Swaraj Koley Mail Id : k.swaraj2017@gmail.com

(In each of the interviews, I told my preference is group theory, ring theory and Field theory)
I have been selected at IIT Bombay, IIT Delhi, IIT Kharagpur and IISER Bhopal among the below institutes.

HRI Allahabad

- Definition of group action and proof of Caley's theorem.
- Prove that $\mathbf{C}$ is algebraically closed using complex analysis.
- Is any finite extension of $\mathbf{Q}$ galois ?
- Is $\mathbf{C}$ is algebraic closure of $\mathbf{Q}$?
- Define normal extension and give an example.
- State primitive element theorem.
- Let E/F is galois and $E =$ split $f(x)$ where $f(x) \in F[x]$ is irreducible. Prove that if $\mathrm{Gal}(E/F)$ is abelian then $E = F(t)$ where t is a root of $f(x)$.
- Difference between continuity and uniform continuity.
- Is every continuous and bounded function on $\mathbf{R}$ uniformly continuous?
- Prove that $f : \mathbf{R} \to \mathbf{R}$ by $f(x) = e^{-x^2}$ is uniformly continuous on $\mathbf{R}$.
- Prove that if $f : \mathbf{R} \to \mathbf{R}$ is cont and lim $f(x)$ exists when $|x| \to \infty$ then f is uniformly continuous on $\mathbf{R}$.
- Prove that if a function's derivative is bounded then the function is uniformly continuous on the domain.
- Define equicontinuity of sequence of functions and state Arzela Ascolli theorem.
- State the Hiene Borel theorem.
- Is every closed and bounded subset of a complete metric space compact?
- If $f_n : [0, 1] \to \mathbf{R}$ is a collection of continuous functions pointwise convergent to f and if f is continuous then convergence is uniform **(True/False)**

IISER Bhopal

- Define field extension with example.
- If K/F is a field extension and R is a ring between K and F what can you say about that ring R ?
- Let $p(x)$ be a polynomial in $F[x]$ and take the extension $F(x)/F(p(x))$, $F(p(x))/F$ and $F(x)/F$. Which are the algebraic extensions, finite extensions? And are they separable extensions?
- Let G be a group and H be a normal subgroup of G such that the set difference i.e. $G - H$ is finite. What can you say about the cardinality of G? To prove your claim is normality of the subgroup needed or not?
- Let S be a subset of R such that every real valued continuous function from S is bounded. What can you say about S ?
- If $f : (0,1) \to \mathbf{R}$ is a continuous map and

  $$\int_0^1 f(x)g(x)\,dx = 0$$

  for all $g \in C(0,1)$ having compact support in $(0,1)$. What can you say about f?

IIT Kharagpur

- Is 5 is a prime element in $\mathbf{Z}[i]$?.
- Find a maximal ideal containing 5 in $\mathbf{Z}[i]$ (I said $< 1 + 2i >$ then they told me to prove why it is a maximal ideal in $\mathbf{Z}[i]$)
- What more you can say about $\mathbf{Z}[i]$ besides being a PID ? (I said it is an ED, then they asked me why it is an ED ?)
- Which prime numbers of $\mathbf{Z}$ are primes of $\mathbf{Z}[i]$ too ? (I said prime numbers in $\mathbf{Z}$ of the form $4k + 3$ are prime elements of $\mathbf{Z}[i]$ too, then they told me to give justification about my claim)
- Can you prove that prime numbers in $\mathbf{Z}$ of the form $4k + 3$ can not be written as the sum of two squares using congruence modulo 4 ?

IIT Bombay

- Let K be a field and H be a finite subgroup of $(K^{\times}, .)$ then what can you say about H ?
- What are the finitely generated subgroups of $(\mathbf{Q}, +)$?
- Is $(\mathbf{Q}^{*}, .)$ is finitely generated ?
- How many homomorphisms are there from $\mathbf{Q}$ to $\mathbf{Z}$?
- Is $(\mathbf{Q}^{*}, .)$ is isomorphic to $(\mathbf{Q}, +)$?
- Is Is $(\mathbf{Q}^{+}, .)$ is isomorphic to $(\mathbf{Q}, +)$?
- Is every finitely generated proper subgroup of $(\mathbf{Q}^{*}, .)$ is cyclic ?
- If $f : [0, 1] \to \mathbf{R}$ is a continuous function such that

  $$\int_0^1 x^n f(x)\, dx = 0$$

  for all $n \in \mathbf{N} \cup \{0\}$. What can you tell about f ?

IIT Madras

- How many elements are there in S_4 which do not fix any element i.e. What is the cardinality of the set $\{f \in S_4 : f(x) \neq x \text{ for all } x \in \{1, 2, 3, 4\}\}$. (They asked me to use group action to find the number of such elements).
- Define separable extension and give an example.
- Is every extension over $\mathbf{Q}$ separbele ?
- Prove every algebraic field extension over an infinite field of char 0 separable.
- Give an example of an inseparable extension. (I said $split_{\mathbf{Z_p(t)}}(x^p - t)$ over $\mathbf{Z_p}(\mathbf{t})$ where t is an indeterminate, then they asked the reason why the extension is inseparable ?)
- Prove that any finite field extension over a finite field is a Galois extension.
- Tell NASC for F_{p^m} to be a subfield of F_{p^n} and prove it.
- What is the degree of the extension F_{p^n} over F_{p^m}.
- Let G be a finite group and H be a subgroup of G such that $[G : H\ ]$ is the smallest prime divisor of the order of G. What can you say about H and prove your claim.

IIT Delhi

1st Stage

- Let $T : \mathbf{R^2} \to \mathbf{R^2}$ be a linear transformation such that $T(v) = v$ for all points on any two chosen lines passing through $(0, 0)$. What can you say about T ?
- Let $T : \mathbf{R^2} \to \mathbf{R^2}$ be a linear transformation.If L_1 is a collection of all points on a line passing through the origin and L_2 is a collection of all points on a different line passing through the origin. And if $T(L_1) \subseteq L_1$ and $T(L_2) \subseteq L_2$. What can you say about eigen values and eigen vectors of T ?
- Give an example of such above linear transformation other than the identity linear transformation ?
- Define differentiability on $\mathbf{R}$
- If f is differentiable on $\mathbf{R}$ and f is periodic then using definition of differentiation prove that f' is also periodic.
- Is there any differentiable function f on $\mathbf{R}$ such that $f'(0) = 0$ and $f'(x) \geq 1$ for all non zero real x ?
  (I said it is not possible by using the Durboux theorem and then they asked me can you prove using the Mean value theorem ?)

2nd Stage

- What can you say about group of order p^2 where p is prime ?
- Prove that $G/Z(G)$ is cyclic if and only if G is abelian.
- Prove group of order p^n has non trivial center.
- If G is a group of order p^3 and $|Z(G)| > p$ then what can you say about G ?
- Is every group of order p^3 abelian ? If not give an example.
- Define field extension, give an example of field extension and define degree of field extension.
- Is every finite extension algebraic ? If yes prove this.
- Is every algebraic extension a finite extension? Give a Counter example. (I gave $\mathbf{Q}(\{2^{1/n} : n \in \mathbf{N}\})$ over $\mathbf{Q}$. Then they asked me to show why it is not a finite extension)
- Give an example of an infinite-dimensional field extension. I said $\mathbf{R}$ over $\mathbf{Q}$. Then they asked me to prove this.
- What is the degree extension of $\mathbf{Q}(x)$ over $\mathbf{Q}$ if x is indeterminate?
- Define quotient ring and prove the well-definedness of + and . operation.
- Is $\{0\}$ a prime ideal of any ring? An example of such a ring where zero ideal is not prime ideal.
- Give an example of an infinite ring where every prime ideal is maximal. (I said any infinite field. Then they told me to give an example of an infinite ring having such properties which is not a field, I told $\mathbf{R}[x]/(x^2)$ then they asked me why)
- What do you mean by a finitely generated $R-$algebra where R is a ring?