IIT Bombay

1. Show that the space of nilpotent matrices is connected
2. Show that if G is a group and $G = H \cup K$, where H and K are subgroups, then either $H \subseteq K$ or $K \subseteq H$.
3. Find homomorphisms from $\mathbb{Q}[x]$ to $\mathbb{C}$.
4. Show that any open set in $\mathbb{R}$ is countable union of disjoint open intervals.

IIT Kanpur

1. Give an example of algebraically closed field, which is countable and of characteristic 0.
2. Show that the subgroup with index of smallest prime dividing the order of the group is normal.
3. What can you say about $\mathbb{R}/\mathbb{Z}$?

IIT Jodhpur

1. Tell about field extensions. Give some examples of various field extensions.
2. State Sylow theorems.

IIT Hyderabad

1. Check whether there is a field of order 30 or not.
2. If a is nilpotent, show that $1+a$ is unit.
3. If x is zero-divisor, can we say that $1+x$ is a unit?
4. $D : P_5[x] \longrightarrow P_5[x]$ is the differentiation map, is it a linear transformation? Is it diagonalizable?

5. Check whether the set of all analytic fns on the open unit disk is an integral domain.

IIT Madras

1. If K/k is a field extension, then will $K \otimes K$ be a field over k? If not, then specify the cases where $K \otimes K$ will be a field.

2. Let p be a prime. For which values of p, will $\frac{\mathbb{Z}_p[x]}{\langle x^3-1\rangle}$ and $\frac{\mathbb{Z}_p[x]}{\langle (x-2)(x-3)(x-5)\rangle}$ are isomorphic?

3. Consider the set $\{ \frac{m}{n} \mid m \in \mathbb{Z}, n = \text{odd integers}\}$. Will it be a PID?

4. Suppose G is a finite abelian group. Let H be a subgroup of G. Will G/H be an isomorphic copy of some subgroup of G?

IISER Pune

1. Suppose $a_1, a_2, a_3 \in R$ where R is an Euclidean domain, and a_1, a_2, a_3 generate R. Can we find a matrix in $GL_3(R)$ whose first row is $[a_1, a_2, a_3]$?

2. How many Sylow 3-subgroups does S_4 have? Can you perform second Sylow theorem on them?

3. What is the difference between normal extension and separable extension? Can you find any relation between them?

4. Show that the space of all real valued continuous fn-s on $[0,1]$, s.t. $\sup_{x\in[0,1]} |f(x)| \leq 1$ is complete.

5. Show that sequentially compactness implies compactness for a metric space.