in \mathcal{M}$ / $\int_{\bigcup_{n=1}^{\infty} A_n} f(x) dx = \sum_{n=1}^{\infty} \int_{A_n} f(x) dx$: $A \subseteq \mathcal{M}$ $\rightarrow$ Difference b/w outer measure & measure?

Q- If f has a measure $\rightarrow$ Difference b/w these 2?
 f is measurable

IIT Ropar (Project)

2018/21

$$u_{xx} + u_{yy} = 0$$

① Is 5-pt diagonal formula consistent? ^{What do you mean} Consistent?

② Why do we not use the Simplex method to solve transportation Pb?

③ In iterative scheme which one you prefer and in SOR, what is your choice for relaxation parameter (ω) to get optimal soln?

IIT J
318/21

① Integro-differential eqn? method to solve I.D.E?

② Can we solve by Laplace T.? What is Laplace T.?

③ Is every norm space is Inner prod space? eg?

④ If norm is given, then can define inner prod?

⑤ Fredholm alternate thm?

⑥ Hilbert mixed thm?

⑦ Incomplete metric space?

⑧ Can we make a incomplete metric space into complete? If yes, How?

⑨ Disadvantage of Newton-Raphson method?

NITJ

02/09/21

- ① 2nd order Quasilinear PDE?
- ② Defn of Quasilinear PDE?
- ③ Superposition of differential operator? or soln?
- ④ $f(x) \in |x|$, $x=0$ cts? differentiable?
- ⑤ $f(x) = |xy|$ ($x, y \in (0,0)$), cts? differentiable?
- ⑥ If a fn. is cts, then is it necessary to exist its partial derivative? if not give e.g.?
- ⑦ Newton Raphson method use?
- ⑧ Non-linear PDE (How do you solve NL PDE)?

MANIT
Bhopal

22/09/21

- ① How to add path in Matlab?
- ② What is symbolic math in Matlab?
- ③ What is vectors?
- ④ Real-life application of convolution theorem?
- ⑤ L.I & L.D vectors mean?
- ⑥ Pseudoconvex fns?
- ⑦ What is convex optimization & concave?
- ⑧ When will the eigenvectors of matrix be orthogonal?
(If matrix is Symmetric, otherwise need not.)

IIT
Roorkee

24/09/21

S.K. Gupta

Q-What had you studied for this Interview?
A-Q.E. emek L.A.

- ① $V.S \in M_{3 \times 3}(\mathbb{R})$ and $W \in \{A_{3 \times 3} \mid A^2 = I\}$: will be Subspace? No
∵ W does not contain zero vector/matrix

What if we include the zero matrix?

Is $W = \{A_{3 \times 3} \mid A^2 = 0\}$ subspace @ not of V?

② Let $A_{3 \times 3} \rightarrow E.V's \in 1, 1, 2$ so what should be the $P(A-I)$ such that A become diagonalizable?

→ For Diagonalizability: $AM = GM$, here $AM(1) = 2$

[17]

② 1. A ^① B_1, B_2 ^{diff} bases of $V \rightarrow \dim(V) = n$

IIT PKD

Proof - $|B_1| = |B_2| = n$

① $\forall V \subseteq V$ has a basis. Yes $\rightarrow$ Proof $\rightarrow$ Zorn's lemma ^{poset + maximal elt.}
 every do you this pt.?

iii) $(A_n) \rightarrow \lambda$ (Eigenvalue), $B = A - \lambda I$, say about B ?

$|B| = |A - \lambda I| = 0 \Rightarrow B$! singular matrix.

$\Downarrow$
 λ is of How $|A - \lambda I|$ will be 0?

~~A has $\dots = 0$~~