PhD Interview questions (Dec 2022)

IIT Kanpur

1. Give an example of a non-zero C^∞ fn f on $\mathbb{R}$ such that $f(x) = 0$, $\forall |x| \geq 1$. Does such a complex analytic fn exist on $\mathbb{C}$.

2. Given a differentiable function f on $\mathbb{R}$, $\lim_{x \to \infty} f(x)$ exists and $\lim_{x \to \infty} f'(x)$ exists.

   Then, $\lim_{x \to \infty} f'(x) = ?$

3. Let X be a Banach space. $T : X \to X$, linear $\ni$ Given, $\exists c > 0$ $\ni$ $\|Tx\| \geq c\|x\|$. Prove that the range of T is closed.

   Is such a function invertible?

4. Relation between $\{x \in X \mid Tx = x\}$ and $\{x \in X \mid T^* x = x\}$, where $\|T\| \leq 1$.

5. Prove that A is diagonalisable if $A^k = I$, for some $k \geq 1$ and $A \in M_n(\mathbb{C})$.

IIT Bombay

1. Given $f : [0,1] \to \mathbb{R}$ is continuous and $\int_0^1 x^n f(x)\,dx = 0$, $\forall n = 0, 1, 2, \ldots$

   What can you tell about f?

   Suppose $f(x) = \sin x$, give a sequence of polynomials that converges to $f(x)$. Why is this conv ~~converges~~ uniform? (Related questions).

2. Find supremum of $\sin n$, $n \in \mathbb{N}$
3. Given f is continuous on $[1,\infty)$ and $\int_1^\infty f(x)dx < \infty$. Then $\lim_{x\to\infty} f(x) = 0$. True/False.
4. Does there exist a finite, ~~base~~ measure μ on the set X, of polynomials of atmost degree n, such that $\int_0^1 p\,d\mu = p'(0), \forall p \in X$

NBHM Interview

1. Is $\mathbb{R}[x]$, the set of polynomials with real coefficients complete?
Example of a complete space that is not finite dimensional. Is $C[0,1]$ seperable? Why?
2. What are the normed linear spaces in which all closed and bounded sets are compact?
Example of a closed and bounded set that is not compact.
3. Is $C[0,1]$ different from $L_p[0,1]$?
4. Given that f is a continuous fn on $\mathbb{R}^2$.
and $f(x) = \frac{1}{4\pi r^2} \int_{B(0,r)} f(y)\,dy, \ \forall r>0$ — (1)
and $f \in L_p(\mathbb{R}^2)$, for some $p<\infty$.
Prove that $f \equiv 0$.
Tell an example of a function that satisfies (1), but doesnot belong to $L_p(\mathbb{R}^2), \forall p<\infty$
5. Prove that a group of order 15 is cyclic.