

HRI

1. Can you give an ^{proof} eg. of weak cgt being strong cgt in ℓ^1 ?
2. $T: \ell^p \rightarrow \ell^1$ one-one, onto, conti $\rightarrow$ possible?
3. T is compact operator (inf dim space).
give Proof that T^{-1} can't exist.
4. Spectrum of self adj operator belongs in real line $\rightarrow$ proof (by hint proved when T also compact)
5. Define fin. rank operator.
What is the closure of fin rank operators?
~~Prove~~ Give eg of $T_n \rightarrow T \rightarrow$ compact
 $\swarrow$
fin rank
6. Field of ~~order~~ order 10 $\rightarrow$ possible?
 $\mathbb{Z}$ ring? $\mathbb{Z}/n\mathbb{Z}$ ring?
ideals of $\mathbb{Z}$?
7. Commutator group, how many grps of order 6,
 $\text{Aut}(S_3)$? questions on automorphism

~~II T M~~ II T M

1) Kernel of a linear functional.

2) Kernel + ~~dimension~~ elt $\notin$ kernel
expression possible?

3) $(f_n) \in X'$. is it convergent?

Is the limit bdd? Is the limit unique

4) Weak topology. Prove weak $ca \Rightarrow bdd$.

5) T compact. dimension of kernel of $\ker(T - \lambda I)$
 $\lambda \in \mathbb{C}$

LISERB

i) Prove $f \in C[0,1]$

$$\forall g \in C[0,1], \quad \int_0^1 f g = 0$$

st $g(0)=0$
 $g(1)=1$

Then $f = ?$

ii) Compact metric space defⁿ?

iii) Isometry? It given that isometry
is ~~one~~ bijective, prove it is homeomorphism
X compact,

iv) Now Prove Isometry surjective

v) $GL(2, \mathbb{Z}_p)$ group questions.

IIT Roorkee

1. $F = \{f: \mathbb{R} \rightarrow \mathbb{R} \mid f \text{ real fun}^n\}$
is F a vector space?

2. $L^1[a, b], L^\infty[a, b], C[a, b], C^1[a, b],$
 $P(x), P_n(x) \rightarrow$ Construct a set inclusion

3. Prove Isometry injective. Is it surjective?
Condition of surjection

4. Eg of Isometry not surjective?

$\rightarrow$ prove if $B = \text{closed disc}$, then $f(B) = B$
 $\nearrow$ isometry

11SERM

1. $A \ B \ C \rightarrow$ Are they homeomorphic?

$\rightarrow$ Defⁿ of connectedness, Continuous map connect $\rightarrow$ connect

2. How are the countable^{connected} metric spaces?
(singleton)

3. Inverse mapping thm, Implicit map thm?

4) Uniform continuity defⁿ?

is $\frac{1}{x}$ uni conti on $(0,1)$?

uni " " $(\epsilon,1)$?

$f: [a,b] \rightarrow \mathbb{R}$ conti $\Rightarrow$ Uni conti?

5) Continuous map maps Cauchy to Cauchy?

No. Take $f(x) = 1/x$. $(0,1) = \times$

$1/n$ Cauchy in $(0,1)$ $f(1/n) = n$ not Cauchy

Completeness needed

6) $\rightarrow$ defⁿ also.
Are all matrices diagonalizable? eg of non diagonalizable. $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ diag?

how did you tell eigenvalues are 0?

GM defⁿ? rank defⁿ?

II T Roparz

Round 1:

Don't remember, few questions from topology.

Round 2: 1) Give eg. of int. dim spaces.

2) Prove $C[a, b]$ ^{int dim} ~~banach~~ space.

3) defⁿ of l^p spaces.

4) value of p ? [~~any~~ $p \geq 1$]

5) why $p < 1$ not possible?

6) Eg of unbdd operator.

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1) X, Y nls. $T: X \rightarrow Y$ bdd. $f \in Y'$.
if $f \circ T: X \rightarrow \mathbb{K}$ bdd?

2) X, Y nls. $T: X \rightarrow Y$ not mentioned.
 $\forall f \in Y', f \circ T$ bdd. $\Rightarrow T$ bdd?

3) Defⁿ of hilbert adjoint operator. How
this T^* is considered? [meaning for a T ,
existence & uniqueness of T^*]

4) $f: \mathbb{C} \rightarrow$ upper half plane.

f entire & nonconstant $\rightarrow$ possible?